NNLO corrections to jet production at hadron colliders

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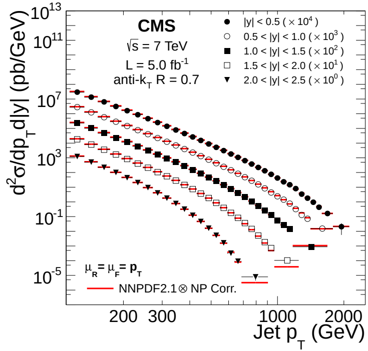
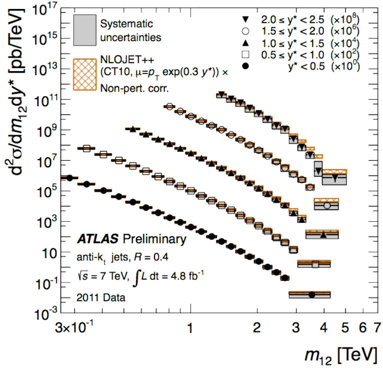
Outline

- Motivation for jet cross sections at NNLO
- Features of the NNLO calculation
- Antenna subtraction method
    - the double-virtual contribution
- Numerical results
- Conclusions and future work

INCLUSIVE JET AND DIJET CROSS SECTIONS

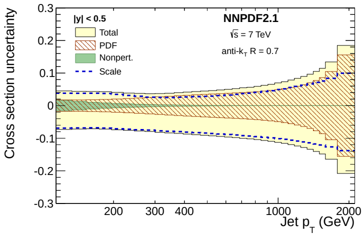


[CMS, arXiv:1212.6660]

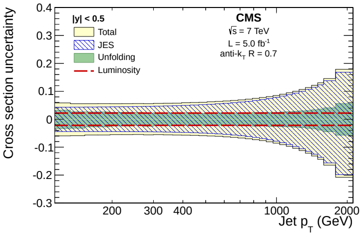


[ATLAS-CONF-2012-021]

- measurements of single jet inclusive jet and dijet observables at the LHC as a function of the jet p_T and rapidity and dijet invariant mass
- probes the basic QCD parton-parton scattering



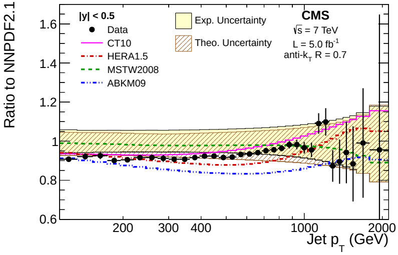
relative theoretical uncertainties for the inclusive jet production (NLO theory input) [CMS, arXiv:1212.6660]



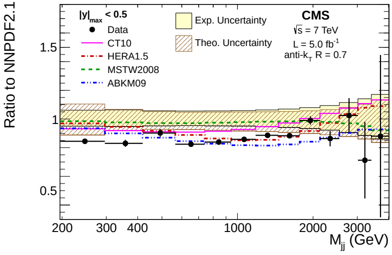
relative experimental uncertainties for the inclusive jet production [CMS, arXiv:1212.6660]

- residual uncertainty due to scale choice at NNLO expected at $\approx$ few percent level
- jet energy scale uncertainty has been determined to less than 5% for central jets $\rightarrow$ expect steady improvement with higher statistics
- theoretical prediction with the same precision as the experimental data $\rightarrow$ need for pQCD predictions at NNLO accuracy

INCLUSIVE JET AND DIJET CROSS SECTIONS



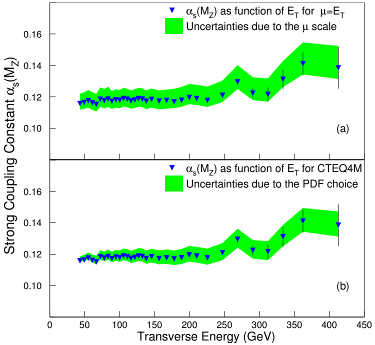
[CMS, arXiv:1212.6660]



