

# Scattering Physics in $b$ -space, LHC and Above

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## Introduction:

The elastic amplitude in b-space is defined

$$\alpha_{el}(s, b) = \frac{1}{2\pi} \int d^2 q e^{-iq \cdot b} f_{el}(s, t).$$

For simplicity lets discuss the unitarity equation in a diagonal representation

$$2 \operatorname{Im} \alpha_{el}(s, b) = |\alpha_{el}(s, b)|^2 + G''(s, b)$$

$$\Omega_{tot}(s, b) = \Omega_{el}(s, b) + \Omega_{in}(s, b).$$

A general solution may be written as

$$\alpha_{el}(s, b) = i \left( 1 - e^{-\frac{\Omega(s, b)}{2}} \right).$$

Generality is maintained as long as  $\Omega$  is not constrained.

In the eikonal approximation  $\Omega$  is real (i.e.  $\alpha_{el}$  is imaginary) and equals the imaginary part of the input elastic amplitude. The interpretation is that the unitarity correction to the input  $\alpha_{el}$  is obtained from a sequence of repeated elastic re-scatterings of the interacting hadronic projectiles.

The implicit assumption of the above one dimensional representation is that  $\sigma_{\text{diff}} \ll \sigma_{\text{el}}$ . In a non diagonal representation diffractive rescatterings are included resulting in multi dimensional unitarity equations.

Most of the qualitative results of interest can be obtained in the diagonal representation. More elaborate, fine tuned, output requires a multi channel approach which will be discussed in the continuation.

## Unitarity vs. Black Disc Bounds:

The black disc bound is defined to be

$|\alpha_{ee}(s, b)| = 1$ . The unitarity bound depends on the properties of  $\mathcal{J}_2$ . We shall check two extreme options.

i)  $\mathcal{J}_2$  is real, as in the eikonal model, i.e.  $\alpha_{ee}$  is imaginary. In this approximation  $|\alpha_{ee}(s, b)| \leq 1$ . The black disc and unitarity bounds co-incide. As a consequence we obtain in the high energy limit:  $\alpha_{ee}(s, b) \xrightarrow{s \rightarrow 1}$

$$\begin{aligned} \text{Re } \alpha_{ee}(s, t=0) &\xrightarrow{s \rightarrow 0} 0 \\ \text{Im } \alpha_{ee}(s, t=0) \end{aligned}$$

In general:

$$\frac{\sigma_{el}}{\sigma_{tot}} \leq \frac{1}{2}$$

In a multichannel model:  $\frac{\sigma_{el} + \sigma_{diff}}{\sigma_{tot}} \leq \frac{1}{2}$ .

The survival probability of inelastic diffractive final states (to be discussed):  $S^2 \xrightarrow{s \rightarrow 0}$ .

2)  $\mathcal{J}_2$  and, thus,  $\alpha_{el}$  are complex.

We get:  $|\alpha_{el}(s, b)| \leq 2$ . i.e. the unitarity bound is twice as large as the black disc bound. Accordingly we have in the high energy limit:

$$\sigma_{el} \xrightarrow{s} \sigma_{tot}, \quad g_{in} \xrightarrow{s} 0$$

$$\frac{\operatorname{Re} \alpha_{el}(s, t=0)}{\operatorname{Im} \alpha_{el}(s, t=0)} \xrightarrow{s} 1$$

$$s^2 \xrightarrow{s} 1.$$

The rate of approach to the high energy limits of the above two options depends on the specific  $s$  and  $b$  dependences of  $\alpha_{el}(s, b)$  input.

Conveniently, this can be written as

$$\mathcal{J}_2(s, b) = f(s) \Gamma(s, b).$$

A model with a complex  $\alpha_{el}(s, b)$  and  $g_{in}(s, b) \xrightarrow{s} 0$  has been promoted by Troshin and Tyurin. The model reproduces existing data well. Its predictions just above the Tevatron are fast going wild!

