Dipole form of the BFKL kernel in QCD and its supersymmetric generalizations

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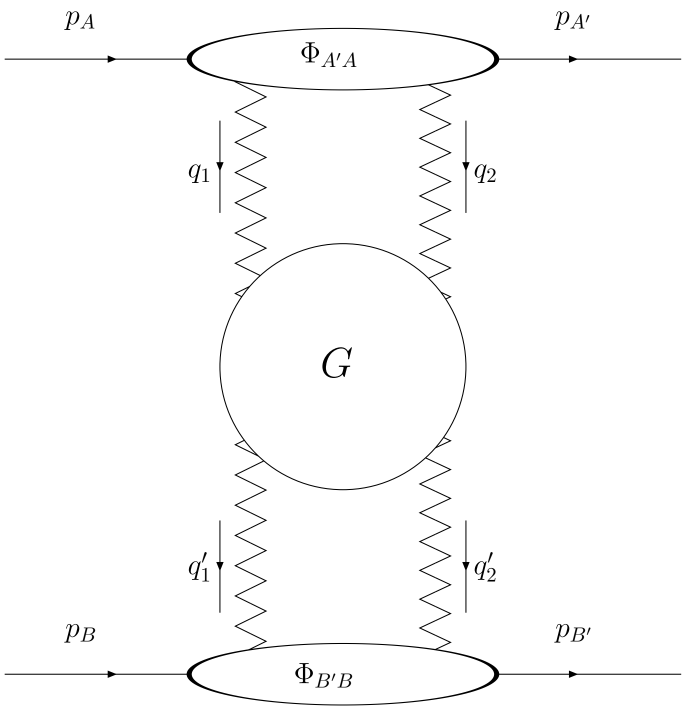
Introduction

The BFKL approach usually is formulated in the momentum space. Scattering amplitudes $\mathcal{A}_{AB}^{A'B'}$ are presented in the form :

$$\Phi_{A'A} \otimes G \otimes \Phi_{B'B}.$$



All dependence from properties of scattering particles is contained in the impact factors $\Phi_{A'A}$ and $\Phi_{B'B}$ describing transitions $A \to A'$ and $B \to B'$.

Green's function G for two interacting Reggeized gluons holds all energy dependence.

The impact factors and the kernel of the BFKL equation for the Green function are defined in the transverse momentum space.

The kernel is known now in the NLO for $t \neq 0$ and all possible t-channel colour states.

Introduction

The most important for phenomenological applications is the colour singlet state (Pomeron channel).

In the following only this channel is considered.

Its distinctive feature is the cancellation of the infrared divergencies in the BFKL kernel.

Further, for scattering of colourless objects the kernel in the LO can be written

L.N. Lipatov, 1986

in the Möbius invariant form.

The Möbius invariance can be made evident by transformation from the transverse momentum to the transverse coordinate representation.

Moreover, in the Möbius invariant form the LO BFKL kernel in the coordinate representation coincides with the kernel of the colour dipole approach

N.N. Nikolaev and B.G. Zakharov, 1994,

A. H. Mueller, 1994.

It makes quite interesting finding of the dipole (Möbius) form of the BFKL kernel in the NLO.

The reasons:

To gain a better insight into conformal properties of the kernel.
Evidently, the conformal invariance is violated by renormalization. One may wonder, however, whether the renormalization is the only source of the violation. If so, one can expect the conformal invariance of the NLO BFKL kernel in supersymmetric extensions of QCD.
To gain a better understanding of the relation between the BFKL and colour dipole approaches. It should help in further development of the theoretical description of small-x processes.
The complexity of the NLO BFKL kernel in the momentum representation. The colour singlet kernel for $t \neq 0$ is found in the NLO in the form of the intricate two-dimensional integrals. Its simplification is extremely desirable.