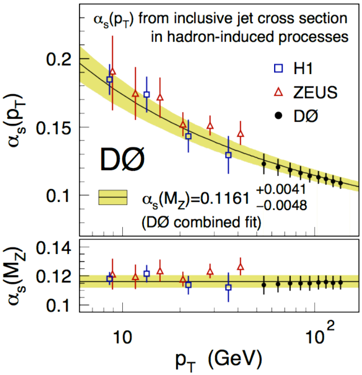
[CMS, arXiv:1212.6660]

- data can be used to constrain parton distribution functions
- size of NNLO correction important for precise determination of PDF's
- inclusion of jet data in NNLO parton distribution fits requires NNLO corrections to jet cross sections

MEASUREMENTS OF α_s AT HADRON COLLIDERS



[CDF, PRL **88, 042001 (2002)**]



[D0, arXiv: 0911.2710]

- CDF run I data gives

$$\alpha_s(M_Z) = 0.1178 \pm 0.0001(\text{stat})^{+0.0081}_{-0.0095}(\text{sys})\ ^{+0.0071}_{-0.0047}(\text{scale}) \pm 0.0059(\text{pdf})$$

- α_s determination from hadronic jet observables limited by the unknown higher order corrections

NNLO INGREDIENTS

- QCD jet cross section perturbative expansion at hadron colliders

$$d\sigma = \sum_{i,j} \int \left[d\hat{\sigma}_{ij}^{LO} + \left(\frac{\alpha_s}{2\pi}\right) d\hat{\sigma}_{ij}^{NLO} + \left(\frac{\alpha_s}{2\pi}\right)^2 d\hat{\sigma}_{ij}^{NNLO} + \mathcal{O}(\alpha_s^3) \right] f_i(x_1) f_j(x_2) dx_1 dx_2$$

- NNLO m-jet corrections contains three contributions:

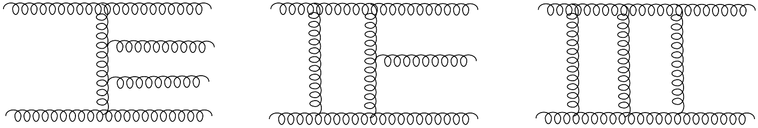
$$\begin{aligned} d\hat{\sigma}_{NNLO} \sim & \int \left[\langle \mathcal{M}^{(0)} | \mathcal{M}^{(0)} \rangle \right]_{m+4} d\Phi_{m+2} \, J_m^{(m+2)} \\ & + \int \left[\langle \mathcal{M}^{(0)} | \mathcal{M}^{(1)} \rangle + \langle \mathcal{M}^{(1)} | \mathcal{M}^{(0)} \rangle \right]_{m+3} d\Phi_{m+1} \, J_m^{(m+1)} \\ & + \int \left[\langle \mathcal{M}^{(1)} | \mathcal{M}^{(1)} \rangle + \langle \mathcal{M}^{(0)} | \mathcal{M}^{(2)} \rangle + \langle \mathcal{M}^{(2)} | \mathcal{M}^{(0)} \rangle \right]_{m+2} d\Phi_{m} \, J_m^{(m)} \end{aligned}$$

- $[\langle \mathcal{M}^{(i)} | \mathcal{M}^{(j)} \rangle]_M$ is the interference of M-particle i-loop and j-loop amplitudes

- NNLO PDF's [MSTW, ABKM, NNPDF]

- NNLO DGLAP evolution [Moch, Vermaseren, Vogt '04]

$pp \to 2j$ at NNLO: gluonic contributions



$A_6^{(0)}(gg \to gggg)$ $\quad$ $A_5^{(1)}(gg \to ggg)$ $\quad$ $A_4^{(2)}(gg \to gg)$

