High energy evolution from the wave-functional approach

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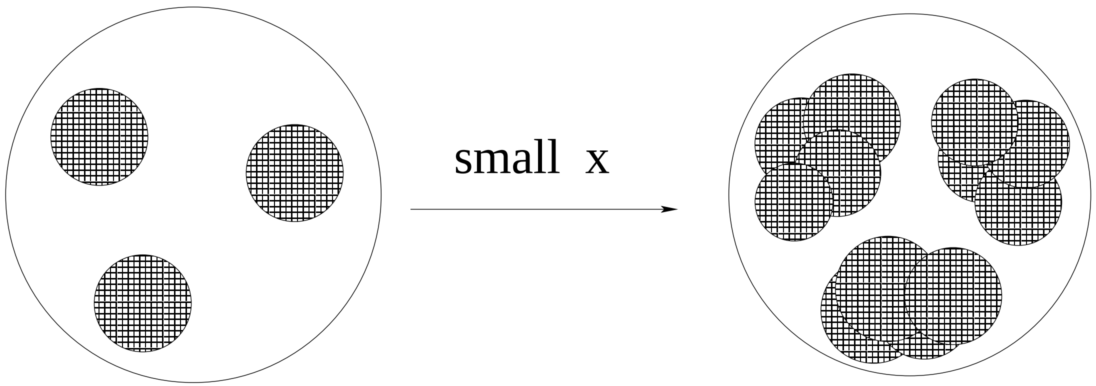
based on: Alex Kovner, M.L., Urs Wiedemann, JHEP 0706:075,2007

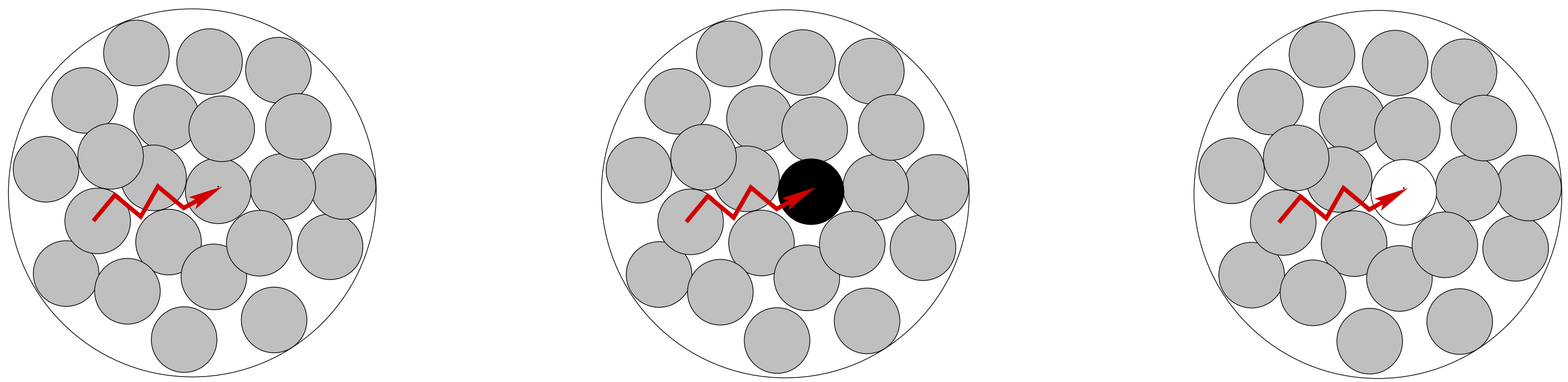
Dilute regime: $\delta\rho \sim \rho \quad \rightarrow \quad \rho \simeq e^{\omega Y}$ BFKL



Dense regime: (1) Emission probability is independent of density

(2) "Bleaching of color"

Random walk $\rho \sim \sqrt{Y}$



Multiple rescatterings of any gluon on an external probe (target) Eikonal approximation

- Linear gluon emission without Multiple Rescatterings - BFKL Pomeron
  Dilute on Dilute - dipole-dipole ($\gamma^* - \gamma^*$)

- Non-Linear gluon emission without Multiple Rescatterings
  This is justified when scattered on dilute object. Dense on Dilute (DIS)
  - BFKL Pomeron "fan" diagrams (upward)
  or a bit more generally JIMWLK evolution[1]

