Summing Pomeron loops

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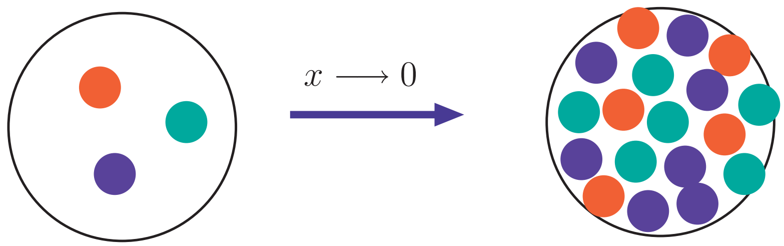
E. Levin, J. Miller , A. Prygarin : arXiv:0706.2944 [hep-ph]

Outline



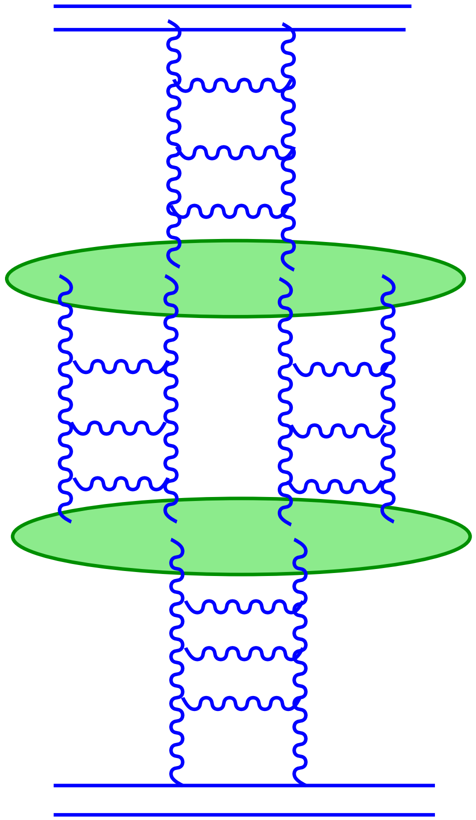
- **The BFKL Pomeron calculus: ups and downs;**
- **Overlapping singularities;**
- **Main idea and simple example of the toy-model;**
- **The improved Mueller-Patel-Salam-Iancu approximation:**
- **The solution for the simplified BFKL kernel;**
- **Resume.**

The BFKL Pomeron calculus

Outline:

- **Everything that has been done during the past three years is nothing more than understanding of the BFKL Pomeron calculus** (Kozlov,E.L. & Prygarin; Bondarenko);

- **The news: The Pomeron interaction generates a new state with the intercept large than intercept of two BFKL Pomerons** (Hatta & Mueller; E.L, Miller & Prygarin);



