Overview of saturation

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(Saclay)

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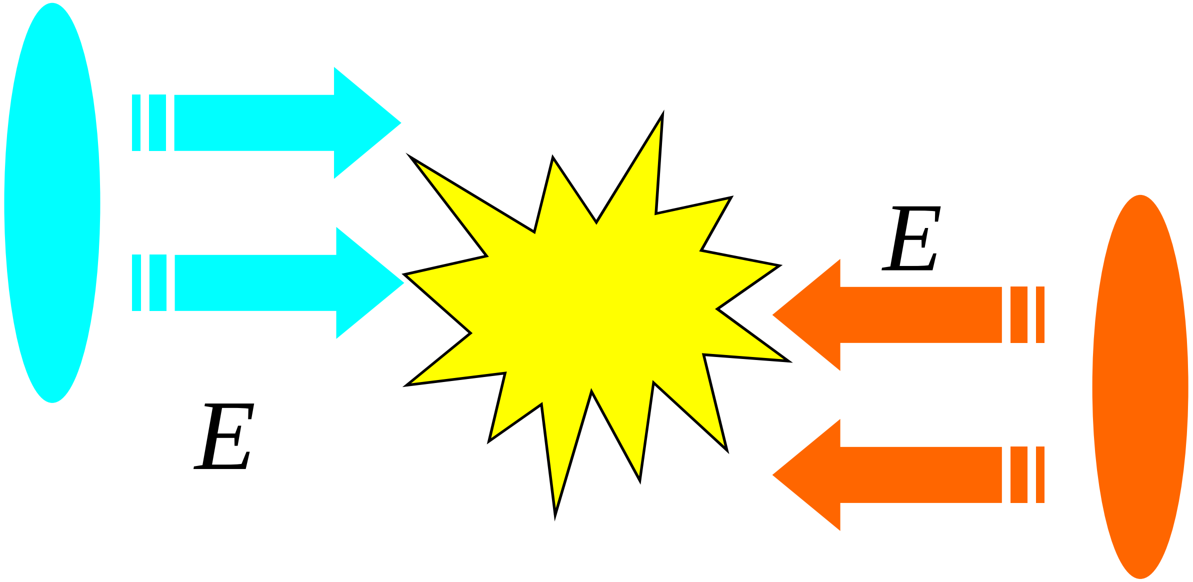
Outline

- BFKL and beyond
- Saturation momentum
- The BK-JIMWLK equation
- Fluctuations & Correlations

Regge limit of QCD

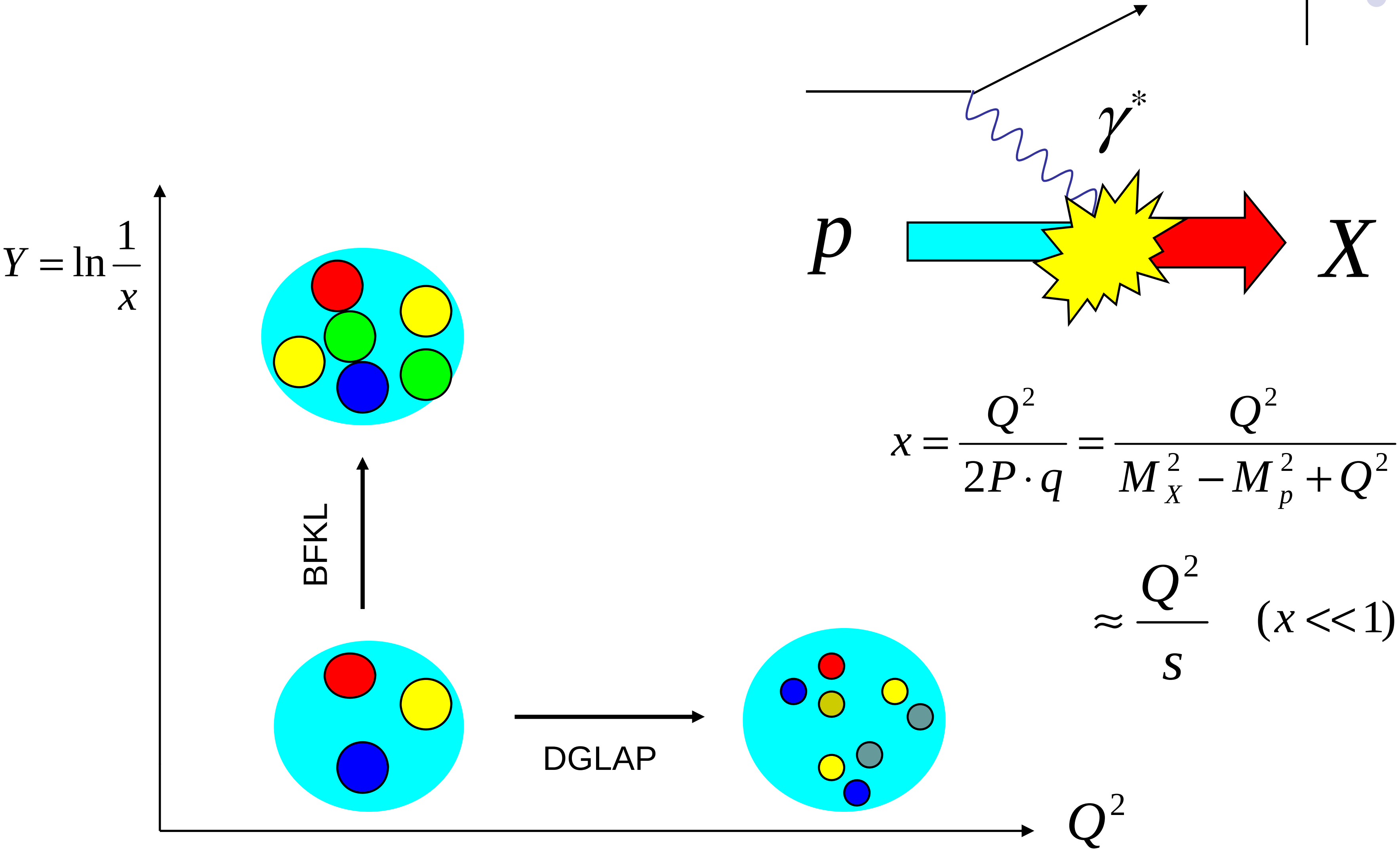
One of the most challenging problems in QCD
is the understanding of its high energy limit.



$$\sqrt{s} = 2E >> \sqrt{-t}, M, p_T, \ldots$$

Can we compute hadron-hadron (nucleus) cross sections
in this regime from first principles?

