GAUGE INVARIANCE AT WORK IN FOUR DIMENSIONAL REGULARIZATION:

$H \to \gamma\gamma$

@1-LOOP IN R_ξ-GAUGE

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FOUR DIMENSIONAL REGULARIZATION

F D R

F D R

a method for multi-loop calculations in QFT

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INTRODUCTION

- PRECISION PHYSICS is crucial in the quest of NEW PHYSICS
- multi-loop calculation essential for Higgs studies
- **D**imensional **R**egularization forces a HUGE ANALYTICAL WORK!

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PRIORITY: developing new techniques allowing for fast and automated calulations of RADIATIVE CORRECTIONS

F D R

- FINITE
- 4-DIM.
- GAUGE INVARIANT

FDR

- **FINITE** → NO NEED OF RENORMALIZATION
- **4-DIM.** → NUMERICAL INTEGRATION
- **GAUGE INVARIANT** → STRONG TEST ~ EXACT RATIONAL TERM

F D R

FINITE

4-DIM.

GAUGE INVARIANT

RE-INTERPRETATION OF THE LOOP INTEGRAL

UV infinities are vacuum bubbles
to be dropped at the integrand level

THE METHOD

notation
$D = q^2 - m^2$

partial fraction identity $\frac{1}{D} = \frac{1}{q^2}\left(1 + \frac{m^2}{D}\right)$

$$\frac{1}{D^2}$$

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IR divergent!

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IR safe!

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notation	add a small mass μ	partial fraction identity
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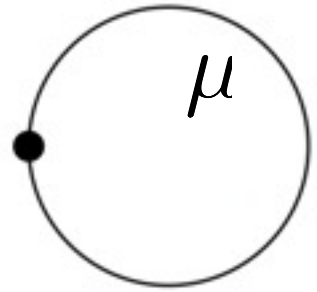

VACUUM BUBBLE
- divergent in UV
- regular in IR
- universal

FINITE PART
- convergent in 4D
- contains all kinematics

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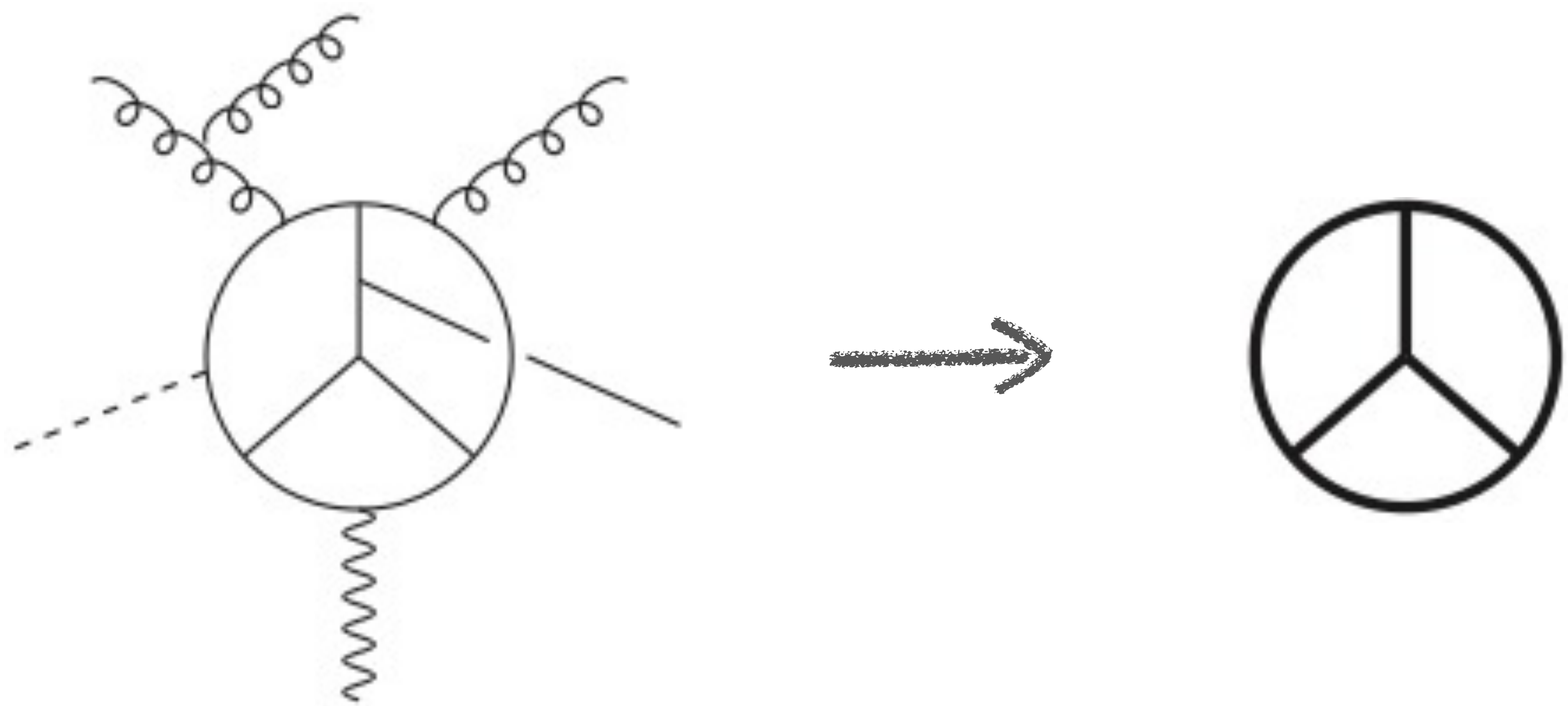
FDR INTEGRAL