- $A\,(1P) \;=\; \bar{\alpha}_S^2\, s^{\Delta};$

$\Delta \;=\; C_1 \bar{\alpha}_S \;+\; C_2 \bar{\alpha}_S^2;$

- $A\,(2P) \;=\; \bar{\alpha}_S^4\, s^{2\,\Delta};\; Y \;=\; \ln\, s$

- **BFKL Pomeron ($A\,(1P) \;\ll\; A\,(2P)$) :**

$$1 \ll \; \bar{\alpha}_S\, Y \;\ll\; \ln(1/\bar{\alpha}_S^2);$$

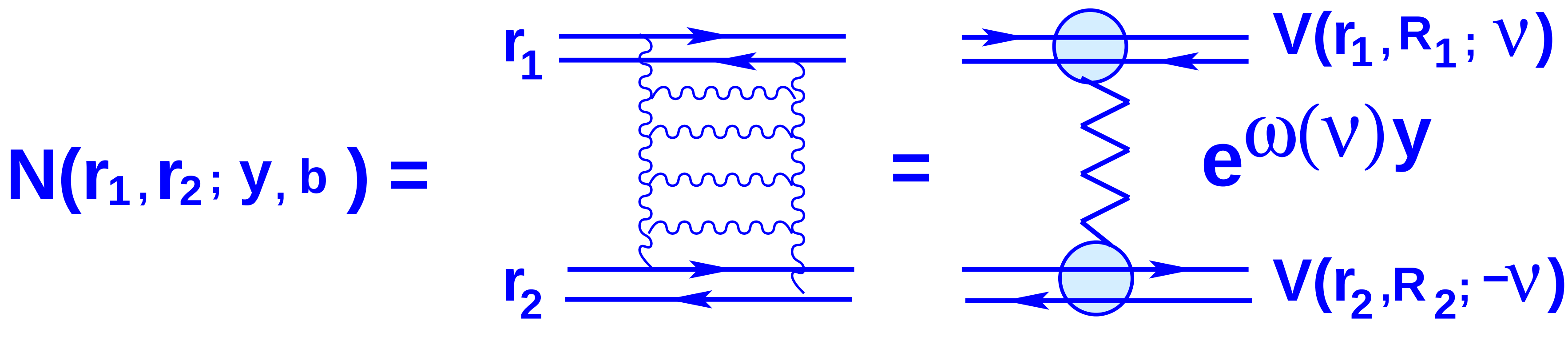
- **Pomeron interactions ($A\,(1P) \approx A\,(2P)$):**

$$\ln(1/\bar{\alpha}_S^2) \;\ll\; \bar{\alpha}_S\, Y \ll\; 1/\bar{\alpha}_S;$$

- **Pomeron interaction + NLO**

BFKL + ... : $1/\bar{\alpha}_S \ll \bar{\alpha}_S Y;$

BFKL Pomeron exchange

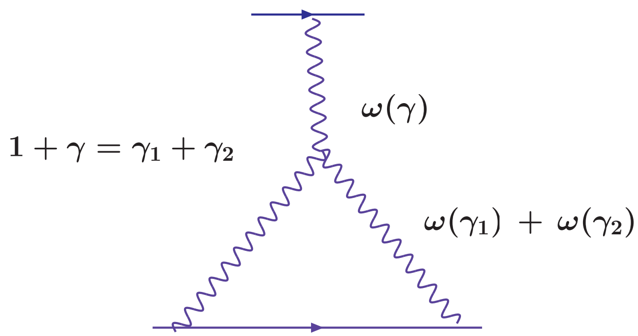


$$N\left(r_1, r_2; Y, b\right) \;=\; \int \frac{d\,\omega\, d\gamma d^2 R_1 d^2 R_2}{(2\pi i)^2}\, \frac{e^{\omega\, Y}}{\omega\; - \;\omega(\gamma)}\, \delta^{(2)}\left(\vec{R}_1 - \vec{R}_2 - \vec{b}\right)$$

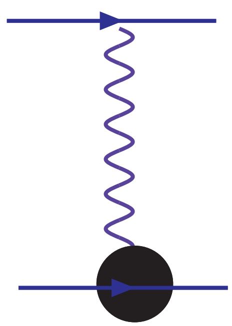
$$V\left(r_1, R_1, \gamma\right)\; V\left(r_2, R_2, 1 - \gamma\right)$$

$\gamma \;=\; 1/2 \;+\; i\,\nu$ - **anomalous dimension**

Overlapping singularities

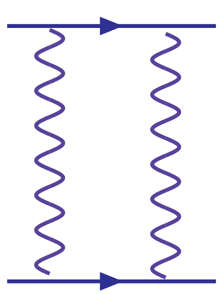


- $$A \propto \frac{1}{2\pi i} \int_{\epsilon - i\infty}^{\epsilon + i\infty} d\omega \; e^{\omega\, Y} \frac{1}{\omega - \omega(\gamma)} \frac{1}{\omega - \omega(\gamma_1) - \omega(\gamma_2)}$$