Deep Inelastic Scattering

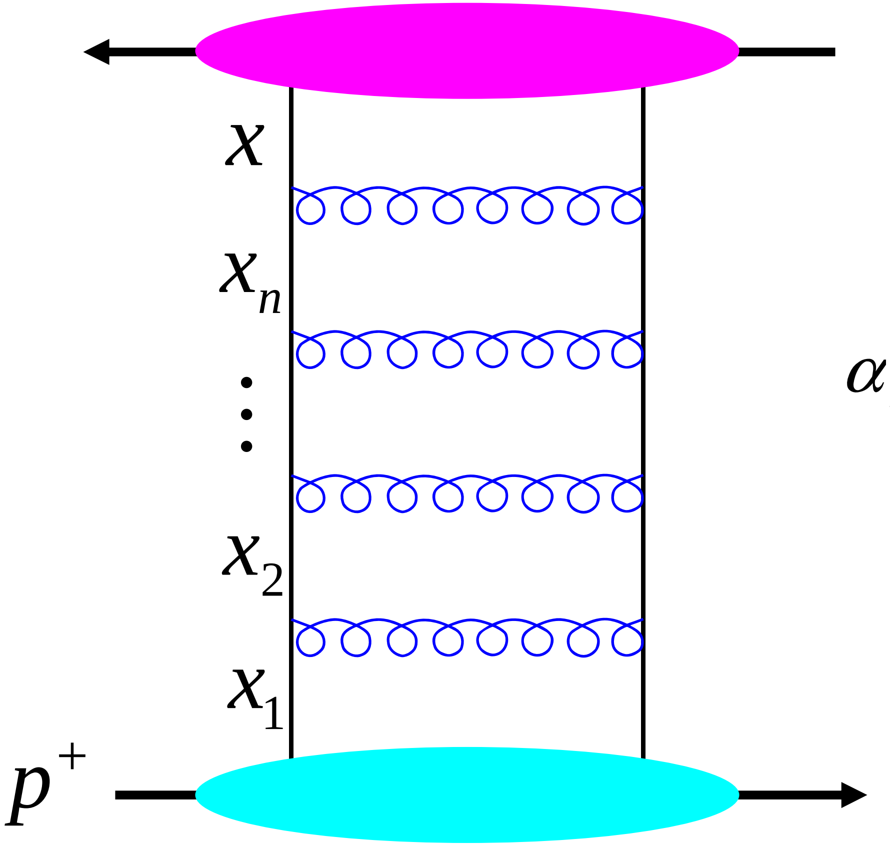


$$x = \frac{Q^2}{2P \cdot q} = \frac{Q^2}{M_X^2 - M_p^2 + Q^2}$$

$$\approx \frac{Q^2}{s} \quad (x << 1)$$

The BFKL Pomeron

Balitsky, Fadin, Kuraev, Lipatov, `75~`78



Rapidity ordering

$$1 >> x_1 >> \cdots x_n >> x$$

$$\alpha_s^n \int_x^1 \frac{dx_1}{x_1} \cdots \int_x^{x_{n-1}} \frac{dx_n}{x_n} = \frac{1}{n!}\left(\alpha_s \ln \frac{1}{x}\right)^n$$

$$\downarrow$$

$$\sigma_{tot}^{\gamma^* p} \propto x^{-4\ln 2\bar{\alpha}_s} = s^{4\ln 2\bar{\alpha}_s}$$

Ways to go — beyond BFKL

LLA BFKL predicts indefinite growth of the gluon number.

Infrared diffusion invalidates perturbative treatment
in the middle of the ladder.

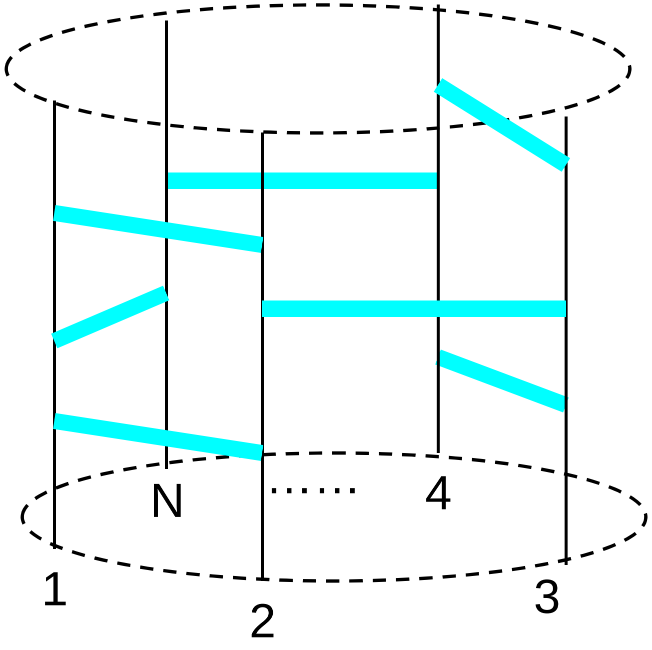
Must be tamed (How?)

- Many reggeized gluon exchange à la BKP
- Next to leading-log approximation (NLLA)
- Nonlinear equations from gluon saturation

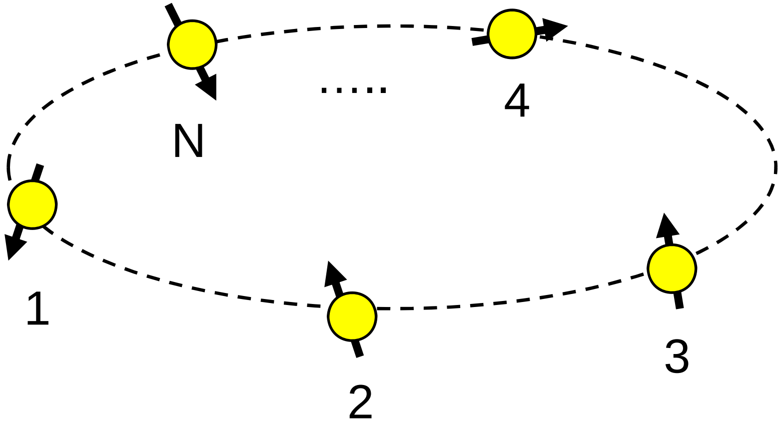
High energy QCD as an integrable model

Lipatov, `93
Faddeev & Korchemsky, `94

BKP equation at large Nc



$s = 0$ Heisenberg spin chain



$$H_{BKP} = H + \overline{H}$$

$$H = \sum_{n=1}^{N} \left(2\psi(1) - \psi(-J_{n,n-1}) - \psi(J_{n,n-1}+1)\right)$$

The BKP Hamiltonian is identical to that of an exactly solvable system.
Eigenvalue ω (→energy dependence s^{ω}) obtained by the Bethe ansatz

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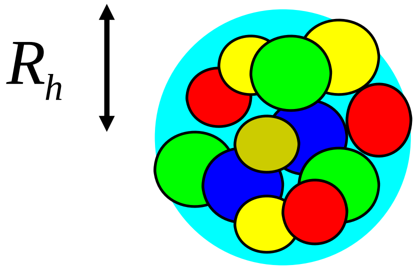
Saturation criterion : hadron

Collinear or kt factorization :

One parton from each hadron scatters.
Partons inside a hadron do not interact.



