Chiral medium in magnetic field

Consider a chiral medium (massless fermionic constituents)

Electric current flowing along external magnetic field B:

$$\mathbf{j}_{el} = \frac{\mu_5}{2\pi^2}\mathbf{B}$$

where μ_5 is the chiral chemical potential, $\mu_L = 1/2(\mu_L - \mu_R)$

The point not to miss: no such current in Maxwell eqns

The reason: a macroscopic manifestation
of the chiral (triangle) anomaly

$j_{el} = (const)\mu_5 B$, brief history

- Introduced many years ago by Vilenkin along with other chiral effects; in particular, chiral vortical effect:

$$\vec{j}_5 = (const)\vec{\Omega}$$

- Manifestations in heavy-ion collisions (Kharzeev...)

$$\mu_5 \sim \sqrt{|Q_{top}|} \sim V_{tot}^{-1/2}$$

- Theoretical developments (non-renormalization...)
- Turn to condensed matter, applications

Recent volume of Lecture Notes on Physics

Theoretical developments/challenges

- Non-renormalization theorems in hydrodynamics (Son & Surowka..)
- No dissipation (Kharzeev & Yee)
- Condensed matter (semimetals, chiral batteries)

The outcome (say, Trento workshop, Nov 2012)
"Chiral Superconductivity":

From general considerations,
there exists dissipation-free, equilibrium current,
no apparent limitations on the temperature.
(However, no explicit microscopic mechanism known)

Most recent: further scrutiny; probably, doubts

Chiral particles in magnetic field, naive picture

Let us begin with massive, spinning particles.
Then, because of the interaction

$$H_{Pauli} = -\vec{\mu} \cdot \vec{B}$$

Spin of positively charged particles looks along $\vec{B}$, negatively charged particles look in opposite direction.

Now let particles move (they are massless) and assume that all particles, say, left-handed, i.e. move along their spins

Then charges get separated and electric current arises

Chiral magnetic effect, non-interacting particles

For non-interacting particles explicit Landau levels are known and produce:

$$\vec{J}_{el} = \frac{\mu_5}{2\pi^2}\vec{B}$$

The whole effect is due to zero modes, i.e. non-perturbative

Instead of “naive picture” :
magnetic moment for a particle near the Fermi sphere,

$$\mu_B \sim 1/\mu_5$$

with factor μ_5^2 coming from the area of the Fermi surface.
No generalization yet for $T \neq 0$

Non-renormalizability by strong interactions

Consider hydrodynamics as
an effective theory (expansion in derivatives)
Introduce external fields with $(E \cdot B) \neq 0, \quad E \to \ 0$

Current: $j_\mu \ = \ nu_\mu + \xi_B B_\mu + ...$
$(B_\mu = 1/2\epsilon_{\mu\nu\alpha\beta}u^\nu F^{\alpha\beta}))$

Entropy current: $s_\mu \ = \ su_\mu + D_B B_\mu + ...$

Imposing $\partial_\mu j^\mu = (anomaly), \quad \partial_\mu s^\mu \geq 0$
solve for ξ_B, D_B in terms of the anomaly

The result is protected by the A-B theorem
Current persists in equilibrium

Non-renormalizability cnt'd

Temperature-dependent chiral vortical effect

$$\vec{j}_5 \sim T^2 \vec{\Omega}$$

is calculable in equilibrium via static correlator

$$\sigma_\Omega = \lim_{k_z \to 0} \frac{i}{k_z} < j_5^x, \; T^{0y} >$$

Corrections mostly cancel
(variation of the Hill-Coleman theorem)

A well-defined log correction (Son & Yamamoto...) survives

Anyhow, up to logs chiral effect grows with temperature

No Dissipation!

The current

$$\vec{j} = \sigma_B \vec{B}$$

is not related to any dissipation.

The argumentation:
An equilibrium current; Magnetic field produces no work.
Hence no dissipation is possible.

Note similarity to superconductivity.
However, the chiral current is fixed,
no additional degree of freedom.
Rather quantum ballistic transport than quantum collective flow.

Another proof

Eqns $\vec{j}_{el} = \sigma_E \vec{E}$ and $\vec{j}_{el} = \sigma_B \vec{B}$

change with opposite sign under time reflection

$$t \rightarrow -t$$

Conclusion: dissipation is forbidden by time invariance
No such selection rule in case of, say, superfluidity

Implication: superfluidity might be not perfect
$(\eta/s \geq 1/4\pi)$
while superconductivity is protected by time invariance
(Stodolsky)

Condensed matter, very briefly

Materials with "relativistic-like" spectrum

$$\epsilon = v_F \cdot p$$

(Similar to superfluidity but for fermions)
Most famous, graphene. For our purpose, need 3d materials, so called semi-metals

Chiral battery (Kharzeev & Yee):

- Switch on $(E \cdot B) \neq 0$, create $\mu_5 \neq 0$
- Switch off fiellds and carry on the battery anywhere
- Switch on magnetic field, get dissipation-free current

(also variations)

Reservations, concerns. Bremsstrahlung.

There exist reservations and concerns about loopholes

A rather obvious one:
massless charged particles are a headache
because of bremsstrahlung

Now, first detailed study exists:
Radiative corrections to chiral separation effect in QED
E.V. Gorbar, et al. ,arXiv:1304.4606 [hep-ph]

The main effect is renormalization of chemical potential

Most probably, the problems are not so serious in case of condensed matter (important for applications)

UV vs IR sensitivity of chiral effects

Evaluation of chiral conductivity,

$$\sigma \sim \lim_{k_z \to 0} \frac{1}{k_z} < j^x_{el} j^y_{el} >$$

is in fact saturated by an UV sensitive polynomial, $\epsilon_{xyz} k_z$ (see Landsteiner et al. 1207.5808)
As a result, chiral effects turn to be independent on mass, intuitively, does not fit

"Views of the Chiral Magnetic Effect"
Kenji Fukushima, arXiv:1209.5064

Towards infrared-sensitive renormalization

First attempt to get rid of heavy particles is
A. Vilenkin (1980)
"Cancellation Of Equilibrium Parity Violating Currents"
But here, the effect is subtracted, independent of mass m_f

"Modern trend" is let the effect survive for $m_f = 0$
V.Z., 1210.2186, D. Kharzeev, Trento Conf, Nov. 2012...
But no well-defined, commonly accepted procedure yet

One of techniques: expand two-point function in chemical potential μ_5, μ and reduce it to the anomalous triangular graph (H.Liu et al., 1103.2035)

Instability of $\mu_5 \neq 0$ state?

Chiral magnetic effect can be imitated by effective action

$$\delta S = \frac{1}{2}\int d^3x \epsilon_{ijk} A^i \partial^j A^k$$

where A^i is the electromagnetic potential
In case of magnetostatics, we come then to Beltrami eqn

$$curl\ B = \sigma\ B$$

as a result of the back-reaction of the medium to $j \sim B$.
The extra piece corresponds to an imaginary topological mass of photon signaling instability of external magnetiic field in the medium. Most probably, magnetic field produced also spontaneously (V. Kirilin et al. (2013))

Conclusions

- beautiful effects, macroscopic manifestations of anomaly, have been derived
- there could be far-reaching applications in condensed-matter physics
- seemingly, in case of ChME theory supported by the data on heavy-ion collisions
- if so, the experiment might be even faster than theory which continues to scrutinize the derivations