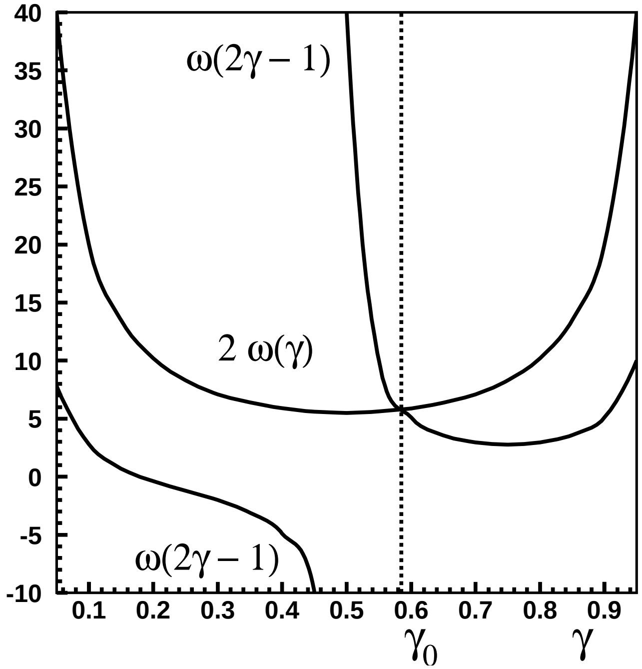
- $$\omega = \omega(\gamma)$$

$$A \propto \frac{e^{\omega(\gamma)\, Y}}{\omega(\gamma) - \omega(\gamma_1) - \omega(\gamma_2)}$$



- $$\omega = \omega(\gamma_1) + \omega(\gamma_2)$$

$$A \propto \frac{e^{(\omega(\gamma_1) + \omega(\gamma_2))\, Y}}{\omega(\gamma_1) + \omega(\gamma_2) - \omega(\gamma)}$$



- $\omega(2\gamma_0 - 1) = 2\,\omega(\gamma_0)$
- $\omega(\gamma_0) > \omega(\gamma = 1/2)$
- $A \propto Y\, e^{2\,\omega(\gamma_0)\,Y}$

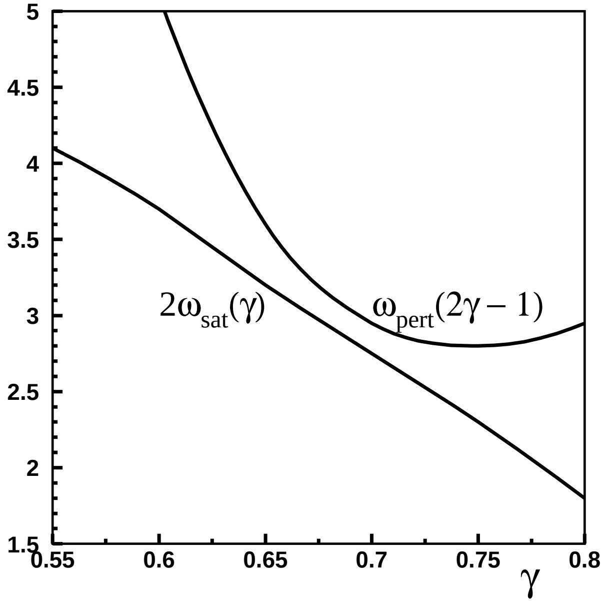
The sad truth: we have to start from the very beginning not only in summing Pomeron loops but also in MFA ? !

Our way out

$\gamma_0 \,>\, 1/2$, therefore

$\omega(2\gamma_0 - 1)$ – outside the saturation domain

but $\omega(\gamma_0)$ - inside the saturation domain



- γ_{cr}: $\frac{\omega(\gamma_{cr})}{1-\gamma_{cr}} = -\frac{d\omega(\gamma_{cr})}{d\gamma_{cr}}$
- $2\,\omega(\gamma_0) > \omega(\gamma = 1/2)$
- For $\gamma > \gamma_{cr}$ (Bartels & E.L. (1992))