⟶ exponential growth in number

$$\frac{\alpha_s}{Q^2} xG(x,Q^2) \approx \pi R_h{}^2$$

recombination
cross section

# of gluons
computed in BFKL

When $Q^2 < Q_s^2(x)$ gluons begin to overlap…

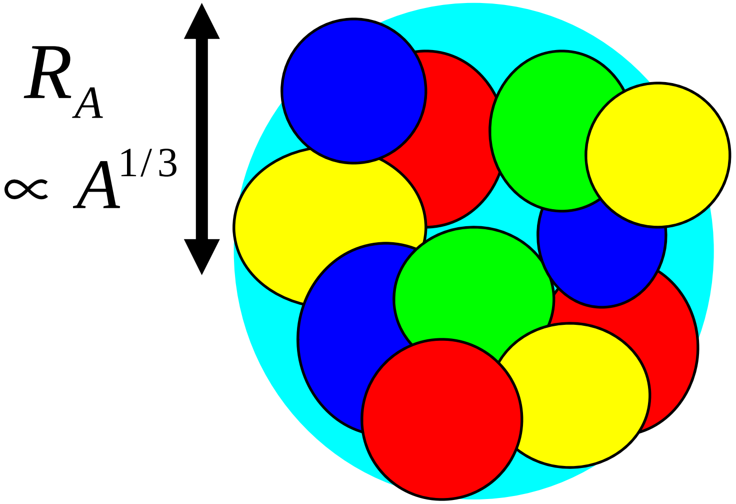
Saturation criterion : nucleus

A large nucleus ($A \longrightarrow \infty$) is a dense system from the beginning.

McLerran & Venugopalan `94

$$\frac{\alpha_s}{Q^2} AxG(x,Q^2) \approx \pi R_A{}^2$$

# of gluons computed in
lowest order perturbation theory



R_A

$\propto A^{1/3}$

$$Q_s^2 \propto A^{1/3}$$

At the same time, multiple scattering becomes unsuppressed. (Glauber—Mueller)

The saturation momentum at LLA

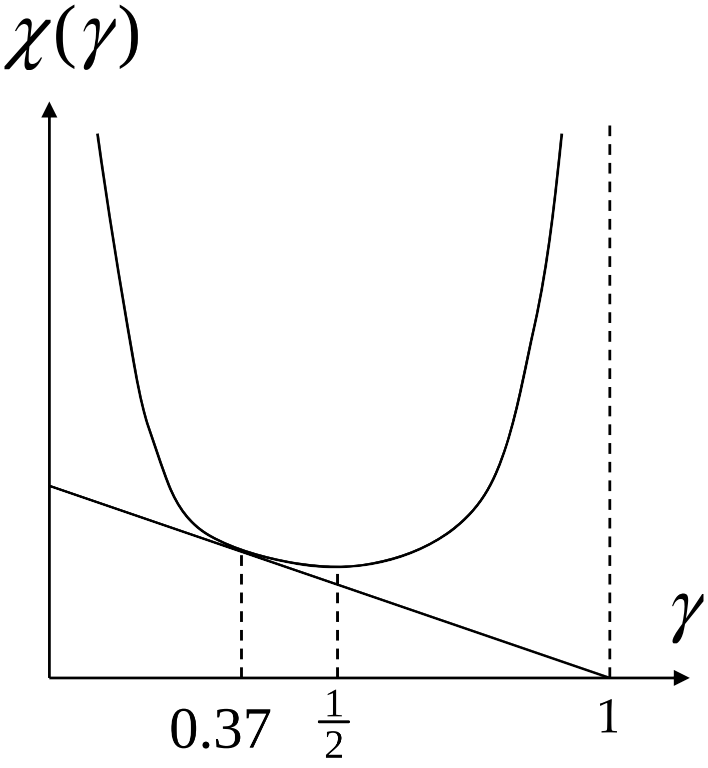
Gribov, Levin, Ryskin, `83

Find the line of constant amplitude in the BFKL solution.

$$xG(x,Q^2)/Q^2$$

$$\propto \int d\gamma \exp\{\underset{\text{increase with } Y}{\chi(\gamma)}\ Y - \underset{\text{decrease with } Q^2}{(1-\gamma)} \ln Q^2/\Lambda^2\}$$

