

# **Collinear improvement of the BFKL kernel in the electroproduction of two light vector mesons**

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# Plan of the talk

## Introduction

- BFKL approach
- **collinear** improvement of the NLA Green's function

## NLA improved amplitude for the process $\gamma^* \gamma^* \rightarrow VV$

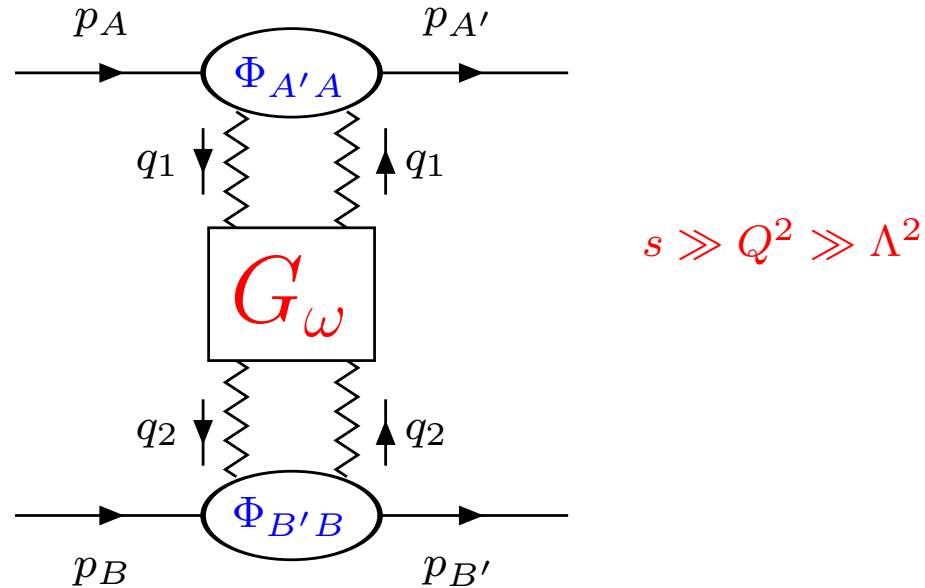
- Construction of the amplitude
- Representations of the amplitude

## Numerical results

- Symmetric kinematics
- Asymmetric kinematics

## Conclusions

# BFKL forward amplitude



$$\mathcal{I}m_s(\mathcal{A}) = \frac{s}{(2\pi)^2} \int \frac{d^2 \vec{q}_1}{\vec{q}_1^2} \int \frac{d^2 \vec{q}_2}{\vec{q}_2^2} \Phi_1(\vec{q}_1, s_0) \Phi_2(-\vec{q}_2, s_0) G(\vec{q}_1, \vec{q}_2, s, s_0)$$

$$G(\vec{q}_1, \vec{q}_2, s, s_0) = \int_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{2\pi i} \left( \frac{s}{s_0} \right)^\omega G_\omega(\vec{q}_1, \vec{q}_2)$$

# Troubles with the NLA corrections to the Green's function

● BFKL equation:

$$\delta^2(\vec{q}_1 - \vec{q}_2) = \omega G_\omega(\vec{q}_1, \vec{q}_2) - \int d^2\vec{q} \mathcal{K}(\vec{q}_1, \vec{q}) G_\omega(\vec{q}, \vec{q}_2)$$

$\mathcal{K}(\vec{q}_1, \vec{q}_2)$  is known in the NLA

$$\int d^2\vec{q}_2 \mathcal{K}_{LLA}(\vec{q}_1, \vec{q}_2) (\vec{q}_2^2)^{\gamma-1} = \bar{\alpha}_s \chi(\gamma) (\vec{q}_1^2)^{\gamma-1}, \quad \gamma = i\nu + 1/2$$

$$\int d^2\vec{q}_2 \mathcal{K}_{NLA}(\vec{q}_1, \vec{q}_2) (\vec{q}_2^2)^{\gamma-1} = \left( \bar{\alpha}_s (\vec{q}_1^2) \chi(\gamma) + \bar{\alpha}_s^2 (\vec{q}_1^2) \chi^{(1)}(\gamma) \right) (\vec{q}_1^2)^{\gamma-1}$$

● NLA corrections to the Green's function turn out to be **very large and negative**