The dipole representation in the LO

In the operator form

$$\delta(\vec{q}_A - \vec{q}_B) disc_s \mathcal{A}_{AB}^{A'B'} = \frac{i}{4(2\pi)^{D-2}} \langle A'\bar{A}| e^{Y\widehat{\mathcal{K}}} \frac{1}{\hat{\vec{q}}_1^{\,2} \hat{\vec{q}}_2^{\,2}} |\bar{B}'B\rangle$$

$\langle A'\bar{A}|$ and $|\bar{B}'B\rangle$ — t–channel states related to the impact factors,
$Y = \ln(s/s_0)$, $\widehat{\mathcal{K}}$ – BFKL kernel,

$$\widehat{\mathcal{K}} = \widehat{\Omega} + \widehat{\mathcal{K}}_r$$

$\widehat{\Omega} = \omega(\hat{\vec{q}}_1) + \omega(\hat{\vec{q}}_2)$ — the “virtual” part, $\langle \vec{q}_i | \hat{\omega}_i | \vec{q}_i{}' \rangle = \delta(\vec{q}_i - \vec{q}_i{}')\omega(\vec{q}_i)$,
$\omega(\vec{q})$– the gluon Regge trajectory;
$\widehat{\mathcal{K}}_r$ — the “real” part.

$$\langle \vec{q}_1, \vec{q}_2 | \hat{\mathcal{K}}_r | \vec{q}_1{}', \vec{q}_2{}' \rangle = \delta(\vec{q} - \vec{q}\,') \frac{1}{\vec{q}_1^{\,2} \vec{q}_2^{\,2}} \mathcal{K}_r(\vec{q}_1, \vec{q}_1{}'; \vec{q}),\ \vec{q} = \vec{q}_1 + \vec{q}_1{}' = \vec{q}_2 + \vec{q}_2{}'.$$

The dipole representation in the LO

In the leading order at $D = 4 + 2\epsilon$:

$$\omega^{(1)}(\vec{q}) = -\frac{g^2 N_c \Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \frac{2}{\epsilon} (\vec{q})^{\epsilon}, \quad \mathcal{K}_r^B(\vec{q}_1, \vec{q}_2; \vec{q}) = \frac{g^2 N_c}{(2\pi)^{D-1}} \left(\frac{\vec{q}_1^{\,2} \vec{q}_2^{\,\prime 2} + \vec{q}_2^{\,2} \vec{q}_1^{\,\prime 2}}{(\vec{q}_1 - \vec{q}_1^{\,\prime}\,)^2} - \vec{q}^{\,2} \right).$$

The direct Fourier transform of the BFKL kernel gives

$$\langle \vec{r}_1 \vec{r}_2 | \widehat{\mathcal{K}} | \vec{r}_1^{\,\prime} \vec{r}_2^{\,\prime} \rangle = \langle \vec{r}_1 \vec{r}_2 | \widehat{\mathcal{K}}_{dip} | \vec{r}_1^{\,\prime} \vec{r}_2^{\,\prime} \rangle$$

$$-\frac{g^2 N_c \Gamma^2(1+\epsilon)}{8\pi^{3+2\epsilon}} \left[\frac{\delta(\vec{r}_{11'})}{\vec{r}_{12'}^{\,2(1+2\epsilon)}} + \frac{\delta(\vec{r}_{22'})}{\vec{r}_{21'}^{\,2(1+2\epsilon)}} - 2 \frac{\delta(\vec{r}_{1'2'}) \vec{r}_{11'} \vec{r}_{22'}}{\vec{r}_{11'}^{\,2(1+\epsilon)} \vec{r}_{22'}^{\,2(1+\epsilon)}} \right],$$

where $\langle \vec{r}_1 \vec{r}_2 | \widehat{\mathcal{K}} | \vec{r}_1^{\,\prime} \vec{r}_2^{\,\prime} \rangle$ – the kernel of the dipole approach:

$$\langle \vec{r}_1 \vec{r}_2 | \hat{\mathcal{K}}_{dip} | \vec{r}_1^{\,\prime} \vec{r}_2^{\,\prime} \rangle = \frac{\alpha_s N_c}{2\pi^2} \int d^2\rho \frac{\vec{r}_{12}^{\,2}}{\vec{r}_{1\rho}^{\,2} \vec{r}_{2\rho}^{\,2}} \left(\delta(\vec{r}_{11'}) \delta(\vec{r}_{2\rho}) + \delta(\vec{r}_{22'}) \delta(\vec{r}_{1\rho}) - \delta(\vec{r}_{11'}) \delta(\vec{r}_{22'}) \right).$$

The dipole representation in the LO

The BFKL kernel is not equivalent to the dipole one.
Actually the first one is more general than the second.
This is clear, because the BFKL kernel can be applied not only in the case of scattering of colourless objects.