FINITE PART

- convergent in 4D
- contains all kinematics



in **UV limit**
all physical scales are irrelevant
with respect to loop momenta:
the diagram behaves as a **vacuum bubble**!

FDR

$$\int [\mathrm{d}^4 q] \frac{1}{\overline{D}^2} = \lim_{\mu\to 0} \int \mathrm{d}^4 q \left(\frac{2m^2}{\bar{q}^4 \overline{D}} + \frac{m^4}{\bar{q}^4 \overline{D}^2} \right)\Bigg|_{\mu=\mu_R}$$

has nice properties

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JUST AN INTEGRAL!

- linear
- shift-invariant

F D R

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no μ dependence

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JUST AN INTEGRAL!

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- **GAUGE INVARIANT**

4D

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GAUGE INVARIANCE

H → γγ @ 1-loop

$$M^{\mu\nu}(\beta,\eta) \propto \frac{T^{\mu\nu}}{M_W}\left(F_W(\beta) + \sum_f N_c Q_f^2 F_f(\eta)\right)$$

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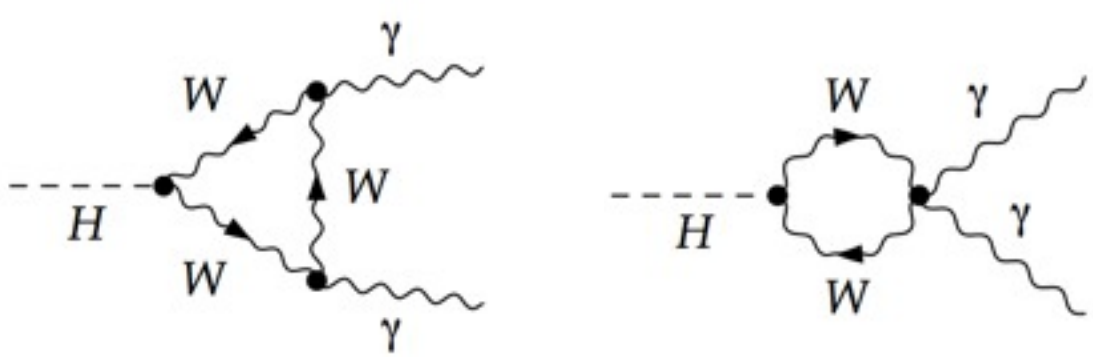
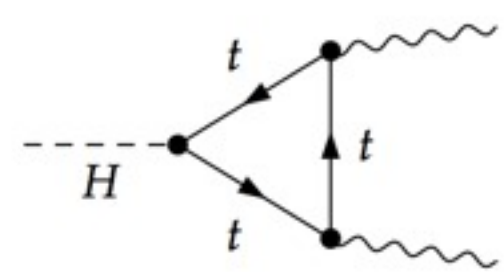
BOSONIC CONTRIBUTION



