# Asymptotics of traveling waves with running coupling and fluctuations

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# Introduction



Two types of effects are not included in the Balitsky and JIMWLK equations for the saturation, but seem to be relevant:

1. Pomeron loops, with in particular gluon number fluctuations.
2. NLL and higher order effects, with in particular running coupling.

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1. Pomeron loops, with in particular gluon number fluctuations.
2. NLL and higher order effects, with in particular running coupling.

- Fixed coupling without fluctuations: $\Rightarrow$ Geometric scaling in $Y$.
- Running coupling without fluctuations: $\Rightarrow$ Geometric scaling in $\sqrt{Y}$.
- Fixed coupling with fluctuations: $\Rightarrow$ Diffusive scaling in $Y$.
- Running coupling with fluctuations: $\Rightarrow$ ?.

# Introduction



Aim: implement running coupling in the following studies of fluctuations at fixed coupling:

Brunet, Derrida (1997),

Brunet, Derrida, Mueller, Munier (2006),

Marquet, Soyez, Xiao (2006)

# The model



1-D model for gluon number fluctuations:

$$\partial_Y \mathcal{N}(L,Y) = \bar{\alpha}\Big[\chi(-\partial_L)\mathcal{N}(L,Y) - \mathcal{N}^2(L,Y) + \sqrt{\kappa\alpha_s^2\,\mathcal{N}(L,Y)}\,\eta(L,Y)\Big]\,,$$

and $\langle \eta(L,Y)\,\eta(L',Y')\rangle = \frac{4}{\bar{\alpha}}\,\delta(L-L')\delta(Y-Y')\,.$

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and $\langle \eta(L,Y)\,\eta(L',Y')\rangle = \dfrac{4}{\bar{\alpha}}\,\delta(L-L')\delta(Y-Y')\,.$

Choice: we take the running coupling at the saturation scale:

$$\bar{\alpha} \equiv \left(b\;\log\left(Q_s^2(Y)/\Lambda^2\right)\right)^{-1} \equiv \left(b\;\rho_s(Y)\right)^{-1}$$

# The model



Ansatz: $\rho_s(Y) \equiv v(t)\, t$, with the *effective time* $t = \sqrt{Y}$.

$$\frac{b\, v(t)}{2}\, \partial_t N(L,t) \quad = \quad \chi(-\partial_L) N(L,t) - N^2(L,t) + \sqrt{\frac{\bar{\kappa}\, N(L,t)}{v(t)\, t^2}}\, \nu(L,t)\ .$$

with $$\langle \nu(L,t)\, \nu(L',t') \rangle = \delta(L-L')\delta(t-t')\ .$$

By consistency, the traveling wave solution must have a velocity $v(t)$, *i.e.* a scaling in $\xi \equiv L - v(t)t$.

