

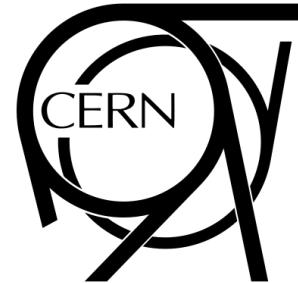
Measurement of γ from $B \rightarrow D K$ decays at LHCb

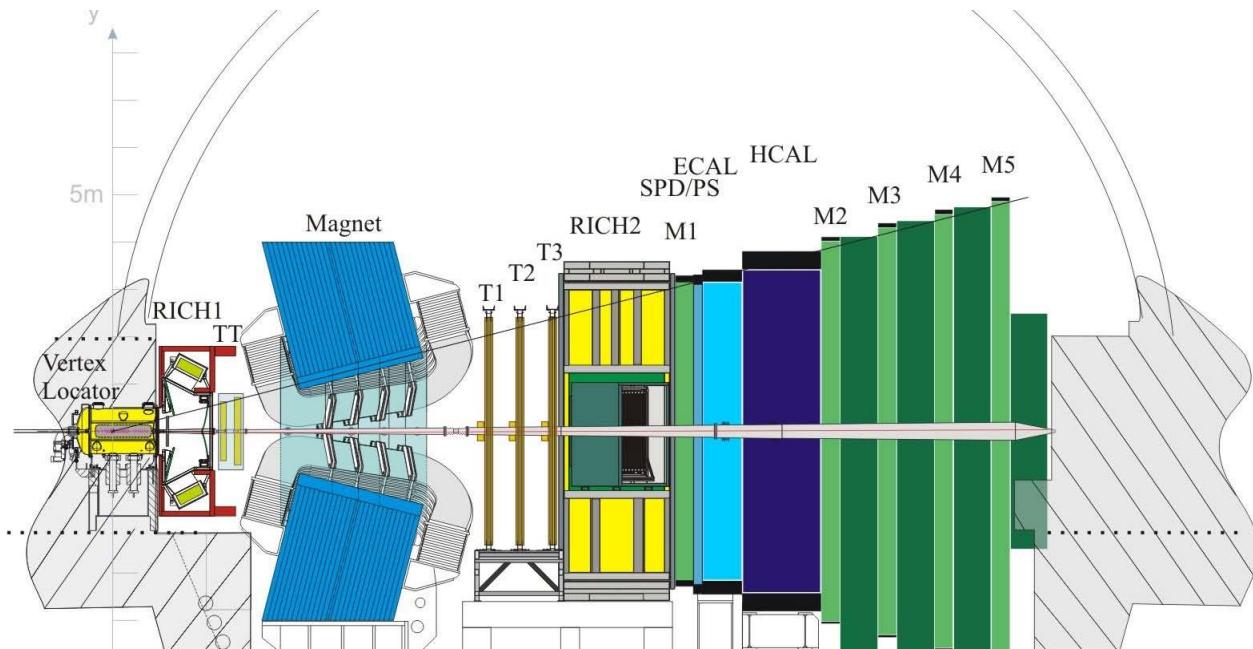
Large Hadron Collider Physics Conference 2013
13-18th May 2013, Barcelona

Maximilian Schlupp
on behalf of the LHCb collaboration

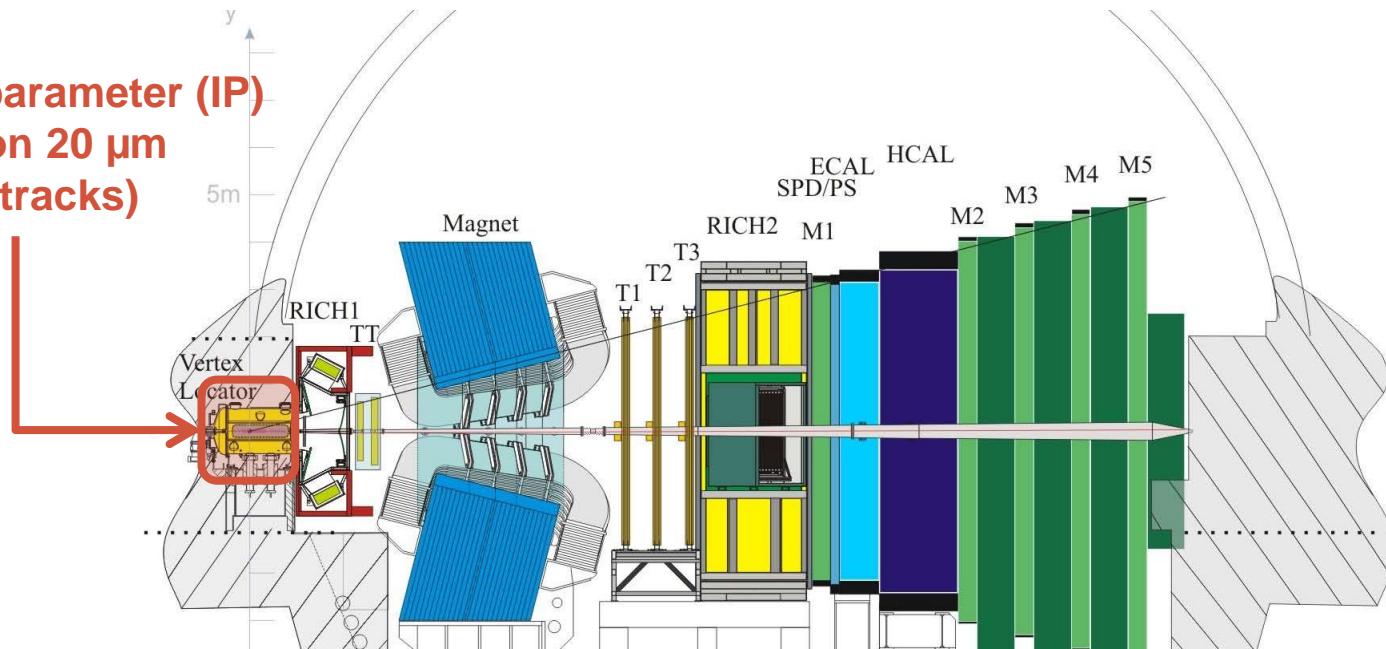


Bundesministerium
für Bildung
und Forschung



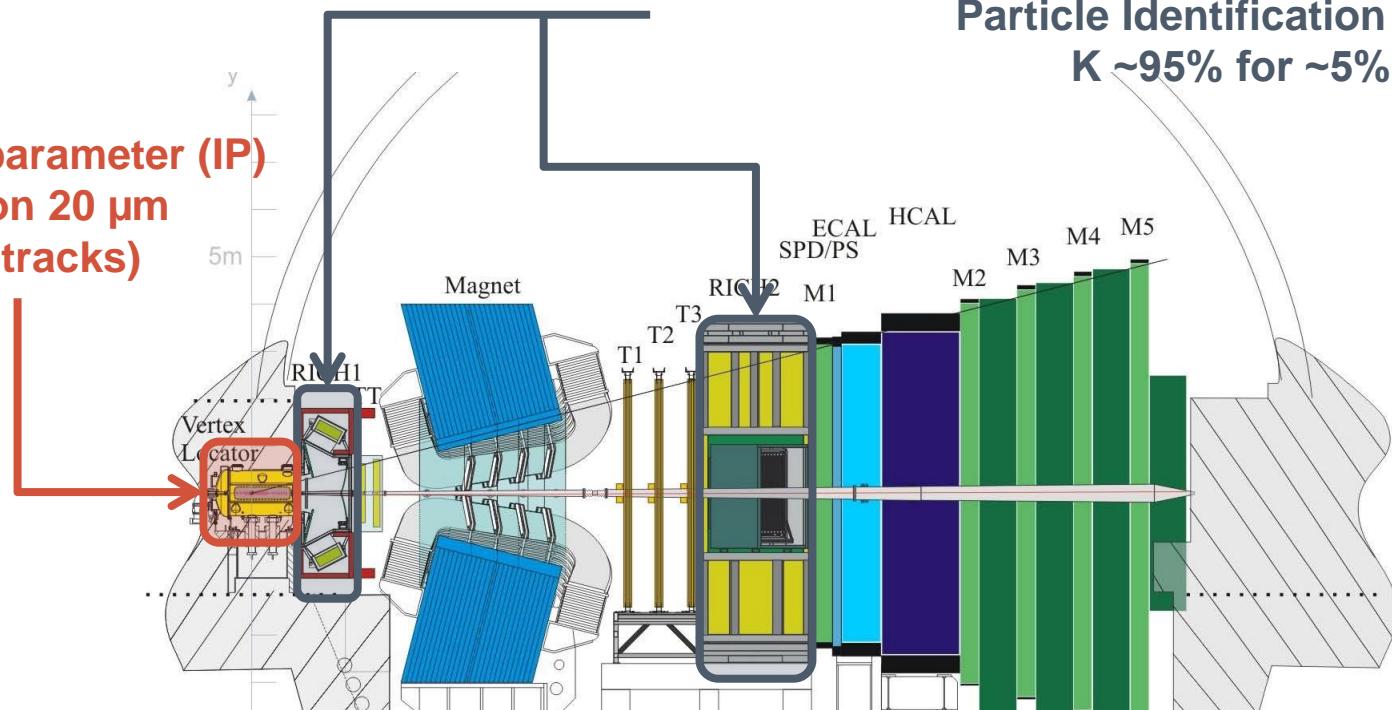


VELO:
Impact parameter (IP)
resolution 20 μm
(high p_T tracks)