⇒

Expectation: NNLA corrections large and of the opposite sign

⇒

We try to infer the essential subleading dynamics by the collinear approach

# Compatibility with DGLAP

🔴 Solving the BFKL equation at the **leading order** one obtains

$$G(\vec{q}_1, \vec{q}_2, s, s_0) = \frac{1}{\pi q_1 q_2} \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left( \frac{s}{s_0} \right)^\omega \int \frac{d\gamma}{2\pi i} \left( \frac{\vec{q}_1^2}{\vec{q}_2^2} \right)^{\gamma-\frac{1}{2}} \frac{1}{\omega - \omega(\gamma)}$$

$\Rightarrow$  where  $\omega(\gamma) \equiv \bar{\alpha}_s \chi(\gamma) = \bar{\alpha}_s (2\psi(1) - \psi(\gamma) - \psi(1-\gamma))$

$$\begin{aligned} G(\vec{q}_1, \vec{q}_2, s) &= \frac{1}{\pi q_1 q_2} \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left( \frac{s}{q_1 q_2} \right)^\omega \int \frac{d\gamma}{2\pi i} \left( \frac{\vec{q}_1^2}{\vec{q}_2^2} \right)^{\gamma-\frac{1}{2}} \frac{1}{\omega - \omega(\gamma)} \\ &= \frac{1}{\pi q_1 q_2} \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left( \frac{s}{q_1^2} \right)^\omega \int \frac{d\gamma}{2\pi i} \left( \frac{\vec{q}_1^2}{\vec{q}_2^2} \right)^{\gamma-\frac{1}{2}+\frac{\omega}{2}} \frac{1}{\omega - \omega(\gamma)} \\ &= \frac{1}{\pi q_1 q_2} \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left( \frac{s}{q_1^2} \right)^\omega \int \frac{d\gamma}{2\pi i} \left( \frac{\vec{q}_1^2}{\vec{q}_2^2} \right)^{\gamma-\frac{1}{2}} \frac{1}{\omega - \omega(\gamma - \frac{\omega}{2})} \end{aligned}$$

# RG-improvement of the BFKL kernel in the LLA

● In collinear limit  $\gamma \sim 0 \longrightarrow \omega \sim \frac{\bar{\alpha}_s}{\gamma}$

$$\omega \sim \frac{\bar{\alpha}_s}{\gamma - \frac{\omega}{2}} \longrightarrow \omega \sim \frac{\bar{\alpha}_s}{\gamma} + \frac{\bar{\alpha}_s^2}{2\gamma^3} + \sum_{n=2}^{\infty} \frac{(2n)!}{2^n n! (n+1)!} \frac{(\bar{\alpha}_s)^{n+1}}{\gamma^{2n+1}}$$

⇒ The second term cancels with the NLA cubic pole

⇒ The other subleading terms, numerically large, are **not allowed by DGLAP**

⇒ **Solution** [*G.P. Salam, M. Ciafaloni, D. Colferai*]: **redefine** the original kernel

$$\omega^{new}(\gamma) = \omega \left( \gamma + \frac{\omega}{2} \right)$$

$$\begin{aligned} G(\vec{q}_1, \vec{q}_2, s) &= \frac{1}{\pi q_1 q_2} \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left( \frac{s}{q_1 q_2} \right)^{\omega} \int \frac{d\gamma}{2\pi i} \left( \frac{\vec{q}_1^2}{\vec{q}_2^2} \right)^{\gamma - \frac{1}{2}} \frac{1}{\omega - \omega^{new}(\gamma)} \\ &= \frac{1}{\pi q_1 q_2} \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left( \frac{s}{q_1^2} \right)^{\omega} \int \frac{d\gamma}{2\pi i} \left( \frac{\vec{q}_1^2}{\vec{q}_2^2} \right)^{\gamma - \frac{1}{2}} \frac{1}{\omega - \omega(\gamma)} \end{aligned}$$

# RG-improvement of the BFKL kernel in the NLA

- $\omega$ -shift at the next-to-leading order [G.P. Salam, A. Sabio Vera]

$$\omega = \bar{\alpha}_s(1 + a\bar{\alpha}_s) \left( 2\psi(1) - \psi\left(\gamma + \frac{\omega}{2} - b\bar{\alpha}_s\right) - \psi\left(1 - \gamma + \frac{\omega}{2} + b\bar{\alpha}_s\right) \right)$$