# The method



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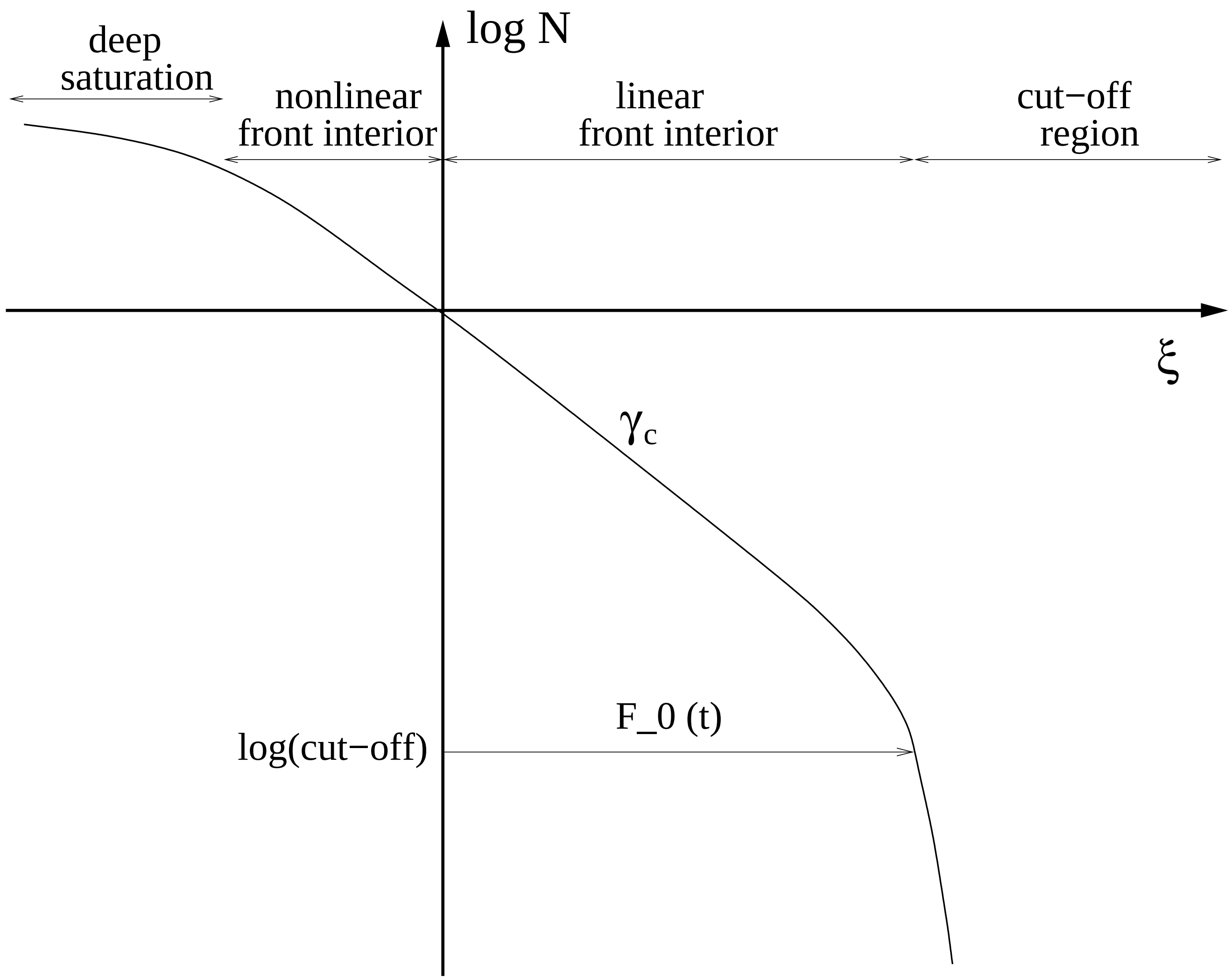
   $\longrightarrow$ Discreteness of the gluons.

2. Large but rare fluctuations: cannot be treated in a deterministic framework, and provide a random shift to the wave front.

   Stochastic evolution $\Rightarrow$ Random saturation scale.

# Front with cut-off





# Results with cut-off



Approximate geometric scaling in $\xi = L - v(t)t$ in the region $0 \leq \xi \leq F_0(t)$

$$\mathcal{N}(L, Y = t^2) = \frac{AF_0(t)}{\pi} \sin\left(\frac{\pi\xi}{F_0(t)}\right) e^{-\gamma_c \xi}$$

$$v(t) = v_c - \frac{\pi^2 \chi''(\gamma_c)}{2b\gamma_c v_c F_0^2(t)}$$

$$F_0(t) = \frac{2}{\gamma_c} \log t = \frac{1}{\gamma_c} \log Y$$

and weak violation of geometric scaling by $F_0(t)$.