increase with Y decrease with Q^2

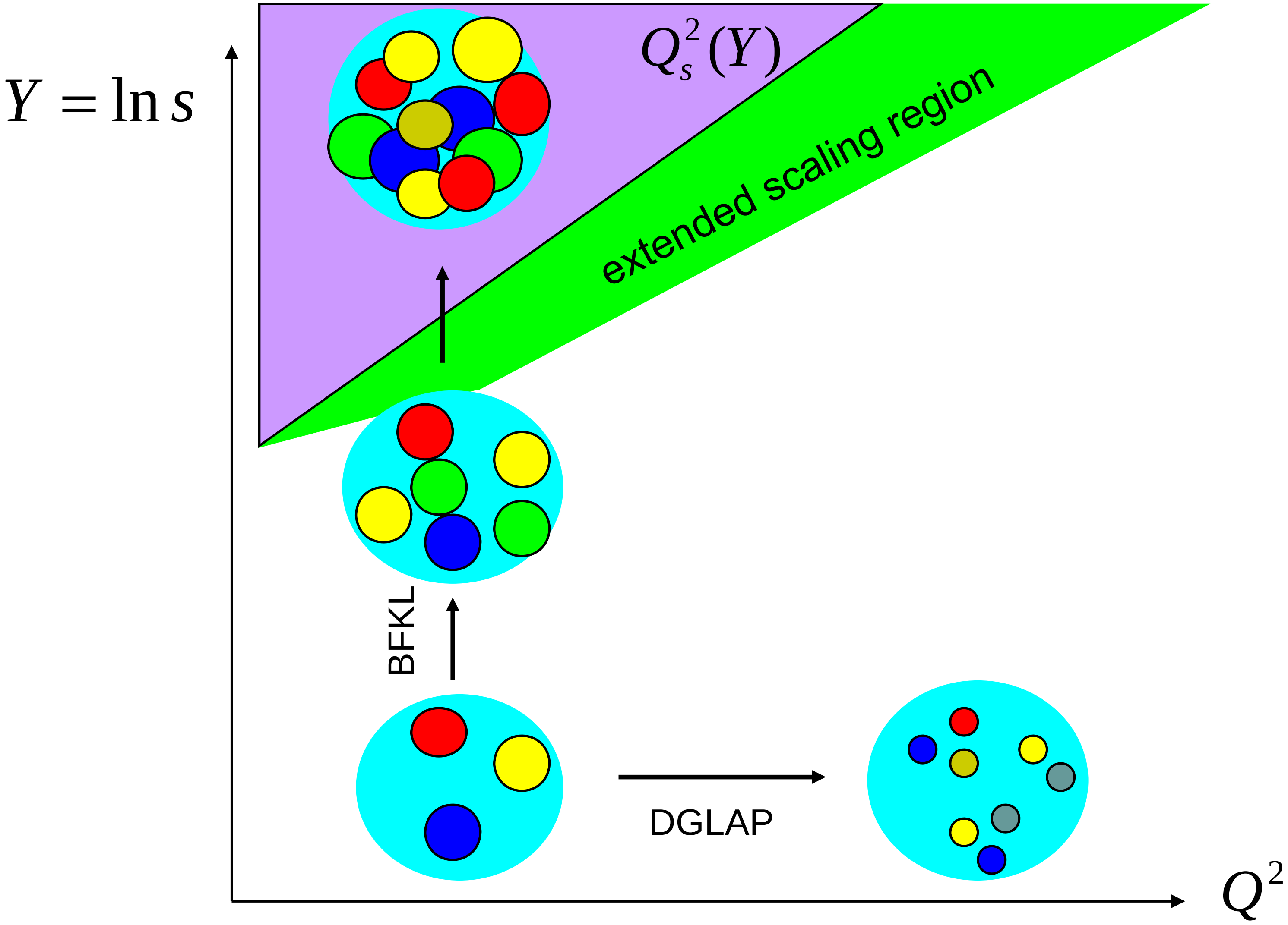


$$Q_s^2(Y) = \Lambda_{QCD}^2 e^{4.9\bar{\alpha}_s Y}$$ (fixed coupling)

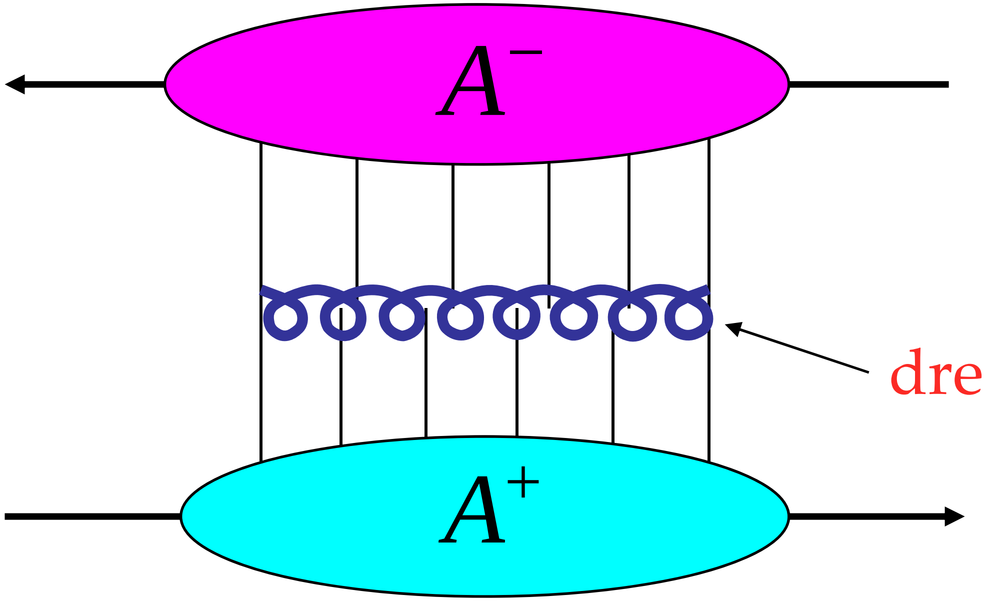
$$= \Lambda_{QCD}^2 e^{\sqrt{9.8Y/\beta}}$$ (running coupling)

Calculations at (improved) NLO available Triantafyllopoulos `03 Khoze et al. `04

The new phase of QCD



Evolution in the presence of saturation

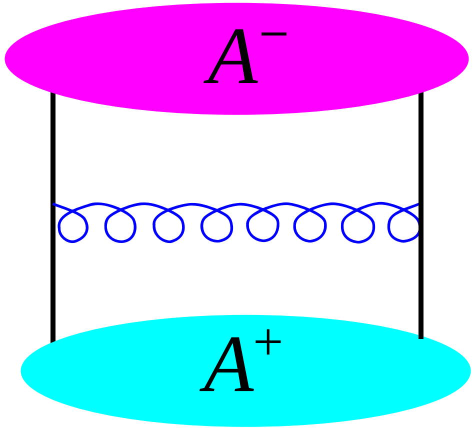


dressed propagator

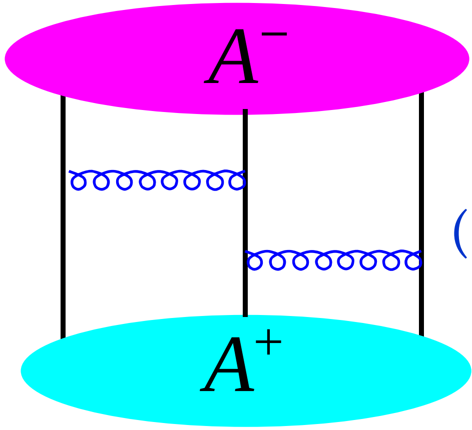
Incoming hadrons (nuclei) are replaced by non-Abelian Weizsacker-Williams fields of strength $\sim 1/g$

Need to sum all orders in gA^+, or gA^-, or both in a single step of evolution

The long and winding road to unitarity…

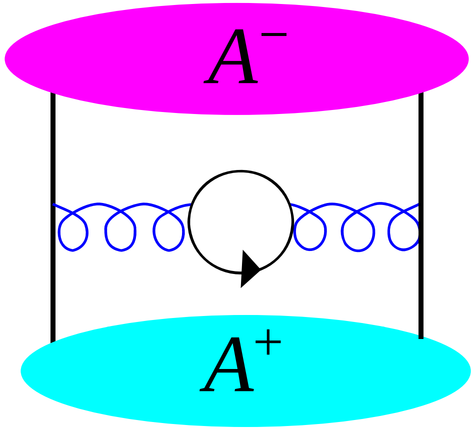


BFKL
(1975~78)