However, when it is applied to the latter case, one can use the “dipole” and “gauge invariance” properties of targets and projectiles
L. N. Lipatov, 1989,
and omit the terms in the kernel proportional to $\delta(\vec{r}_{1'2'})$, as well as change the terms independent either of $\vec{r}_1$ or of $\vec{r}_2$ in such a way that the resulting kernel becomes conserving the “dipole” property, i.e. the property which provides vanishing of cross-sections for scattering of zero-size dipoles.

The coordinate representation of the kernel obtained in such a way is what we call the dipole form of the BFKL kernel.

The dipole representation in the LO

Indeed, for colourless objects the impact factors in the representation

$$\delta(\vec{q}_A - \vec{q}_B) disc_s \mathcal{A}_{AB}^{A'B'} = \frac{i}{4(2\pi)^{D-2}} \langle A'\bar{A}| e^{Y\widehat{\mathcal{K}}} \frac{1}{\hat{\vec{q}}_1^{\,2} \hat{\vec{q}}_2^{\,2}} |\bar{B}'B\rangle$$

are "gauge invariant":

$$\langle A'\bar{A}|\vec{q}, 0\rangle = \langle A'\bar{A}|0, \vec{q}\rangle = 0 \; .$$

Therefore $\langle A'\bar{A}|\Psi\rangle = 0$ if $\langle \vec{r}_1, \vec{r}_2|\Psi\rangle$ does not depend either on $\vec{r}_1$ or on $\vec{r}_2$.
$\langle A'\bar{A}|\widehat{\mathcal{K}}$ is "gauge invariant" as well, because $\langle \vec{q}_1, \vec{q}_2|\widehat{\mathcal{K}}_r|\vec{q}_1^{\;'}, \vec{q}_2^{\;'}\rangle$ vanishes at $\vec{q}_1^{\;'} = 0$ or $\vec{q}_2^{\;'} = 0$.
It means that we can change $|In\rangle \equiv \left(\hat{\vec{q}}_1^{\,2} \hat{\vec{q}}_2^{\,2}\right)^{-1} |\bar{B}'B\rangle$ for $|In_d\rangle$, where $|In_d\rangle$ has the "dipole " property $\langle \vec{r}, \vec{r}|In_d\rangle = 0$.
After this one can omit the terms in the kernel proportional to $\delta(\vec{r}_{1'2'})$, as well as change the terms independent either of $\vec{r}_1$ or of $\vec{r}_2$ in such a way that the resulting kernel becomes conserving the "dipole " property.

The form of the NLO kernel in the dipole representation

In the NLO the dipole form can be written as

$$\langle \vec{r}_1 \vec{r}_2 | \hat{\mathcal{K}}_d^{NLO} | \vec{r}_1{}' \vec{r}_2{}' \rangle = \frac{\alpha_s^2(\mu) N_c^2}{4\pi^3} \Bigg[\delta(\vec{r}_{11'}) \delta(\vec{r}_{22'}) \int d\vec{\rho}\, g^0(\vec{r}_1, \vec{r}_2; \vec{\rho})$$

$$+ \delta(\vec{r}_{11'}) g(\vec{r}_1, \vec{r}_2; \vec{r}_2{}') + \delta(\vec{r}_{22'}) g(\vec{r}_2, \vec{r}_1; \vec{r}_1{}') + \frac{1}{\pi} g(\vec{r}_1, \vec{r}_2; \vec{r}_1{}', \vec{r}_2{}') \Bigg]$$

with the functions g turning into zero when their first two arguments coincide.

The first three terms contain ultraviolet singularities which cancel in their sum, as well as in the LO, with account of the “dipole” property of the “target” impact factors. The coefficient of $\delta(\vec{r}_{11'})\delta(\vec{r}_{22'})$ is written in the integral form in order to make the cancellation evident.

The term $g(\vec{r}_1, \vec{r}_2; \vec{r}_1{}', \vec{r}_2{}')$ is absent in the LO because the LO kernel in the momentum space does not contain terms depending on all three independent momenta simultaneously.