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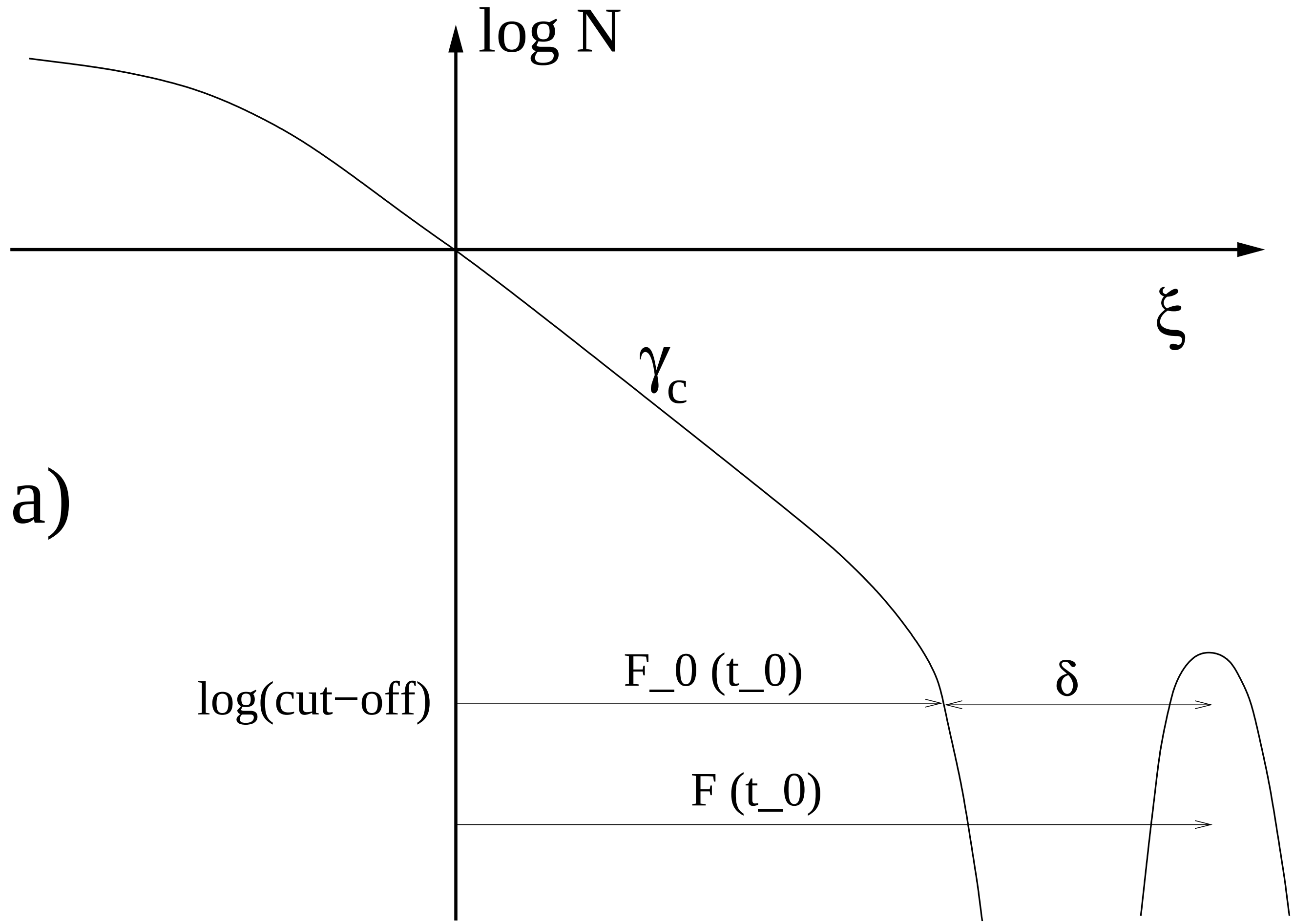
Result valid only if $F_0(t) \gg 1$,
*i.e.* asymptotic result, at very large rapidity.

# Relaxation of a large fluctuation





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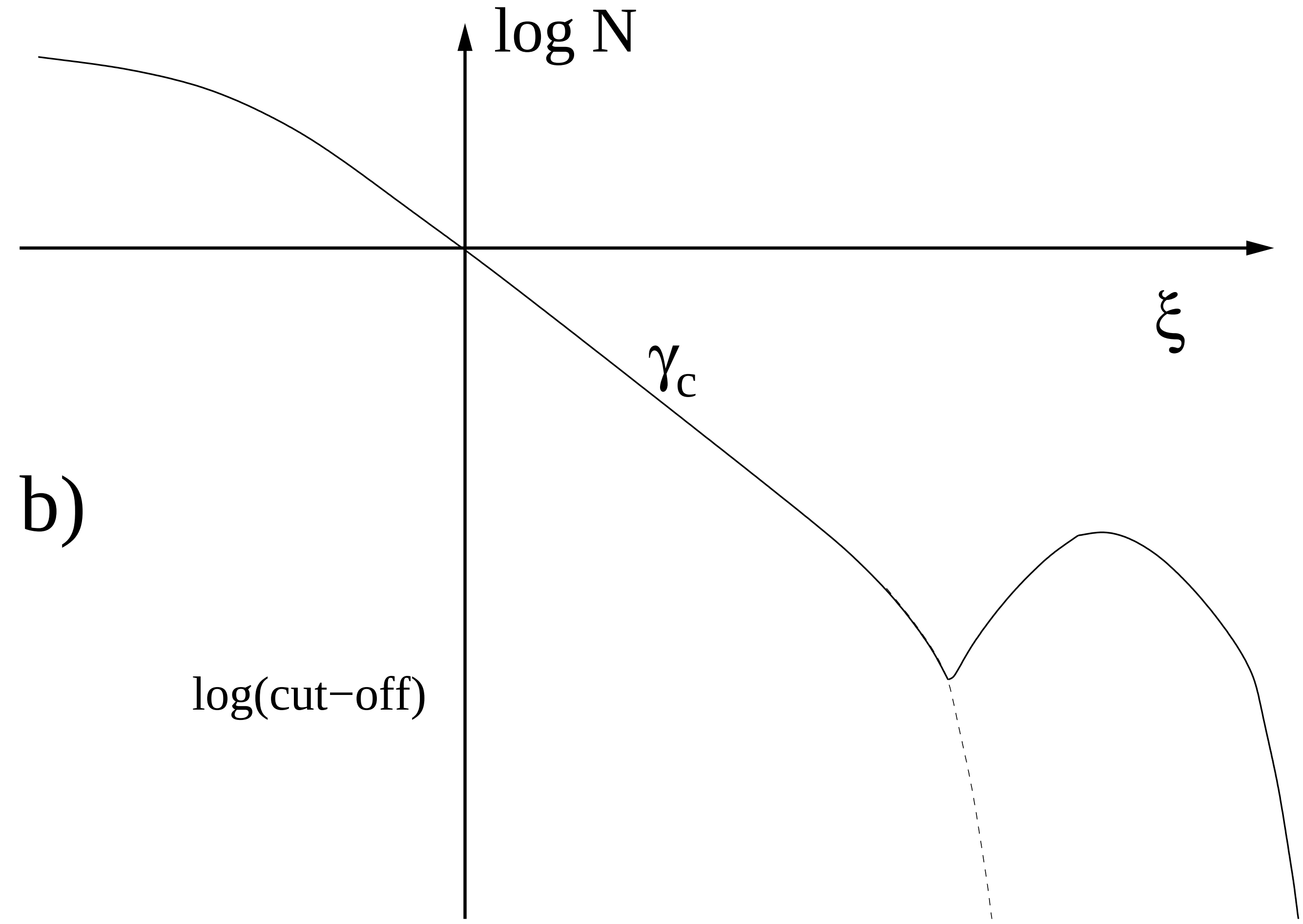