$\omega_{sat}(\gamma) = \frac{\omega(\gamma_{cr})}{1-\gamma_{cr}}\,(1-\gamma)$

- Equation

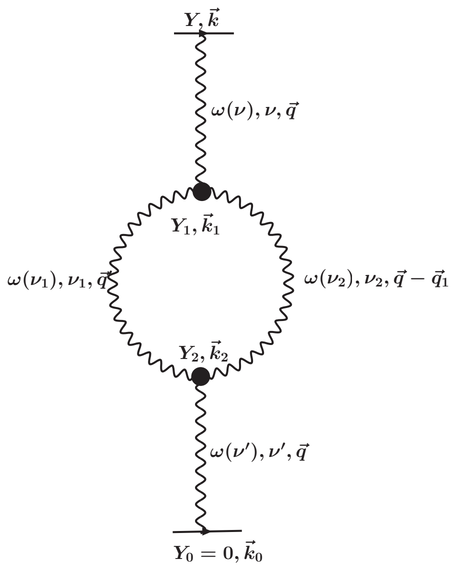
$2\omega_{sat}(\gamma_0) \;=\; \omega_{pert}(2\gamma_0 - 1)$

has no solution

Main idea and simple examples of the toy model.

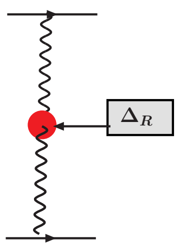
$$1 \approx \bar{\alpha}_S Y \leq \ln 1/\bar{\alpha}_S^2 \leq \bar{\alpha}_S Y \leq \bar{\alpha}_S Y \leq 1/\bar{\alpha}_S$$

- $1 \approx \bar{\alpha}_S Y \leq \ln 1/\bar{\alpha}_S^2 \longrightarrow$ **LO BFKL Pomeron**
- $\ln 1/\bar{\alpha}_S^2 \leq \bar{\alpha}_S Y \leq 1/\bar{\alpha}_S \longrightarrow$ **BFKL Pomeron calculus**
- $1/\bar{\alpha}_S \leq \bar{\alpha}_S Y \longrightarrow$ **NLO BFKL Pomeron and non-linear QCD**



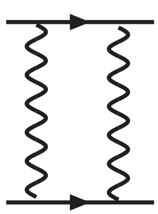
- $$A \propto \int \frac{d\omega}{2\pi i} \frac{1}{\omega - \omega(\gamma)} \frac{1}{\omega - \omega(\gamma_1) - \omega(\gamma_2)} \frac{1}{\omega - \omega(\gamma')}$$

- $$\omega = \omega(\gamma) = \omega(\gamma')$$



$$A \propto \frac{Y\, e^{\omega(\gamma)\, Y}}{\omega(\gamma) - \omega(\gamma_1) - \omega(\gamma_2)}$$

