Longitudinal WW scattering in light of the "Higgs" discovery
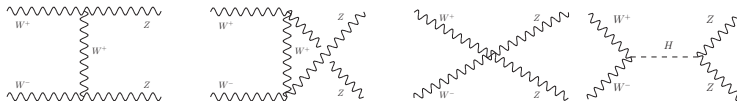
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D.E., B. Yencho, PRD 87, 055017 (2013), arXiv:1212.4158
D.E., F. Mescia and B. Yencho, in preparation

LHCP2013, May 2013

Motivation

We know that in the SM the Higgs boson unitarizes $W_L W_L$ scattering. Consider e.g. $W_L^+ W_L^- \to Z_L Z_L$

If any of these couplings are different from SM values, the careful balance necessary for perturbative unitarity is lost
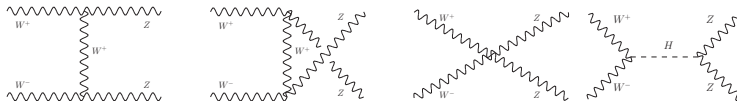Furthermore, having new effective operators typically spoils unitarity too.

$$\mathcal{L}_{SM} \to \mathcal{L}_{SM} + \sum_i a_i \mathcal{O}_i$$

New physics may produce either type of modifications
What can the unitarity of longitudinal WW scattering tell us about anomalous couplings in EW sector?

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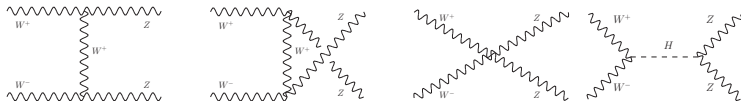
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Parametrizing 'new' physics

A light "Higgs boson" with mass $M_H \sim 125$ GeV is coupled to the EW bosons according to

$$
\begin{aligned}
\mathcal{L}_{\text{eff}} \supset\; & -\frac{1}{2}\text{Tr}W_{\mu\nu}W^{\mu\nu} - \frac{1}{4}\text{Tr}B_{\mu\nu}B^{\mu\nu} + \mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{FP}} + \sum_i \mathcal{L}_i \\
& + \left[1 + 2a\left(\frac{h}{v}\right) + b\left(\frac{h}{v}\right)^2\right]\frac{v^2}{4}\text{Tr}D_\mu U^\dagger D^\mu U - V(h)
\end{aligned}
$$

where the Goldstone bosons are in the nonlinear representation

$$
\begin{aligned}
U &= \exp(i\,\omega\cdot\tau/v) \\
D_\mu U &= \partial_\mu U + \frac{1}{2}igW_\mu^i\tau^i U - \frac{1}{2}ig'B_\mu^i U\tau^3
\end{aligned}
$$

and additional gauge-invariant operators are encoded in $\mathcal{L}_i$.
Setting $a = b = 1$ (and $\mathcal{L}_i = 0$) reproduces the SM interactions

d=4 operators

The 14 $\mathcal{L}_i$ are a full set of C, P, and $SU(2)_L \times U(1)_Y$ gauge invariant, $d = 4$ operators that parameterize the *low-energy effects* of the *model-dependent high-energy EWSB sector* along with *a,b*. The two most relevant *custodial-symmetry preserving* operators are

$$\mathcal{L}_4 = a_4 \left(\operatorname{Tr}[V_\mu V_\nu]\right)^2 \qquad\qquad \mathcal{L}_5 = a_5 \left(\operatorname{Tr}[V_\mu V^\mu]\right)^2$$

where $V_\mu = (D_\mu U) U^\dagger$

- For example:

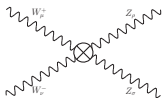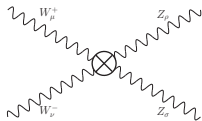|		*Heavy Higgs*	*Technicolor*
a_4	$=$	0	$-2a_5$
a_5	$=$	$\frac{v^2}{8M_H^2}$	$\frac{N_{TC}}{96\pi^2}$

(up to logarithmic corrections)

$$ig^4 \left[a_4 \left(g^{\mu\sigma}g^{\nu\rho} + g^{\mu\rho}g^{\nu\sigma}\right) + 2a_5 g^{\mu\nu}g^{\rho\sigma}\right]$$