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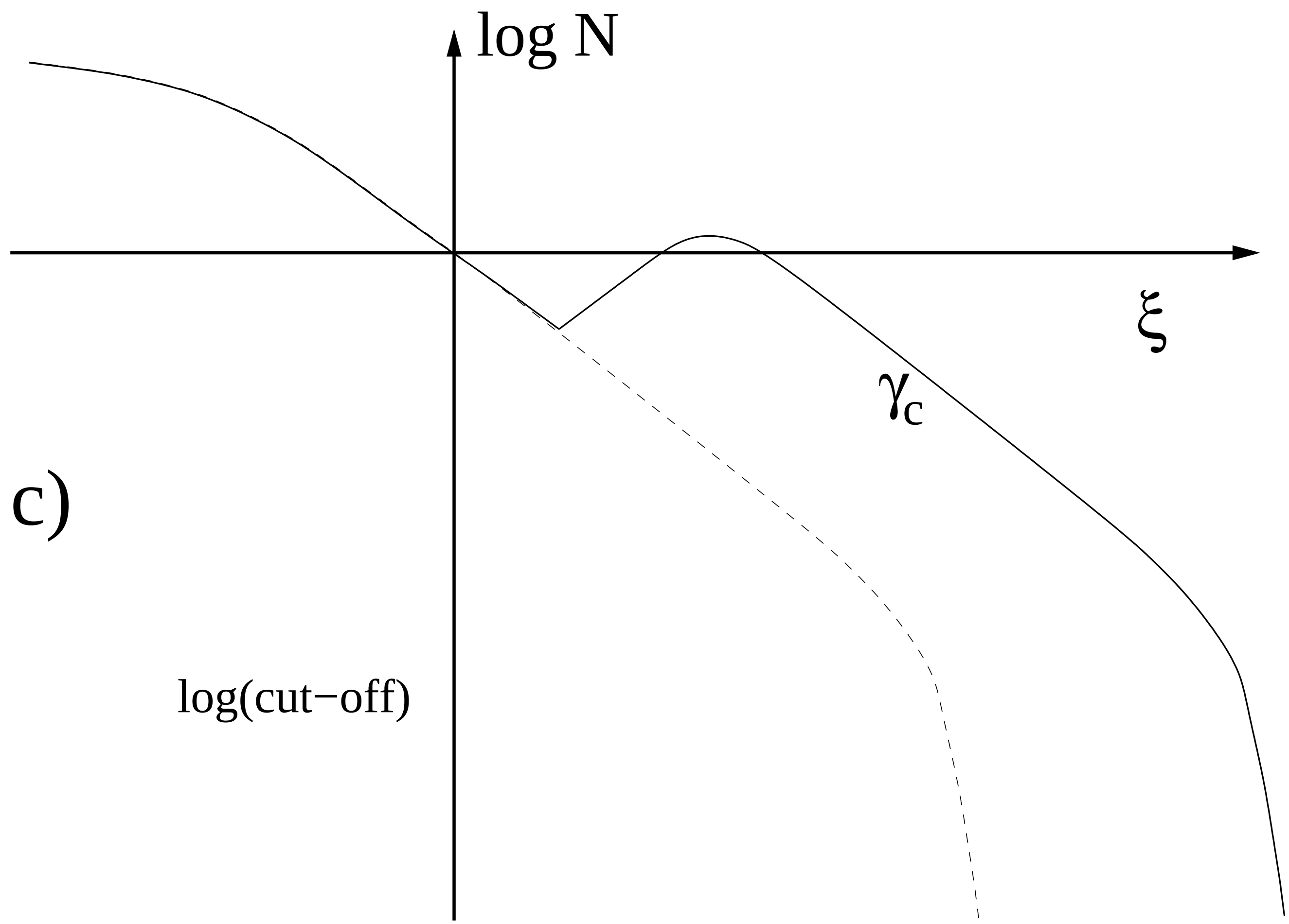