- The solution must match the NLA accuracy of the original BFKL calculation
- The explicit solution of this equation gives these new terms

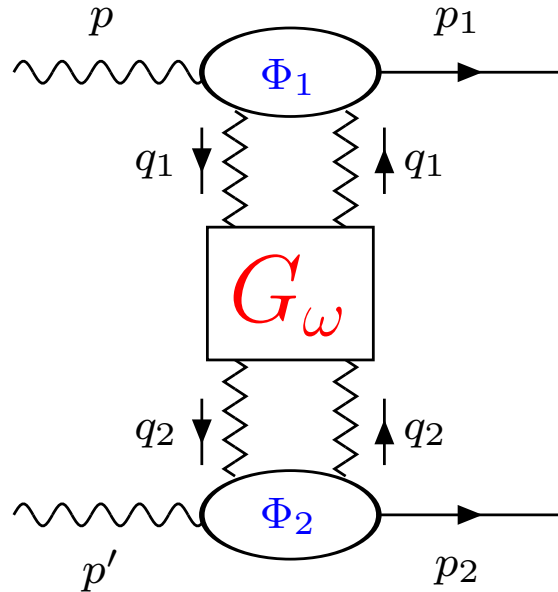
$$\begin{aligned} \chi_{RG}(\gamma) = & 2\Re \left\{ \sum_{m=0}^{\infty} \left[ \left( \sum_{n=0}^{\infty} \frac{(-1)^n (2n)!}{2^n n! (n+1)!} \frac{(\bar{\alpha}_s + a\bar{\alpha}_s^2)^{n+1}}{(\gamma + m - b\bar{\alpha}_s)^{2n+1}} \right) \right. \right. \\ & \left. \left. - \frac{\bar{\alpha}_s}{\gamma + m} - \bar{\alpha}_s^2 \left( \frac{a}{\gamma + m} + \frac{b}{(\gamma + m)^2} - \frac{1}{2(\gamma + m)^3} \right) \right] \right\} \end{aligned}$$

$$\Rightarrow \int d^2 \vec{q}_2 \mathcal{K}(\vec{q}_1, \vec{q}_2) (\vec{q}_2^2)^{\gamma-1} = \left( \bar{\alpha}_s \chi(\gamma) + \bar{\alpha}_s^2 \chi^{(1)}(\gamma) + \chi_{RG}(\gamma) \right) (\vec{q}_1^2)^{\gamma-1}$$

- $\chi_{RG}(\gamma)$  is  $\mathcal{O}(\bar{\alpha}_s^3)$



# Amplitude for the $\gamma^* \gamma^* \rightarrow VV$ forward scattering



$$s \gg Q_{1,2}^2 \gg \Lambda_{QCD}^2$$

$$\mathcal{I}m_s (\mathcal{A}) = \frac{s}{(2\pi)^2} \int \frac{d^2 \vec{q}_1}{\vec{q}_1^2} \Phi_1(\vec{q}_1, s_0) \int \frac{d^2 \vec{q}_2}{\vec{q}_2^2} \Phi_2(-\vec{q}_2, s_0) \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left( \frac{s}{s_0} \right)^\omega G_\omega(\vec{q}_1, \vec{q}_2)$$

# Transverse momentum representation

- We define the transverse momentum representation:

$$\hat{\vec{q}}|\vec{q}_i\rangle = \vec{q}_i|\vec{q}_i\rangle ,$$

$$\langle\vec{q}_1|\vec{q}_2\rangle = \delta^{(2)}(\vec{q}_1 - \vec{q}_2) ,$$

$$\langle A|B\rangle = \langle A|\vec{k}\rangle\langle\vec{k}|B\rangle = \int d^2k A(\vec{k})B(\vec{k})$$

- The kernel operator  $\hat{K}$  is

$$\mathcal{K}(\vec{q}_2, \vec{q}_1) = \langle\vec{q}_2|\hat{K}|\vec{q}_1\rangle$$