After the 'Higgs' discovery

There are solid indications that the "Higgs" couples to the W, Z
very similarly to the SM rules

$$\mathcal{L}_{\text{eff}} \simeq \mathcal{L}_{\text{SM}} + a_4 \left(\text{Tr} \left[V_\mu V_\nu \right] \right)^2 + a_5 \left(\text{Tr} \left[V_\mu V^\mu \right] \right)^2$$

Then a_4 and a_5 represent anomalous 4-point couplings of the W
bosons due to an extended EWSBS that however does not manifest
with $O(p^2)$ couplings *noticeably different to the ones in the SM*
These operators will lead to violations of perturbative unitarity at
loop level $(\sim g^4)$

$$\sim \left(\frac{s}{v^2} \right)^2$$

Violations of unitarity in strongly interacting theories are cured by
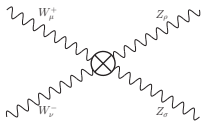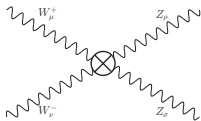*the appeareance of new resonances*
We can now use well-understood unitarization techniques to
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To take home...

- The Higgs unitarizes these amplitudes in SM (where $a = b = \{a_i\} = 0$)
- The theory is renormalizable *without* the $\{a_i\}$ if $a = b = 0$
- The $\{a_i\}$ will then be finite non-running parameters.

We would like to

- Determine how much room is left for the a_i
- Find possible additional resonances imposed by unitarity
- Should we have already seen any?
- To what extent an extended EWSBS is excluded?

Yes, there are new resonances with relatively light masses
No, we should not have seen them yet. Their signal is too weak
Looking for the resonances is an efficient (albeit indirect) way of
setting constrains on aTGC and aQGC

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The method

Partial waves

We will assume $e = 0$ (*no e.m.*) i.e. the custodial limit $c_w = 1$.
The *WW* scattering amplitudes can then be deconstructed into
amplitudes of *fixed isospin* T_I

$$
\begin{aligned}
T_0 &= 3A^{+-00} + A^{++++} \\
T_1 &= 2A^{+-+-} - 2A^{+-00} - A^{++++} \\
T_2 &= A^{++++}
\end{aligned}
$$

where $A^{+-00} = A(\omega^+\omega^- \to \omega^0\omega^0)$ and all others may be expressed
in terms of this amplitude through isospin and crossing symmetries
These can then be written in terms of *partial waves*

$$
t_{I\,J}(s) = \frac{1}{64\pi} \int_{-1}^{1} d(\cos\theta) P_J(\cos\theta) T_I
$$

which are constrained by unitarity at high energies to be $|t_{I\,J}| < 1$.
Most discussions based on unitarity are based on this simple
constraint (*tree-level unitarity*)

Inverse Amplitude method

Partial wave unitarity **requires**

$$\text{Im } t_{I\,J}(s) \;=\; \sigma(s)|t_{I\,J}(s)|^2 \quad + \quad \sigma_H(s)|t_{H,I\,J}(s)|^2$$

<div align="center">

Elastic   Inelastic

$WW \to WW$ $WW \to hh$

</div>

where σ and σ_H are phase space factors.

Given an expansion

$$t_{I\,J} \quad \approx \quad t_{I\,J}^{(2)} \quad + \quad t_{I\,J}^{(4)} \quad + \quad \cdots$$

<div align="center">

tree   one-loop

$+\ a_i$ terms

</div>

we can require unitarity to hold (up to second order) by defining

$$t_{I\,J} \approx \frac{t_{I\,J}^{(2)}}{1 - t_{I\,J}^{(4)}/t_{I\,J}^{(2)}}$$

(Several mild assumptions concerning the analyticity are implied)

New resonances

The unitarization of the amplitudes may result in the appearance of *new heavy resonances* associated with the high-energy theory

$$
\begin{aligned}
t_{00} &\rightarrow \text{Scalar isoscalar} \\
t_{11} &\rightarrow \text{Vector isovector} \\
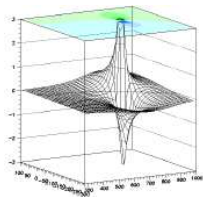t_{20} &\rightarrow \text{Scalar isotensor}
\end{aligned}
$$

Will search for poles in $t_{I\,J}(s)$ up to $(4\pi v) \sim 3$ TeV (domain of applicability)

True resonances will have the phase shift pass through $+\pi/2$

$$
\delta_{IJ} = \tan^{-1}\left(\frac{\operatorname{Im} t_{IJ}}{\operatorname{Re} t_{IJ}}\right)
$$

This method is known to work remarkably well in strong interactions: $\pi\pi$ scattering $\Rightarrow \sigma$ and ρ meson masses and widths

In hadronic physics

Is this unitarization method unique?