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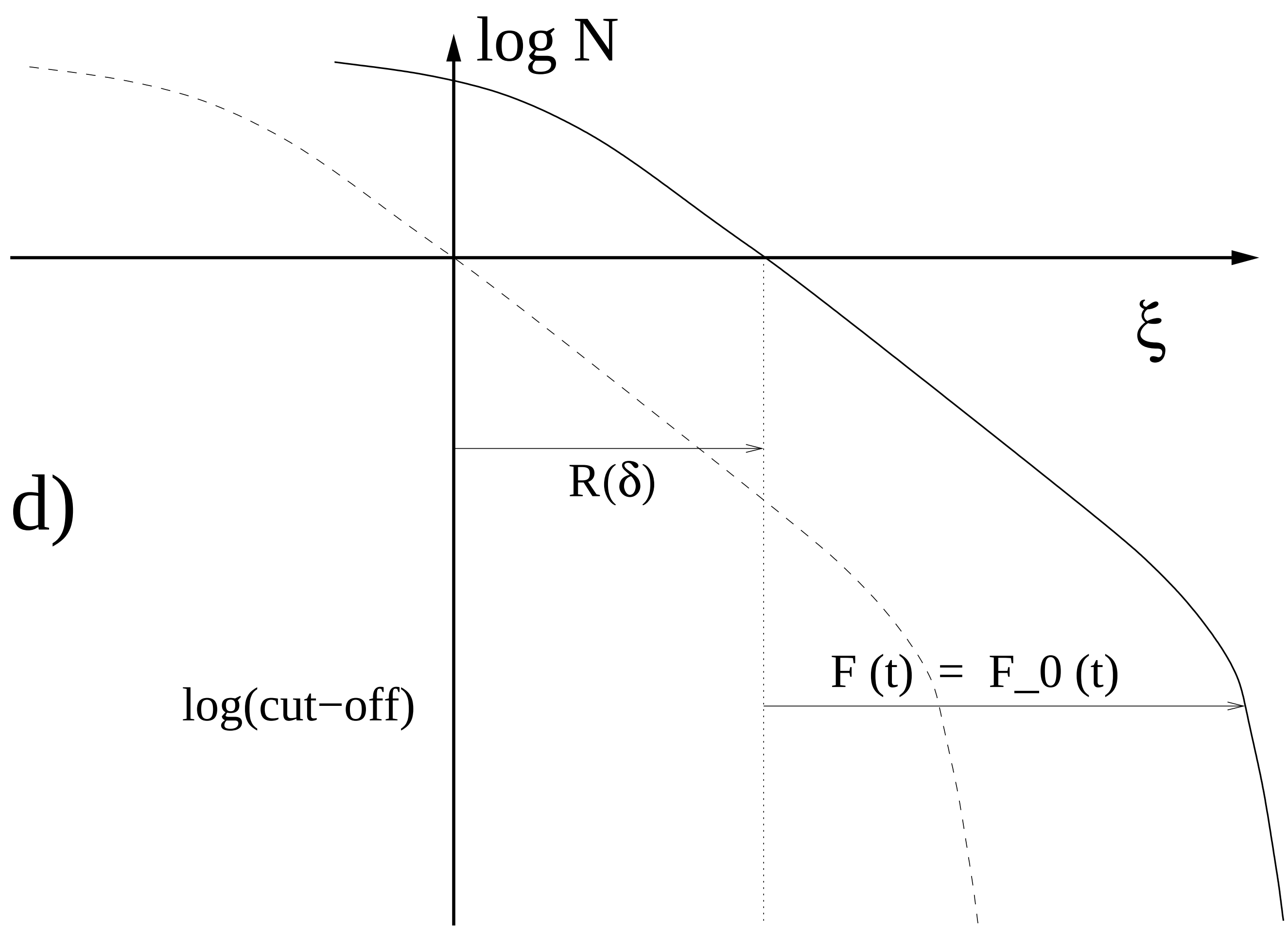

