Multi-particle production in the CGC framework

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The Color Glass Condensate

- the CGC: an effective theory to describe the saturation regime

McLerran and Venugopalan (1994)

lifetime of the fluctuations in a hadron or nucleus wave function ~ $xP^+/k_\perp^2$ $\Rightarrow$ $\begin{cases} \text{high-x partons} \equiv \text{static sources } \rho \\ \text{low-x partons} \equiv \text{dynamical fields } \mathcal{A} \end{cases}$

$$\left|\mathrm{hadron}\right\rangle = \left|qqq\right\rangle + \left|qqqg\right\rangle + \ldots + \left|qqq\ldots\ldots\underbrace{ggggg}_{\text{short-lived fluctuations}}\right\rangle \Rightarrow \left|\mathrm{hadron}\right\rangle = \int D\rho\, \Phi_x[\rho]\left|\rho\right\rangle \equiv \left|\mathrm{CGC}\right\rangle$$

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short-lived fluctuations

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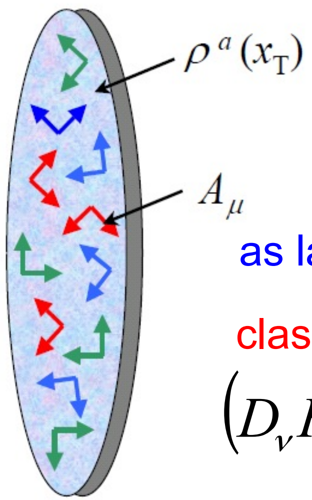


valence partons as static random color source

small x gluons as large classical fields

classical Yang-Mills equations

$$\left(D_\nu F^{\nu\mu}\right)^a = \delta^{\mu+}\delta(x^-)\rho^a(x_\perp)$$

effective wave function for the dressed hadron

separation between the long-lived high-x partons and the short-lived low-x gluons

the evolution of $\left|\Phi_x[\rho]\right|^2$ with *x* is a renormalization group equation

which sums both $\begin{cases} \alpha_s^n \ln^n(1/x) \\ g_s^n \mathcal{A}^n \end{cases}$

this effective description of the hadronic wave function applies only to the small-x part

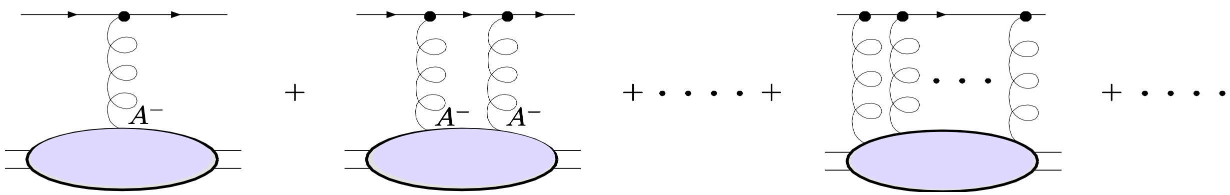
Scattering off the CGC

- this is described by Wilson lines

scattering of a quark:

$$W_F[\alpha](\mathbf{x}) = \mathcal{P}\,\text{exp}\left\{ig_s\int dx^+ T^c \alpha_c(x^+,\mathbf{x})\right\}$$

α dependence kept implicit in the following



in the CGC framework, any cross-section is determined by colorless combinations of Wilson lines $S[\alpha]$, averaged over the CGC wave function $\langle S\rangle_x = \int D\alpha\;\left|\Phi_x[\alpha]\right|^2 S[\alpha]$

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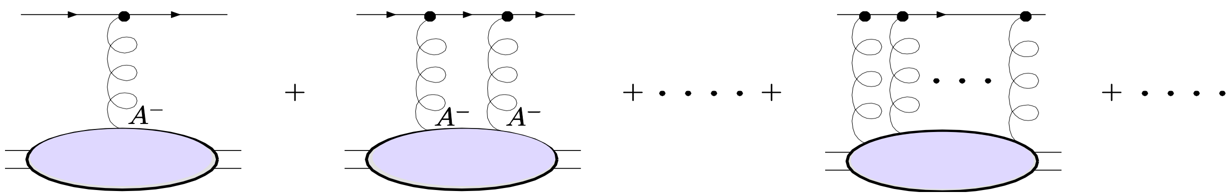

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- the 2-point function or dipole amplitude

the $q\bar{q}$ dipole scattering amplitude:

$$T_{q\bar{q}}(\mathbf{x},\mathbf{y}) = 1 - \frac{1}{N_c} Tr\left(W_F^+(\mathbf{y}) W_F(\mathbf{x})\right)$$