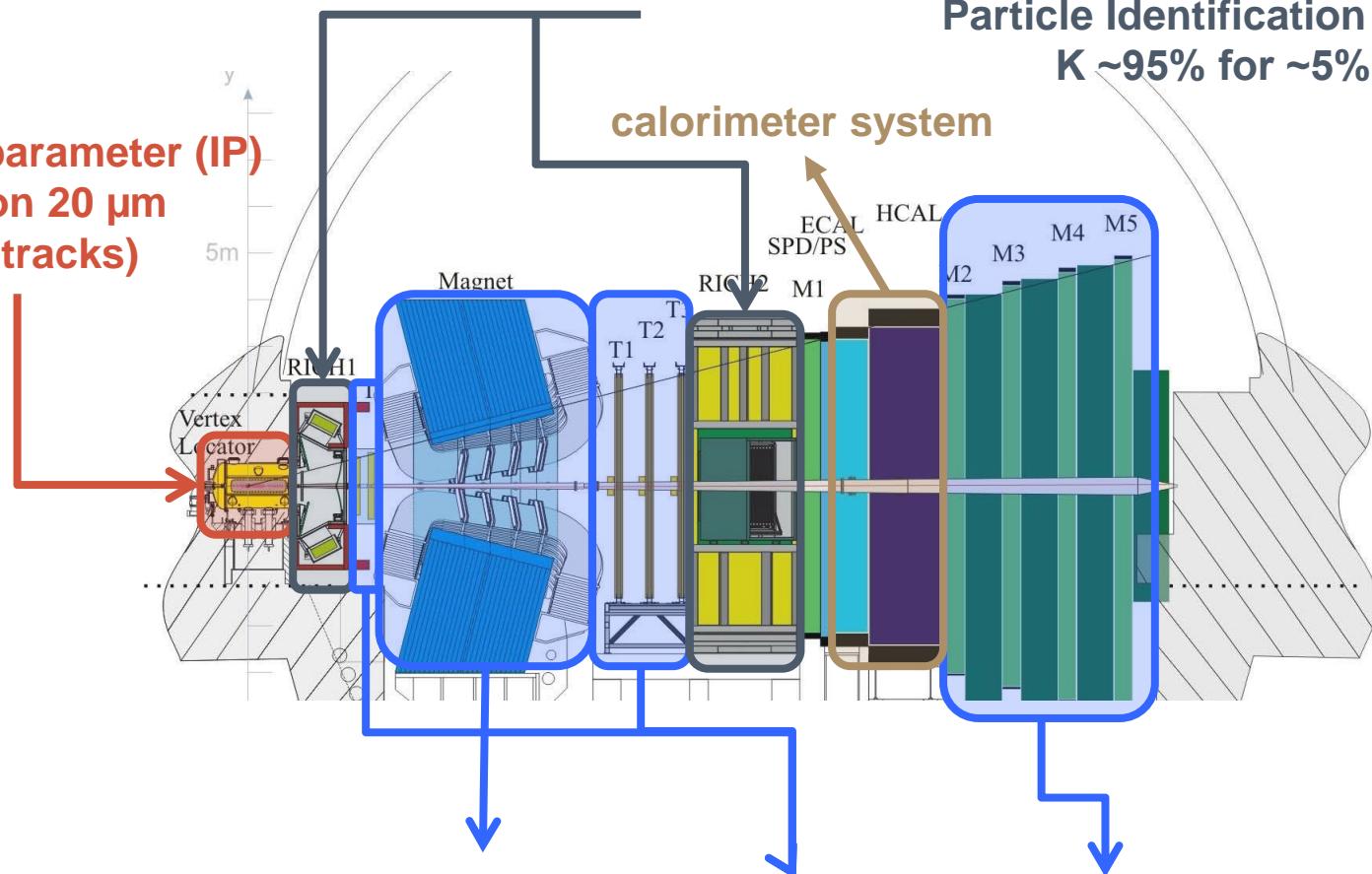
Ring Imaging Cherenkov (RICH) detectors
Particle Identification efficiencies:
 $K \sim 95\%$ for $\sim 5\% \pi \rightarrow K$ mis-id

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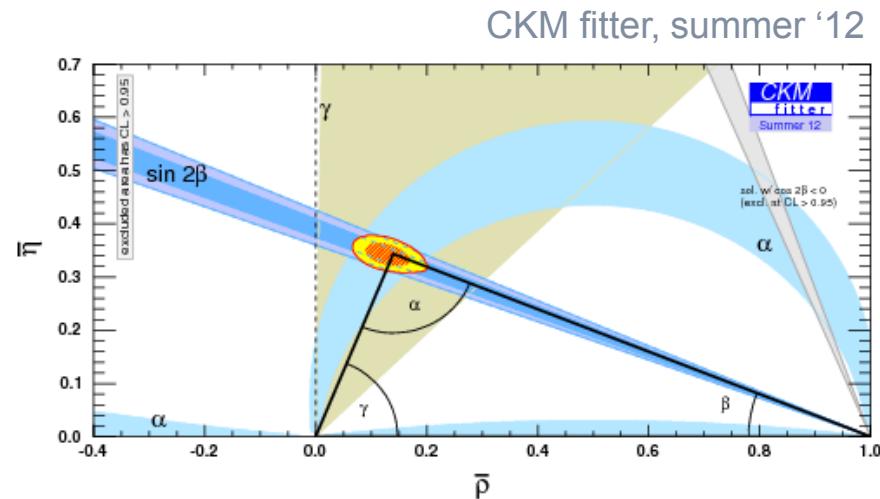
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Dipole-Magnet, Tracking stations, Muon stations
Momentum resolution
 $\Delta p/p = (0.4 - 0.6)\%$ at $(5 - 100) \text{ GeV}/c$

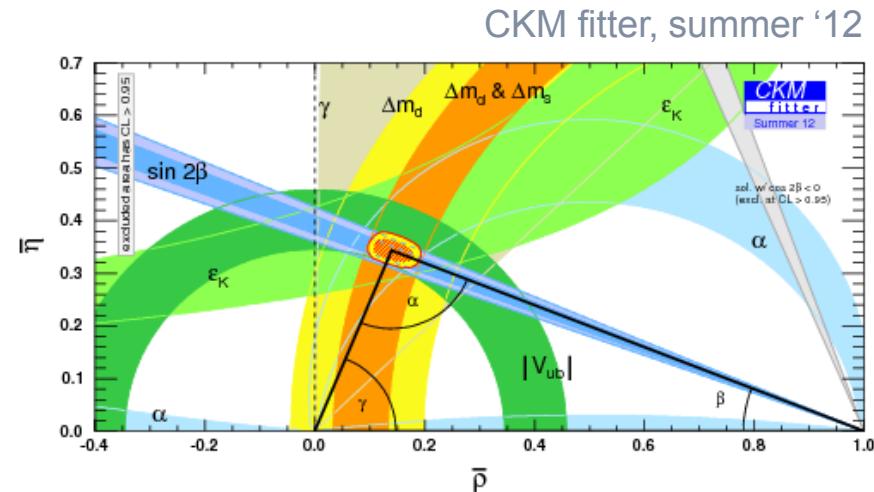
$$\gamma = \arg \left(-\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} \right)$$

- least-well determined CKM angle
- direct Measurements
 - BaBar $\gamma = (69^{+17}_{-16})^\circ$
[Phys. Rev. D 87, 052015 (2013)]
 - Belle $\gamma = (68^{+15}_{-14})^\circ$
(arXiv:1301.2033)



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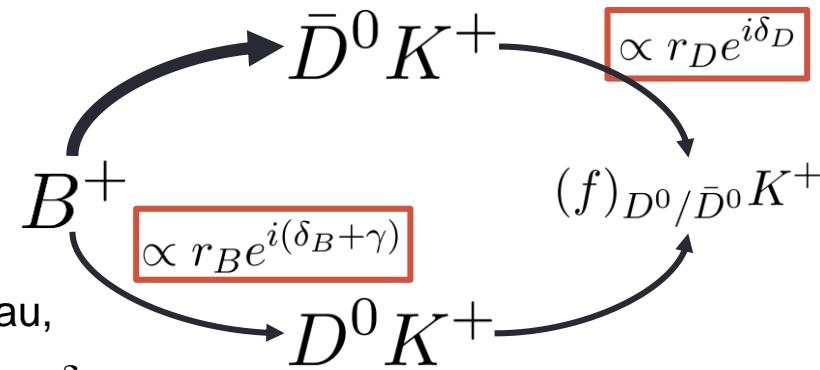
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- Standard Model (SM) fit
 - full constraint $\gamma = (67.7^{+4.1}_{-4.6})^\circ$ (CKM fitter , summer '12)
- discovery potential for new physics (NP) in comparison of:
 - direct measurement and SM fits
 - tree-level (SM) and loop (SM+NP) γ measurements