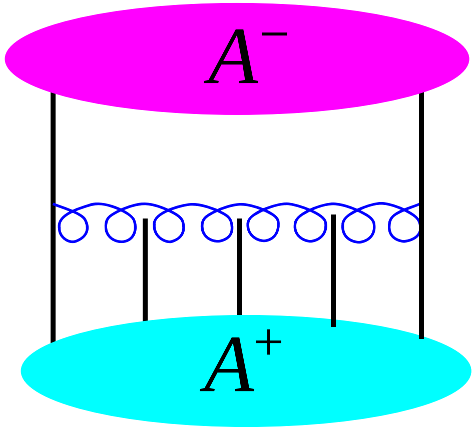


BKP
(1978~80)

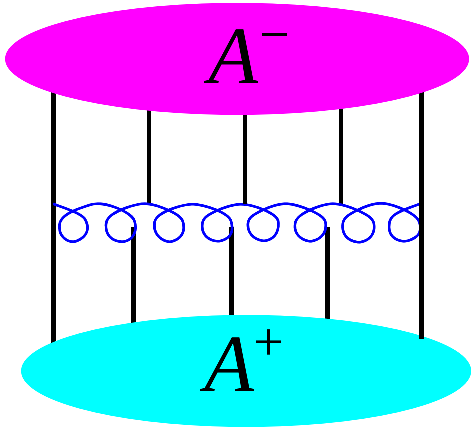
NLO BK
(Coming soon !)



NLO BFKL
(1996~98)



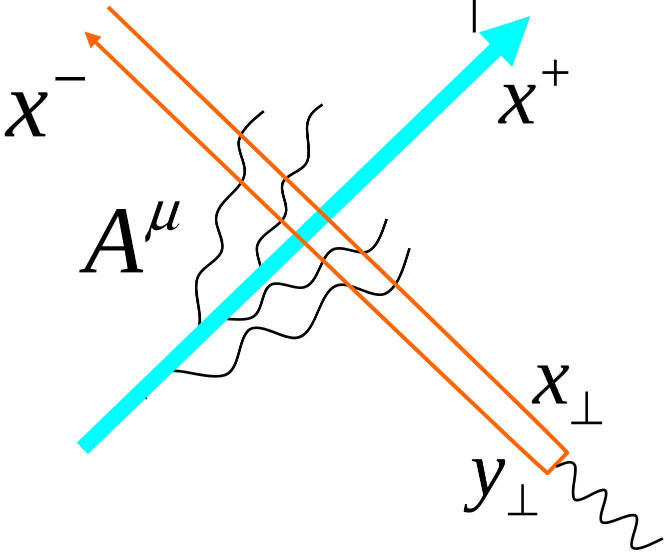
BK-JIMWLK
(1996~2001)



Pomeron loop
(~20XX)

The BK-JIMWLK equation

Dipole-nucleus scattering (subprocess of DIS)



$$T_Y(x,y) = 1 - \frac{1}{N_c} < \mathrm{Tr}[V_x^+ V_y] >_Y$$

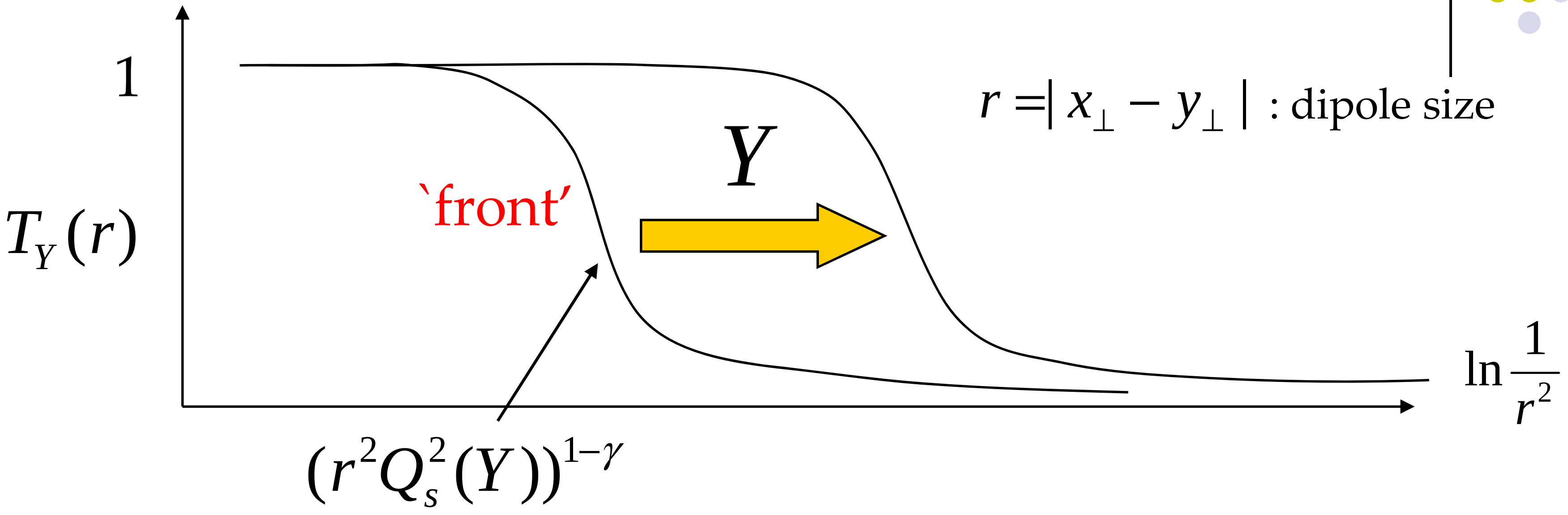
$$V_x = P\exp\{ig\int_{-\infty}^{\infty} dx^+ A^-(x^+, x_\perp)\}$$

$$\frac{\partial}{\partial Y} T_Y(x,y) = \frac{\bar{\alpha}_s}{2\pi}\int d^2z \frac{(x-y)^2}{(x-z)^2(z-y)^2}$$

$$\times\{\underbrace{T(x,z)+T(z,y)-T(x,y)}_{\text{BFKL}} - \underbrace{\langle T(x,z)T(z,y)\rangle}_{\text{gluon recombination}}\}_Y$$

\`Mean field' approximation $\langle T(x,z)T(z,y)\rangle \approx T(x,z)T(z,y)$

The geometric scaling



$r = |x_\perp - y_\perp|$: dipole size

The scaling persists even when $Q^2 >> Q_s^2$ Iancu, Itakura & McLerran, `02

Mapping onto the traveling wave solution of the FKPP equation

$$\partial_t f = \partial_x^2 f + f - f^2$$

Munier & Peschanski, `03

Some indication in the HERA data

Stasto, Golec-Biernat & Kwiecinski, `01
Marquet & Schoeffel, `06 (diffraction)

Application to other processes

- Diffractive DIS

→ Talk by Marquet

Inclusive & exclusive

Saturation prediction $\sigma_{diff}/\sigma_{tot} \propto 1/\ln Q^2$ Independent of x !