- tree level $2 \to 4$ matrix elements [Berends, Giele '87], [Mangano, Parke, Xu '87], [Britto, Cachazo, Feng '06]
- 1-loop $2 \to 3$ matrix elements [Bern, Dixon, Kosower '93]
- 2-loop $2 \to 2$ matrix elements [Anastasiou, Glover, Oleari, Tejeda-Yeomans '01], [Bern, De Freitas, Dixon '02]

$$\mathrm{d}\hat{\sigma}_{NNLO} = \int_{\mathrm{d}\Phi_4} \mathrm{d}\hat{\sigma}_{NNLO}^{RR} + \int_{\mathrm{d}\Phi_3} \mathrm{d}\hat{\sigma}_{NNLO}^{RV} + \int_{\mathrm{d}\Phi_2} \mathrm{d}\hat{\sigma}_{NNLO}^{VV}$$

$$\begin{aligned}
\mathrm{d}\hat{\sigma}_{NNLO}^{RR} &= \mathcal{N}\ \mathrm{d}\Phi_4(p_3,p_4,p_5,p_6;p_1,p_2)|\mathcal{M}^{(0)}_{gg\to gggg}|^2\ J_2^{(4)}(p_3,p_4,p_5,p_6) \\
\mathrm{d}\hat{\sigma}_{NNLO}^{RV} &= \mathcal{N}\ \mathrm{d}\Phi_3(p_3,p_4,p_5;p_1,p_2) \\
&\quad \left(\mathcal{M}^{(0)\star}_{gg\to ggg}\mathcal{M}^{(1)}_{gg\to ggg} + \mathcal{M}^{(0)}_{gg\to ggg}\mathcal{M}^{(1)\star}_{gg\to ggg}\right)\ J_2^{(3)}(p_3,p_4,p_5) \\
\mathrm{d}\hat{\sigma}_{NNLO}^{VV} &= \mathcal{N}\ \mathrm{d}\Phi_2(p_3,p_4;p_1,p_2) \\
&\quad \left(\mathcal{M}^{(2)\star}_{gg\to gg}\mathcal{M}^{(0)}_{gg\to gg} + \mathcal{M}^{(0)}_{gg\to gg}\mathcal{M}^{(2)\star}_{gg\to gg} + |\mathcal{M}^{(1)}_{gg\to gg}|^2\right)\ J_2^{(2)}(p_3,p_4)
\end{aligned}$$

- explicit infrared poles from loop integrations
- implicit poles in phase space regions for single and double unresolved gluon emission
- procedure to extract the infrared singularities and assemble all the parts

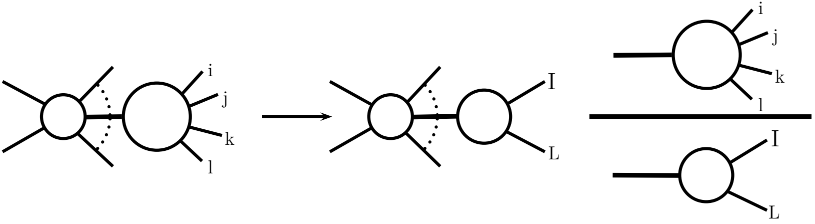
NNLO ANTENNA SUBTRACTION

$$
\begin{aligned}
\mathrm{d}\hat{\sigma}_{NNLO} &= \int_{\mathrm{d}\Phi_{m+2}} \left(\mathrm{d}\hat{\sigma}^{RR}_{NNLO} - \mathrm{d}\hat{\sigma}^{S}_{NNLO}\right) \\
&+ \int_{\mathrm{d}\Phi_{m+1}} \left(\mathrm{d}\hat{\sigma}^{RV}_{NNLO} - \mathrm{d}\hat{\sigma}^{T}_{NNLO}\right) \\
&+ \int_{\mathrm{d}\Phi_{m}} \left(\mathrm{d}\hat{\sigma}^{VV}_{NNLO} - \mathrm{d}\hat{\sigma}^{U}_{NNLO}\right)
\end{aligned}
$$

- $\mathrm{d}\hat{\sigma}^{S}_{NNLO}$: real radiation subtraction term for $\mathrm{d}\hat{\sigma}^{RR}_{NNLO}$
- $\mathrm{d}\hat{\sigma}^{T}_{NNLO}$: one-loop virtual subtraction term for $\mathrm{d}\hat{\sigma}^{RV}_{NNLO}$
- $\mathrm{d}\hat{\sigma}^{U}_{NNLO}$: two-loop virtual subtraction term for $\mathrm{d}\hat{\sigma}^{VV}_{NNLO}$
- contribution in each of the square brackets is finite, well behaved in the infrared singular regions and can be evaluated numerically
- subtraction terms constructed using the antenna subtraction method at NNLO for hadron colliders $\rightarrow$ presence of initial state partons to take into account