*No, it is not. Many methods exist: IAM, K-matrix approach, N/D expansions, Roy equations,...*

While the quantitative results differ slightly, the gross picture does not change

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Real problem: one-loop calculation *extremely difficult*
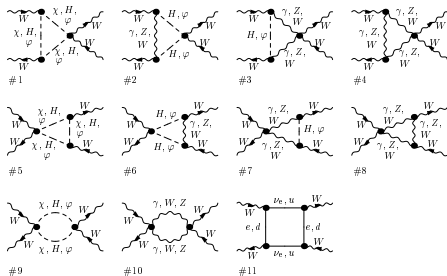
+ *more than 1000 other diagrams!*

Denner & Hahn (1998) [hep-ph/9711302]

Shortcut

We can take a shortcut:

$$t_{IJ}^{(4)} = \mathrm{Re}\, t_{IJ}^{(4)} + \mathrm{Im}\, t_{IJ}^{(4)}$$

The **Optical Theorem** implies the *perturbative* relation

$$\mathrm{Im}\, t_{I\,J}^{(4)}(s) = \underset{\text{one-loop}}{\sigma(s)|t_{I\,J}^{(2)}(s)|^2} + \underset{\text{tree}}{\sigma_H(s)|t_{H,I\,J}^{(2)}(s)|^2}$$

For *real part*, note that

$$\begin{aligned}
\mathrm{Re}\, t_{IJ}^{(4)} &= a_i\text{-dependent terms} \quad + \quad \text{real part of loop calculation} \\
&\approx a_i\text{-dependent terms} \\
&\quad \text{(for large } s,\, a_i\text{)}
\end{aligned}$$

We approximate *real part of loop contribution* with one-loop Goldstone boson amplitudes using the Equivalence Theorem

Summary of the method

To summarize:

* In all cases, a single fundamental amplitude A^{+-00} is calculated
* It is used to construct the isospin amplitudes T_I using isospin/crossing relations
* It is then expressed as the lowest order partial wave in each isospin channel (t_{00}, t_{11}, and t_{20}), where

$$
\begin{array}{lcl}
t_{IJ}^{(2)} & \rightarrow & \text{calculated from tree-level amplitude with } W_L \\[1mm]
\operatorname{Re} t_{IJ}^{(4)} & \rightarrow & \text{calculated from } a_i\text{-dependent terms with } W_L + \\
& & \text{real part of one-loop Goldstone boson scattering} \\[1mm]
\operatorname{Im} t_{IJ}^{(4)} & \rightarrow & \left\{ \begin{array}{ll} \sigma(s)|t_{IJ}^{(2)}|^2 + \sigma_H(s)|t_{H,IJ}^{(2)}|^2 & \text{if } I = 0 \\ \sigma(s)|t_{IJ}^{(2)}|^2 & \text{otherwise} \end{array} \right.
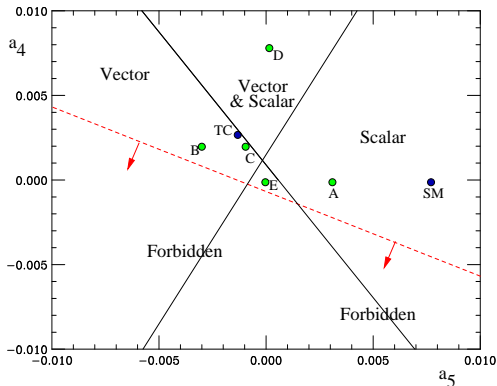\end{array}
$$