S. M. Troshin and N.E. Tyurin: EPJC 35,  
435 (2005)

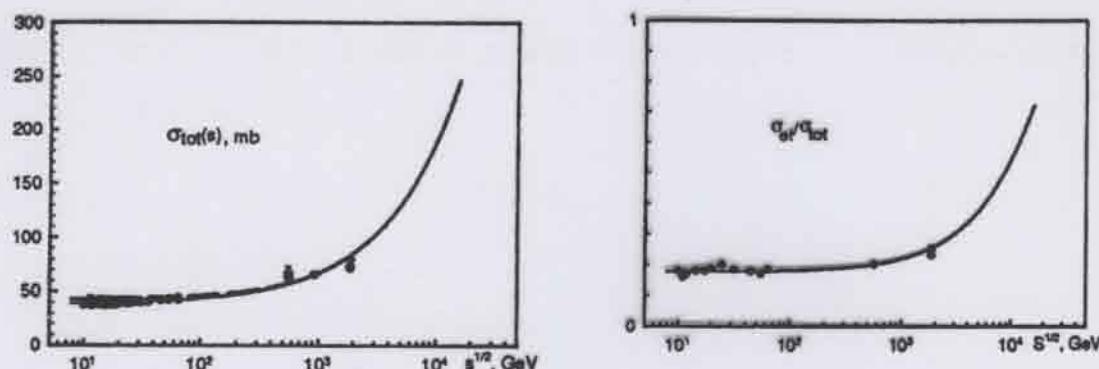


Figure 1: Total and ratio of elastic to total cross-sections of  $pp$  and  $\bar{p}p$ -interaction

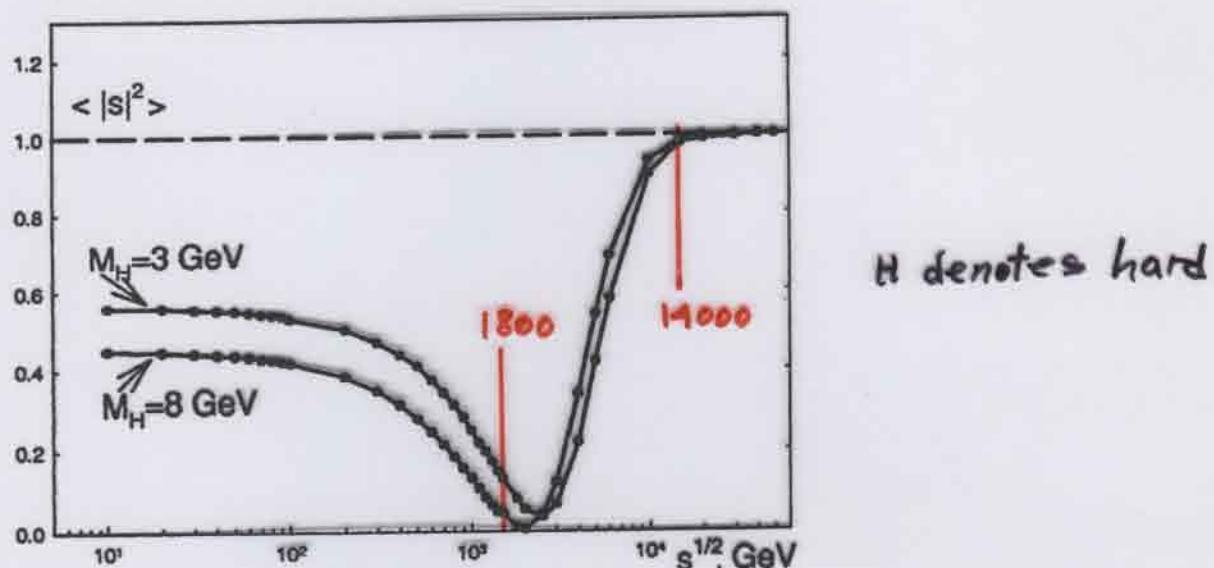


Figure 3: Energy dependence of gap survival probability  $\langle |S|^2 \rangle$

### LHC Predictions:

$$\sigma_{\text{tot}} \simeq 250 \text{ mb}$$

$$\sigma_{ee} \simeq 190 \text{ mb} \simeq 0.75 \sigma_{\text{tot}}$$

$$\frac{\text{Re } \alpha_{ee}(t=0)}{\text{Im } \alpha_{ee}(t=0)} \simeq 0.85 - 0.90$$