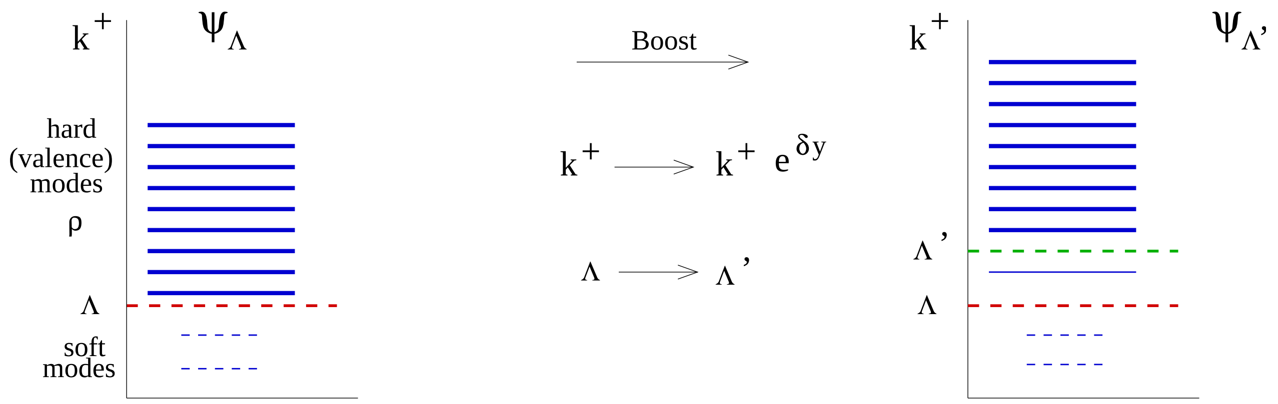
- Linear gluon emission with Multiple Rescatterings
  Important for scattering on dense objects. Dilute on Dense (DIS)
  - BFKL Pomeron "fan" diagrams (downward)
  or a bit more generally KLWMIJ evolution Alex Kovner and ML (2005)

  Dense Dilute Duality (DDD) relates JIMWLK and KLWMIJ

- Non-Linear gluon emission with Multiple Rescatterings
  - Everything including Pomeron Loops (pp, pA, AA, ...)
  - True limit of QCD at high energies. Reggeon Field Theory (RFT).
  - Should restore both s- and t- channel unitarity whatever it means.

[1] In fact, JIMWLK does partially account for multiple rescatterings

High energy evolution of hadronic wavefunction



Hard particles with $k^+ > \Lambda$ scatter off the target. In the eikonal approximation, the scattering amplitude is independent of k^+. Hard (valence) modes are described by the valence density $\rho(x_\perp)$.

Soft modes are not many. They do not contribute much to the scattering amplitude.

The boost opens a window above Λ with the width $\sim \delta y$. The window is populated by soft modes, which became hard after the boost. These newly created hard modes do scatter off the target.

In the dilute limit $\rho \sim g$; gluon emission $\sim g\,\rho$. 1 gluon in LO, 2 gluons in NLO

In the dense limit $\rho \sim 1/g$, we have $g\,\rho \sim 1$, and the number of gluons in the window can be very large.

Hadron wave function in the gluon Fock space

$$|\Psi\rangle \;=\; \Psi[a_i^{\dagger a}(x)]\,|0\rangle \qquad\qquad |\Psi\rangle \;=\; |v\rangle$$

The evolved wave function

$$|\Psi\rangle_{\Delta Y} \;=\; \Omega_{\Delta Y}(\rho,\; a)\,|v\rangle\;; \qquad\qquad |v\rangle \;=\; |v\rangle\;\otimes\;|0_a\rangle$$

Gluon cloud operator in the dilute limit

$$\mathcal{C}_{\Delta Y}\;\equiv\;\Omega_{\Delta Y}(\rho\;\rightarrow\;0)\;=\;\mathrm{Exp}\left\{i\int_z b_i^a(z)\int_{e^{-\Delta Y}\Lambda}^{\Lambda}\frac{dk^+}{|k^+|^{1/2}}\left[a_i^a(k^+,z)\;+\;a_i^{\dagger a}(k^+,z))\right]\right\}\;.$$