Unitarity checks
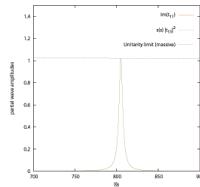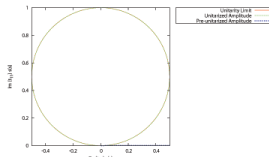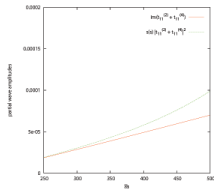
Vector channel ($a_4 = 0.008, \quad a_5 = 0$)
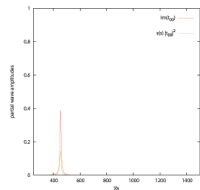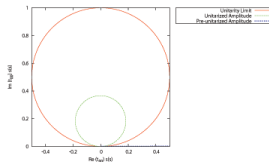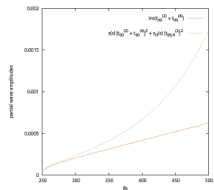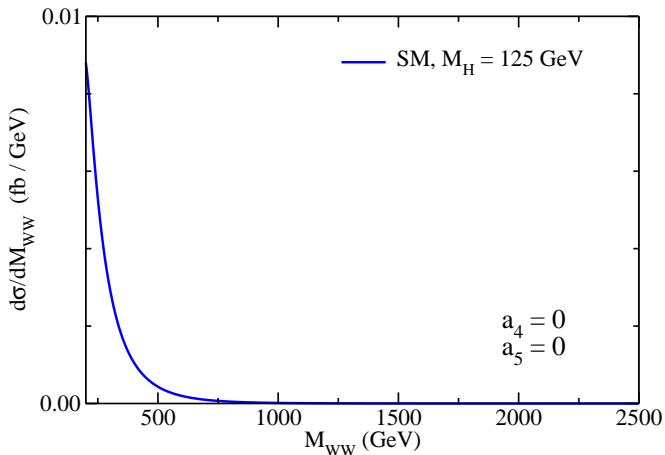

Scalar channel ($a_4 = 0.008, \quad a_5 = 0$)

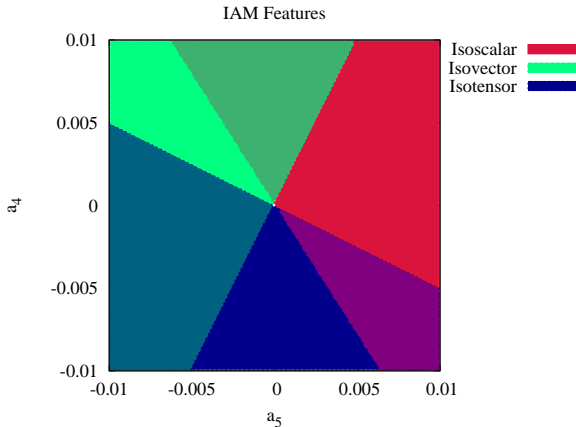# The results

Before the 'Higgs' discovery: $M_H^2 >> s$

Butterworth, Cox, & Forshaw (2002) [hep-ph/0201098]
$(\mu = M_H = 1 \text{ TeV})$

Forbidden region has isotensor "resonances" in which the phase δ_{20} goes through $-\pi/2$ (*violates causality*)

The SM

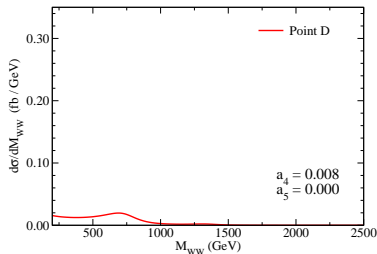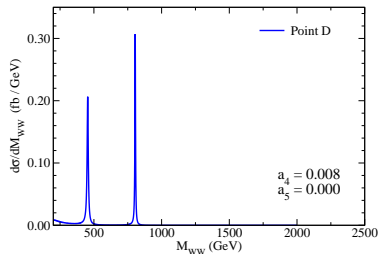For SM ($a_4 = a_5 = 0$) there are *no additional resonances*.

IAM Features

The regions are similar to before, but resonances below 3 TeV now appear in *even more of the space*.

However, the properties of the resonances are quite different.