$$S^2_{\text{Higgs}} \simeq 1.0$$

Quality of data fit (up to Tevatron): Good!  
The "drama" starts just above, where we have  
no data!

There is no unique mathematical solution to the unitarity equation even when  $a_{ee}$  is imaginary. Therefore, the approach of  $a_{ee}(s, b) \xrightarrow{s} 1$  is model dependent. It is easy to get misled by the observation that  $\frac{da_{ee}}{dt}$  is exponential in  $t$  in the forward cone, and extend the assumed exponentiality to high  $t$ .

Regardless, assume this approximation is O.K. Its  $b$ -transform is a central Gaussian for which we have

$$a_{ee}(s, b=0) = \frac{\sigma_{tot}}{4\pi B_{tot}}.$$

This was used by CDF (PRD 50, 5550 (1994)) from data extending in the narrow range of  $0.09 \leq |t| \leq 0.25 \text{ GeV}^2$  leading to an estimate  $a_{ee}(1800, b=0) = 0.989 \pm 0.016$  quoted by many (myself included!).

In my opinion this estimate can not be trusted!

Low  $\times 07$

The above approximation for  $a_{ee}(s, b=0)$  was recently used by Frankfurt, Hyde-Wright, Strikman, Weiss (PRD 75, 054009 (2007)), who predict that  $a_{ee}(LHC, b=0) \approx 1$ .

Taking the above at face value we obtain a very strong prediction that  $\frac{\sigma_{tot}}{B_{ee}}$  is a constant from LHC and up to the Planck scale!

## Higgs Survival Probability:

The main observable signature of s-channel unitarity at present energies is a severe suppression, called LRG survival probability, of inelastic diffraction (soft and hard). This phenomenon is observed as of the relatively low ISR energies.

There is an obvious interest in the theoretical predictions of  $S_H^2$  corresponding to exclusive central diffractive Higgs production ( $p+p \rightarrow p+LRG+H+LRG+p$ ) at the LHC.

This is a prime example where the fine tuning of the theoretical calculation is crucially important!

In a single channel model

$$S^2 = \frac{\int d^2 b |M_{\text{diff}}|^2 P(s, b)}{\int d^2 b |M_{\text{diff}}|^2} = \frac{\sigma_{\text{diff}}(\text{out})}{\sigma_{\text{diff}}(\text{in})} .$$

In the eikonal approximation  $P(s, b) = e^{-\Omega(s, b)}$ .  $\Omega(s, b)$  is determined from a fit to a global soft scattering data base.

In the GLM two channel model diffractive re-scatterings are included along side the elastic re-scattering option.

In the simplest approximation diffraction at a vertex is regarded as a single hadronic state. We have, thus, two orthonormal vertex wave functions  $\langle \Psi_h | \Psi_D \rangle = 0$ , corresponding is a  $2 \times 2$  interaction operator  $T$ . Assume that  $\Psi_1$  and  $\Psi_2$  are diagonal with respect to  $T$ . We have

$$\Psi_h = \alpha \Psi_1 + \beta \Psi_2$$

$$\Psi_D = -\beta \Psi_1 + \alpha \Psi_2$$

$$\alpha^2 + \beta^2 = 1$$

and 4 amplitudes  $a_{i,\ell}$  for which we have a diagonal unitarity equation

$$2 \operatorname{Im} a_{i,\ell} = |a_{i,\ell}|^2 + G_{i,\ell}^{in}$$

and a probability  $P_{i,\ell} = e^{-G_{i,\ell}}$ .