$$F_W(\beta) = 2 + 3\beta + 3\beta(2-\beta)f(\beta)$$

FERMIONIC CONTRIBUTION



$$F_f(\eta) = 2\eta\left[1 + (1-\eta)f(\eta)\right]$$

H → γγ @ 1-loop

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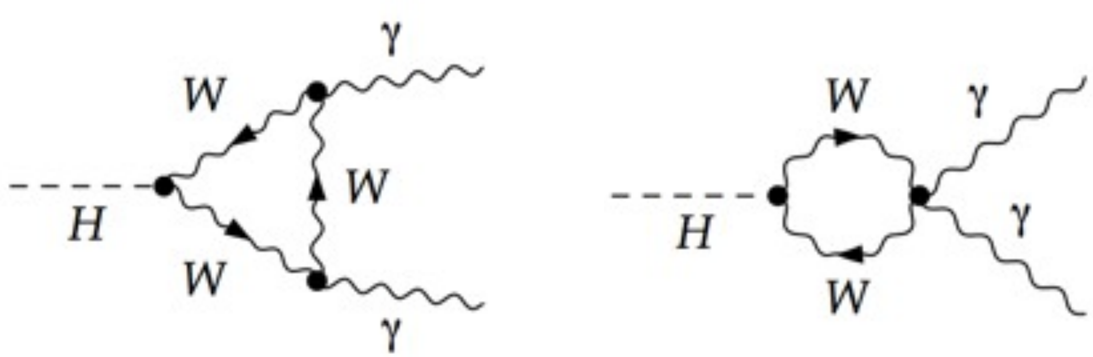

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rational term!

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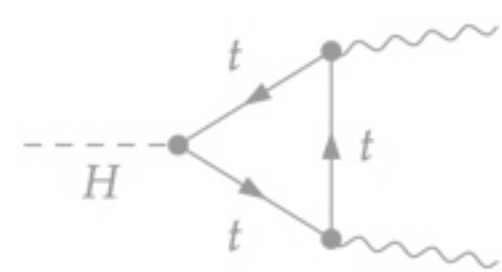

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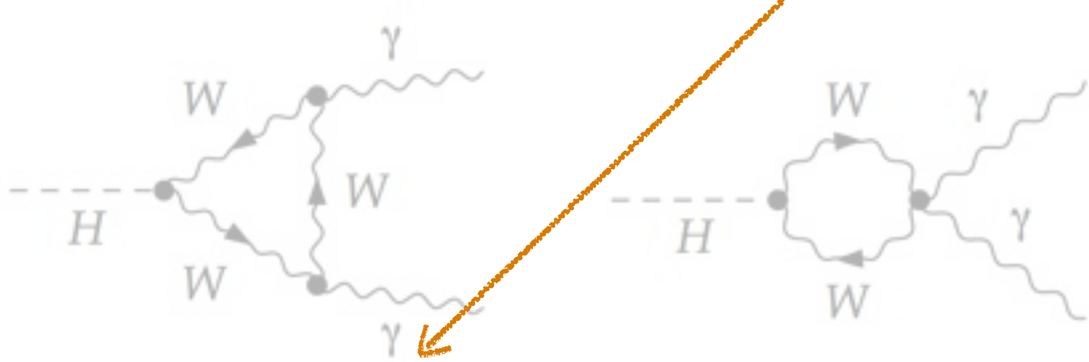
FDR
subtraction
of vacuum configurations
before integrating

?