# Large fluctuations: results



1. Probability of occurence of a large fluctuation:

$$\mathcal{P}(\delta)\ \mathrm{d}\delta\ \mathrm{d}t = C_1 e^{-\gamma_c \delta}\ \mathrm{d}\delta\ \mathrm{d}t\,.$$

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2. Shift of the front induced by a fluctuation:

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3. Relaxation time of a large fluctuation $\ll$ time between 2 large fluctuations.

# Random saturation scale



- Without large fluctuations: deterministic saturation scale.
- With large fluctuations: random saturation scale, with a probability law determined by it's cumulants:

$$\partial_t \langle \rho_s(Y=t^2) \rangle = v_c - \frac{\pi^2 \chi''(\gamma_c)}{2b\gamma_c v_c F_0^2(t)} + \frac{3\pi^2 \chi''(\gamma_c) \log F_0(t)}{b\gamma_c^2 v_c (F_0(t))^3}$$

$$\partial_t \langle \rho_s^n(Y=t^2) \rangle_c = \frac{n!\zeta(n)\pi^2\chi''(\gamma_c)}{\gamma_c^{n+1} b v_c \, (F_0(t))^3} \quad \text{for} \quad n \geq 2\,.$$

Similar to the fixed coupling case, but for asymptotics:

$F_0(t) = \frac{1}{\gamma_c} \log Y \gg 1.$

# Integration of the cumulants



For $\log Y_0 \gg \gamma_c$:

$$\langle \rho_s(Y) \rangle = \langle \rho_s(Y_0) \rangle + v_c \left( \sqrt{Y} - \sqrt{Y_0} \right) - \frac{\gamma_c \pi^2 \chi''(\gamma_c)}{8 b v_c} \left[ \text{Li}\left(\sqrt{Y}\right) - \frac{\sqrt{Y}}{\log \sqrt{Y}} - \text{Li}\left(\sqrt{Y_0}\right) + \frac{\sqrt{Y_0}}{\log \sqrt{Y_0}} \right] + \frac{3 \gamma_c \pi^2 \chi''(\gamma_c)}{8 b v_c} \int_{\sqrt{Y_0}}^{\sqrt{Y}} \mathrm{d}t \, \frac{\log \log t}{\log^3 t}$$

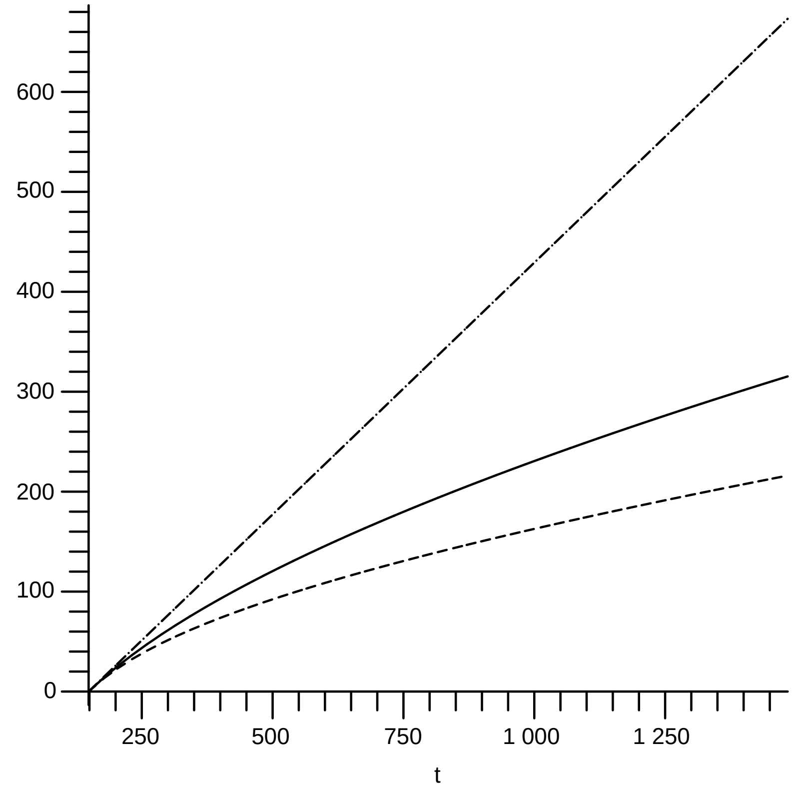
$$\langle \rho_s^n(Y) \rangle_c = \langle \rho_s^n(Y_0) \rangle_c + \frac{n! \zeta(n) \pi^2 \chi''(\gamma_c)}{16 \gamma_c^{n-2} b v_c} \left[ \text{Li}\left(\sqrt{Y}\right) - \frac{\sqrt{Y}}{\log \sqrt{Y}} - \frac{\sqrt{Y}}{\left(\log \sqrt{Y}\right)^2} - \text{Li}\left(\sqrt{Y_0}\right) + \frac{\sqrt{Y_0}}{\log \sqrt{Y_0}} + \frac{\sqrt{Y_0}}{\left(\log \sqrt{Y_0}\right)^2} \right]$$