- The equation for the Green's function reads

$$\hat{1} = (\omega - \hat{K})\hat{G}_\omega$$

$\Rightarrow$  its solution being

$$\hat{G}_\omega = (\omega - \hat{K})^{-1}$$

# Application of the RG-improved kernel

$$\hat{K} = \bar{\alpha}_s \hat{K}^0 + \bar{\alpha}_s^2 \hat{K}^1 + \hat{K}_{RG}$$

$\Rightarrow$   $\hat{K}_{RG}$  includes the RG-generated terms, which are  $\mathcal{O}(\bar{\alpha}_s^3)$

- The solution for the  $\hat{G}_\omega$  with NLA accuracy is

$$\hat{G}_\omega = (\omega - \bar{\alpha}_s \hat{K}^0)^{-1} + (\omega - \bar{\alpha}_s \hat{K}^0)^{-1} \left( \bar{\alpha}_s^2 \hat{K}^1 + \hat{K}_{RG} \right) (\omega - \bar{\alpha}_s \hat{K}^0)^{-1} + \mathcal{O} \left[ \left( \bar{\alpha}_s^2 \hat{K}^1 \right)^2 \right]$$

- Basis of eigenfunctions of the LLA kernel

$$\hat{K}^0 |\nu\rangle = \chi(\nu) |\nu\rangle, \quad \langle \vec{q} | \nu \rangle = \frac{1}{\pi \sqrt{2}} (\vec{q}^2)^{i\nu - \frac{1}{2}}$$

$$\langle \nu' | \nu \rangle = \int \frac{d^2 \vec{q}}{2\pi^2} (\vec{q}^2)^{i\nu - i\nu' - 1} = \delta(\nu - \nu')$$

# Application of the RG-improved kernel

- The action of the modified BFKL kernel on these functions may be expressed as follows:

$$\begin{aligned}\hat{K}|\nu\rangle &= \bar{\alpha}_s(\mu_R)\chi(\nu)|\nu\rangle + \bar{\alpha}_s^2(\mu_R)\left(\chi^{(1)}(\nu) + \frac{\beta_0}{4N_c}\chi(\nu)\ln(\mu_R^2)\right)|\nu\rangle \\ &+ \bar{\alpha}_s^2(\mu_R)\frac{\beta_0}{4N_c}\chi(\nu)\left(i\frac{\partial}{\partial\nu}\right)|\nu\rangle + \chi_{RG}(\nu)|\nu\rangle\end{aligned}$$

$\Rightarrow$  where we have taken into account the **running of the coupling constant**

$\chi_{RG}(\nu)$  is the solution of the  $\omega$ -shift equation

# Impact factors

- Expansion in  $\alpha_s$  of the impact factors [*D.Yu. Ivanov, M.I. Kotsky, A. Papa*]:

$$\Phi_{1,2}(\vec{q}) = \alpha_s D_{1,2} \left[ C_{1,2}^{(0)}(\vec{q}^2) + \bar{\alpha}_s C_{1,2}^{(1)}(\vec{q}^2) \right], \quad D_{1,2} = -\frac{4\pi e_q f_V}{N_c Q_{1,2}} \sqrt{N_c^2 - 1}$$

- $|\nu\rangle$  representation for the impact factors:

$$c_1(\nu) = \int d^2 \vec{q} C_1^{(0)}(\vec{q}^2) \frac{(\vec{q}^2)^{i\nu - \frac{3}{2}}}{\pi \sqrt{2}}, \quad c_2(\nu) = \int d^2 \vec{q} C_2^{(0)}(\vec{q}^2) \frac{(\vec{q}^2)^{-i\nu - \frac{3}{2}}}{\pi \sqrt{2}},$$
$$c_1^{(1)}(\nu) = \int d^2 \vec{q} C_1^{(1)}(\vec{q}^2) \frac{(\vec{q}^2)^{i\nu - \frac{3}{2}}}{\pi \sqrt{2}}, \quad c_2^{(1)}(\nu) = \int d^2 \vec{q} C_2^{(1)}(\vec{q}^2) \frac{(\vec{q}^2)^{-i\nu - \frac{3}{2}}}{\pi \sqrt{2}},$$