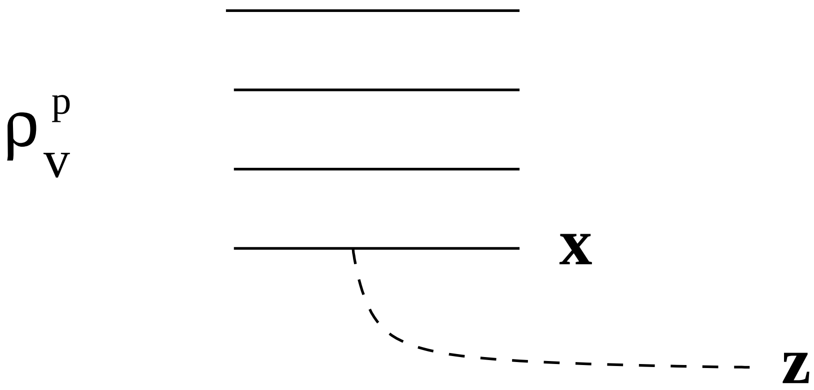
The classical WW field

$$b_i^a(z)\;=\;\frac{1}{2\pi}\;\int d^2x\frac{(z-x)_i}{(z-x)^2}\;\;\rho^a(x)$$

The emission amplitude is proportional to the density of emitters - BFKL evolution

The coherent operator $\mathcal{C}$ dresses the valence wave function by the cloud of the Weizsacker-Williams gluons:

$$\mathcal{C}^{\dagger}\, A_i^a(k^+, z)\, \mathcal{C} \;=\; A_i^a(k^+, z) + \frac{i}{k^+} b_i^a(z)$$



Given the evolution of hadronic wave function one can calculate the evolution of an arbitrary observable $\hat{\mathcal{O}}[\rho]$ which depends on the color charge density

$$\langle v|\, \hat{\mathcal{O}}[\rho]\, |v\rangle \;=\; \int D\rho\, W[\rho]\;\mathcal{O}[\rho]$$

The evolution of the diagonal matrix element[2]

$$\partial_Y\,\langle v|\hat{\mathcal{O}}|v\rangle \;=\; \lim_{\Delta Y\to 0}\frac{\langle v|\Omega_Y^\dagger\,\hat{\mathcal{O}}[\rho+\rho_{soft}]\,\Omega_Y|v\rangle \;-\; \langle v|\hat{\mathcal{O}}[\rho]|v\rangle}{\Delta Y} = -\int D\rho\, W[\rho]\, H^{RFT}[\rho]\,\mathcal{O}[\rho]$$

Charge density due to newly produced soft gluons

$$\rho^a_{soft}(x) \;=\; \int_{e^{-\Delta Y}\Lambda}^{\Lambda} dk^+\, a_i^{\dagger b}(k^+, x)\, T^a_{bc}\, a_i^c(k^+, x)$$

The charge density shift operator

$$\hat{R}_a \;=\; \exp\left[\int d^2z\,\rho^c_{soft}(z)\,\frac{\delta}{\delta\rho^c(z)}\right], \qquad\qquad \hat{R}_a\;\hat{\mathcal{O}}[\rho] \;=\; \hat{\mathcal{O}}[\rho\,+\,\rho_{soft}]$$

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[2] This can be extended to non-diagonal matrix elements A. Kovner, ML and H. Weigert (2006)

$$H^{RFT}\left[\rho, \frac{\delta}{\delta\rho}\right] = - \lim_{\Delta Y \to 0} \frac{\langle 0_a | \Omega^{\dagger}_{\Delta Y}(\rho, a) \left(\hat{R}_a - 1\right) \Omega_{\Delta Y}(\rho, a) | 0_a \rangle}{\Delta Y} .$$

Two main ingredients:

Ω takes into account non-linear gluon emission (gluon saturation in the wavefunction)

R takes into account multiple rescatterings of newly produced soft modes

Dilute limit (KLWMIJ)

Gluon saturation in Ω is neglected, $\Omega \to \mathcal{C}$.
Only one gluon is emitted per one step in rapidity
Evolution of the gluon density is given by the BFKL: $\rho^2_Y = \rho^2 e^{\omega Y}$.
Multiple Rescatterings (R) are fully taken into account

$$H^{RFT}(\rho \to 0) \to H^{KLWMIJ} = \int_z b^a_z[\rho]\,(1 - R_z)^{ab}\, b^b_z[\rho]$$

Dual Wilson line for a single emitted gluon

$$R^{ab}_z = \left[\mathcal{P} \exp \int dz^- \, T^c \frac{\delta}{\delta\rho^c(z, z^-)}\right]^{ab}$$

Dense limit (JIMWLK)

Non-Linear emission is accounted for by complete $\Omega = \mathcal{C}\,\mathcal{B}$ (not known previously).
Here $\mathcal{B} \sim Exp[a\,M\,a]$ stands for a Bogoliubov-type operator
$\rho_Y^2 = \rho^2 + \kappa\,Y$ (the density does not really saturate).