for $n \geq 2$.

# Rise of the cumulants

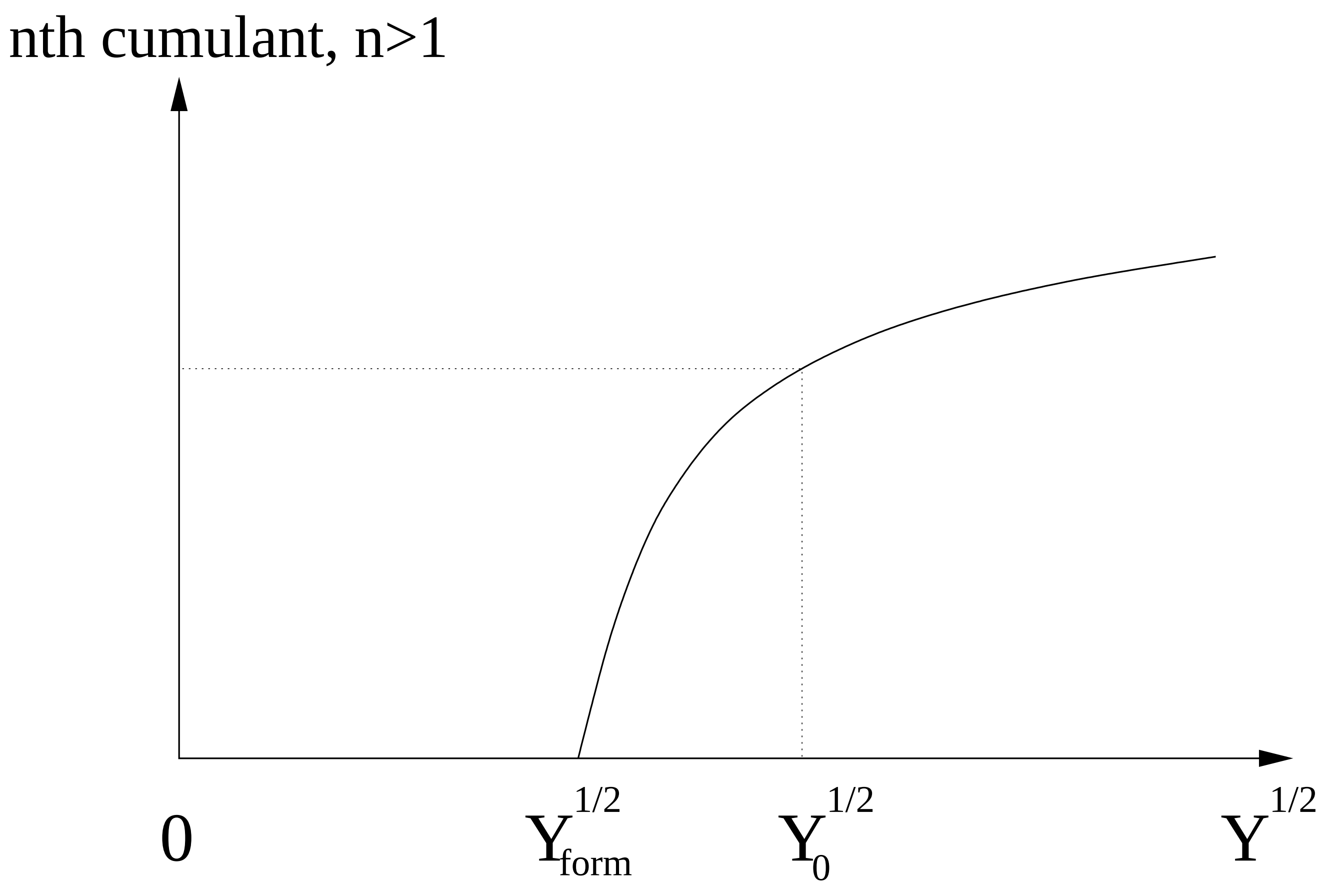


$\sqrt{b}\left(\langle\rho_s^2(Y)\rangle_c - \langle\rho_s^2(Y_0)\rangle_c\right)$ as a function of $t = \sqrt{Y}$.