# "Exponentiated" representation of the amplitude

● We convolute the improved Green's function with the impact factors

$$\begin{aligned} \frac{\mathcal{I}m_s(\mathcal{A})}{D_1 D_2} &= \frac{s}{(2\pi)^2} \int_{-\infty}^{+\infty} d\nu \left( \frac{s}{s_0} \right)^{\bar{\alpha}_s(\mu_R)\chi(\nu) + \bar{\alpha}_s^2(\mu_R) \left( \bar{\chi}(\nu) + \frac{\beta_0}{8N_c} \chi(\nu) \left[ -\chi(\nu) + \frac{10}{3} \right] \right)} + \chi_{RG}(\nu) \\ &\times \alpha_s^2(\mu_R) c_1(\nu) c_2(\nu) \left[ 1 + \bar{\alpha}_s(\mu_R) \left( \frac{c_1^{(1)}(\nu)}{c_1(\nu)} + \frac{c_2^{(1)}(\nu)}{c_2(\nu)} \right) \right. \\ &+ \left. \bar{\alpha}_s^2(\mu_R) \ln \left( \frac{s}{s_0} \right) \frac{\beta_0}{8N_c} \chi(\nu) \left( i \frac{d \ln \left( \frac{c_1(\nu)}{c_2(\nu)} \right)}{d\nu} + 2 \ln(\mu_R^2) \right) \right] \end{aligned}$$

where  $c_1(\nu) c_2(\nu) = \frac{1}{Q_1 Q_2} \left( \frac{Q_1^2}{Q_2^2} \right)^{i\nu} \frac{9 \pi^3 (1+4\nu^2) \sinh(\pi\nu)}{32 \nu (1+\nu^2) \cosh^3(\pi\nu)}$

# Series representation of the amplitude

- Another possible representation of the amplitude closer to the original idea of the BFKL approach, is the “series” representation

$$\begin{aligned} \frac{Q_1 Q_2}{D_1 D_2} \frac{\text{Im}_s \mathcal{A}}{s} &= \frac{1}{(2\pi)^2} \alpha_s(\mu_R)^2 \\ &\times \left\{ b_0 + a_0 \ln \left( \frac{s}{s_0} \right) + \sum_{n=1}^{\infty} \bar{\alpha}_s(\mu_R)^n \left[ a_n \ln \left( \frac{s}{s_0} \right)^{n+1} \right. \right. \\ &+ \left. \left. b_n \left( \ln \left( \frac{s}{s_0} \right)^n + d_n(s_0, \mu_R) \ln \left( \frac{s}{s_0} \right)^{n-1} \right) \right] \right\}, \\ \frac{b_n}{Q_1 Q_2} &= \int_{-\infty}^{+\infty} d\nu \, c_1(\nu) c_2(\nu) \frac{\chi^n(\nu)}{n!}, \quad \frac{a_n}{Q_1 Q_2} = \int_{-\infty}^{+\infty} d\nu \, c_1(\nu) c_2(\nu) \chi_{RG}(\nu) \frac{\chi^n(\nu)}{n!} \end{aligned}$$

$\Rightarrow$  we stress that  $\chi_{RG}$  is  $\mathcal{O}(\bar{\alpha}_s^3)$

# Series representation of the amplitude

⇒ NLA coefficients

$$\begin{aligned}
 d_n = & n \ln \left( \frac{s_0}{Q_1 Q_2} \right) + \frac{\beta_0}{4N_c} \left( (n+1) \frac{b_{n-1}}{b_n} \ln \left( \frac{\mu_R^2}{Q_1 Q_2} \right) - \frac{n(n-1)}{2} \right. \\
 & \left. + \frac{Q_1 Q_2}{b_n} \int_{-\infty}^{+\infty} d\nu (n+1) f(\nu) c_1(\nu) c_2(\nu) \frac{\chi^{n-1}(\nu)}{(n-1)!} \right) \\
 & + \frac{Q_1 Q_2}{b_n} \left( \int_{-\infty}^{+\infty} d\nu c_1(\nu) c_2(\nu) \frac{\chi^{n-1}(\nu)}{(n-1)!} \left[ \frac{\bar{c}_1^{(1)}(\nu)}{c_1(\nu)} + \frac{\bar{c}_2^{(1)}(\nu)}{c_2(\nu)} + (n-1) \frac{\bar{\chi}(\nu)}{\chi(\nu)} \right] \right)
 \end{aligned}$$