DR
subtraction
of the UV poles $1/\epsilon$
after integrating



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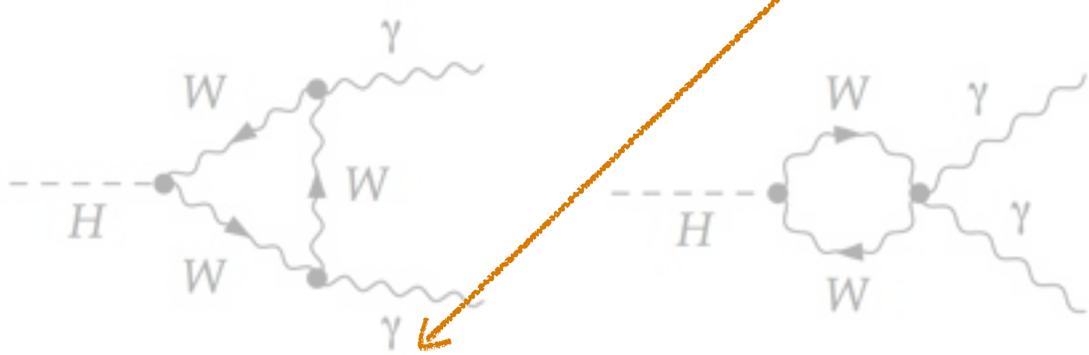
FDR
subtraction
of vacuum configurations
before integrating

↔ **gauge invariance!**

DR
subtraction
of the UV poles 1/ϵ
after integrating



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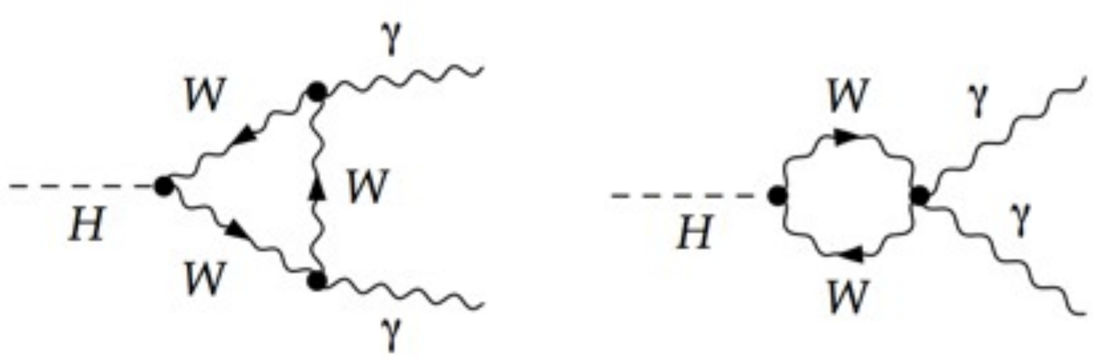
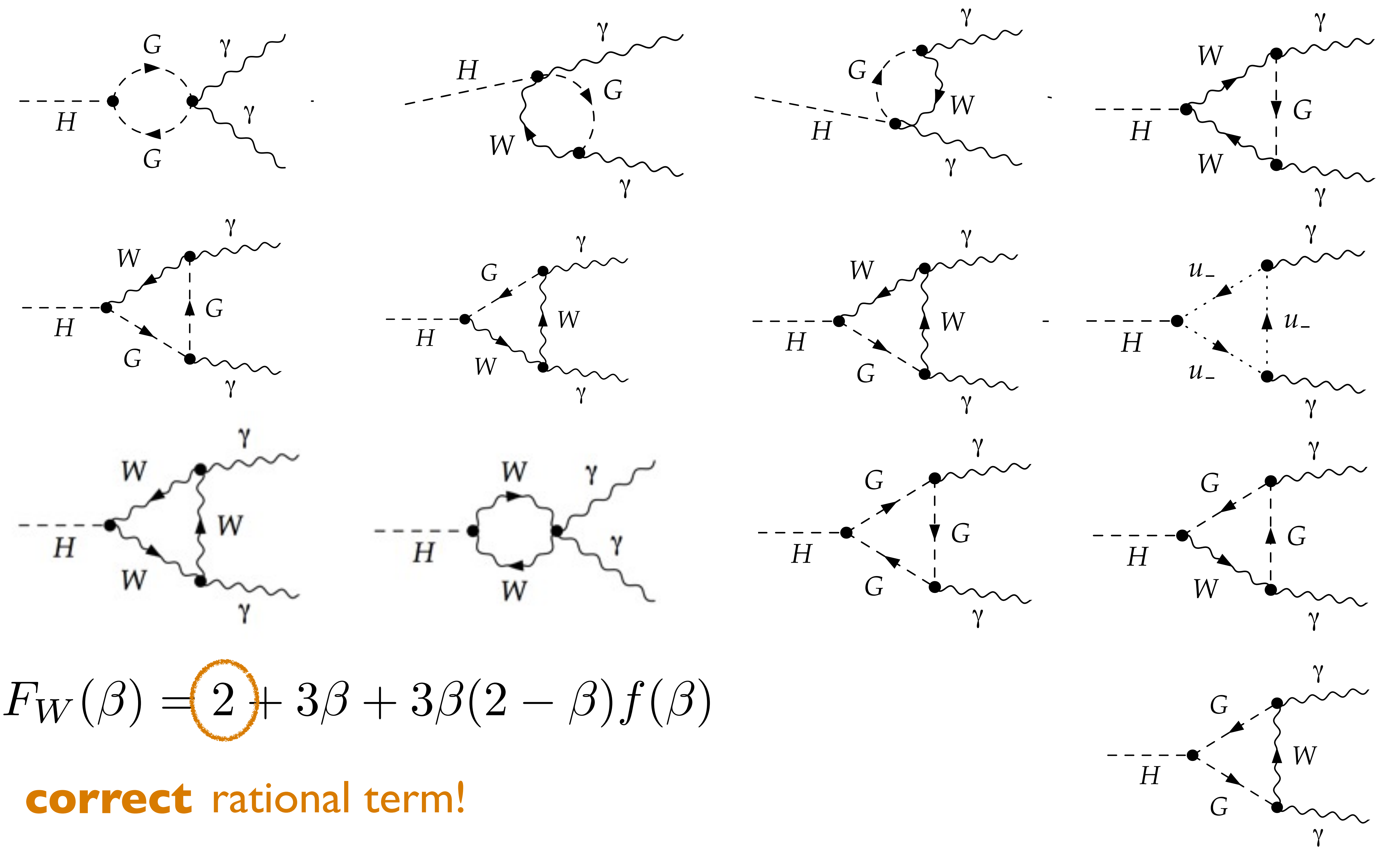