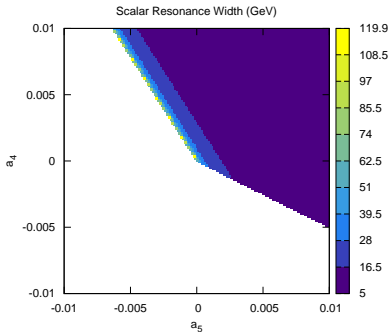# Comparison

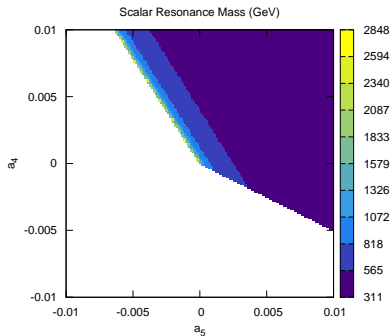No light Higgs   With light Higgs

Here $\sqrt{s} = 8$ TeV

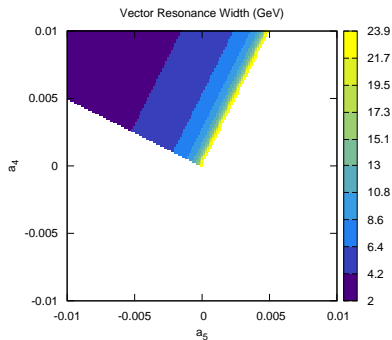Compare before/after for same point (ex: Point D $a_4 = 0.008$, $a_5 = 0.000$)

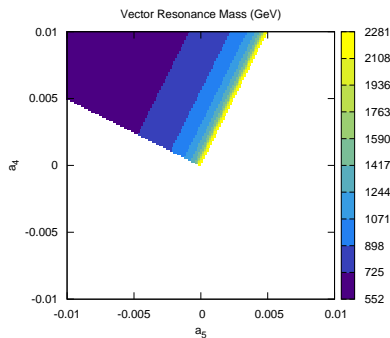- Different continuum
- Masses have changed positions
- Widths are *much narrower*

Scalar Resonance Mass (GeV)

Scalar Resonance Width (GeV)

- $M_S \sim 300 - 3000$ GeV
- $\Gamma_S \sim 5 - 120$ GeV

Vector Resonance Mass (GeV)

Vector Resonance Width (GeV)

- $M_V \sim 550 - 2300$ GeV
- $\Gamma_V \sim 2 - 24$ GeV

We can estimate how observable these signals are by comparing to a heavy SM Higgs of the same mass → *look at LHC Higgs search data*

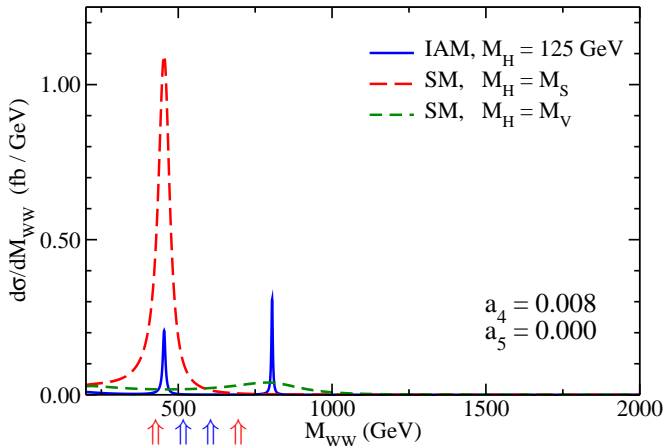For a resonance of mass M_R and width Γ_R, let

$$
\begin{aligned}
\sigma^{peak} &\equiv \int_{M_R-2\Gamma_R}^{M_R+2\Gamma_R} \left[dM_{WW} \times \frac{d\sigma}{dM_{WW}} \right] \\
\sigma_{SM}^{peak} &\equiv \int_{M_H-2\Gamma_H}^{M_H+2\Gamma_H} \left[dM_{WW} \times \frac{d\sigma_{SM}}{dM_{WW}} \right]
\end{aligned}
$$

Then for a heavy Higgs with $M_H \to M_R$ and $\Gamma_H(M_H \to M_R)$

$$
R \equiv \left(\frac{\sigma^{peak}}{\sigma_{SM}^{peak}} \right)
$$
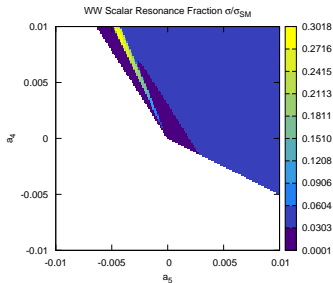
compares the strength of the resonance regions of the same mass.