$\langle T_{q\bar{q}}(\mathbf{x},\mathbf{y})\rangle_x$ or $\langle T_{q\bar{q}}(\mathbf{r},\mathbf{b})\rangle_x$

$\mathbf{x}$: quark transverse coordinate

$\mathbf{y}$: antiquark transverse coordinate

this is the most common Wilson-line average

The dipole scattering amplitude

a fundamental quantity to study high-energy scattering in QCD

- deep inelastic scattering at small *x* :

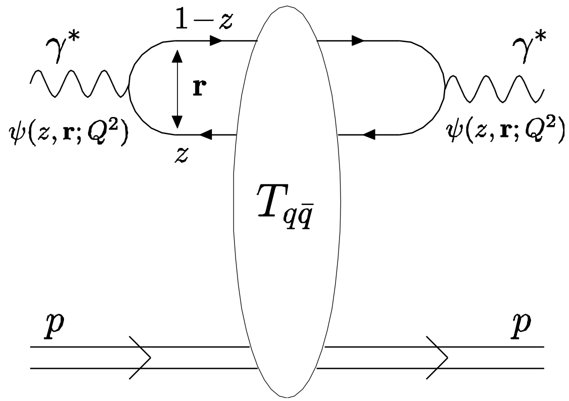
$$\sigma_{T,L}^{\gamma^* p\to X} = 2\int d^2r\, dz\, |\psi_{T,L}(z,\mathbf{r};Q^2)|^2 \underbrace{\int d^2b\, T_{q\bar{q}}(\mathbf{r},\mathbf{b},x_B)}$$

overlap of $\gamma^* \to q\bar{q}$ splitting functions

r = dipole size

dipole-hadron cross-section computed in the CGC

resums powers of $g_S A$ and powers of $\alpha_S \ln(1/x_B)$



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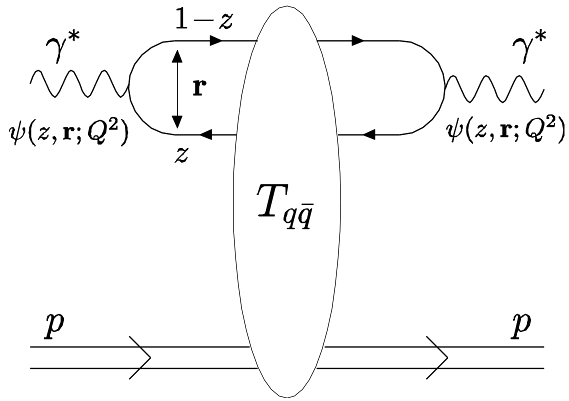


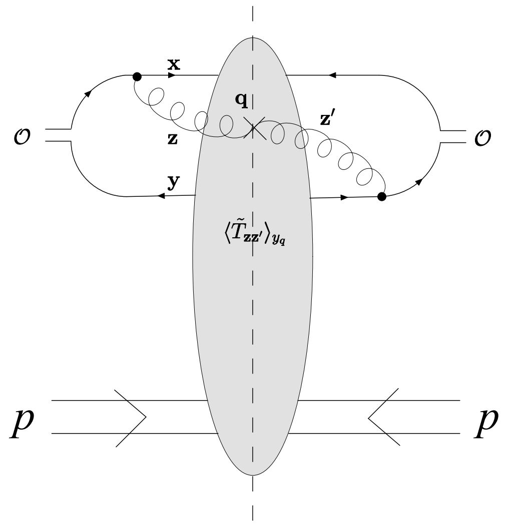
- particle production at forward rapidities:

$$q^2 \frac{d\sigma}{d^2q d^2b} \propto \int \frac{d^2r}{(2\pi)^2} \, e^{-i\mathbf{q}\cdot\mathbf{r}} [1 - T_{gg}(\mathbf{r}, \mathbf{b}, x)]$$