Multiple Rescatterings of newly produced soft modes are neglected:
R is expended to second order (two gluon exchange): $R \sim \frac{\delta}{\delta\rho}\frac{\delta}{\delta\rho}$

$$H^{RFT}(\rho \to \infty) \to H^{JIMWLK} = \int_z b_z^a \left[\frac{\delta}{\delta\rho}\right] (1 - S_z)^{ab}\, b_z^b \left[\frac{\delta}{\delta\rho}\right]$$

In the light cone gauge ($A^- = 0$) the large field component is A^+ ($\Delta\, A^+ “=” \rho$)
The Wilson line S stands for single gluon S-matrix

$$S(x) = \mathcal{P} \exp\left\{ i \int dx^-\, T^a\, A^{+\,a}(x, x^-) \right\} .$$

No wavefunctional derivation of the JIMWLK has been previously available.

Light Cone Hamiltonian

$$H = \int_{k^+>0} \frac{dk^+}{2\pi}\, d^2x\, \left(\frac{1}{2}\Pi_a^-(k^+,x)\,\Pi_a^-(-k^+,x) + \frac{1}{4}G_a^{ij}(k^+,x)\,G_a^{ij}(-k^+,x)\right)\,,$$

The electric and magnetic pieces have the form

$$\Pi_a^-(x^-,x) = -\frac{1}{\partial^+}\left(D^i\partial^+A_i\right)^a(x^-,x)\,,$$

$$G_a^{\mu\nu}(x^-,x) = \partial^\mu A_\nu^a(x^-,x) - \partial^\nu A_\mu^a(x^-,x) - g f^{abc} A_\mu^b(x^-,x)\,A_\nu^c(x^-,x)$$

We split the modes into hard and soft: The hard modes act as an external current $j^+ = \delta(x^-)\,\rho(x)$ for the soft modes.

The Hamiltonian for the soft modes is then given by the substitution

$$\Pi_a^-(k^+,x) = \frac{-i}{k^+}\partial^i\partial^+A_i^a(k^+,x) + \frac{i}{k^+}\rho^a(x) + \frac{g\,i}{k^+}\,f^{abc}\int\frac{dp^+}{2\pi}A_i^b(k^+-p^+,x)\,(-ip^+)\,A_i^c(p^+,x)\,.$$

The soft fields A are defined only below the longitudinal momentum cutoff Λ

The main problem: diagonalize the Hamiltonian for the soft modes for a given distribution of valence charges ρ.

We solve the problem for any ρ in the range

$$\begin{array}{ccccc} & g \ \leq \ \rho \ \leq \ 1/g & \\ KLWMIJ & & JIMWLK \end{array}$$

For any ρ we keep terms which are order one in the coupling constant g. The remaining corrections are strictly perturbative and vanish in the limit $g \rightarrow 0$.

Very serious complication: the valence densities do not commute, but form algebra

$$[\rho^a(x),\ \rho^b(y)] \ = \ i\, g\, f^{abc}\, \rho^c(x)\, \delta^2(x-y)$$

Cannot be neglected neither in the dilute limit when the commutator is not suppressed nor in the dense limit when it is subleading but of order one.

Perturbation theory in external (strong) non-commutative background field.
Additional subtlety: residual gauge fixing results in constraints (Dirac quantization)

$$A_i^a(x^-, x) \; = \; \theta(x^-)\, b_i^a(x) \; + \; c_i^a(x^-, x)$$

$$\begin{aligned} \partial_i \, b_i^a(x) \;&=\; \rho^a(x) \,, \\ \partial_i \, b_j^a(x) \; - \; \partial_j \, b_i^a(x) \; - \; g \, f^{abc} \, b_i^b(x) \, b_j^c(x) \;&=\; 0 \end{aligned}$$

The classical fields b's do not commute, because ρ's do not.