This Talk: Tree level $B_{(s)} \rightarrow D_{(s)} K$ transition

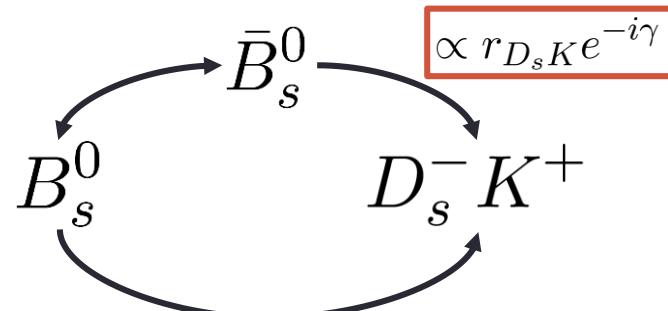
Time-Independent: charged B decays

- method of use depends on D final state f
 - f = CP eigenstates (e.g. KK , $\pi\pi$): Gronau, London, Wyler (GLW) \rightarrow no phase difference δ_D
 - f = quasi-flavour-specific states (e.g. $K\pi$, $K\pi\pi\pi$): Atwood, Dunietz, Soni (ADS) \rightarrow decay suppression levels (large asymmetries)
 - f = self-conjugate multi-body-state (e.g. $K_s\pi\pi$, K_sKK): Giri, Grossman, Soffer, Zupan (GGSZ) \rightarrow only Cabibbo allowed decays



Time-Dependent: neutral B decays

- neutral B oscillations
- e.g. $B_s \rightarrow D_s K$, $B_s \rightarrow D_s K\pi\pi$

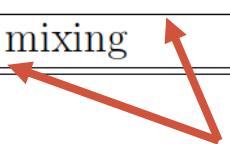


- single GLW / ADS measurement not yet sensitive enough
→ combine observables with GGSZ to increase precision on γ

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- inputs for the γ combination

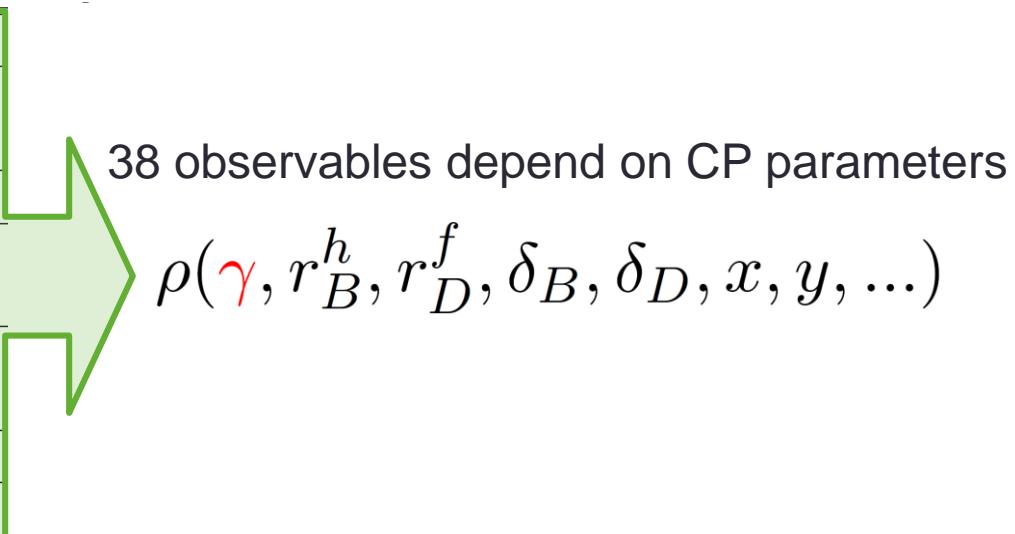
Analysis	N_{obs}
$B^+ \rightarrow D h^+$, $D \rightarrow h h$, GLW/ADS	13
$B^+ \rightarrow D K^+$, $D \rightarrow K_s^0 h^+ h^-$, GGSZ	4
$B^+ \rightarrow D h^+$, $D \rightarrow K \pi \pi \pi$, ADS	7
CLEO $D^0 \rightarrow K \pi$, $D^0 \rightarrow K \pi \pi \pi$ [Phys.Rev.D80:031105,2009]	9
CP violation in the charm system	2
charm mixing	3

Account for possible CPV & mixing in the charm system (not covered here)



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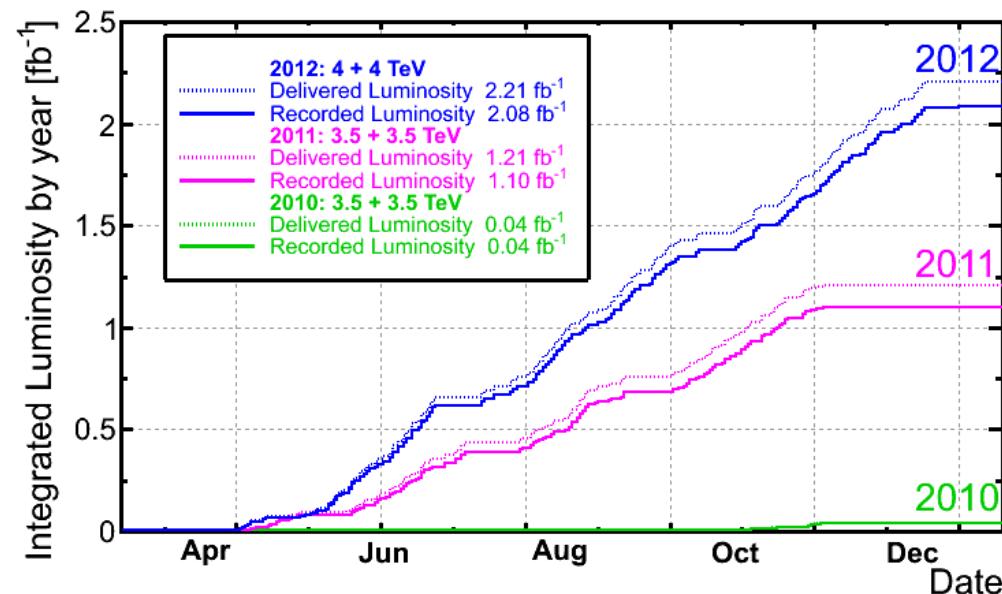
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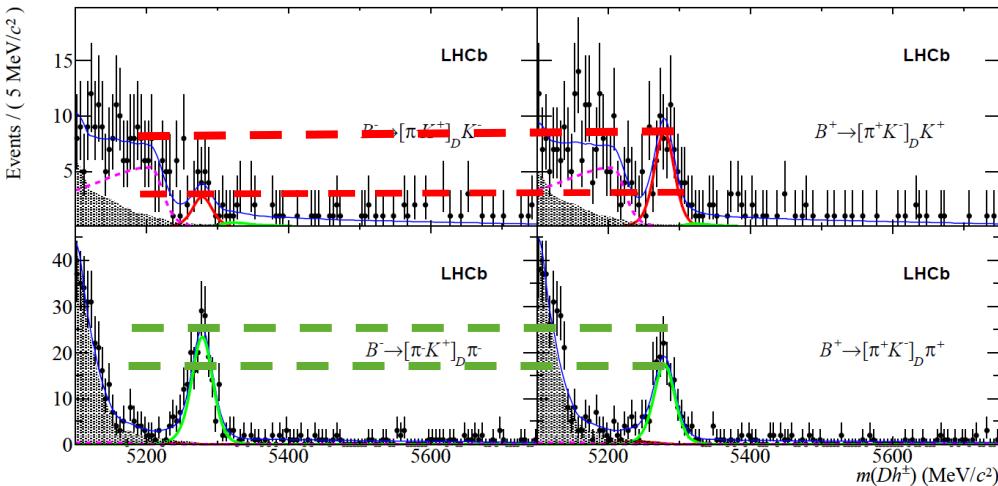


- ADS (two & four body) / GLW results based on 1 fb^{-1} data taken in 2011 at $\sqrt{s} = 7 \text{ TeV}$
- GGSZ results based on 2 fb^{-1} data taken in 2012 at $\sqrt{s} = 8 \text{ TeV}$

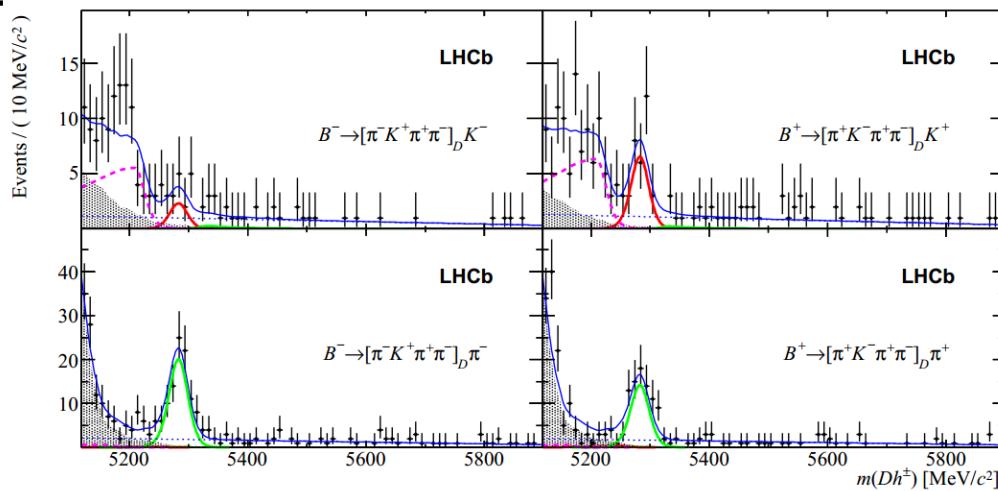
- with bachelor $h = K$ or π
- D final states: KK , $K\pi$, $\pi\pi$
- first observation of
 - $B \rightarrow [\pi K]_D K$
 - $B \rightarrow [\pi K \pi \pi]_D K$