- pA (and AA) collisions at RHIC

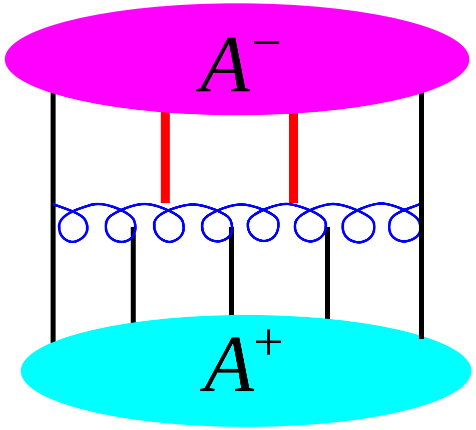
Multiplicity, pt distribution, heavy quark, R_{pA}, limiting fragmentation, etc.

Saturation models confront RHIC data, doing well.

- Odderon

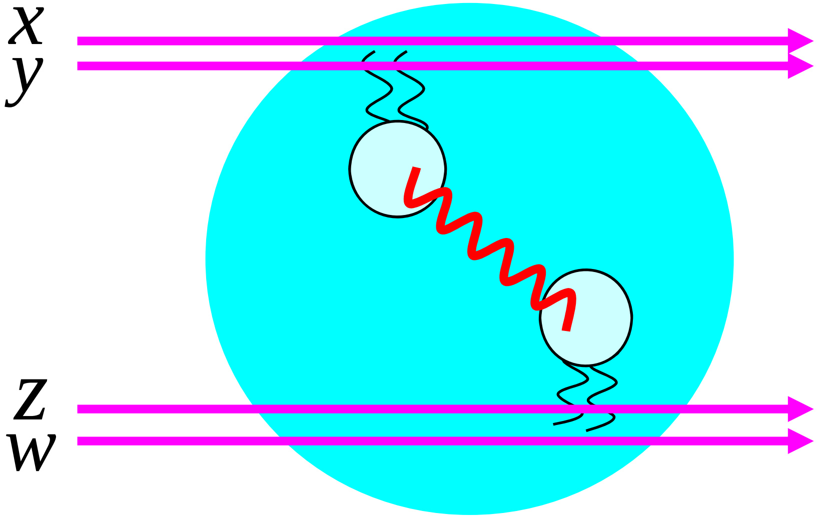
The BLV solution : constant in energy
Saturation effects tend to suppress the odderon amplitude.

Beyond BK—JIMWLK



What's missing in the BK-JIMWLK?

Not symmetric w.r.t. the target and projectile,
Saturation effect of the projectile missing.

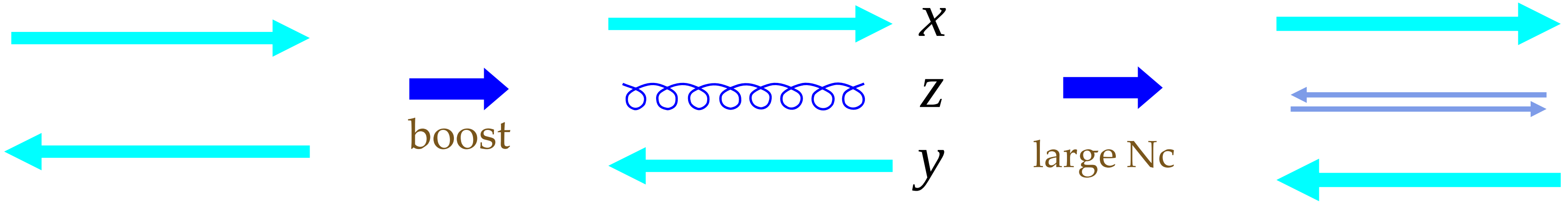


A kind of mean field approximation,
neglecting fluctuations and correlations
in the target wavefunction.

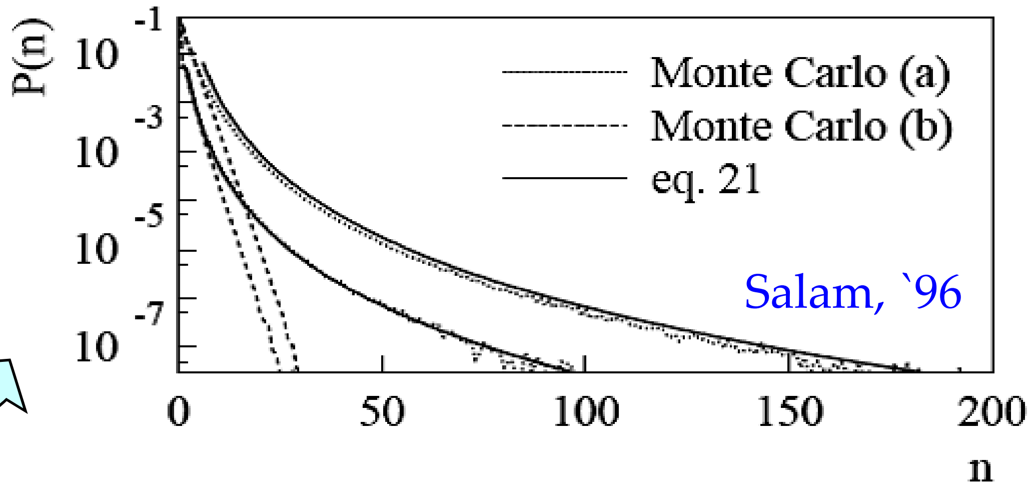
$$\left\langle T(x,y)T(z,w)\right\rangle = \left\langle T(x,y)\right\rangle\left\langle T(z,w)\right\rangle + O(1/N_c^2)$$