$$F_W(\beta) = 2 + 3\beta + 3\beta(2-\beta)f(\beta)$$

correct rational term!

working in arbitrary **Rξ-gauge**



$$F_W(\beta) = 2 + 3\beta + 3\beta(2-\beta)f(\beta)$$

correct rational term!

WHAT GUARANTEES GAUGE INVARIANCE?

global treatment of μ : promoting $q^2 \rightarrow \bar{q}^2 = q^2 - \mu^2$ everywhere

$$\int [\mathrm{d}^4 q] \frac{\bar{q}^2}{\bar{D}^2} = \int [\mathrm{d}^4 q] \frac{1}{\bar{D}} + \int [\mathrm{d}^4 q] \frac{m^2}{\bar{D}^2}$$

⇒ usual simplifications between numerator and denominator

$$F_W(\beta) = 2 + 3\beta + 3\beta(2-\beta)f(\beta)$$

correct rational term!

CONCLUSIONS

first test of **FDR** in the Standard Model:

it works!

it is gauge-invariant!

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F D R

FINITE

LAGRANGIAN
UNTOUCHED
~
NO CANCELLATIONS
TO BE VERIFIED

4-DIM.

NUMERICAL
INTEGRATION
~
IDENTITIES IN 4D

GAUGE INVARIANT

STRONG TEST
~
EXACT
RATIONAL TERM

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F D R

- FINITE
- 4-DIM.
- GAUGE INVARIANT

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F D R

FINITE

4-DIM.

GAUGE INVARIANT

EASIER CALCULATIONS !

AUTOMATION !

F D R

FINITE

4-DIM.

GAUGE INVARIANT

new insights on renormalization ?

predictivity of non-renormalizable theories ?

EASIER CALCULATIONS !

AUTOMATION !