ADS suppressed modes

→ combined 2-body GLW / ADS:
 5.8σ significance of direct CP violation (first observation)



Phys. Lett. B 712 (2012) 203,
arXiv: 1203.3662



arXiv:1303.4646

- D final state: $K_s \pi\pi, K_s KK$
- $B^+ \rightarrow [K_s hh]_D K^+$ amplitude

$$\mathcal{A}_B(m_{K_s h^+}^2, m_{K_s h^-}^2) = \mathcal{A}_{\bar{D}^0 \rightarrow K_s h^+ h^-}(m_{K_s h^+}^2, m_{K_s h^-}^2) + r_B e^{i(\delta_B + \gamma)} \mathcal{A}_{D^0 \rightarrow K_s h^+ h^-}(m_{K_s h^+}^2, m_{K_s h^-}^2)$$

- $(m_{K_s h^+}^2, m_{K_s h^-}^2)$ is a point in the $D \rightarrow K_s hh$ Dalitz space

LHCb-CONF-2013-004
2 fb⁻¹, 8 TeV, 2012

→ binned Dalitz plot analysis

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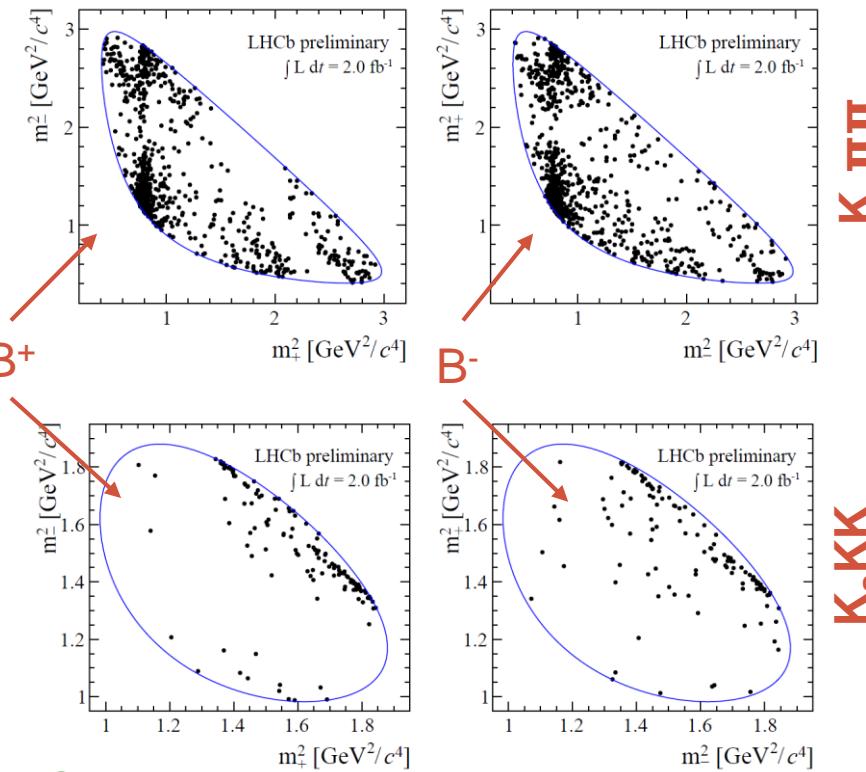
→ binned Dalitz plot analysis

$$\gamma = (57 \pm 16)^\circ \quad (3 \text{ fb}^{-1})$$

→ world's most precise single measurement

- crucial input for the γ combination

LHCb-CONF-2013-004
2 fb^{-1} , 8 TeV, 2012



- combine Observables using a frequentist approach
- more precise than simple averaging (different measurements sensitive to same parameters)
- inputs for the γ combination

Analysis	N_{obs}	Parameters
$B^+ \rightarrow Dh^+$, $D \rightarrow hh$, GLW/ADS	13	$\gamma, r_B, \delta_B, r_B^\pi, \delta_B^\pi, R_{K/\pi}, r_{K\pi}, \delta_{K\pi}, A_{CP}^{D \rightarrow KK}, A_{CP}^{D \rightarrow \pi\pi}$
$B^+ \rightarrow DK^+$, $D \rightarrow K_s^0 h^+ h^-$, GGSZ	4	γ, r_B, δ_B
$B^+ \rightarrow Dh^+$, $D \rightarrow K\pi\pi\pi$, ADS	7	$\gamma, r_B, \delta_B, r_B^\pi, \delta_B^\pi, R_{K/\pi}, r_{K3\pi}, \delta_{K3\pi}, \kappa_{K3\pi}$
CLEO $D^0 \rightarrow K\pi$, $D^0 \rightarrow K\pi\pi\pi$	9	$x_D, y_D, \delta_{K\pi}, \delta_{K3\pi}, \kappa_{K3\pi}, r_{K\pi}, r_{K3\pi}, \mathcal{B}(K\pi), \mathcal{B}(K\pi\pi\pi)$
CP violation in the charm system	2	$A_{CP}^{D \rightarrow KK}, A_{CP}^{D \rightarrow \pi\pi}$
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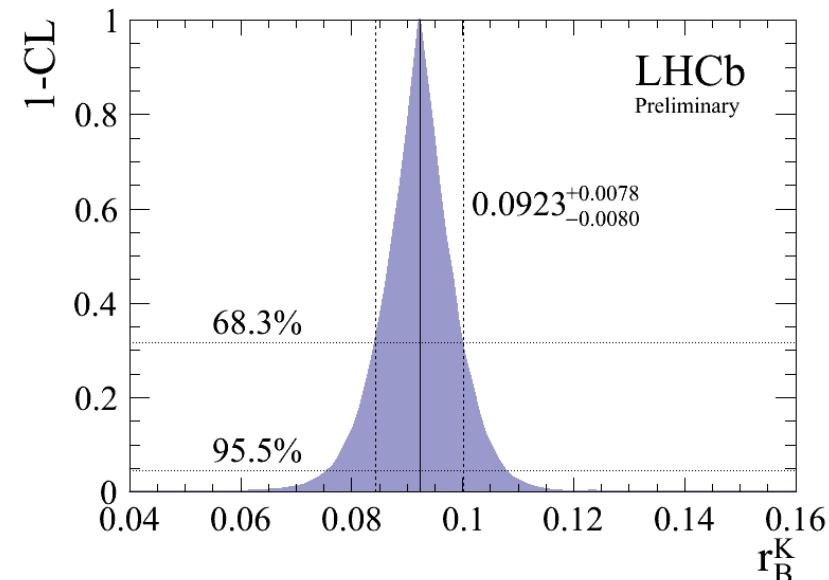
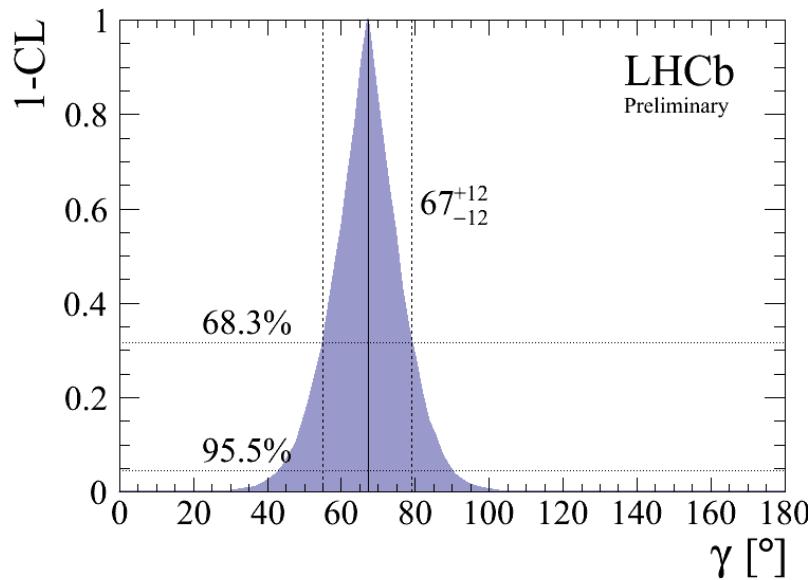
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LHCb measurements sensitive to γ

External inputs

LHCb-CONF-2013-006, prel.
LHCb-PAPER-2013-020, in prep.