In a  $p\bar{p}$  ( $\bar{p}p$ ) interaction  $a_{b,2} = a_{2,1}$ .

The corresponding elastic and diffractive amplitudes are

$$a_{ee}(s,b) = \alpha^4 a_{1,1}(s,b) + 2\alpha^2 \beta^2 a_{1,2}(s,b) + \beta^4 a_{2,2}(s,b)$$

$$a_{ed} = \alpha \beta [\alpha^2 a_{11} + (\alpha^2 - \beta^2) a_{1,2} + \beta^2 a_{2,2}]$$

$$a_{dd} = \alpha^2 \beta^2 [a_{1,1} - 2a_{1,2} + a_{2,2}]$$

In a two amplitude approximation  $a_{dd} = 0$  and we can eliminate  $a_{2,2} = 2a_{1,2} - a_{1,1}$ .

We make, thus, the distinction between 1CH, 2CH (2Amp), 2CH (3Amp) models.

Our 2CH (3Amp) LHC output is:

$$\sigma_{\text{tot}} = 110.5 \text{ mb}$$

$$\sigma_{ee} = 25.3 \text{ mb}$$

$$\sigma_{ss} = 11.6 \text{ mb}$$

$$\sigma_{dd} = 4.9 \text{ mb}$$

$$B_{ee} = 20.5 \text{ GeV}^{-2}$$

$$B_{ss} = 15.9 \text{ GeV}^{-2}$$

$$B_{dd} = 13.5 \text{ GeV}^{-2}$$

$$\frac{\text{Re } \alpha_{ee}(b=0)}{\text{Im } \alpha_{ee}(b=0)} = 0.127 .$$

The importance of improved  $S_\pi^2$  calculations is demonstrated in the comparison:

$$S_\pi^2(1\text{CH}) = 0.06 \quad \text{1st generation}$$

$$S_\pi^2(2\text{CH}(2\text{Amp})) = 0.027 \quad \text{2d generation}$$

$$S_\pi^2(2\text{CH}(3\text{Amp})) = 0.007 \quad \text{3d generation}$$

Our 1st and 2d  $S_\pi^2$  generations are comparable to the corresponding calculations of Durham and Frankfurt et. al.

Our recent 3d generation is almost identical to Frankfurt et. al.

Even though both GLM and Durham are two channel models, they are dynamically different. GLM formalism relate to the diversity of the soft re-scatterings for which we have different amplitudes  $a_{ij,k}$  with a different  $P_{ik}$  probability. In the Durham formalism the two channels relate to two different dynamical options of the screened diffractive process under consideration. Durham has not published for a while an improved estimate of  $S_H^2$ .

## The Approach Toward the Black Disc Bound:

$\sqrt{s}$ TeV	$\sigma_{tot}^{DL}$ mb	$\sigma_{tot}$ mb	$\sigma_{el}$ mb	$\sigma_{sd}$ mb	$\sigma_{dd}$ mb	$B_{el}$ $GeV^{-2}$	$R_{el}$	$R_D$	$\frac{\sigma_{diff}}{\sigma_{el}}$
1.8	73.0	78.0	16.3	9.6	3.8	16.8	0.21	0.38	0.83
14	101.7	110.5	25.3	11.6	4.9	20.5	0.23	0.38	0.65
30	115.0	124.8	29.7	12.2	5.3	22.0	0.24	0.38	0.59
60	128.6	139.0	34.3	12.7	5.7	23.4	0.25	0.38	0.54
120	143.9	154.0	39.6	13.2	6.1	24.9	0.26	0.38	0.49
250	162.0	172.0	45.9	13.6	6.6	26.5	0.27	0.38	0.44
500	181.2	190.0	52.7	14.0	7.0	28.1	0.28	0.39	0.40
1000	202.7	209.0	60.2	14.3	7.4	29.8	0.29	0.39	0.10
$10^{11}$	3970.0	1070.0	451.2	21.6	19.5	109.9	0.42	0.46	0.09
$1.22 \cdot 10^{19}$ (Planck)	26400.0	1970.0	871.4	25.5	27.7	202.6	0.44	0.47	0.06

$D = el + diff$

Table 2: Cross sections and elastic slope in Model B(2).  $\sigma_{tot}^{DL}$  is presented for comparison.