- $$\omega = \omega(\gamma_1) + \omega(\gamma_2)$$

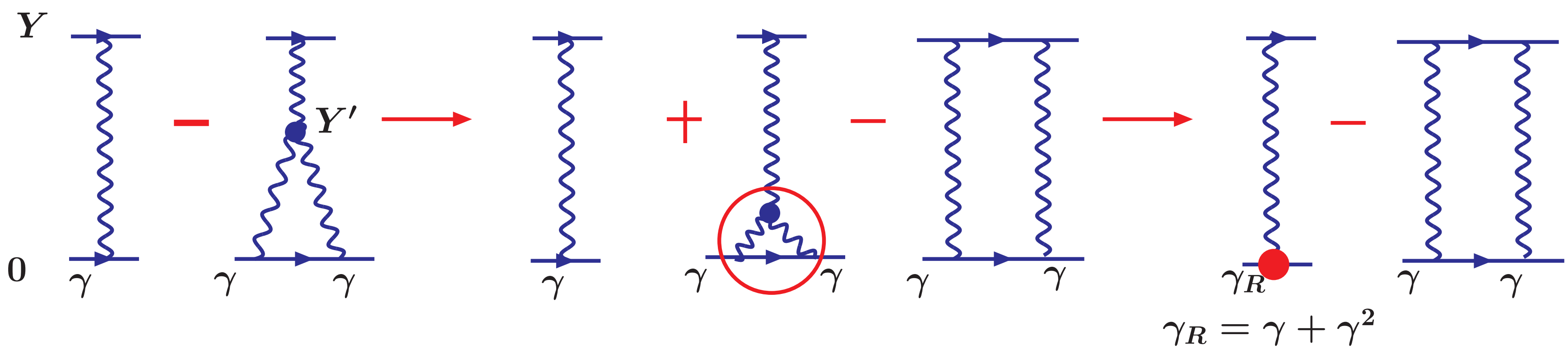


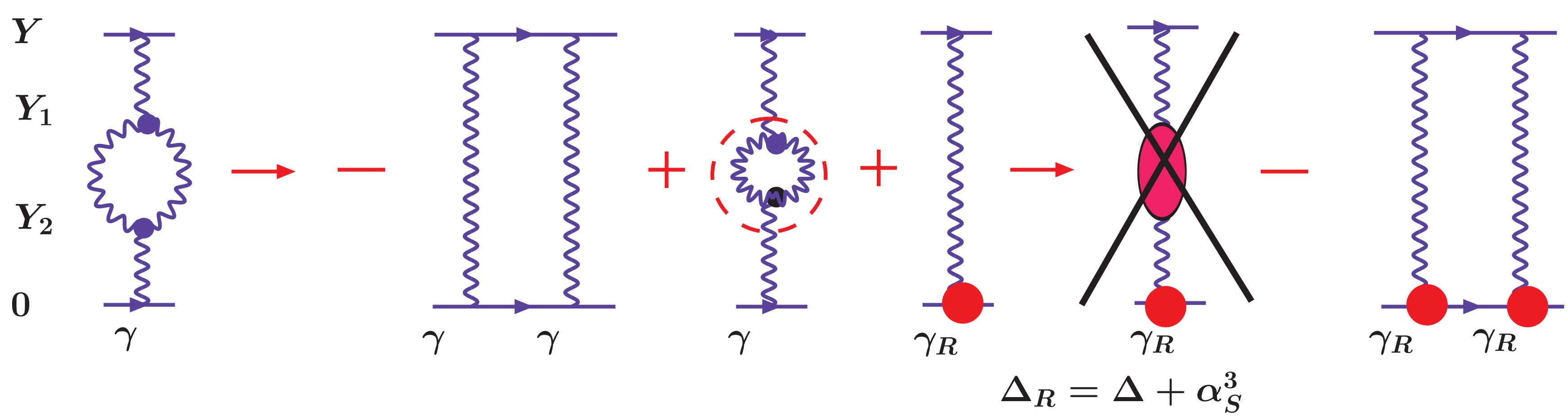
$$A \propto \frac{Y\, e^{(\omega(\gamma) + \omega(\gamma_2)Y}}{(\omega(\gamma) + \omega(\gamma_2 - \omega(\gamma))^2}$$

The scenario:

- We can neglect the overlapping singularities;
- We are dealing with the system of the non-interacting BFKL Pomerons;
- For summing Pomeron loops we can use the Mueller-Patel -Salam-Iancu approximation, improved by the renormalization of the scattering amplitude at low energies;

An example:





Our prescription:

1. To find the solution in the MFA for arbitrary initial conditions.

$$N\left(Y - Y', [\gamma_R(k_i, b_i)]\right) =$$

$$= \sum_{n=1}^{\infty} \int \prod_{i=1}^{n} d^2 k_i \; (-1)^n \; C_n(k) \; P\left(k, k_i; b_i | Y - Y'\right) \gamma_R(k_i, b_i)$$

where γ_R is an arbitrary function of k ;