γ from $B \rightarrow D\bar{K}$ ADS(1 fb^{-1})/GLW(1 fb^{-1})/GGSZ(3 fb^{-1}) combined:

$$\gamma = (67 \pm 12)^\circ$$

world's most precise

$$r_B^K = 0.092 \pm 0.008$$

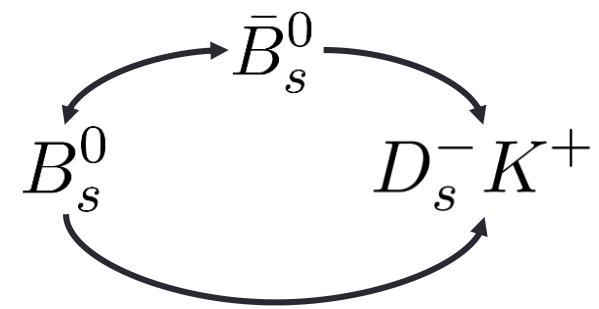
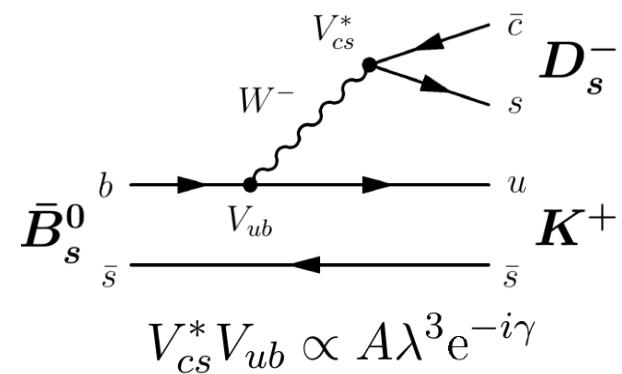
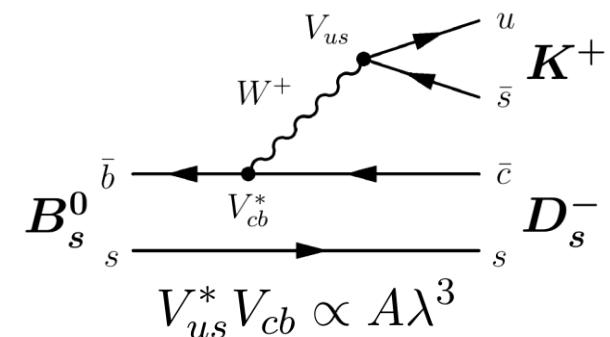
- $B_s \rightarrow D_s K$ exclusive to LHCb (time resolution, PID, B_s yields)
- interfering amplitudes of the same order of magnitude in the Wolfenstein parameter λ

- → large interference expected

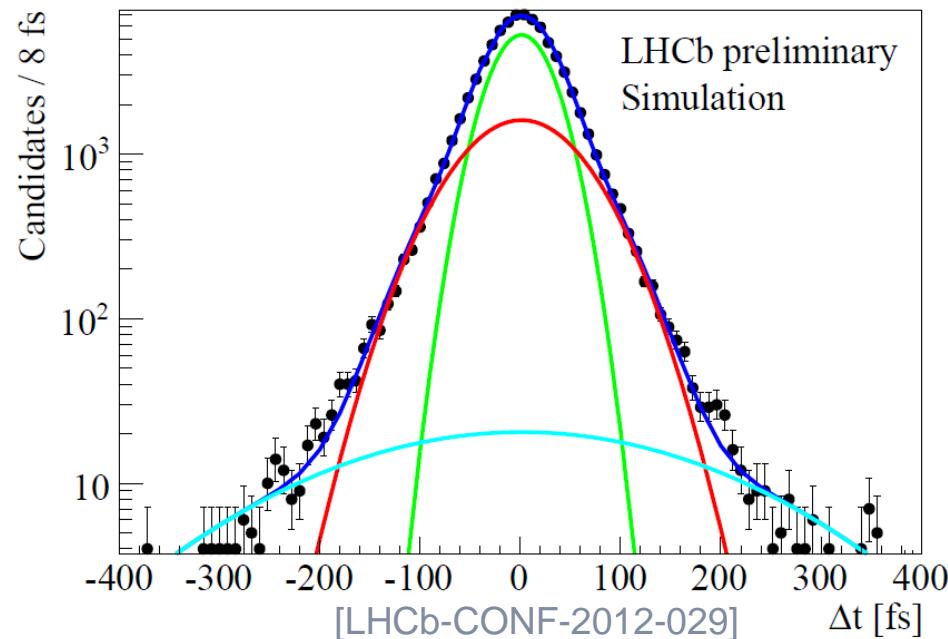
$$r_{D_s K} = \left| \frac{\mathcal{A}(\bar{B}_s \rightarrow D_s^- K^+)}{\mathcal{A}(B_s \rightarrow D_s^- K^+)} \right| \approx 0.37$$

- time-dependent measurement using tagged & untagged events

$$\begin{aligned} \frac{d\Gamma_{B_s \rightarrow D_s^- K^+}(t)}{dt e^{-\Gamma_s t}} &\propto |\mathcal{A}_f|^2 (1 + |r_{D_s K}|^2) \\ &\times \left[\cosh\left(\frac{\Delta\Gamma_s t}{2}\right) - \mathcal{D}_f \sinh\left(\frac{\Delta\Gamma_s t}{2}\right) \right. \\ &\quad \left. + \mathcal{C}_f \cos(\Delta m_s t) - \mathcal{S}_f \sin(\Delta m_s t) \right] \end{aligned}$$

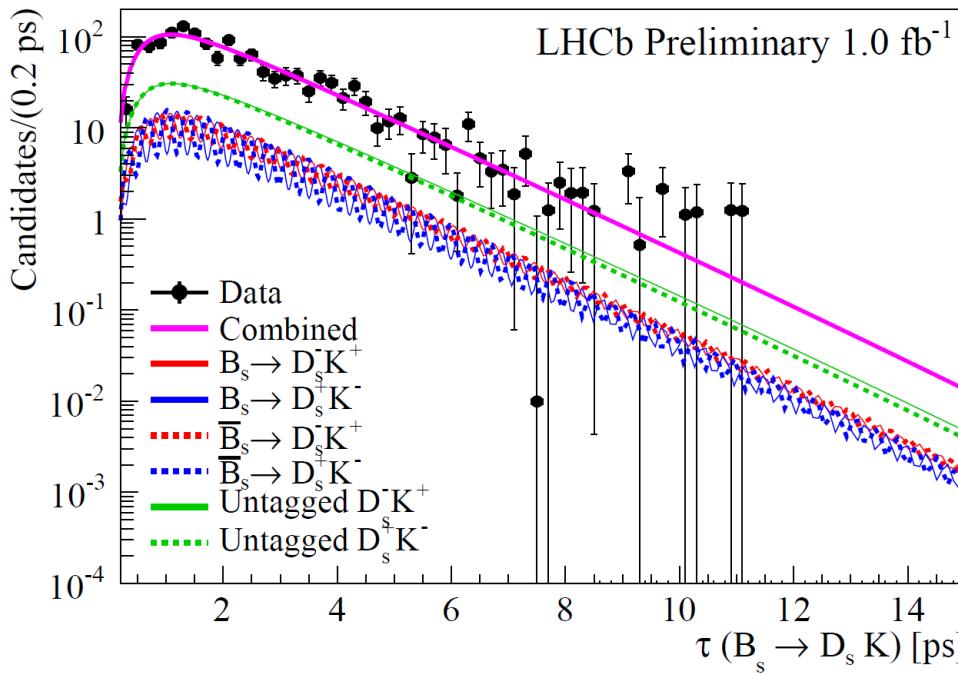


- time acceptance & time resolution crucial for time-dependent B_s measurements
- time acceptance:
 - shape from simulations
 - parameter from fit to a $D_s \pi$ data sample (unbinned maximum likelihood)
 - scale parameter with factor accounting for the differences between $D_s K$ and $D_s \pi$ simulations
- decay time resolution:
 - triple Gaussian function is fit to signal MC (unbinned maximum likelihood)
 - effective decay time resolution ~ 50 fs in data



- combined weighted unbinned maximum likelihood fit of the decay rates to the time distribution

[LHCb-CONF-2012-029]
1 fb^{-1} , 7 TeV, 2011



	stat.	syst.
$C =$	$1.01 \pm 0.50 \pm 0.23$	
$S_f =$	$-1.25 \pm 0.56 \pm 0.24$	
$S_{\bar{f}} =$	$0.08 \pm 0.68 \pm 0.28$	
$D_f =$	$-1.33 \pm 0.60 \pm 0.26$	
$D_{\bar{f}} =$	$-0.81 \pm 0.56 \pm 0.26$	