Ambiguity of the NLO BFKL kernel

The discontinuities

$$\delta(\vec{q}_A - \vec{q}_B) disc_s \mathcal{A}_{AB}^{A'B'} = \frac{i}{4(2\pi)^{D-2}} \langle A'\bar{A}| e^{Y\widehat{\mathcal{K}}} \frac{1}{\hat{\vec{q}}_1^{\,2} \hat{\vec{q}}_2^{\,2}} |\bar{B}'B\rangle$$

are invariant under the operator transformation

$$\widehat{\mathcal{K}} \to \widehat{\mathcal{O}}^{-1} \widehat{\mathcal{K}} \widehat{\mathcal{O}},\ \langle A'\bar{A}| \to \langle A'\bar{A}|\widehat{\mathcal{O}},\ \frac{1}{\hat{\vec{q}}_1^{\,2} \hat{\vec{q}}_2^{\,2}} |\bar{B}'B\rangle \to \widehat{\mathcal{O}}^{-1} \frac{1}{\hat{\vec{q}}_1^{\,2} \hat{\vec{q}}_2^{\,2}} |\bar{B}'B\rangle\ .$$

In the LO the kernel can be fixed by the requirement of the Möbius invariance of its dipole form. But even after this transformations with $\hat{\mathcal{O}} = 1 + \hat{O}$, where $\hat{O} \sim g^2$, are still possible. At that

$$\widehat{\mathcal{K}} \to \widehat{\mathcal{K}} - [\widehat{\mathcal{K}}^B \hat{O}].$$

These transformations rearrange NLO corrections to the kernel and impact factors. They can be used for simplification of the dipole form.

The quark contribution

The simplest piece of the NLO BFKL kernel is the “non-Abelian” (leading in N_c) part of the quark contribution. It is known at arbitrary D (V.S. F., R. Fiore, A. Papa, 1999).
Its dipole form is found
V.S. F., R. Fiore, A. Papa, 2006
at arbitrary D as well.
However, the transformation is rather complicated. In the physical space-time dimension $D = 4$ the dipole form can be obtained in a much easier way, starting from the renormalized BFKL kernel at $D = 4$ in a specific form
V.S. F., R. Fiore, A. Papa, 2006.
It occurs that the dipole form of the original “non-Abelian” part contains the term $g(\vec{r}_1, \vec{r}_2; \vec{r}_1{}', \vec{r}_2{}')$ and is not very simple. However, the operator transformation

$$\widehat{\mathcal{K}} \to \widehat{\mathcal{K}} - [\widehat{\mathcal{K}}^B \hat{O}_Q]$$

The quark contribution

with

$$\hat{O}_Q = \frac{\alpha_s(\mu)}{8\pi}\frac{2}{3}n_f \ln\left(\hat{\vec{q}}_1^{\,2}\hat{\vec{q}}_2^{\,2}\right)$$

do away this term and simplify the dipole form considerably. After this transformation we remain with

$$g_Q(\vec{r}_1, \vec{r}_2; \vec{\rho}) = -g_Q^0(\vec{r}_1, \vec{r}_2; \vec{\rho}) = \frac{n_f}{3N_c}\left(\frac{\vec{r}_{12}^{\,2}}{\vec{r}_{1\rho}^{\,2}\vec{r}_{2\rho}^{\,2}}\ln\frac{\vec{r}_Q^{\,2}}{\vec{r}_{12}^{\,2}} + \frac{\vec{r}_{1\rho}^{\,2} - \vec{r}_{2\rho}^{\,2}}{\vec{r}_{1\rho}^{\,2}\vec{r}_{2\rho}^{\,2}}\ln\frac{\vec{r}_{1\rho}^{\,2}}{\vec{r}_{2\rho}^{\,2}}\right) ,$$

where

$$\ln \vec{r}_Q^{\,2} = -\frac{5}{3} + 2\psi(1) - \ln\frac{\mu^2}{4}.$$

The result agrees with J. Balitsky, 2006.

The quark contribution

In the momentum representation the “Abelian” contribution was calculated many years ago in the framework of QED.