Here: $Y_0 = e^{10}$.

# Discussion



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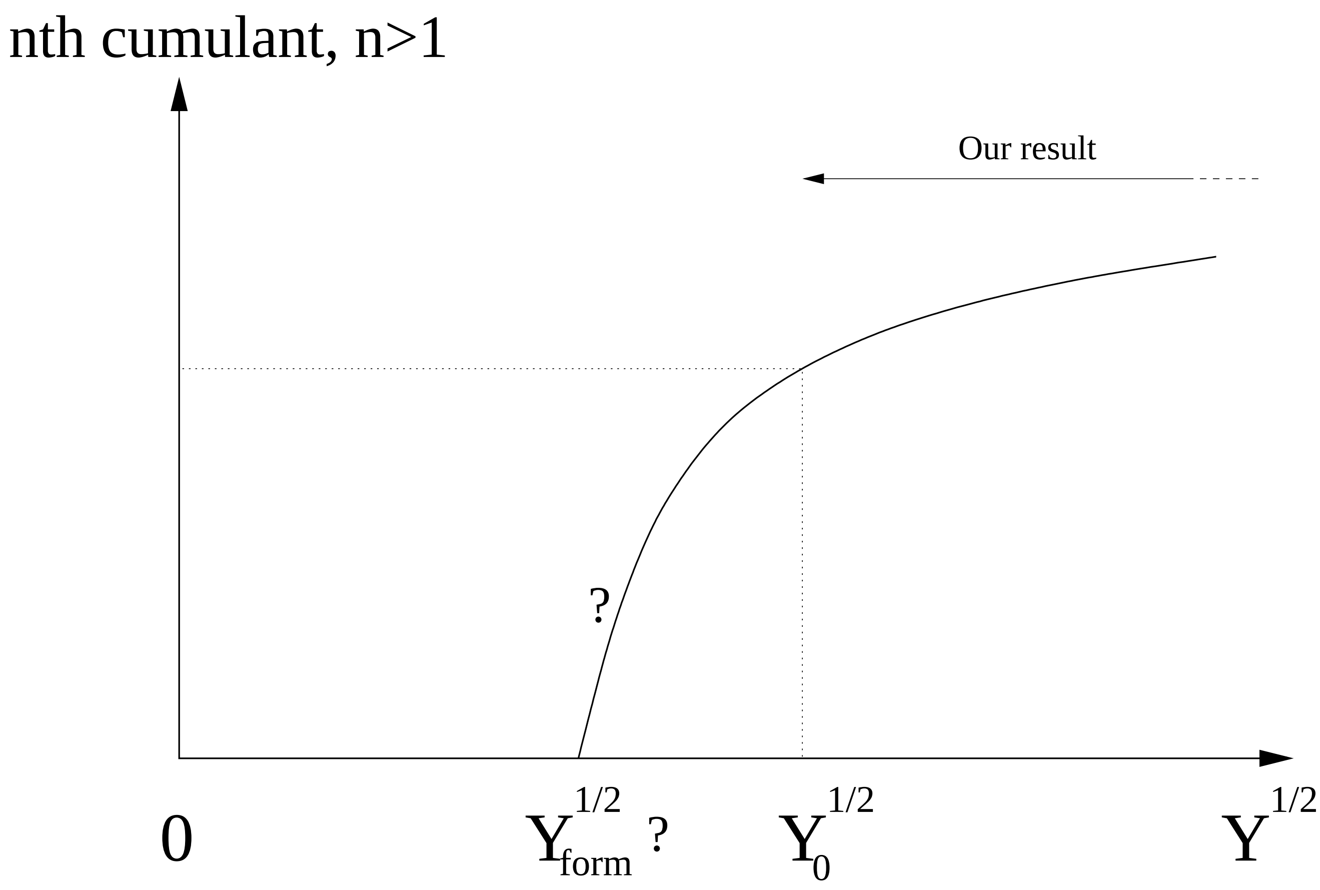

# Conclusion



We have calculated the asymptotic behavior of the solution to the saturation problem, with running coupling and fluctuation effects.

- Exact asymptotic behavior of the distribution of the random saturation scale have been derived.
- Diffusive scaling has been proven in the high rapidity limit, in the running coupling case.

Can we obtain analytical results at lower rapidity?