Tension to Planck mass.

An instant conclusion is derived from this table:

- 1) Up to  $\sqrt{s} \approx 10^4$  TeV there is no real difference between the non screened DL parametrization and the unitarized GLM model output of  $\sigma_{tot}$  (and to a lesser extent  $\sigma_{el}$ ). Conclusion: Unitarity induced taming of the elastic amplitude in the GLM model is very slow and becomes effective only at the  $\sqrt{s} \approx 10^4$  TeV range.
- 2) The above conclusion is supported by the observation that  $R_{el} = \frac{\sigma_{el}}{\sigma_{tot}}$  grows very slowly up to the same  $\sqrt{s} \approx 10^4$  TeV range. In this range  $R_D = \frac{\sigma_{el} + \sigma_{diff}}{\sigma_{tot}} = 0.38-0.39$ .
- 3)  $S^2 \rightarrow 0$ . Indeed  $\frac{\sigma_{diff}}{\sigma_{tot}}$  gets smaller with  $\sqrt{s}$ . Is smaller than 0.1 above  $10^4$  TeV.

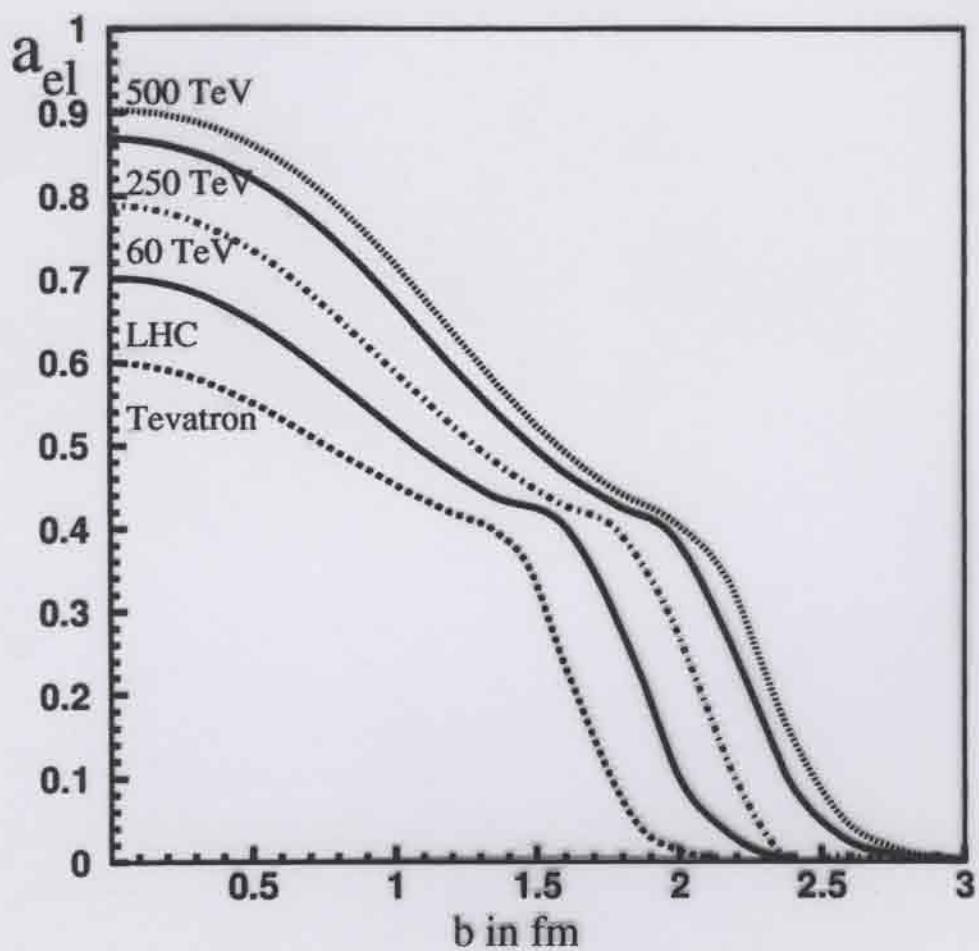
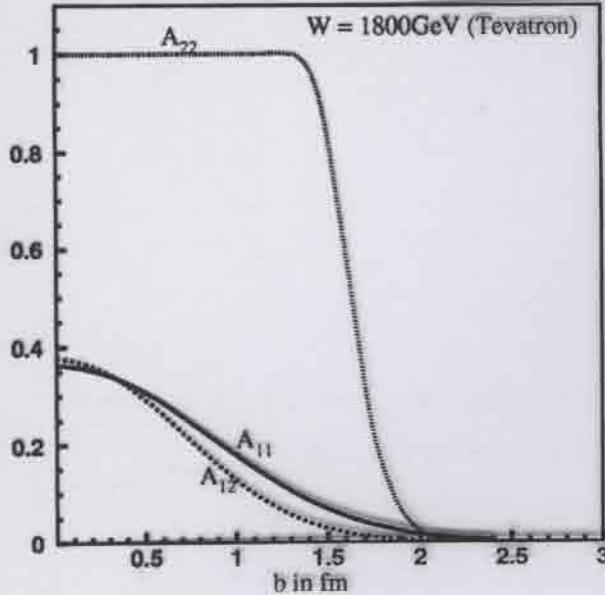
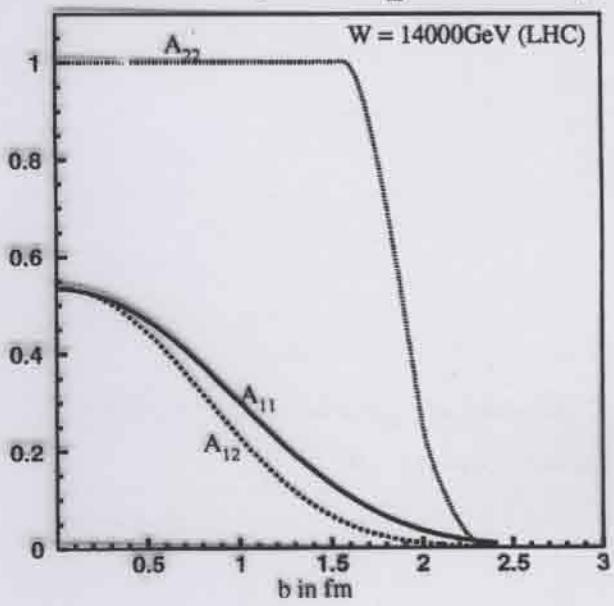
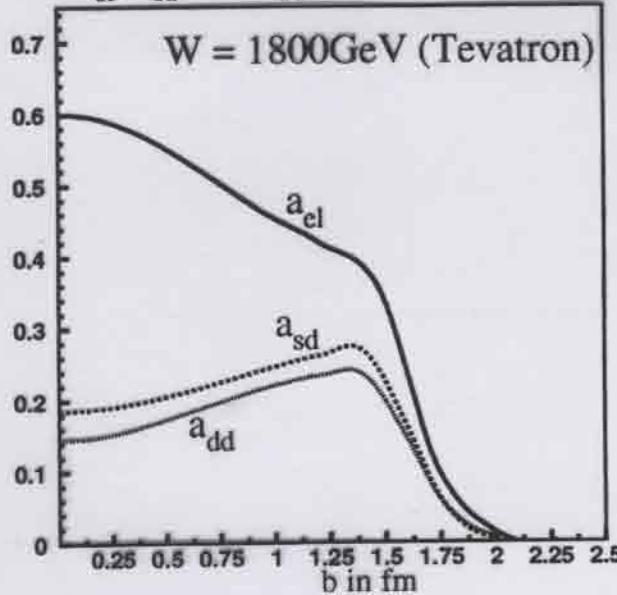
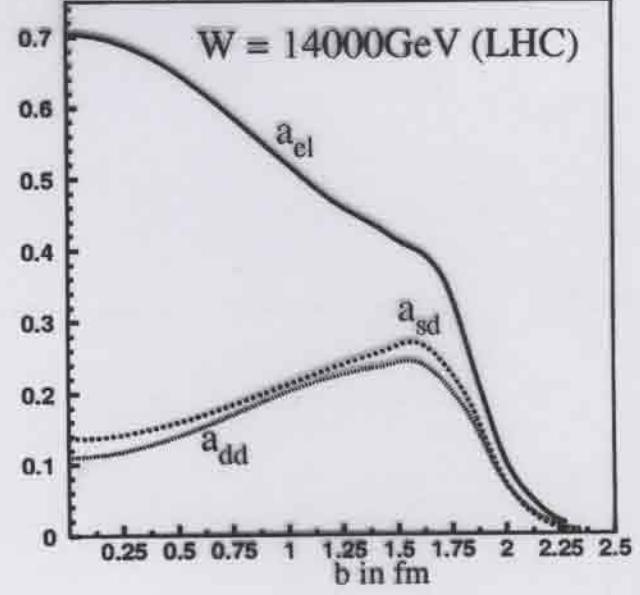
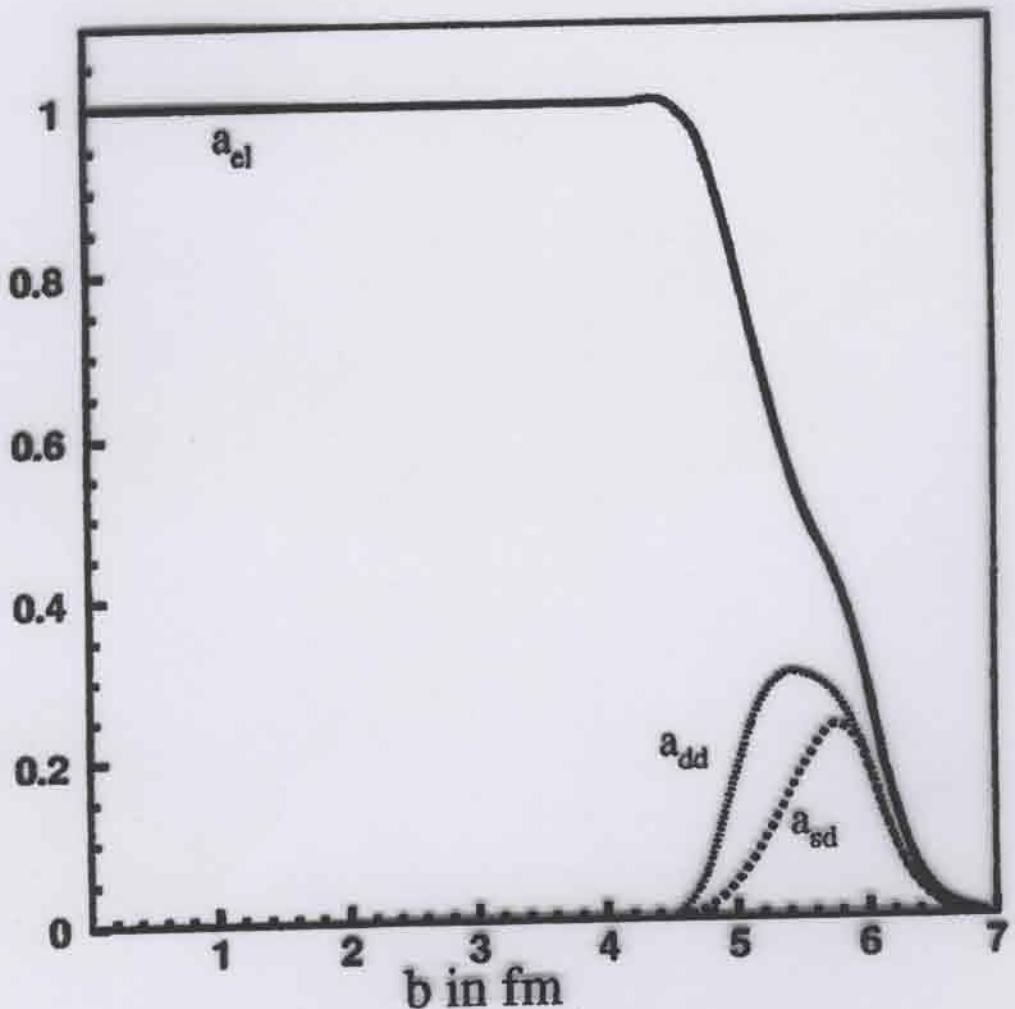