H. Cheng, T.T. Wu, Phys. Rev. D10 (1970) 2775

V.N. Gribov, L.N. Lipatov, G.V. Frolov, Yad. Fiz 12 (1970) 994

and is extremely complicated.

It turns out, however, that the dipole form of the Abelian” part of the quark contribution is quite simple. This part contributes only to $g_(\vec{r}_1, \vec{r}_2; \vec{r}_1{}', \vec{r}_2{}')$:

$$g_Q(\vec{r}_1, \vec{r}_2; \vec{r}_1{}', \vec{r}_2{}') = \frac{n_f}{N_c^3} \frac{1}{\vec{r}^{\,4}_{1'2'}} \left[\left(\frac{\vec{r}^{\,2}_{12'}\vec{r}^{\,2}_{1'2} + \vec{r}^{\,2}_{11'}\vec{r}^{\,2}_{22'} - \vec{r}^{\,2}_{12}\vec{r}^{\,2}_{1'2'}}{2\left(\vec{r}^{\,2}_{12'}\vec{r}^{\,2}_{1'2} - \vec{r}^{\,2}_{11'}\vec{r}^{\,2}_{22'}\right)} \ln \frac{\vec{r}^{\,2}_{12'}\vec{r}^{\,2}_{1'2}}{\vec{r}^{\,2}_{11'}\vec{r}^{\,2}_{22'}} - 1 \right) \right].$$

It coincides with the corresponding part of the quark contribution to the dipole kernel I. Balitsky, 2006.

Moreover, it is conformal invariant.

It could be especially interesting for the QED Pomeron.

The gluon contribution

Note that the transformation of the kernel

$$\widehat{\mathcal{K}} \to \widehat{\mathcal{K}} - [\widehat{\mathcal{K}}^B \hat{O}]$$

must be accompanied by corresponding transformation of the impact factors.
The transformation with

$$\hat{O}_Q = \frac{\alpha_s(\mu)}{8\pi}\frac{2}{3}n_f \ln\left(\hat{\vec{q}}_1^{\,2}\hat{\vec{q}}_2^{\,2}\right)$$

do away quark parts of NLO corrections to impact factors, so that all related to quarks corrections turn out to be included in the kernel.
The reason is the simplicity of the quark corrections.
They are related to the charge renormalization only.
For the gluon contribution transformations which include all corrections to the kernel are not known.
It would be a miracle if they existed at all.

The gluon contribution

It seems reasonable to perform the transformation related to the charge renormalization, i.e. with

$$\hat{O}_G = \frac{\alpha_s(\mu)}{8\pi}(-\frac{11}{3}N_c)\ln\left(\hat{\vec{q}}_1^{\,2}\hat{\vec{q}}_2^{\,2}\right).$$

With this tranformation the result

V.S. F, R. Fiore, A.V. Grabovsky, A. Papa, 2007

for the total gluon contributions is the following.

$$g^0_G(\vec{r}_1,\vec{r}_2;\rho) = \frac{3}{2}\frac{\vec{r}_{12}^{\,2}}{\vec{r}_{1\rho}^{\,2}\vec{r}_{2\rho}^{\,2}}\ln\left(\frac{\vec{r}_{1\rho}^{\,2}}{\vec{r}_{12}^{\,2}}\right)\ln\left(\frac{\vec{r}_{2\rho}^{\,2}}{\vec{r}_{12}^{\,2}}\right) - \frac{11}{12}\left[\frac{\vec{r}_{12}^{\,2}}{\vec{r}_{1\rho}^{\,2}\vec{r}_{2\rho}^{\,2}}\ln\left(\frac{\vec{r}_{1\rho}^{\,2}\vec{r}_{2\rho}^{\,2}}{r_G^4}\right)\right.$$

$$\left. + \left(\frac{1}{\vec{r}_{2\rho}^{\,2}} - \frac{1}{\vec{r}_{1\rho}^{\,2}}\right)\ln\left(\frac{\vec{r}_{2\rho}^{\,2}}{\vec{r}_{1\rho}^{\,2}}\right)\right],$$