The gluon number fluctuation

QCD dipole model



$$dP = \bar{\alpha}_s \frac{(x-y)^2}{(x-z)^2(z-y)^2} d^2 z dY$$



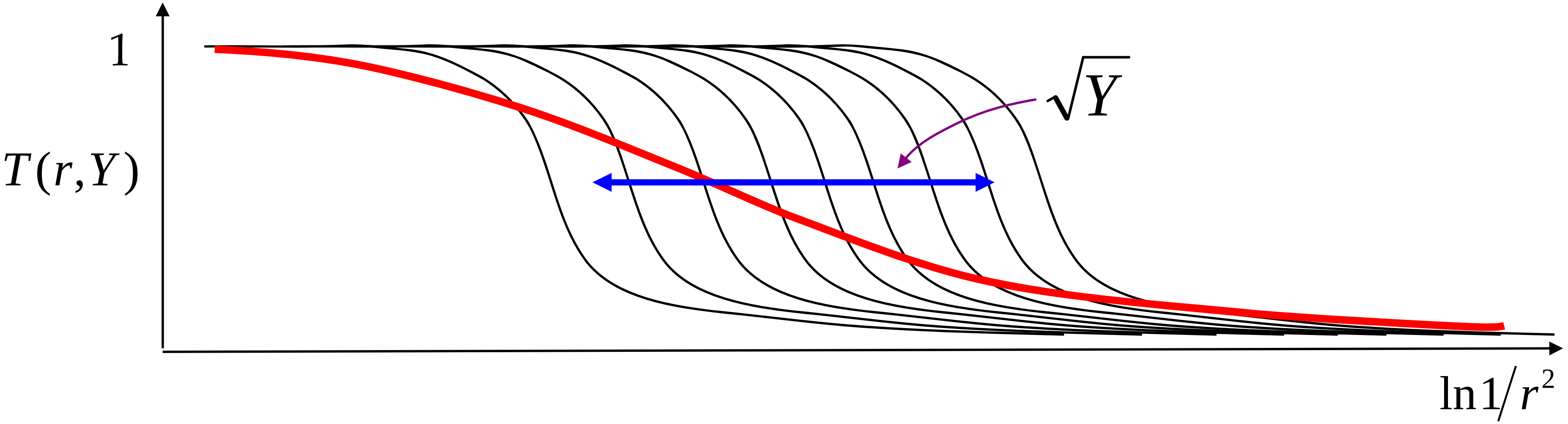
Significant consequences in the approach to unitarity.
Deep connection to the stochastic FKPP equation in statistical physics.

$$\partial_t f = \partial_x^2 f + f - f^2 + \varepsilon\sqrt{f} \cdot \xi$$

Universal behavior of the sFKPP equation

Iancu, Mueller, Munier, `04

Front position (saturation scale Q_s^2) becomes a random variable



Observed amplitude is obtained after averaging over events.

Each event shows geometric scaling, but the average does not !

Factorization `maximally' violated $\langle TTT\cdots\rangle \approx \langle T\rangle$

Updating the phase diagram

LHC,
cosmic ray ??

HERA ?