Figure 10: Impact parameter dependence of  $a_{el}$  in Model B(2) at different energies.

Profiles of Amplitudes  $A_{ik}$  for Model B(2)Profiles of Amplitudes  $A_{ik}$  for Model B(2)Figure 11: Impact parameter dependence of  $A_{i,k}$  at the Tevatron and LHC in Model B(2). $a_{el}, a_{sd}$  and  $a_{dd}$  for Model B(2) $a_{el}, a_{sd}$  and  $a_{dd}$  for Model B(2)Figure 12: Impact parameter dependence of  $a_{el}$ ,  $a_{sd}$  and  $a_{dd}$  at the Tevatron and LHC in Model B(2).



**Figure 15:**  $b$  dependence of  $a_{el}$ ,  $a_{sd}$ ,  $a_{dd}$  at the Planck mass in Model B(2).

We conclude that the approach toward the black disc bound in the GLM model is very slow, reading

$W = \sqrt{s}$ TeV	$a_{el}(b = 0)$	$a_{el}$ black core radius fm
1.8	0.62	
14	0.71	
60	0.79	
250	0.87	
500	0.90	
$10^3$	0.93	
$3 \cdot 10^4$	1.00	0.5
$6 \cdot 10^4$	1.00	0.8
$3 \cdot 10^8$	1.00	2.3
$10^{11}$	1.00	3.0
$1.22 \cdot 10^{19}$	1.00	4.6
(Planck)		

it well above the GZK energy ( $\sim 550$  TeV) above which we do not expect to collect significant cosmic rays data.

Is this a peculiar feature of the GLM model or a general feature ???

## Unitarization in the Hard Sector:

This issue is at the core of a very active research program on high density QCD. I wish to focus on a relatively simple problem with far reaching implications: How reliable are the updated editions of pdf parametrization when applied to LHC analysis in a new kinematical domain -  $x < 10^{-4}$ ,  $P_T^2 = O(2-10 \text{ GeV}^2)$ .

Presentation of the problem in DIS is straight forward

$$\sigma_{\text{dipole}}(r_T^2, x) = \int db^2 \text{Im} \alpha_{ee}^{\text{dipole}}(r_T^2, x; b)$$

In LL DGLAP

$$\frac{\partial F_2(x, \alpha^2)}{\partial \ln \alpha^2} = \frac{2\zeta_5}{9\pi} \times G^{\text{DGLAP}}(x, \alpha^2) = \\ = \frac{\alpha^2}{3\pi^3} \sigma_{\text{dipole}}(r_T^2, x)$$

$$r_T^2 = \frac{4}{\alpha^2} \text{ and } \alpha_{ee}^{\text{dipole}}(r_T^2, x; b) \leq 1.$$

At the time there was a ping pong game between the pdf parametrization producers (GRV, MRST, CTEQ) and the promoters of eikonalization (GLM, Porto Alegre, ...)

Reminder: Each new pdf edition has an improved data base + improved pQCD calculations. An updated pdf edition is an excellent post dictum. Is it a good prediction?