The gluon contribution

$$g_G(\vec{r}_1,\vec{r}_2;\vec{r}_2^{\,\prime}) \;= \frac{11}{6}\frac{\vec{r}_{12}^{\,2}}{\vec{r}_{22'}^{\,2}\vec{r}_{12'}^{\,2}}\ln\left(\frac{\vec{r}_{12}^{\,2}}{r_G^2}\right)+\frac{11}{6}\left(\frac{1}{\vec{r}_{22'}^{\,2}}-\frac{1}{\vec{r}_{12'}^{\,2}}\right)\ln\left(\frac{\vec{r}_{22'}^{\,2}}{\vec{r}_{12'}^{\,2}}\right)$$

$$+\frac{1}{2\vec{r}_{22'}^{\,2}}\ln\left(\frac{\vec{r}_{12'}^{\,2}}{\vec{r}_{22'}^{\,2}}\right)\ln\left(\frac{\vec{r}_{12}^{\,2}}{\vec{r}_{12'}^{\,2}}\right)-\frac{\vec{r}_{12}^{\,2}}{2\,\vec{r}_{22'}^{\,2}\vec{r}_{12'}^{\,2}}\ln\left(\frac{\vec{r}_{12}^{\,2}}{\vec{r}_{22'}^{\,2}}\right)\ln\left(\frac{\vec{r}_{12}^{\,2}}{\vec{r}_{12'}^{\,2}}\right),$$

$$\ln r_G^2 = 2\psi(1)-\ln\frac{\mu^2}{4}-\frac{3}{11}\left(\frac{67}{9}-2\zeta(2)\right).$$

Both $g_G^0(\vec{r}_1,\vec{r}_2;\vec{\rho})$ and $g_G(\vec{r}_1,\vec{r}_2;\vec{\rho})$ vanish at $\vec{r}_1=\vec{r}_2$. Then, these functions turn into zero for $\vec{\rho}^{\,2}\to\infty$ faster than $(\vec{\rho}^{\,2})^{-1}$ to provide the infrared safety. The ultraviolet singularities of these functions at $\vec{\rho}=\vec{r}_2$ and $\vec{\rho}=\vec{r}_1$ cancel on account of the “dipole” property of the “target” impact factors.

The most complicated is the structure which is absent in the LO:

The gluon contribution

$$g_G(\vec{r}_1,\vec{r}_2;\vec{r}_1{}',\vec{r}_2{}') = \left[\frac{(\vec{r}_{22'}\ \vec{r}_{12})}{\vec{r}_{11'}^{\,2}\vec{r}_{22'}^{\,2}\vec{r}_{1'2'}^{\,2}} - \frac{2\,(\vec{r}_{22'}\ \vec{r}_{11'})}{\vec{r}_{11'}^{\,2}\vec{r}_{22'}^{\,2}\vec{r}_{1'2'}^{\,2}} + \frac{2\,(\vec{r}_{22'}\ \vec{r}_{12'})\,(\vec{r}_{11'}\ \vec{r}_{12'})}{\vec{r}_{11'}^{\,2}\vec{r}_{22'}^{\,2}\vec{r}_{1'2'}^{\,2}\vec{r}_{12'}^{\,2}}\right]\ln\left(\frac{\vec{r}_{12'}^{\,2}}{\vec{r}_{1'2'}^{\,2}}\right)$$

$$+\frac{1}{2\vec{r}_{1'2'}^{\,2}}\left[\frac{(\vec{r}_{11'}\,\vec{r}_{22'})}{\vec{r}_{11'}^{\,2}\,\vec{r}_{22'}^{\,2}} + \frac{(\vec{r}_{21'}\,\vec{r}_{12'})}{\vec{r}_{21'}^{\,2}\,\vec{r}_{12'}^{\,2}} - \frac{2(\vec{r}_{22'}\ \vec{r}_{21'})}{\vec{r}_{22'}^{\,2}\vec{r}_{21'}^{\,2}}\right]\ln\left(\frac{\vec{r}_{11'}^{\,2}\,\vec{r}_{22'}^{\,2}}{\vec{r}_{1'2'}^{\,2}\vec{r}_{12}^{\,2}}\right) + \frac{(\vec{r}_{11'}\,\vec{r}_{22'})}{2\vec{r}_{11'}^{\,2}\,\vec{r}_{22'}^{\,2}\vec{r}_{1'2'}^{\,2}}\ln\left(\frac{\vec{r}_{21'}^{\,2}\,\vec{r}_{12'}^{\,2}}{\vec{r}_{11'}^{\,2}\,\vec{r}_{22'}^{\,2}}\right)$$