The shift is achieved with the aid of the Coherent operator

$$\mathcal{C} = \exp\left[2\, i \int d^2x \, b_i^a(x) \, A_i^a(x^- = 0, x)\right]$$

The light cone canonical quantization

$$A_i^a(x^-, x) = \int_0^\infty \frac{dk^+}{2\pi} \frac{1}{\sqrt{2k^+}} \left\{ a_i^a(k^+, x)\, e^{-ik^+x^-} + a_i^{a\,\dagger}(k^+, x)\, e^{ik^+x^-} \right\},$$

$$\left[a_i^a(k^+, x), a_j^{b\,\dagger}(p^+, y)\right] = (2\pi)\, \delta^{ab}\, \delta_{ij}\, \delta(k^+ - p^+)\, \delta^{(2)}(x - y)$$

We need to eliminate the zero mode ($k^+ = 0$), which is constrained by the Gauss law

Quadratic Hamiltonian

$$H = -\frac{1}{2} \int dx^- \, d^2x \, \Big[\theta(-x^-) \, c_i^a(x^-, x) \, \partial^2 \, c_i^a(x^-, x) \; + \; \theta(x^-) \, c_i^a(x^-, x) \, D^{2\,ab}[b] \, c_i^b(x^-, x) \Big] \, .$$

plus terms subleading in g.

Valence degrees act as a shock wave.

Almost free Hamiltonian for $x^- < 0$.   Pure 2d gauge background for $x^- > 0$
c is $almost$ canonical, but has a non-trivial commutation relation due to non-commutativity of b.

Diagonalizing the Hamiltonian with Bogoliubov transformation

$$\mathcal{B} \; = \; \exp\left[\, c \; M \; c \,\right]$$

$$\mathcal{B}^\dagger \; H \; \mathcal{B} \; = \; \int_{p^-, q} p^- \;\; \beta^\dagger_{p^-, q} \;\; \beta_{p^-, q} \, .$$

$$c \; = \; \kappa_1 \, \beta \; + \; \kappa_2 \, \beta^\dagger$$

The final result for the evolution operator Ω re-expressed in terms of the original field A:

$$\Omega \; \equiv \; \mathcal{C}[A] \;\; \mathcal{B}[A]$$

$$\Omega^\dagger \tilde{A}_i^a(x^-,x)\Omega = c_i^a(x^-,x)+\epsilon(x^-)\left[b_i^a(x)+2\int_y\left\{D_i\frac{1}{D\partial}D_j - D_i\frac{1}{\partial D}\partial_j\right\}^{ab}(x,y)c_j^b(0,y)\right]$$

$$+\epsilon(x^-)\int_{y,z}\left\{g\left[D_i\frac{1}{\partial D}\right]^{ab}(xz)f^{bcd}j^d(z)\int dy^-\partial^+\mathcal{A}_j^e(y^-,y)\frac{\delta\mathcal{A}_j^e(y^-,y)}{\delta j^c(z)}\right.$$

$$\left.+\frac{2i}{3}[d_{ij}^{ab}(x,y),b_k^c(z)]c_j^b(0,y)c_k^c(0,z)\right\}\ ,$$

$$\Omega^\dagger j^a(x)\Omega = j^a(x)+2\int_y\left\{\left(\partial D\frac{1}{D\partial}-1\right)D_j\right\}^{ab}(x,y)c_j^b(0,y)$$

$$+gf^{acd}j^d(x)\int dy^-d^2y\partial^+\mathcal{A}_j^b(y^-,y)\frac{\delta\mathcal{A}_j^b(y^-,y)}{\delta j^c(x)}$$

$$+2i\int_{y,z}\left[\left\{\left(\partial D\frac{1}{D\partial}-1\right)D_j\right\}^{ab}(x,y),b_k^c(z)\right]c_j^b(0,y)c_k^c(0,z)\ .$$

Outlook

•We have been able to derive the evolution operator Ω for arbitrary valence density ρ.

• In the dense and dilute limits we reproduced the JIMWLK and KLWMIJ evolution Hamiltonians

• In order to obtain the complete Hamiltonian we need to compute the following expression without applying any expansions.

$$H^{RFT}\left[\rho, \frac{\delta}{\delta\rho}\right] = -\lim_{\Delta Y \to 0} \frac{\langle 0_a | \Omega^{\dagger}_{\Delta Y}(\rho, a, a^{\dagger}) \left(\hat{R}_a - 1\right) \Omega_{\Delta Y}(\rho, a, a^{\dagger}) | 0_a \rangle}{\Delta Y}.$$

• We intend to apply our results to semi-inclusive processes (jets, diffractions, ...) at the LHC, TeVatron and RHIC.