very first measurement of CP observables in $B_s \rightarrow D_s K$

- important step towards extraction of γ in $B_s \rightarrow D_s K$

- time-independent measurements of CP observables using GLW, ADS, GGSZ

- combination of the tree-level $B \rightarrow D\bar{K}$ analyses

- world's most precise measurement & combination of γ

GGSZ: $\gamma = (57 \pm 16)^\circ$ combination: $\gamma = (67 \pm 12)^\circ$

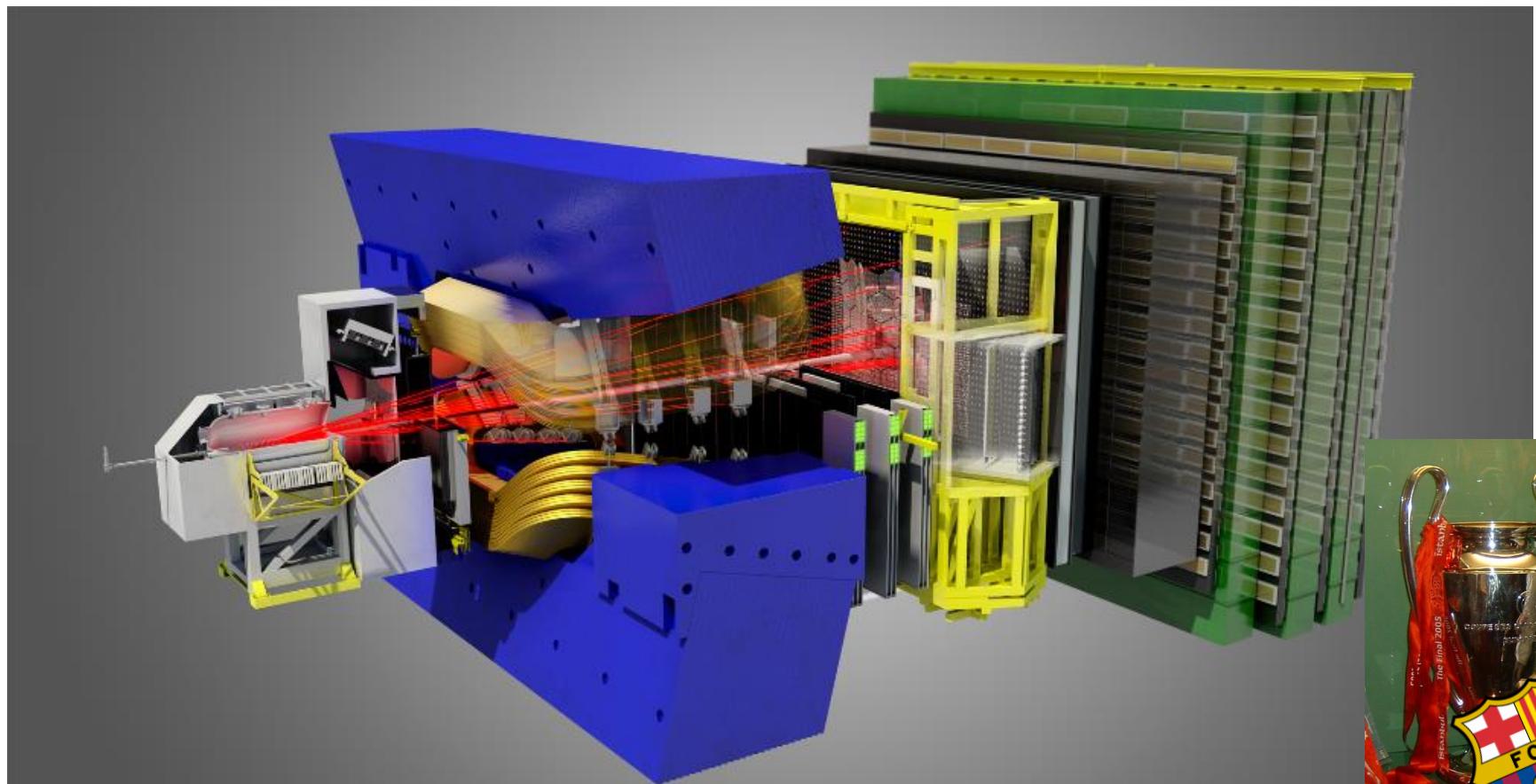
- First time-dependent measurement of CP observables in $B_s \rightarrow D_s K$
 - important step towards extraction of γ

- more to come in the next years:

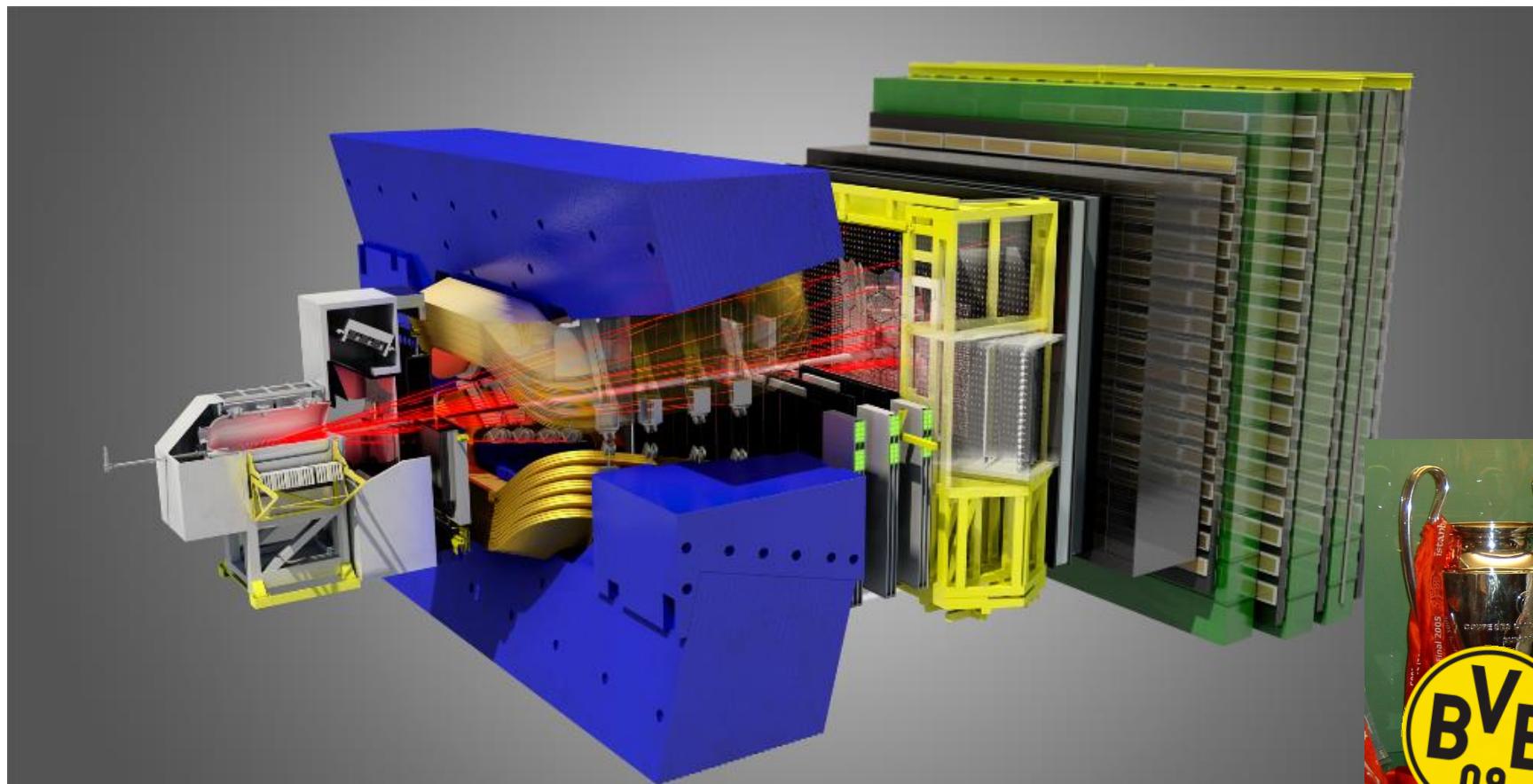
- update analyses to full dataset (3 times the statistics)
 - until 2018: collect up to $\sim 10 \text{ fb}^{-1}$
 - γ from loop induced processes

- prospects for the LHCb upgrade (2018+, $\sim 50 \text{ fb}^{-1}$) [arXiv:1208.3355]
 - γ combined: $\delta\gamma \sim \mathcal{O}(1^\circ)$

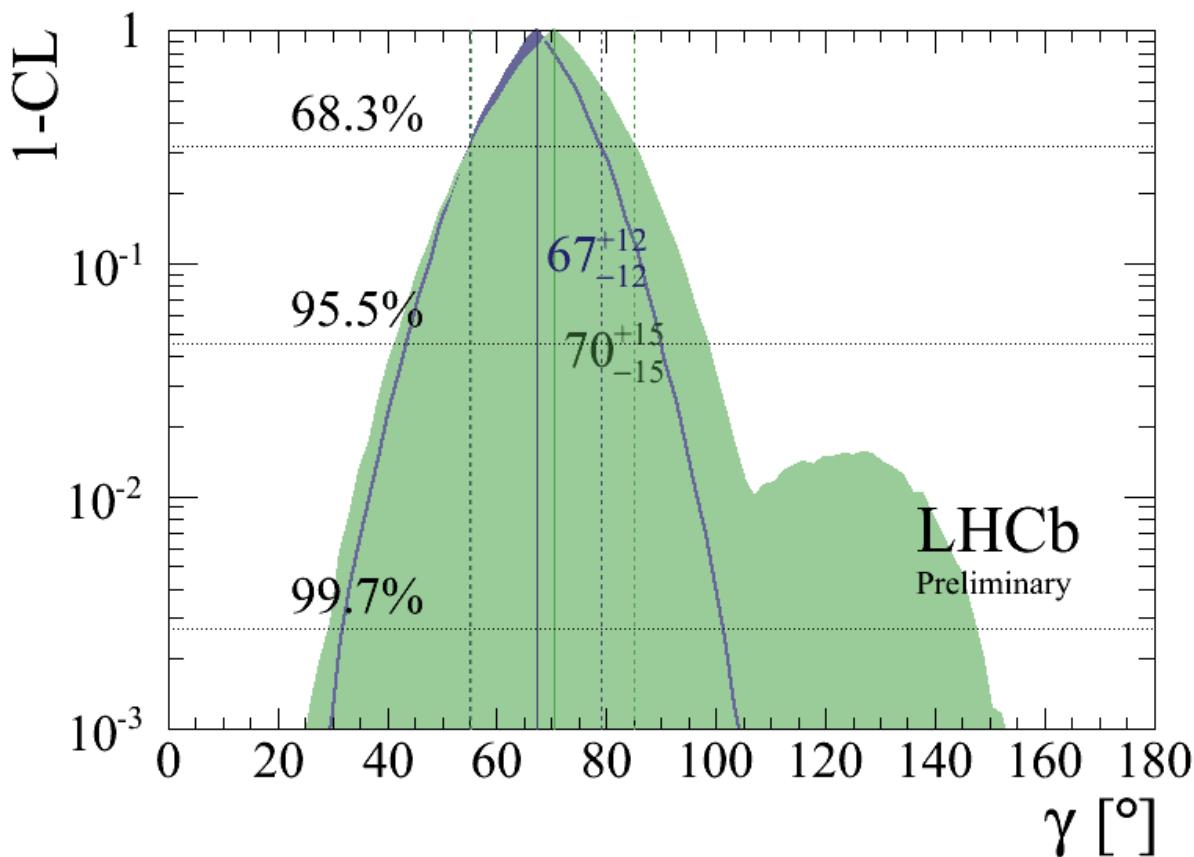
LHCb is on it's way to a precision measurement of γ