⇒  $\bar{c}_{1,2}^{(1)}(\nu)$  represent the contribution without the terms depending on  $s_0$  and on  $\beta_0$

$$f(\nu) = \frac{5}{3} + \psi(3 + 2i\nu) + \psi(3 - 2i\nu) - \psi\left(\frac{3}{2} + i\nu\right) - \psi\left(\frac{3}{2} - i\nu\right)$$



# Numerical study

- We have studied the  $s$ -dependence of the  $\mathcal{I}m_s(\mathcal{A})Q_1Q_2/(s D_1 D_2)$  using the **PMS method**

⇒ minimal sensitivity to the change of  $\mu_R$  and  $s_0$

- In the previous determinations [*D.Yu. Ivanov, A. Papa*] the optimal choice for  $\mu_R$  and  $s_0$  turned out to be **very far** from the kinematical scales of the process!

⇒ Does the RG-improvement lead to more "natural" values for  $\mu_R$  and  $s_0$  ?

- If true, this would demonstrate that the RG-generated terms **catch the essential subleading dynamics**

⇒ the perturbative series of the amplitude should be more stable

# Numerical results: symmetric kinematics

● We consider here  $Q_1^2 = Q_2^2 \equiv Q^2 = 24 \text{ GeV}^2$ ,  $n_f = 5$

● We start with the “**exponentiated**” representation, putting

$$\ln \left( \frac{s}{s_0} \right) = Y - Y_0 \quad Y = \ln(s/Q^2) \quad Y_0 = \ln(s_0/Q^2)$$

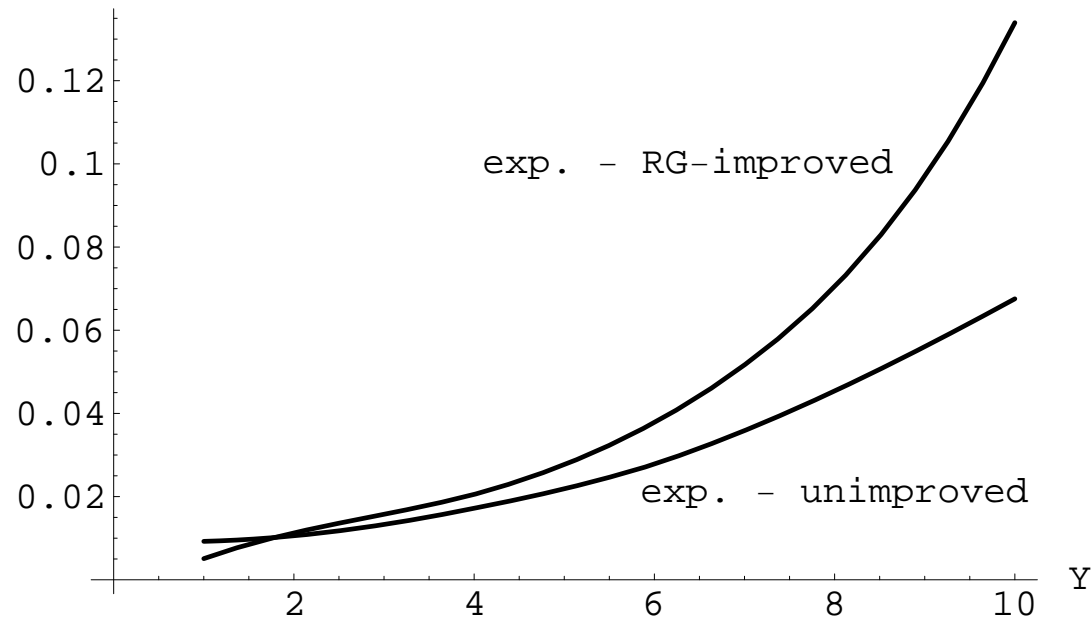
● In practice, for each fixed value of  $Y$  we have determined the optimal choice of  $\mu_R$  and  $Y_0$  for which the amplitude is the least sensitive to their variation

$\Rightarrow$  typically  $Y_0 \simeq 2$  and  $\mu_R \simeq 3Q$

● In the previous determinations  $Y_0 \simeq 2$  and  $\mu_R \simeq 10Q$

● We see that there is a remarkable move toward “naturalness” for  $\mu_R$ !