2. To find the solution in terms of the renormalized vertices.

$$N\left(Y - Y' = 0, [\gamma_R(k, b)]; b\right) = N([\gamma(k, b)])$$

3. To use the MPSI formula to take into account the Pomeron loops. Using

$$P\left(k, k_i; b_i | Y - Y'\right) \bigotimes \gamma^{BA}\left(k_i, k_j, \underline{b} - \underline{b}_i - \underline{b}_k\right) \bigotimes P\left(k_0, k_j; b_j | Y'\right)$$

$$= \bar{\alpha}_S^2 \, P\left(k, k_0; b | Y\right)$$

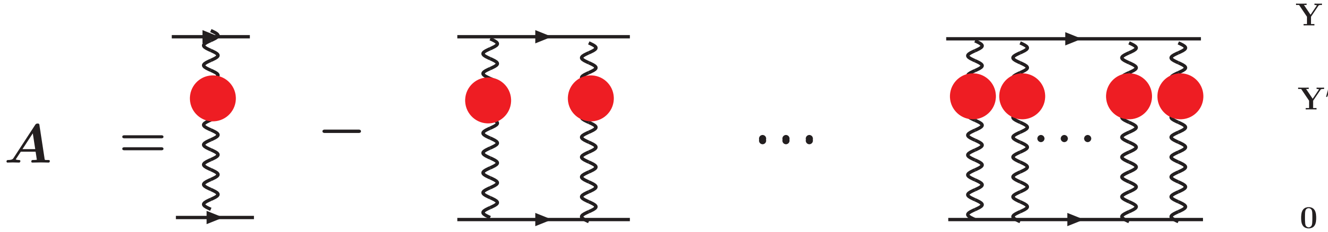
one can rewrite it in the form

$$N^{IMPSI}\left(k, k_0; b, Y - 0\right) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n!} \prod_{i=1,j=1}^{n} d^2k_i d^2k_j \, d^2b_i d^2b_j \, \frac{\delta}{\delta\,\gamma_R^{(1)}(k_i,b_i)} \frac{\delta}{\delta\,\gamma_R^{(2)}(k_j,b_j)}$$

$$N^{MFA}\left(Y - Y', [\gamma_R^{(1)}(k_i, b_i)]\right) N^{MFA}\left(Y', [\gamma_R^{(2)}(k_j, b_j)]\right)|_{\gamma_R^{(1)}\gamma_R^{(2)}=0} \left(\bar{\alpha}_S^2\, P\left(k, k_0; b | Y\right)\right)^n$$

$$= 1 - \exp\left\{ - \bar{\alpha}_S^2\, P\left(k, k_0; b | Y\right) \frac{\partial}{\partial \gamma_P^{(1)}} \frac{\partial}{\partial \gamma_P^{(2)}} \right\} N^{MFA}\left(\gamma_P^{(1)}\right) N^{MFA}\left(\gamma_P^{(2)}\right)|_{\gamma_P^{(1)} = \gamma_P^{(2)} = 0}$$

where $\gamma_P^{(1)} = P\left(k, k_i; b_i | Y - Y'\right) \gamma_R(k_i, b_i)$ and $\gamma_P^{(2)} = P\left(k_0, k_i; b_i | Y'\right) \gamma_R(k_i, b_i)$




γ^{BA} low energy amplitude $\gamma_R \to \gamma^{BA}$

Solution:

For model BFKL kernel

$$\omega(\gamma) = \bar{\alpha}_S \begin{cases} \frac{1}{\gamma} & \text{for } r^2 Q_s^2 \ll 1 - \text{ summing } \left(\bar{\alpha}_S \ln(1/(r^2 \Lambda^2_{QCD}))\right)^n; \\ \frac{1}{1-\gamma} & \text{for } r^2 Q_s^2 \gg 1 - \text{ summing } \left(\bar{\alpha}_S \ln(r^2 Q_s^2)\right)^n; \end{cases}$$

$$N(z) = 1 - e^{-\zeta(z)} \text{ where } z = \ln\left(k^2/Q_s^2(Y,b)\right)$$

Solution: (E. L. & Tuchin (2001))

- $$z = \sqrt{2} \int_{\zeta_0(b)}^{\zeta} \frac{d\zeta'}{\sqrt{\zeta' + (\exp(-\zeta') - 1)}}$$

Boundary condition:

For $z > 0$ only one BFKL Pomeron contributes

$$z \rightarrow 0^{+} \qquad P\,(z,\gamma) = \gamma\, e^{-\frac{1}{2}\,z}\, e^{-b^2/R^2} = \gamma(b)\, e^{-\frac{1}{2}\,z}$$

$$z \rightarrow 0^{-} \qquad N\,(z) \longrightarrow P\,(z,\gamma)$$

1. For small ζ

$$\ln\frac{\zeta}{\zeta_0(b)} = -\,\frac{1}{2}\,z \quad \text{or} \quad \zeta = \zeta_0(b)\,e^{-\,\frac{1}{2}\,z}$$

therefore $\zeta_0(b) = \gamma(b)$

2. For large ζ

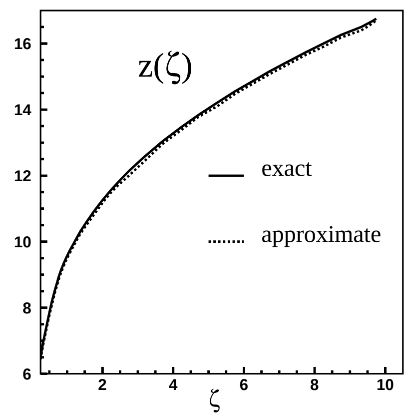
$$\zeta(z) \;=\; \frac{1}{2}\ln^2\left(\gamma(b)\,e^{-\frac{1}{2}\,z}\right)$$

3. Generally

$$\zeta(z) \;=\; \Phi\left(\gamma(b)\,e^{-\frac{1}{2}\,z}\right) \;\equiv\; \Phi\left(\gamma_P(z,b)\right)$$

Indeed

$$-z \;\; + 2\;\ln\gamma(b) = 2\;\ln\left(\gamma(b)\,e^{-\frac{1}{2}\,z}\right) \;\; =$$
$$\sqrt{2}\;\int^{\zeta}_{\zeta_0(b)} \frac{d\zeta'}{\sqrt{\zeta' + \left(\exp\left(-\zeta'\right) - 1\right)}} + \ln\zeta_0(b) =$$
$$\sqrt{2}\;\int^{\zeta}_{a} \frac{d\zeta'}{\sqrt{\zeta' + \left(\exp\left(-\zeta'\right) - 1\right)}} + \ln\left(\zeta_0(b)/a\right)$$



- **Near the saturation scale ($z \to 0$)**

$$N^{IMPSI}\left(x^2, R^2; Y\right) = 1 - \exp\left(-\gamma(b)\, P\left(x^2; R^2; Y\right)\right)$$

$$= 1 - \exp\left(-\gamma(b)\, e^{-\frac{1}{2} z}\right)$$

- **Deeply inside of the saturation domain ($z \ll 0$)**

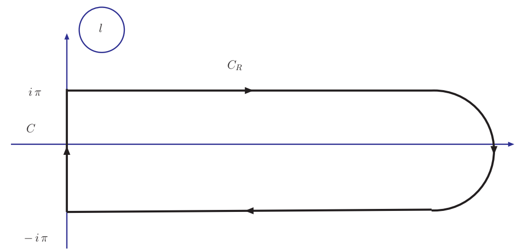
$$N_0^{MPSI}(z) = 1 - \frac{1}{(2\pi i)^2} \oint_{C_R} d\,l \oint_{C_R} d\,l_1 \; \exp\left(\frac{e^l}{P\left(\gamma^{BA}, z\right)}\right)$$

$$\times \left(\frac{e^l}{P\left(\gamma^{BA}, z\right)}\right) \Gamma\left(0, \frac{e^l}{P\left(\gamma^{BA}, z\right)}\right)$$

$$\times Z^{MFA}\left(P\left(e^{\frac{1}{2}l + l_1}\right)\right) Z^{MFA}\left(P\left(e^{\frac{1}{2}l - l_1}\right)\right)$$

$$Z^{MFA} = 1 - N^{NFA}\left(\gamma_P^{(1)}\right) = e^{-\zeta\left(\frac{1}{2}l + l_1\right)}$$

$$Z^{MFA} = 1 - N^{NFA}\left(\gamma_P^{(2)}\right) = e^{-\zeta\left(\frac{1}{2}l - l_1\right)}$$