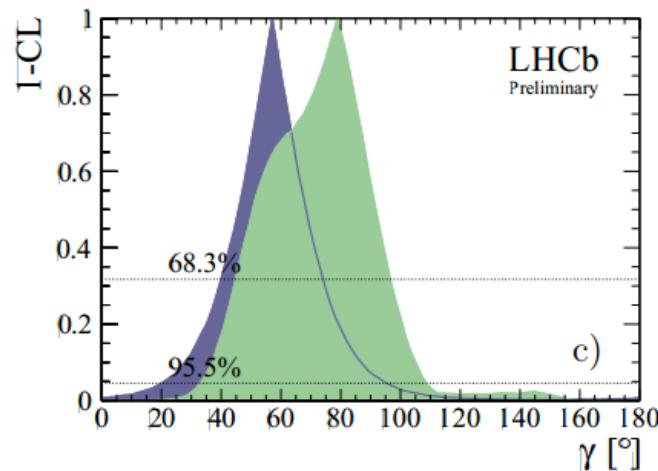
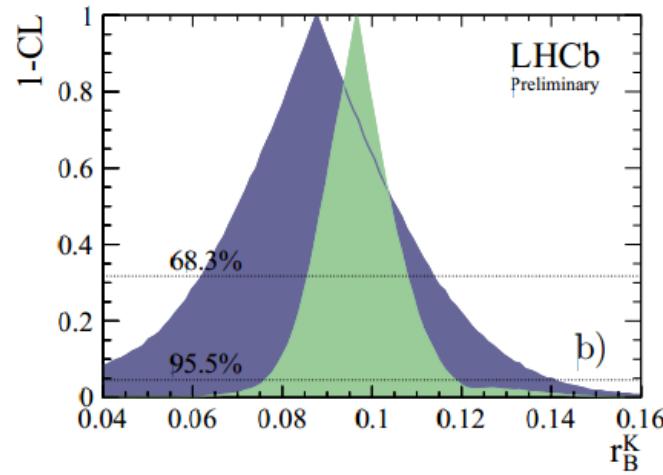
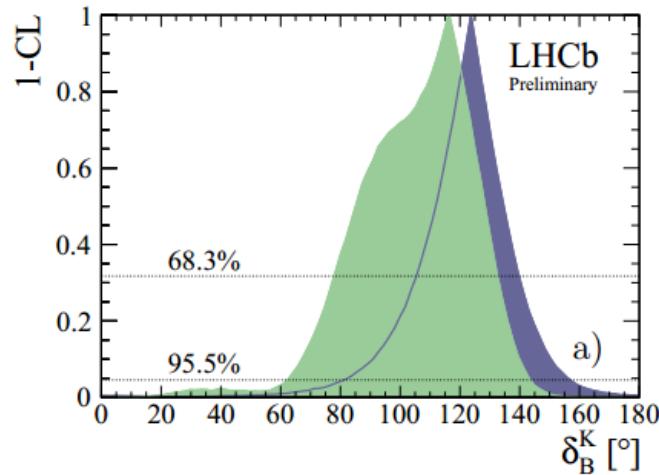
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LHCb-CONF-2013-006, prel.
LHCb-PAPER-2013-020, in prep.

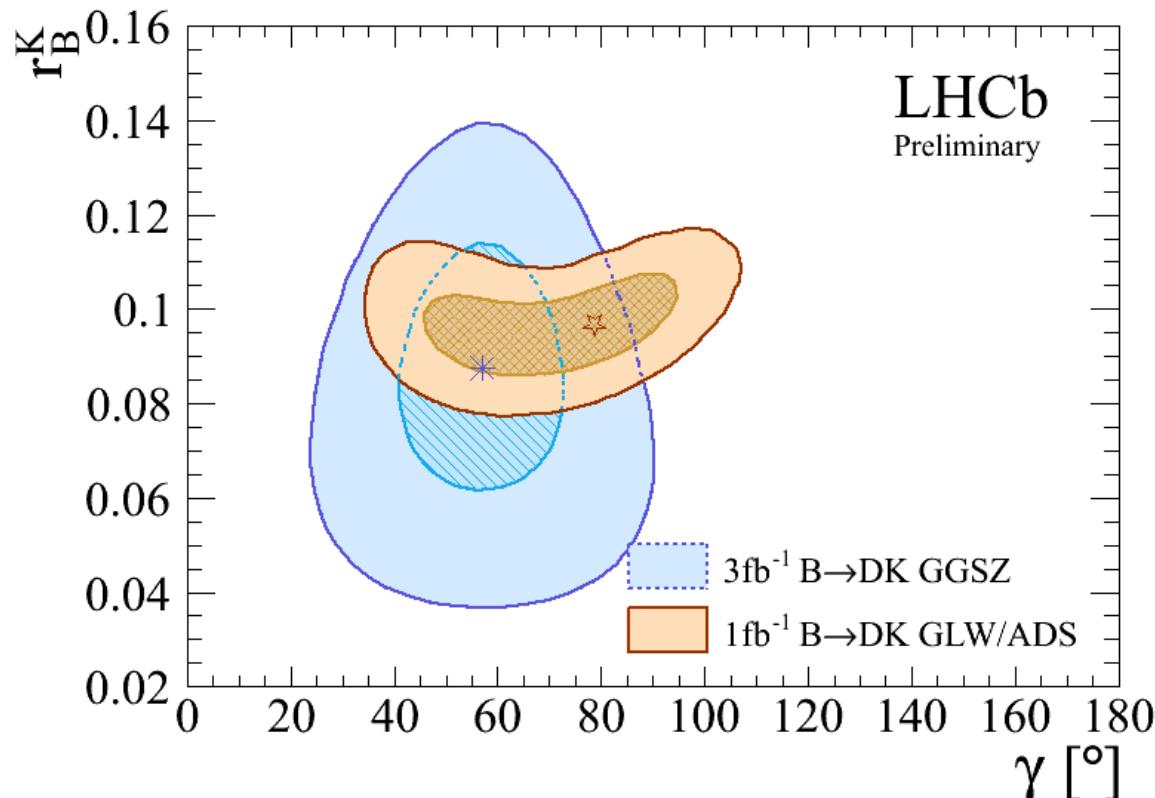
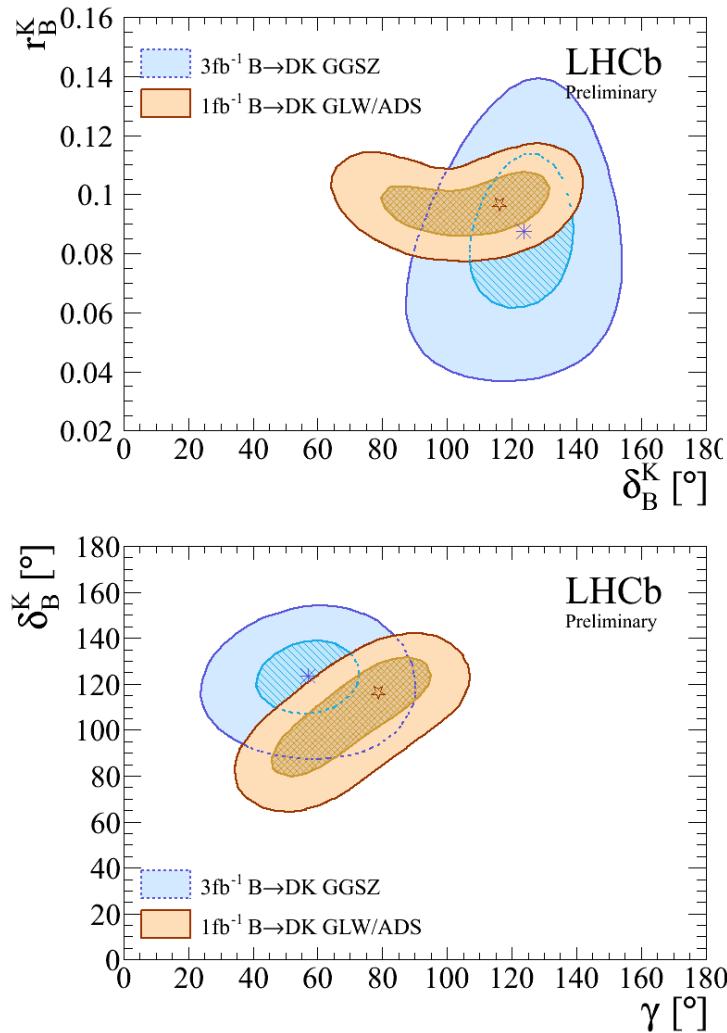


LHCb-CONF-2013-006, prel.
LHCb-PAPER-2013-020, in prep.