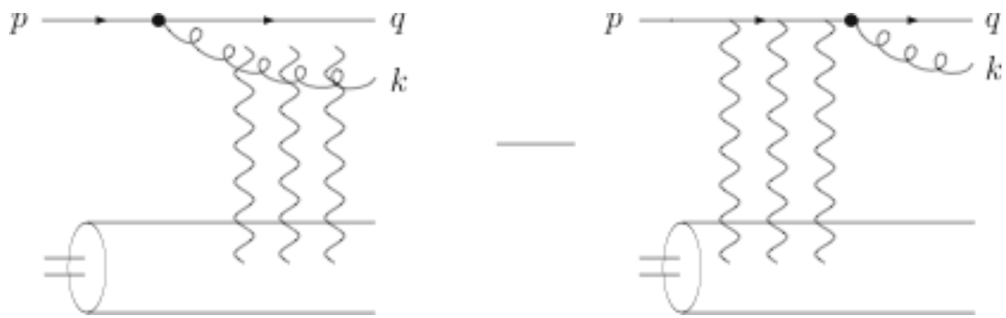
r = z-z'

dipole-hadron scattering amplitude
(adjoint or fundamental)

FT of dipole amplitude ≡
unintegrated gluon distribution



Di-hadron production



b: quark in the amplitude
x: gluon in the amplitude
b': quark in the conj. amplitude
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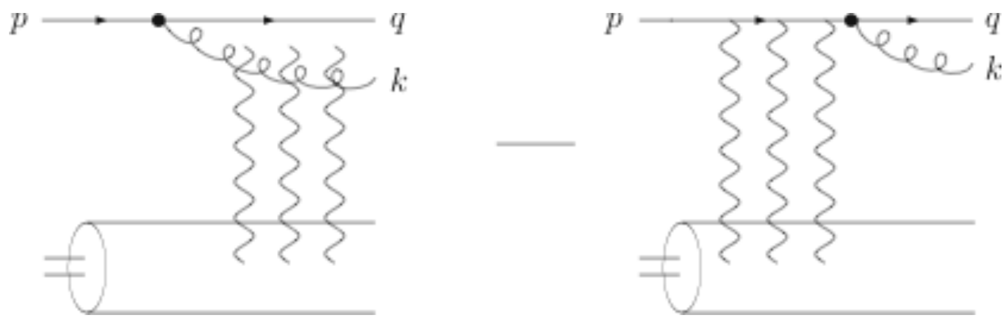

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Fourier transform $k_\perp$ and $q_\perp$ into transverse coordinates

collinear factorization of quark density in deuteron

$$\frac{d\sigma^{dAu\to qgX}}{d^2k_\perp dy_k d^2q_\perp dy_q} = \alpha_S C_F N_c\, x_d q(x_d,\mu^2) \int \frac{d^2x}{(2\pi)^2}\frac{d^2x'}{(2\pi)^2}\frac{d^2b}{(2\pi)^2}\frac{d^2b'}{(2\pi)^2}\, \overbrace{e^{ik_\perp.(\mathbf{x}'-\mathbf{x})}e^{iq_\perp.(\mathbf{b}'-\mathbf{b})}}$$

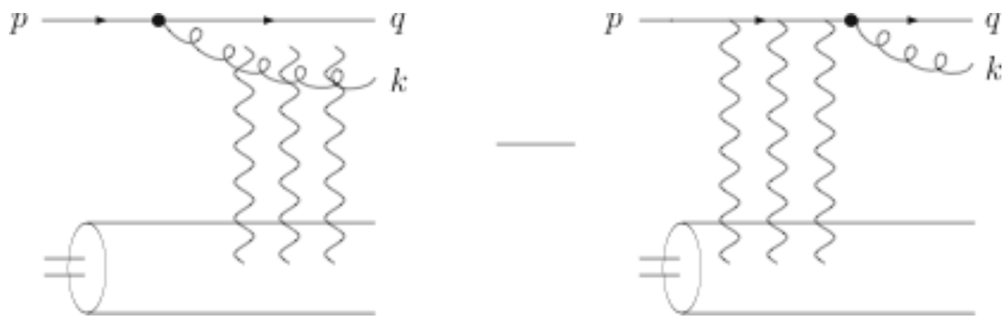
$$\left|\Phi^{q\to qg}(z,\mathbf{x}-\mathbf{b},\mathbf{x}'-\mathbf{b}')\right|^2 \Big\{S^{(4)}_{qg\bar{q}g}[\mathbf{b},\mathbf{x},\mathbf{b}',\mathbf{x}';x_A] - S^{(3)}_{qg\bar{q}}[\mathbf{b},\mathbf{x},\mathbf{b}'+z(\mathbf{x}'-\mathbf{b}');x_A]$$

$$-S^{(3)}_{\bar{q}gq}[\mathbf{b}+z(\mathbf{x}-\mathbf{b}),\mathbf{x}',\mathbf{b}';x_A] + S^{(2)}_{q\bar{q}}[\mathbf{b}+z(\mathbf{x}-\mathbf{b}),\mathbf{b}'+z(\mathbf{x}'-\mathbf{b}');x_A]\Big\}$$

pQCD $q \to qg$ wavefunction

$$z = \frac{|k_\perp|e^{y_k}}{|k_\perp|e^{y_k} + |q_\perp|e^{y_q}}$$

Di-hadron production



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pQCD $q \to qg$ wavefunction

interaction with target nucleus

$$z = \frac{|k_\perp|e^{y_k}}{|k_\perp|e^{y_k} + |q_\perp|e^{y_q}}$$

n-point functions that resums the powers of $g_S A$ and the powers of $\alpha_S \ln(1/x_A)$