# Numerical results: symmetric kinematics



● Good agreement at the lower energies but not for large values of  $Y$

●  $\bar{\alpha}_s(\mu_R)Y \sim 1 \implies Y \sim 6$

around  $Y \sim 6$  the discrepancy is not so pronounced

# Numerical results: symmetric kinematics

- We consider the “series” representation always with  $Q^2=24 \text{ GeV}^2$ ,  $n_f = 5$

PMS  $\Rightarrow$  typically  $Y_0 \simeq 3$  and  $\mu_R \simeq 3Q$

$$b_0 = 17.0664 \quad b_1 = 34.5920 \quad b_2 = 40.7609 \quad b_3 = 33.0618 \quad b_4 = 20.7467$$

$$d_1 = 0.674275 \quad d_2 = -1.73171 \quad d_3 = -7.46518 \quad d_4 = -15.927$$

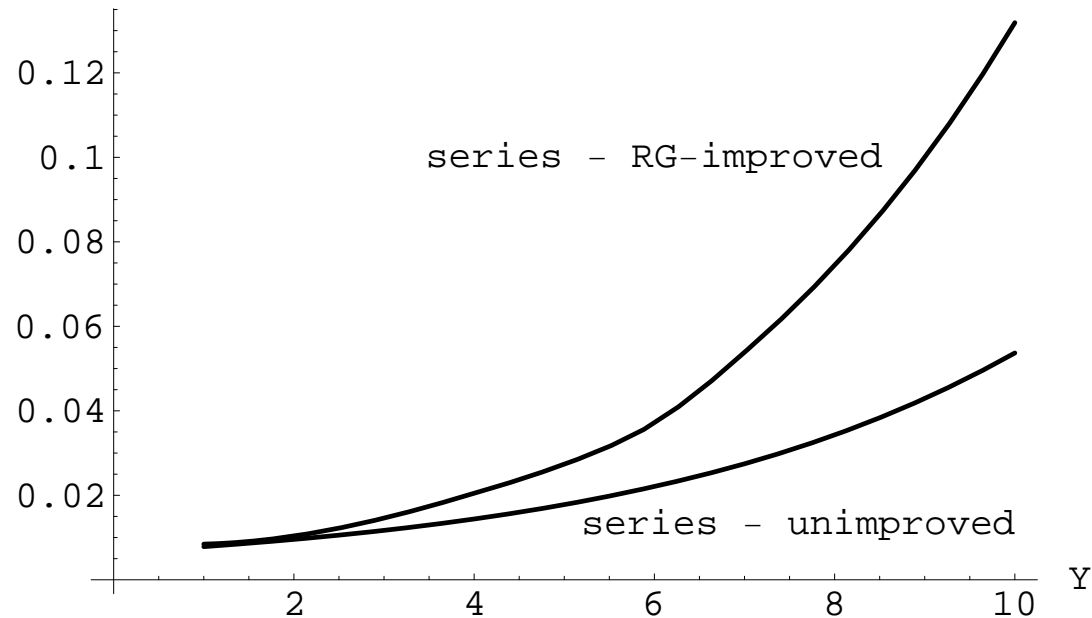
$$a_1 = 5.52728 \quad a_2 = 7.30295 \quad a_3 = 6.42149 \quad a_4 = 4.24011$$

$\Rightarrow$   $a_n$  coefficients cure the bad behavior of the BFKL series

- $a_n$  appear in the amplitude with two more powers of the energy logarithm!

$\Rightarrow$  their effect is not limited to low energies

# Numerical results: symmetric kinematics



- The curves for “exponentiated” and the “series” representations with collinear improvement **fall almost on top of each other**
- Without the collinear improvement there was an **evident discrepancy**



This indicate a better stability, induced by the collinear improvement

# Numerical results: asymmetric kinematics

- When the virtualities of the photons are **strongly ordered** we enter the “**DGLAP**” regime

⇒ collinear effects should come heavily into the game

- Previous attempts to numerically determine the amplitude using unimproved kernels were **unsuccessful** due to severe **instabilities**
- We use the “exponentiated” representation, and we define

$$Y = \ln(s/Q_1 Q_2) \quad Y_0 = \ln(s_0/Q_1 Q_2)$$

# Numerical results: asymmetric kinematics

● We choose  $Q_1=2$  GeV,  $Q_2=12$  GeV  $\implies Q_1 Q_2=24$  GeV<sup>2</sup>