GGSZ, 3 fb^{-1}
ADS/GLW, 1 fb^{-1}

LHCb-CONF-2013-006, prel.
LHCb-PAPER-2013-020, in prep.



- Observables' relations to parameter

$$R_{\pi^-}^{K\pi} = \frac{r_{B(\pi)}^2 + r_D^{K\pi 2} + 2r_{B(\pi)}r_D^{K\pi} \cos(\delta_{B(\pi)} + \delta_D^{K\pi} - \gamma)}{1 + r_{B(\pi)}^2 r_D^{K\pi 2} + 2r_{B(\pi)}r_D^{K\pi} \cos(\delta_{B(\pi)} - \delta_D^{K\pi} - \gamma)},$$

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$$R_{K/\pi}^{K\pi} = R_{\text{cab}} \frac{1 + r_{B(K)}^2 r_D^{K\pi 2} + 2r_{B(K)}r_D^{K\pi} \cos(\delta_{B(K)} - \delta_D^{K\pi}) \cos \gamma}{1 + r_{B(\pi)}^2 r_D^{K\pi 2} + 2r_{B(\pi)}r_D^{K\pi} \cos(\delta_{B(\pi)} - \delta_D^{K\pi}) \cos \gamma}$$

$$R_{K/\pi}^{KK} = R_{\text{cab}} \frac{1 + r_{B(K)}^2 + 2r_{B(K)} \cos \delta_{B(K)} \cos \gamma}{1 + r_{B(\pi)}^2 + 2r_{B(\pi)} \cos \delta_{B(\pi)} \cos \gamma},$$

$$R_{K/\pi}^{\pi\pi} = R_{K/\pi}^{KK},$$

$$A_\pi^{K\pi} = \frac{2r_{B(\pi)}r_D^{K\pi} \sin(\delta_{B(\pi)} - \delta_D^{K\pi}) \sin \gamma}{1 + r_{B(\pi)}^2 r_D^{K\pi 2} + 2r_{B(\pi)}r_D^{K\pi} \cos(\delta_{B(\pi)} - \delta_D^{K\pi}) \cos \gamma},$$

$$A_K^{K\pi} = \frac{2r_{B(K)}r_D^{K\pi} \sin(\delta_{B(K)} - \delta_D^{K\pi}) \sin \gamma}{1 + r_{B(K)}^2 r_D^{K\pi 2} + 2r_{B(K)}r_D^{K\pi} \cos(\delta_{B(K)} - \delta_D^{K\pi}) \cos \gamma},$$

$$A_\pi^{KK} = \frac{2r_{B(\pi)} \sin \delta_{B(\pi)} \sin \gamma}{1 + r_{B(\pi)}^2 + 2r_{B(\pi)} \cos \delta_{B(\pi)} \cos \gamma} + A_{CP}(D \rightarrow KK),$$

$$A_\pi^{\pi\pi} = \frac{2r_{B(\pi)} \sin \delta_{B(\pi)} \sin \gamma}{1 + r_{B(\pi)}^2 + 2r_{B(\pi)} \cos \delta_{B(\pi)} \cos \gamma} + A_{CP}(D \rightarrow \pi\pi),$$

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Phys. Lett. B 712 (2012) 203,
arXiv: 1203.3662

- Results on the observables
- Provide crucial input for the combination

	stat.	syst.
$R_{K/\pi}^{K\pi}$	$0.0774 \pm 0.0012 \pm 0.0018$	
$R_{K/\pi}^{KK}$	$0.0773 \pm 0.0030 \pm 0.0018$	
$R_{K/\pi}^{\pi\pi}$	$0.0803 \pm 0.0056 \pm 0.0017$	
$A_\pi^{K\pi}$	$-0.0001 \pm 0.0036 \pm 0.0095$	
$A_K^{K\pi}$	$0.0044 \pm 0.0144 \pm 0.0174$	
A_K^{KK}	$0.148 \pm 0.037 \pm 0.010$	
$A_K^{\pi\pi}$	$0.135 \pm 0.066 \pm 0.010$	
A_π^{KK}	$-0.020 \pm 0.009 \pm 0.012$	
$A_\pi^{\pi\pi}$	$-0.001 \pm 0.017 \pm 0.010$	
R_K^-	$0.0073 \pm 0.0023 \pm 0.0004$	
R_K^+	$0.0232 \pm 0.0034 \pm 0.0007$	
R_π^-	$0.00469 \pm 0.00038 \pm 0.00008$	
R_π^+	$0.00352 \pm 0.00033 \pm 0.00007$	

- Decay width of the B mesons per Dalitz plot bin ($\pm i$)

$$\Gamma_{\pm i}(B^-) = n^-(K_{\pm i} + r_B^2 K_{\mp i} + 2\sqrt{K_i K_{-i}}(x_- c_i \mp y_- s_i))$$

$$\Gamma_{\pm i}(B^+) = n^+(K_{\mp i} + r_B^2 K_{\pm i} + 2\sqrt{K_i K_{-i}}(x_+ c_i \mp y_+ s_i))$$

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2 fb⁻¹, 8 TeV, 2012

- with the CP Observables

$$x_{\pm} \equiv r_B \cos(\delta_B \pm \gamma), \quad y_{\pm} \equiv r_B \sin(\delta_B \pm \gamma)$$

- the efficiency-corrected yield per bin ($\pm i$), $K_{\pm i}$
- normalized cosine, sine of the strong-phase difference averaged in each bin, c_i , s_i
- a normalisation factor n^{\pm}



External input:
CLEO-c, Phys. Rev. D82 (2010) 112006
arXiv:1010.2817

$$x_+ = (-8.7 \pm 3.1 \pm 1.6 \pm 0.6) \times 10^{-2}$$

$$x_- = (5.3 \pm 3.2 \pm 0.9 \pm 0.9) \times 10^{-2}$$

$$y_+ = (0.1 \pm 3.6 \pm 1.4 \pm 1.9) \times 10^{-2}$$

$$y_- = (9.9 \pm 3.6 \pm 2.2 \pm 1.6) \times 10^{-2}$$

- Statistical procedure

- Define Likelihood $\mathcal{L}(\vec{\alpha}) = \prod_i f_i(\vec{A}_{i,\text{obs}} | \vec{A}_i(\vec{\alpha}_i))$

$\vec{A}_{i,\text{obs}}$ Set of observables per measurement i

$\vec{\alpha}_i$ Set of parameter per measurement i

$\vec{A}_i(\vec{\alpha}_i)$ Truth relation between parameter and observable

$f_i(\vec{A}_{i,\text{obs}} | \vec{A}_i(\vec{\alpha}_i))$ Multivariate Gaussian taking possible correlations into account
(or experimental likelihood, if approximation is not valid)

- Statistical procedure

- to evaluate the confidence level for the parameter of interest (e.g. γ) use

$$\chi^2(\vec{\alpha}) = -2 \ln \mathcal{L}(\vec{\alpha})$$

- say $\vec{\alpha}'$ is the parameter-set minimizing $\chi^2(\vec{\alpha})$
 - and $\vec{\alpha}'_c$ minimizes $\chi^2(\vec{\alpha})$ with fixing $\gamma = \gamma_0$
 - For each value of γ_0 get p-Value:

- calculate $\Delta\chi^2_{\text{data}} = \chi^2(\vec{\alpha}') - \chi^2(\vec{\alpha}'_c)$ given \vec{A}_{obs}
 - generate N_{toy} toy experiments \vec{A}_{toy} with $\vec{\alpha}'_c$
 - calculate $\Delta\chi^2_{\text{toy}} = \chi^2(\vec{\alpha}') - \chi^2(\vec{\alpha}'_c)$ but given \vec{A}_{toy}

$$\Rightarrow p = 1 - \text{CL} = \frac{N(\Delta\chi^2_{\text{toy}} > \Delta\chi^2_{\text{data}})}{N_{\text{toy}}}$$

Conventions:

$$f = D_s^- K^+$$

$$\Delta\Gamma_s = \Gamma_H - \Gamma_L > 0$$

$$\Delta m_s = m_H - m_L > 0$$

- As one example $\Gamma_{B_s \rightarrow D_s^- K^+}(t)$

$$\frac{d\Gamma_{B_s \rightarrow D_s^- K^+}(t)}{dt e^{-\Gamma_s t}} \propto |\mathcal{A}_f|^2 (1 + |\lambda_f|^2) \left[\cosh\left(\frac{\Delta\Gamma_s t}{2}\right) - \mathcal{D}_f \sinh\left(\frac{\Delta\Gamma_s t}{2}\right) + \mathcal{C}_f \cos(\Delta m_s t) - \mathcal{S}_f \sin(\Delta m_s t) \right]$$

with $\left|\frac{q}{p}\right| = 1 \rightarrow r_{D_s K} \equiv |\lambda_f|$

$$\mathcal{C}_{f(\bar{f})} = \frac{1 - r_{D_s K}^2}{1 + r_{D_s K}^2}$$

$$\mathcal{D}_{f(\bar{f})} = \frac{2r_{D_s K} \cos(\Delta \mp (\gamma - 2\beta_s))}{1 + r_{D_s K}^2}$$

$$\mathcal{S}_{f(\bar{f})} = \frac{2r_{D_s K} \sin(\Delta \mp (\gamma - 2\beta_s))}{1 + r_{D_s K}^2}$$

- Δ strong phase difference
- β_s weak B_s CPV mixing phase; small effect [e.g. in $B_s \rightarrow J/\Psi \phi$; LHCb-CONF-2012-002]
- γ weak CPV decay phase