2- 4- and 6-point functions

the scattering off the CGC is expressed through the following correlators of Wilson lines:

if the gluon is emitted before the interaction, four partons scatter off the CGC

$$S^{(4)}_{qg\bar{q}g}(\mathbf{b},\mathbf{x},\mathbf{b}',\mathbf{x}';x_A) = \frac{1}{C_F N_c}\left\langle \mathrm{Tr}\left(W_F(\mathbf{b})W_F^\dagger(\mathbf{b}')T^d T^c\right)\left[W_A(\mathbf{x})W_A^\dagger(\mathbf{x}')\right]^{cd}\right\rangle_{x_A}$$

if the gluon is emitted after the interaction, only the quarks interact with the CGC

$$S^{(2)}_{q\bar{q}}(\mathbf{b},\mathbf{b}';x_A) = \frac{1}{N_c}\left\langle \mathrm{Tr}\left(W_F(\mathbf{b})W_F^\dagger(\mathbf{b}')\right)\right\rangle_{x_A}$$

interference terms, the gluon interacts in the amplitude only (or c.c. amplitude only)

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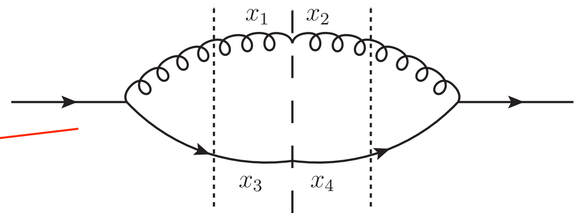
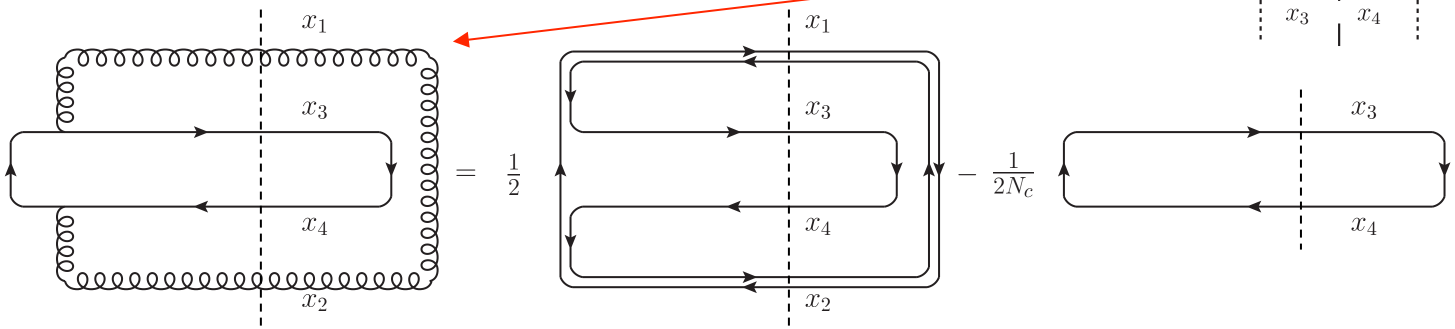
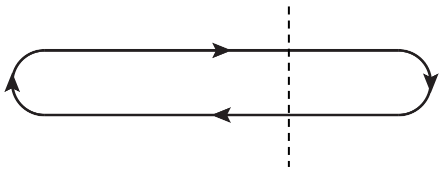
in the large Nc limit, everything is expressed in terms
of dipoles and quadrupoles