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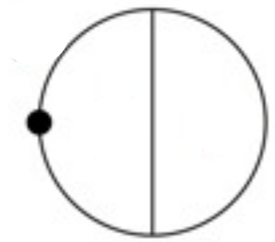
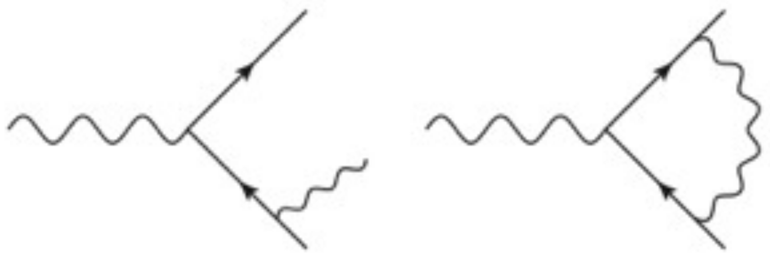
WHAT'S NEXT?

going beyond the 1-loop



studying IR and collinear divergences

(automatically regulated by μ)



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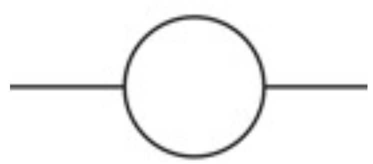
THANK YOU

THE FDR INTEGRAL

$$I_\ell^{\mathrm{FDR}} = \int \prod_{i=1}^{\ell} [\mathrm{d}^4 q_i]\; J(\{q_i^2\}) = \lim_{\mu \to 0} \int \prod_{i=1}^{\ell} \mathrm{d}^4 q_i \; J_F(\{\bar{q}_i^2\}) \Big|_{\mu=\mu_R}$$

1) parametrize in terms of μ
2) decouple vacuum configurations using partial fraction identities
3) drop vacuum configurations
4) integrate in 4 dimensions
5) take the limit $\mu \to 0$ until meeting a log divergence
6) evaluate in $\mu = \mu_R$

The bubble: example of a UV-divergent integral



$$B_0(p, m_0^2, m_1^2) = \int [\mathrm{d}^4 q] \frac{1}{\overline{D}_0 \, \overline{D}_1} = -i\pi^2 \int_0^1 \mathrm{d}\alpha \log \frac{\chi(\alpha)}{\mu_R^2}$$

$$\overline{D}_p = (q+p)^2 - m_p^2 - \mu^2$$

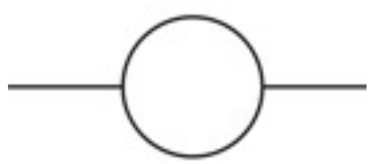
$$\chi(\alpha) = m_0^2 \, \alpha + m_1^2 \, (1-\alpha) + p^2 \, \alpha \, (1-\alpha)$$

μ_R = renormalization scale

@1-loop: 1-1 correspondece between **FDR** and **DR**

$$\frac{1}{\epsilon} \leftrightarrow \log \mu_R$$

The bubble: details of the calculation



$$\int [d^4q] \frac{1}{\bar{D}_0 \bar{D}_1} = \int_0^1 d\alpha \int [d^4q] \frac{1}{[\bar{q}^2 - \chi(\alpha)]^2} \quad *$$

Feynman parametrization
(allowed thanks to shift-invariance)

$$* \int [d^4q] \frac{1}{\bar{D}^2} \equiv \lim_{\mu \to 0} M^2 \int d^4q \left(\frac{1}{\bar{D}^2 \bar{q}^2} + \frac{1}{\bar{D} \bar{q}^4} \right) \Bigg|_{\mu = \mu_R} = -i\pi^2 \ln \frac{M^2}{\mu_R^2}$$

WHAT GUARANTEES GAUGE INVARIANCE?

solitary μ : a μ^2 must be treated as a $\bar{q}^2$ within an integral

$$\int [\mathrm{d}^4 q] \frac{\mu^2}{\bar{D}^2} = i\pi^2 m^2$$

⇒ correct constant part (rational term)

FDR

$$\int [\mathrm{d}^2 q] \frac{\mu^2}{\bar{D}^2} \rightarrow \text{constant}$$

↔

DR

$$\frac{1}{\epsilon} \times O(\epsilon) \rightarrow \text{constant}$$

where can we meet a **solitary μ** ?

during the TENSORIAL REDUCTION.

$$\frac{q^\mu q^\nu}{\bar{D}^2} = \frac{1}{4} g^{\mu\nu} \frac{q^2}{\bar{D}^2} = \frac{1}{4} g^{\mu\nu} \left(\frac{\bar{q}^2}{\bar{D}^2} + \frac{\mu^2}{\bar{D}^2} \right)$$

tensorial reduction (Passarino-Veltman, OPP, ...) works fine in **FDR**

WHAT GUARANTEES GAUGE INVARIANCE?