$$+\frac{1}{d\,\vec{r}_{1'2'}^{\,2}}\left[\frac{(\vec{r}_{1'2'}\,\vec{r}_{12'})\vec{r}_{12}^{\,2}}{\vec{r}_{11'}^{\,2}} + \frac{2(\vec{r}_{22'}\,\vec{r}_{21'})(\vec{r}_{12}\,\vec{r}_{21'})}{\vec{r}_{21'}^{\,2}} + \frac{(\vec{r}_{22'}\,\vec{r}_{12'})(\vec{r}_{11'}\,\vec{r}_{21'})}{\vec{r}_{11'}^{\,2}\,\vec{r}_{22'}^{\,2}}\vec{r}_{1'2'}^{\,2} - \vec{r}_{1'2'}^{\,2}\right]\ln\left(\frac{\vec{r}_{12'}^{\,2}\,\vec{r}_{21'}^{\,2}}{\vec{r}_{11'}^{\,2}\,\vec{r}_{22'}^{\,2}}\right)$$

$$+\frac{1}{2\vec{r}_{1'2'}^{\,4}}\left(\frac{\vec{r}_{11'}^{\,2}\,\vec{r}_{22'}^{\,2}}{d}\ln\left(\frac{\vec{r}_{12'}^{\,2}\,\vec{r}_{21'}^{\,2}}{\vec{r}_{11'}^{\,2}\vec{r}_{22'}^{\,2}}\right) - 1\right) + \frac{1}{\vec{r}_{11'}^{\,2}}\left(\frac{(\vec{r}_{12}\,\vec{r}_{21'})}{\vec{r}_{1'2'}^{\,2}\vec{r}_{21'}^{\,2}} - \frac{(\vec{r}_{11'}\,\vec{r}_{12})}{\vec{r}_{1'2'}^{\,2}\vec{r}_{22'}^{\,2}} - \frac{(\vec{r}_{11'}\,\vec{r}_{21'})}{\vec{r}_{22'}^{\,2}\vec{r}_{21'}^{\,2}}\right)\ln\left(\frac{\vec{r}_{12'}^{\,2}}{\vec{r}_{11'}^{\,2}}\right)$$

$$-\frac{(\vec{r}_{12}\ \vec{r}_{22'})}{\vec{r}_{1'2'}^{\,2}\vec{r}_{22'}^{\,2}\vec{r}_{12'}^{\,2}}\ln\left(\frac{\vec{r}_{11'}^{\,2}}{\vec{r}_{1'2'}^{\,2}}\right) + (1\leftrightarrow 2),$$

$$d = \vec{r}_{12'}^{\,2}\vec{r}_{21'}^{\,2} - \vec{r}_{11'}^{\,2}\vec{r}_{22'}^{\,2}.$$

This term also vanishes at $\vec{r}_1 = \vec{r}_2$, so that it possesses the “dipole” property. It has ultraviolet singularity only at $\vec{r}_{1'2'} = 0$ and tends to zero at large $\vec{r}_1{}'^{\,2}$ and $\vec{r}_2{}'^{\,2}$ suffi ciently quickly in order to provide the infrared safety.

BFKL in SUSY

SUSY extensions of QCD contain gluons and Maiorana fermions in the adjoint representation of the colour group. The gluon contribution does not change. The fermion one can be obtained by change of the group coefficients:

$$n_f \longrightarrow n_M N_c$$

for the "non-Abelian" part, and

$$n_f \longrightarrow -n_M N_c^3$$

for the "Abelian" part; n_M is the number of flavours of Maiorana quarks. For N–extended SUSY $n_M = N$.

At $N > 1$ besides quarks there are n_S scalar particles; $n_S = 2$ at $N = 2$ and $n_S = 6$ at $N = 4$.

Analogously to the quark case it is convenient to divide the contribution of the scalar particles to the BFKL kernel into two parts, with the same colour group coefficients.