Comparison of IAM scalar/vector signal and SM Higgs of *same mass*

Scalar

Vector
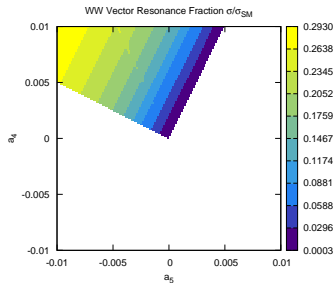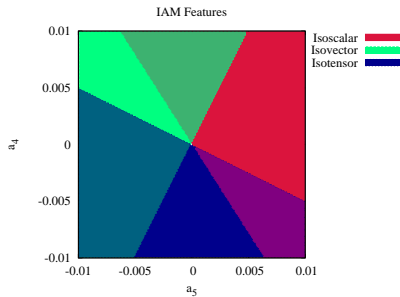
$R \sim 0.0001 - 0.30$
Typically $\mathcal{O}(0.01)$

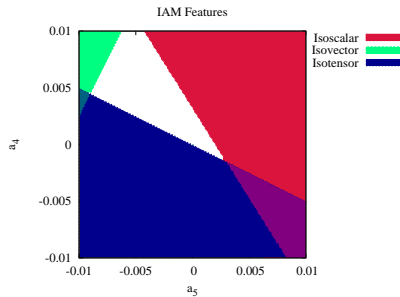$R \sim 0.0003 - 0.3$
Typically $\mathcal{O}(0.1)$

Currently probing, at best, $R \sim 1$

The large contribution that the SM Higgs represents leaves little room for additional resonances. They *could still be there*, but would give a small signal.

All Resonances

$M_R < 600$ GeV

Nothing prevents us from carrying out the same programme

$$
\begin{aligned}
\mathcal{L}_{\text{eff}} &= -\frac{1}{2}\text{Tr}W_{\mu\nu}W^{\mu\nu} - \frac{1}{4}\text{Tr}B_{\mu\nu}B^{\mu\nu} + \sum_{i=0,13}\mathcal{L}_i + \mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{FP}} \\
&+ \left[1 + 2a\left(\frac{h}{v}\right) + b(\frac{h}{v})^2\right]\frac{v^2}{4}\text{Tr}D_\mu U^\dagger D^\mu U + \frac{1}{2}(\partial_\mu h)^2 - \frac{1}{2}M_H^2 h^2 \\
&- d_3(\lambda v)h^3 - d_4\frac{1}{4}h^4
\end{aligned}
$$

This *effective theory* is *non-renormalizable* and the a_i will be required to absorb the divergences

$$
\delta a_4 = \Delta_\epsilon \frac{1}{(4\pi)^2}\frac{-1}{12}(1-a^2)^2
$$

$$
\delta a_5 = \Delta_\epsilon \frac{1}{(4\pi)^2}\frac{-1}{24}\left[(1-a^2)^2 + \frac{3}{2}((1-a^2)-(1-b))^2\right]
$$

We have set $d_3 = d_4 = 1$ for simplicity.

For $a = b = 1$ these results reproduce the SM prediction, i.e. no counterterms (renormalizable theory)

$$\delta a_4 = 0, \qquad \delta a_5 = 0$$

For $a = b = 0$ one gets the 'no Higgs' results (EChL)

$$\delta a_4 = \Delta_\epsilon \frac{1}{(4\pi)^2} \frac{-1}{12}, \qquad \delta a_5 = \Delta_\epsilon \frac{1}{(4\pi)^2} \frac{-1}{24}$$

So far everything suggests $a \simeq 1$ but b is largely unbounded. If $a = 1$

$$\delta a_4 = 0$$

$$\delta a_5 = \Delta_\epsilon \frac{1}{(4\pi)^2} \frac{-1}{16}(1 - b)^2 \quad a_5|_{\text{finite}} \simeq \frac{1}{256\pi^2}(1 - b)^2 \log \frac{f^2}{v^2}$$

We can also determine other interesting counterterms and coefficients such as those relevant for the TGC vertex

Conclusions

- Unitarity is a powerful constraint on scattering amplitudes. Its validity is well tested.

- Even in the presence of a light Higgs, it can help constrain anomalous couplings by helping predict heavier resonances.

- An extended EWSBS would typically have such resonances even in the presence of a light 'Higgs'

- However their properties are radically different from the 'standard lore'

- Current LHC Higgs search results do not yet probe the IAM resonances, but may be possible in near future.

THANK YOU!

Conclusions

- Unitarity is a powerful constraint on scattering amplitudes. Its validity is well tested.

- Even in the presence of a light Higgs, it can help constrain anomalous couplings by helping predict heavier resonances.

- An extended EWSBS would typically have such resonances even in the presence of a light 'Higgs'

- However their properties are radically different from the 'standard lore'

- Current LHC Higgs search results do not yet probe the IAM resonances, but may be possible in near future.

THANK YOU!