μ-prescription in fermionic lines (strings of γ-matrices)

$$\not{q} \to \bar{\not{q}} \equiv q \pm \mu$$

according to the position within the string:

$$(\dots\ \bar{\not{q}}\,\gamma^{\alpha_1}\dots\gamma^{\alpha_n}\,\bar{\not{q}}\,\dots) = (\dots\ (\not{q} \pm \mu)\,\gamma^{\alpha_1}\dots\gamma^{\alpha_n}(\not{q} \mp (-)^n\mu)\dots)$$

⇒ usual simplifications between numerator and denominator

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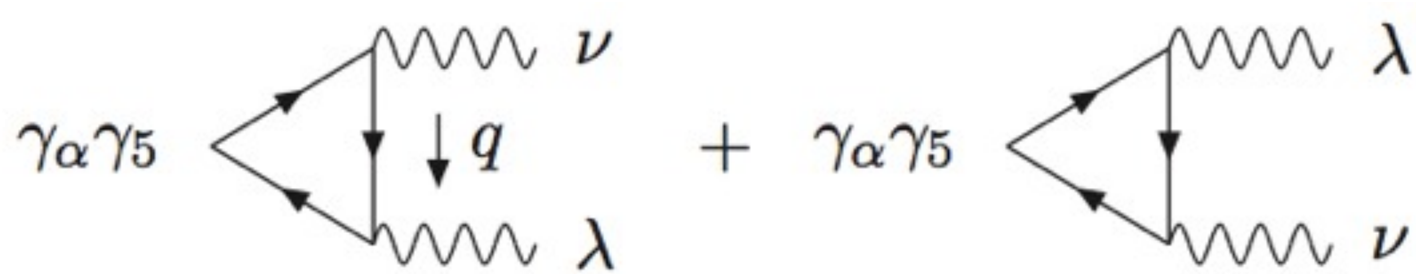
what about **the chiral matrix γ_5** ?

- all γ_5's should be anticommutated at the beginning of the string first
- all γ_5's should be anticommutated next to the non-conserved current, in closed loops

⇒ ABJ anomaly reproduced in **FDR** (JHEP1211)



Passarino-Veltman reduction in FDR

Example:

$$B^{\mu\nu} = B_{00}\, g^{\mu\nu} + B_{11}\, p^\mu p^\nu \qquad \times\ g_{\mu\nu}$$

$$q^2 = \bar{q}^2 + \mu^2$$

$$g_{\mu\nu}\, B^{\mu\nu}(p, m_0^2, m_1^2) = A_0(m_1^2) + m_0^2\, B_0(p, m_0^2, m_1^2) + \underbrace{\int [\, d^4 q] \frac{\mu^2}{(\bar{q}^2 - m_0^2)\,[(q+p)^2 - m_1^2 - \mu^2]}}_{= i\pi^2 M_W^4}$$

correct rational term!

WHAT ABOUT **RENORMALIZATION** ?

CONCEPTUAL JUMP:

it is irrelevant what mechanism nature uses
to wipe out infinities
(renormalization ? absorption into the vacuum ?)

"finite" renormalization remains
(RG equations unchanged in FDR) ⟵ @1-loop: **FDR** and **DR** in 1-1 correspondence $\frac{1}{\epsilon} \leftrightarrow \log \mu_R$

Read more about **FDR**

JHEP 11 (2012) 151 arXiv:1208.5457	"A Four Dimensional approach to Quantum Field Theories" - Roberto Pittau -
JHEP 4 (2013) arXiv:1302.5668	"Gauge Invariance at work in FDR: $H \to \gamma\gamma$" - Alice M. Donati, Roberto Pittau -
arXiv:1304.6346	"Quantum Field Theory in 4 Dimensions" - Roberto Pittau -
arXiv:1305.0419	"On the predictivity of non-renormalizable Quantum Field Theories" - Roberto Pittau -

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