● The amplitude is still quite stable under the variation of the energy parameters

PMS  $\implies$  typically  $Y_0 \simeq 2$  and  $\mu_R \simeq 4\sqrt{Q_1 Q_2}$

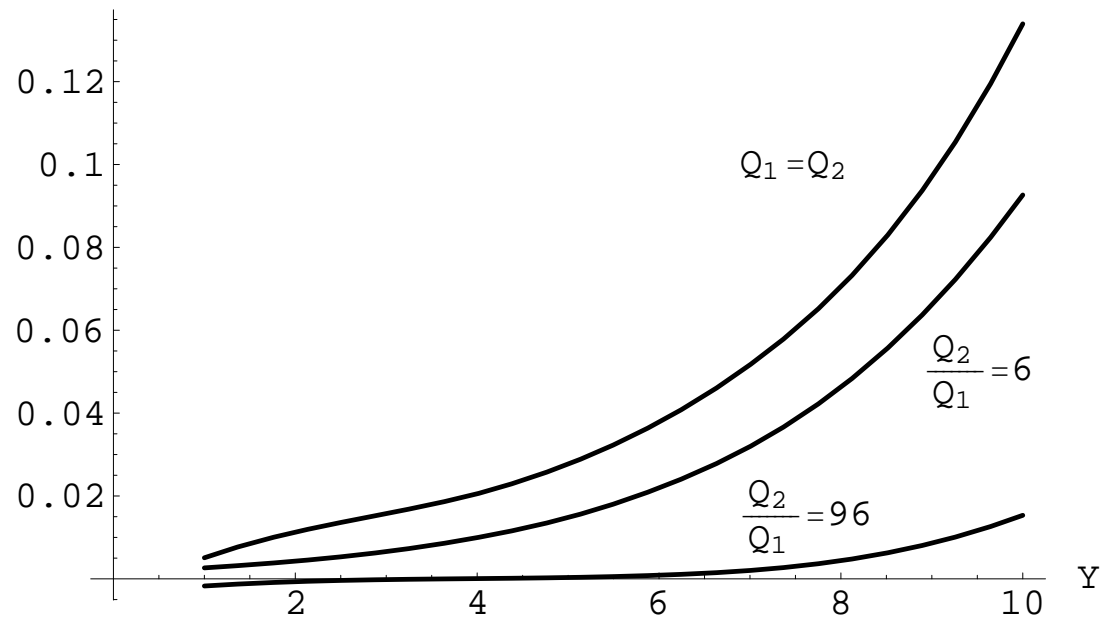
● We choose  $Q_1=0.5$  GeV,  $Q_2=48$  GeV  $\implies Q_1 Q_2=24$  GeV<sup>2</sup>

● The amplitude is still quite stable but the optimal values depend strongly on  $Y$

PMS  $\implies$  for  $Y = 6$  we find  $Y_0 \simeq 7$  and  $\mu_R \simeq 3\sqrt{Q_1 Q_2}$

● If we ridefine  $Y'_0 = \ln(s_0/Q_2^2)$   $\implies Y'_0 \simeq 2.5$

## Numerical results: asymmetric kinematics



- The amplitude becomes **smaller and smaller** when  $Q_2/Q_1$  increases  
 $\Rightarrow$  due to the presence of the factor  $\cos(\nu \log(Q_2^2/Q_1^2))$  in the integration over  $\nu$



# Conclusions

- We have applied a RG-improved kernel to determine the amplitude for the forward transition  $\gamma^* \gamma^* \rightarrow VV$
- We considered both equal and strongly ordered photons' virtualities
- The PMS method has led to results stable in the considered energy interval which allow to predict the energy behavior of the forward amplitude
- The optimal choice of  $s_0$  and  $\mu_R$  are much closer to the kinematical scales of the problem than in previous determinations based on unimproved kernels
  - $\Rightarrow$  the extra-terms coming from collinear improvement, which are subleading to the NLA, catch an important fraction of the NNLA order dynamics
  - $\Rightarrow$  the use of the improved kernel has allowed to obtain the energy behavior of the forward amplitude in the case of strongly ordered photons' virtualities