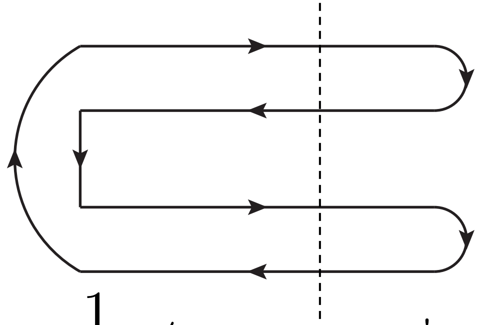




From n to n+1 particles produced

- assume the cross section for the production of n partons is made of only dipoles and quadrupoles





$$S^{(2)} = \frac{1}{N_c} \left\langle \mathrm{Tr}(W_{\mathbf{x}} W_{\mathbf{y}}^{\dagger}) \right\rangle_{x_A} \qquad S^{(4)} = \frac{1}{N_c} \left\langle \mathrm{Tr}(W_{\mathbf{x}} W_{\mathbf{y}}^{\dagger} W_{\mathbf{u}} W_{\mathbf{v}}^{\dagger}) \right\rangle_{x_A}$$

- then one can show that the cross section for the production of n+1 partons is also made of only dipoles and quadrupoles

gluon line added between two different objects

does not increase the total number of color traces (loops in the diagramatic representation) therefore these contributions will be subleading in Nc (they do involve higher-point functions)

gluon line added within same object

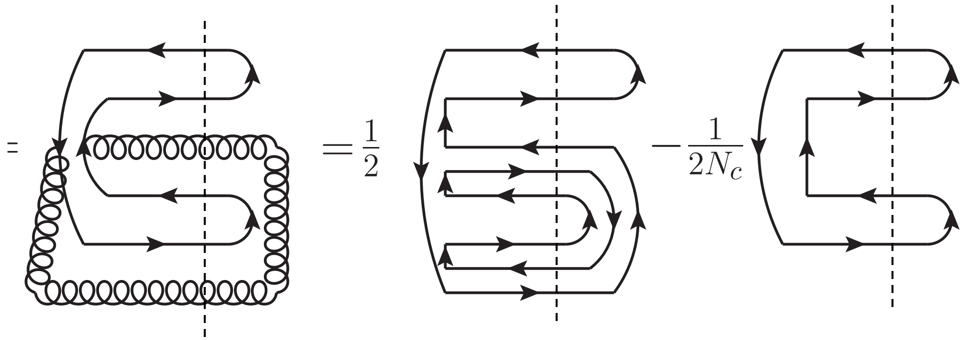
Dominguez, CM, Stasto and Xiao (2013)

this will provide the leading-Nc terms

From n to n+1 particles produced

- gluon line added within the same objects

if the extra gluon interacts both in the amplitude and in the conjugate amplitude
this adds a quadrupole to the expression for the n-parton cross section



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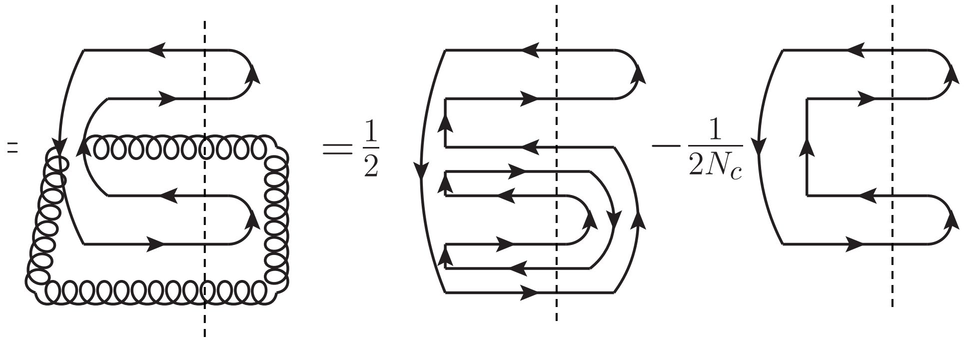

if the extra gluon interacts only in the amplitude (or in the conjugate amplitude)
this adds a dipole to the expression for the n-parton cross section

if the extra gluon interacts in neither the amplitude or the conjugate amplitude
this splits a quadrupole into two dipoles (or adds a Nc factor to a dipole)

- other case to consider

  q-qbar pair added to n-1 partons

Conclusions

- Non-linear small-x evolution well established for dipoles
  - cornerstone: the Balitsky-Kovchegov equation
  - running-coupling corrections necessary for phenomenology

- The non-linear evolution of the quadrupole
  - brought to our attention by recent di-hadron correlation measurements
  - so far the phenomenology relies on (too many?) approximations

- Large-Nc limit: dipoles and quadrupoles sufficient for multi-particle production in pA collisions
  - worth to put some effort to accurately solve the quadrupole evolution
  - need data to constrain the initial condition (e.g. p+Pb at the LHC)