$N(z) \Longrightarrow$

$$\boxed{1 - e^{\frac{1}{2} z}}$$

For MFA

$N(z) \Longrightarrow$

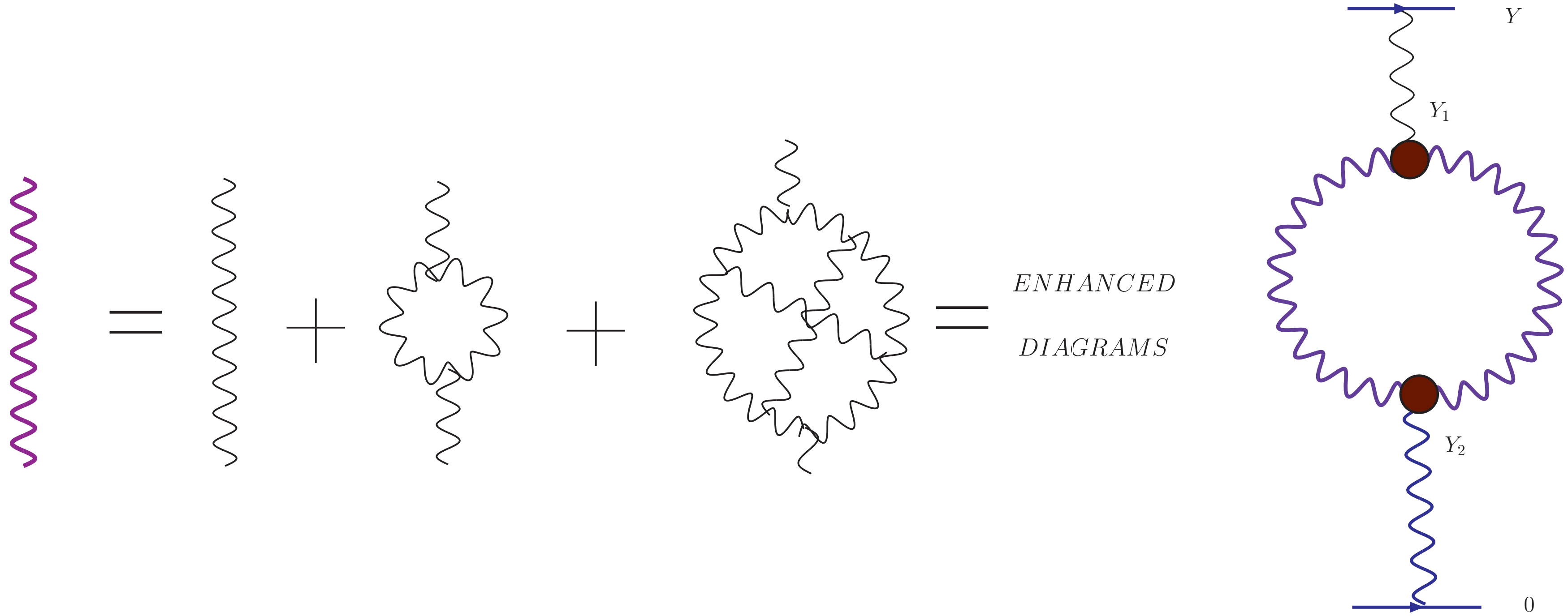
$$1 - e^{-C\, z^2}$$

We obtain:

- **geometrical scaling behaviour;**

- **rather slow approaching the asymptotic value, namely $1 - N \propto \exp(\frac{1}{2}z)$ where $z = \ln(r^2 Q_s^2)$;**

Self-consistency check



Conclusions

- For $\alpha_s Y \ll 1/\alpha_S$ it turns out that the problem reduces to the problem of non-interacting Pomerons;
- We can neglect the overlapping singularities;
- To sum the enhanced diagrams we can use the t-channel unitarity (Mueller-Patel-Salam-Iancu approach);
- The amplitude shows the geometrical scaling behaviour approaching 1 as $N = 1 - \exp[-z/2]$;

“ Once you eleminate the impossible what remains is the solution - no matter how improbable it may seem”