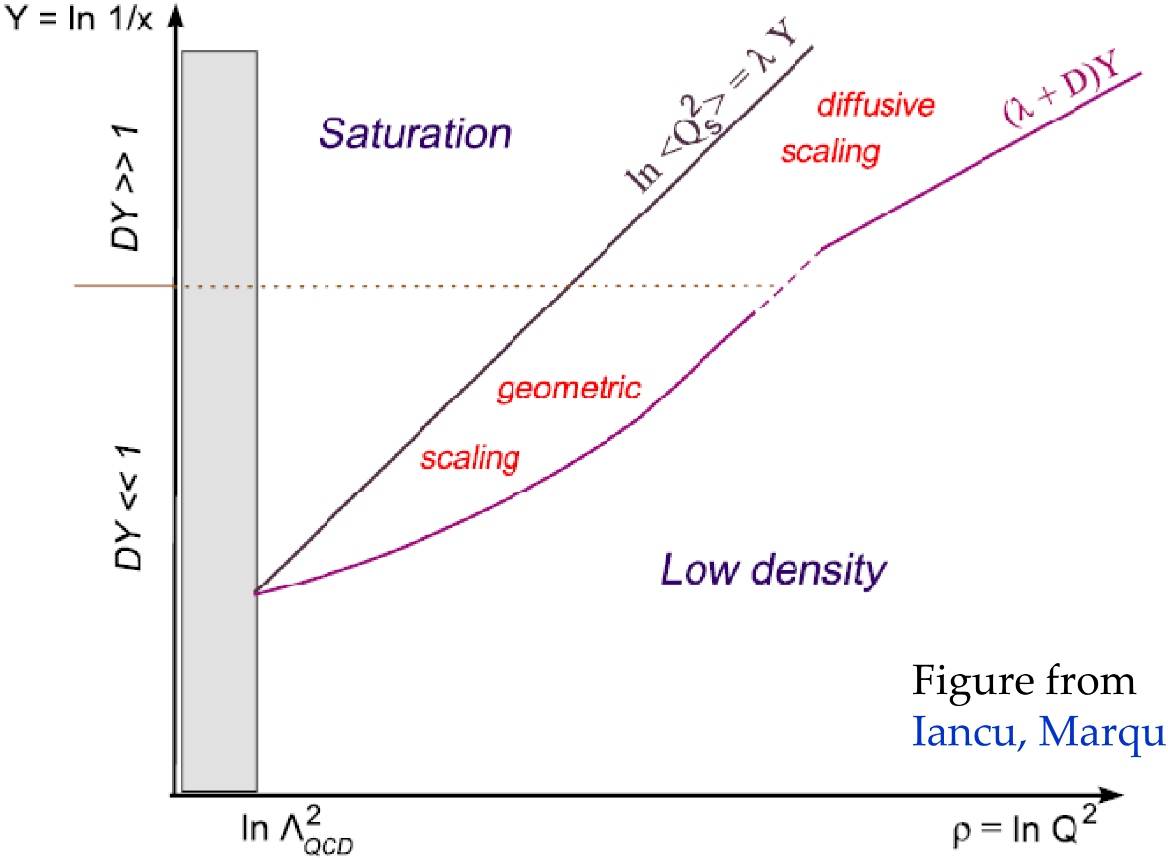


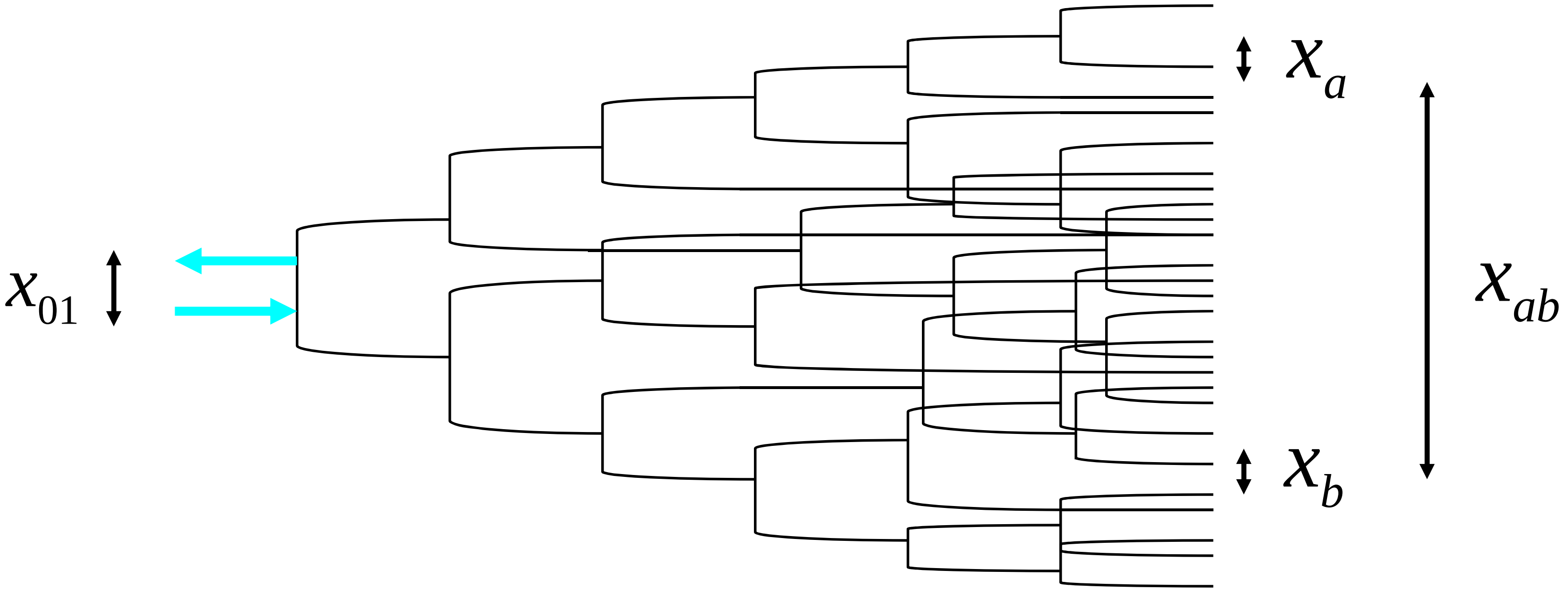
Figure from
Iancu, Marquet, Soyez `06

Caveats: Requires enormous energy for the fluctuation to become significant.
Running coupling may be (very) important.

→ Talks by Beuf, Kozlov, Soyez

Power—law correlation in the transverse plane

Y.H. & Mueller, `07



Small-x gluons are correlated because they come from a common ancestor.

$$dP = \bar{\alpha}_s \frac{(x-y)^2}{(x-z)^2(z-y)^2} d^2 z dY \longrightarrow (correlation) \propto \left(\frac{1}{x_{ab}}\right)^L$$

Determination of the power

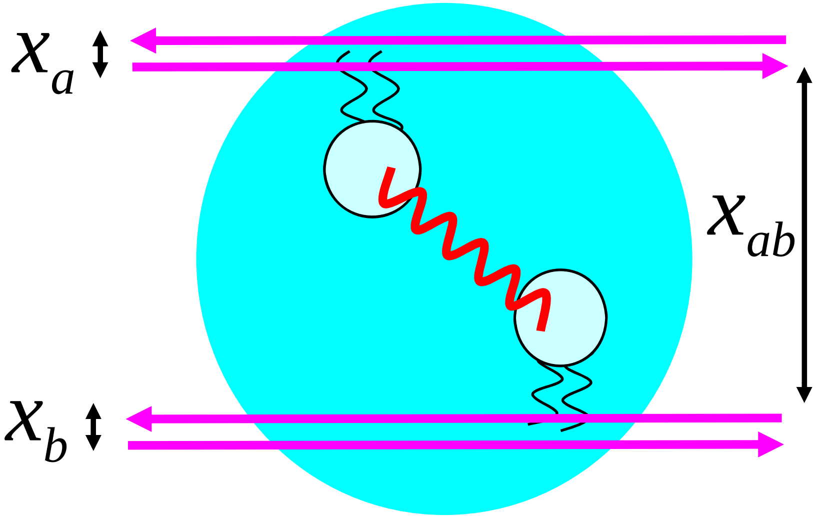
Dipole pair density

$$n_Y^{(2)}(x_{01}, x_{a_0a_1}, x_{b_0b_1}) = \int dh dh_a dh_b \frac{1}{2x_{a_0a_1}^2 x_{b_0b_1}^2} \int_0^Y dy\; e^{\chi(h)y+(\chi(h_a)+\chi(h_b))(Y-y)}$$
$$\times \int d^2x_\alpha d^2x_\beta d^2x_\gamma E^{h,\bar{h}}(x_{0\gamma}, x_{1\gamma}) E^{h_a,\bar{h}_a}(x_{a_0\alpha}, x_{a_1\alpha}) E^{h_b,\bar{h}_b}(x_{b_0\beta}, x_{b_1\beta})$$
$$\times \int \frac{d^2x_2 d^2x_3 d^2x_4}{x_{23}^2 x_{34}^2 x_{42}^2} E^{h,\bar{h}*}(x_{2\gamma}, x_{3\gamma}) E^{h_a,\bar{h}_a*}(x_{2\alpha}, x_{4\alpha}) E^{h_b,\bar{h}_b*}(x_{3\beta}, x_{4\beta}),$$

Peschanski, \`97   Braun & Vacca, \`97

Breakdown of factorization
from BFKL. Explicit !

$$\langle T(x_a)T(x_b)\rangle \propto \langle T(x_a)\rangle\langle T(x_b)\rangle \frac{1}{x_{ab}^{2(2\gamma-\gamma_0)}}$$



Summary

- Quest for unitarity : a difficult but fascinating problem

  Continual efforts & progresses, still many open questions

- BK-JIMWLK equation : the best ‘simple’ equation including nonlinear effects.

- Beyond the BK-JIMWLK : Pomeron loops.

  Hadron wavefunction teems with correlations and fluctuations.

Important theoretical problems

- Full inclusion of Pomeron loops, and its physical consequences. → Talks by Levin and Lublinsky
- Gluon production in AA collision, quantum evolution. Gelis, Lappi, Venugopalan `07
- NLO BK phenomenology

Balitsky `06, Kovchegov, Weigert `06, Balitsky, Chirilli, to appear

- Saturation in AdS/CFT ?

→ Talk by Y.H.