BFKL in SUSY

After the transformation

$$\hat{\mathcal{K}} \to \hat{\mathcal{K}} - [\hat{\mathcal{K}}^{(B)}, \hat{O}] \; ,$$

$$\hat{O} = -\frac{\alpha_s(\mu) N_c}{8\pi} (\frac{11}{3} - \frac{2}{3} n_M - \frac{1}{6} n_S) \ln \left(\hat{\vec{q}}_1^{\,2} \hat{\vec{q}}_2^{\,2} \right) ,$$

the "non-Abelian" part contributes only to $g_S(\vec{r}_1, \vec{r}_2; \vec{\rho})$ and $g_S^0(\vec{r}_1, \vec{r}_2; \vec{\rho})$. At that

$$g_S(\vec{r}_1, \vec{r}_2; \vec{\rho}) = -g_S^0(\vec{r}_1, \vec{r}_2; \vec{\rho}) = \frac{n_S}{12} \left(\frac{\vec{r}^{\,2}_{12}}{\vec{r}^{\,2}_{1\rho} \vec{r}^{\,2}_{2\rho}} \ln \frac{\vec{r}^{\,2}_{S}}{\vec{r}^{\,2}_{12}} + \frac{\vec{r}^{\,2}_{1\rho} - \vec{r}^{\,2}_{2\rho}}{\vec{r}^{\,2}_{1\rho} \vec{r}^{\,2}_{2\rho}} \ln \frac{\vec{r}^{\,2}_{1\rho}}{\vec{r}^{\,2}_{2\rho}} \right) \; ,$$

where $\ln r_S^2 = -\frac{8}{3} + 2\psi(1) - \ln \frac{\mu^2}{4}$.

The "Abelian" part contributes only to $g(\vec{r}_1, \vec{r}_2; \vec{r}_1^{\,\prime}, \vec{r}_2^{\,\prime})$:

$$g_S(\vec{r}_1, \vec{r}_2; \vec{r}_1^{\,\prime}, \vec{r}_2^{\,\prime}) = \frac{n_S}{2} \frac{1}{\vec{r}^{\,4}_{1'2'}} \left[\left(\frac{\vec{r}^{\,2}_{12'} \vec{r}^{\,2}_{1'2}}{\left(\vec{r}^{\,2}_{12'} \vec{r}^{\,2}_{1'2} - \vec{r}^{\,2}_{11'} \vec{r}^{\,2}_{22'} \right)} \ln \frac{\vec{r}^{\,2}_{12'} \vec{r}^{\,2}_{1'2}}{\vec{r}^{\,2}_{11'} \vec{r}^{\,2}_{22'}} - 1 \right) \right] .$$

It is evidently conformal invariant.

BFKL in SUSY

At $N = 4$

$$\frac{11}{3} - \frac{2}{3}n_M - \frac{1}{6}n_S = 0,$$

α_s does not depend on μ and

$$\hat{O} = -\frac{\alpha_s(\mu)N_c}{8\pi}(\frac{11}{3} - \frac{2}{3}n_M - \frac{1}{6}n_S)\ln\left(\hat{\vec{q}}_1^{\,2}\hat{\vec{q}}_2^{\,2}\right) = 0.$$

Unfortunately, cancellation of terms violating conformal invariance is not complete.

However, the hope for conformal invariance still remains.

The reason is the ambiguity of the NLO kernel.

The question requires investigation.

Summary

- The colour singlet BFKL kernel is more general than the dipole one.
- In the case of scattering of colourless objects the BFKL kernel can be written in the dipole form (Möbius representation).
- The dipole form is greatly simplified in comparison with the BFKL kernel in the momentum representation.
- The quark contribution to the dipole form agrees with corresponding contribution to the BK kernel.
- It would be extremely interesting to compare corresponging gluon contributions. Unfortunately, this contribution to the BK kernel is not yet known.
- The "Abelian" part of the quark contribution is conformal invariant. The same is the scalar particle contribution in the SUSY QCD extensions.
- The ambiguity of the NLO kernel reserves the hope for conformal invariance of the dipole form of the colour singlet NLO BFKL kernel at $N = 4$ SUSY.