NNLO ANTENNA SUBTRACTION

- universal factorisation of both colour ordered matrix elements and the $(m+2)$- particle phase space → colour connected unresolved particles



$$|M_{m+4}(\dots,i,j,k,l,\dots)|^2\; J(\{p_{m+4}\}) \;\longrightarrow\; |M_{m+2}(\dots,I,L,\dots)|^2\; J(\{p_{m+2}\})\cdot X_4^0(i,j,k,l)$$

- phase-space factorisation

$$d\Phi_{m+2}(p_a,\dots,p_i,p_j,p_k,p_l,\dots,p_{m+2}) = d\Phi_m(p_a,\dots,p_I,p_L,\dots,p_{m+2})\, d\Phi_{X_{ijkl}}(p_i,p_j,p_k,p_l)$$

- integrated antennae is the inclusive integral

$$\mathcal{X}^0_{ijkl}(s_{ijkl}) = \frac{1}{C(\epsilon)^2}\int d\Phi_{X_{ijkl}}(p_i,p_j,p_k,p_l)X_4^0(i,j,k,l)$$

INTEGRATED ANTENNAE

- antennae integrals are performed once and for all to become universal building blocks for subtraction of IR singularities at NNLO
- massless antennae ($m = 0$)

	NLO	NNLO
final-final	✓[1]	✓[1]
initial-final	✓[2]	✓[3]
initial-initial	✓[2]	✓[4,5,6]

[1] A. Gehrmann-De Ridder, T. Gehrmann and E. W. N. Glover, *JHEP* **09** (2005) 056 [hep-ph/0505111];
[2] A. Daleo, T. Gehrmann and D. Maître, *JHEP* **04** (2007) 016 [hep-ph/0612257];

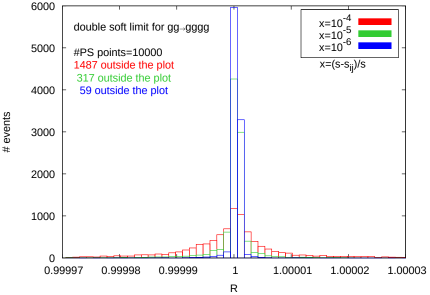
[3] A. Daleo, A. Gehrmann-De Ridder, T. Gehrmann and G. Luisoni, *JHEP* **01** (2010) 118 [0912.0374];

[4] R. Boughezal, A. Gehrmann-De Ridder and M. Ritzmann, *JHEP* **02** (2011) 098 [1011.6631];

[5] T. Gehrmann, P.F. Monni, *JHEP* **12** (2011) 049 [1107.4037];

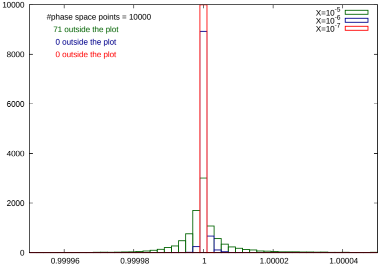
[6] A. Gehrmann-De Ridder, T. Gehrmann and M. Ritzmann, *JHEP* **10** (2012) 047 [1207.5779];

NNLO CORRECTIONS TO $pp \to 2j$



Double-real contribution

- $d\sigma^{RR}_{NNLO} - d\sigma^{S}_{NNLO}$ (gluons only)
- numerical convergence between double-real matrix element $d\sigma^{RR}_{NNLO}$ and antenna subtraction term $d\sigma^{S}_{NNLO}$ tested in all soft and collinear phase space regions [N.Glover, JP]



Real-virtual contribution

- $d\sigma^{RV}_{NNLO} - d\sigma^{T}_{NNLO}$ (gluons only)
- local (pointwise) analytic cancellation of explicit ϵ-poles
- numerical convergence tested in all soft or collinear phase space regions [A.Gehrmann-De Ridder, N.Glover, JP]

DOUBLE-VIRTUAL CONTRIBUTION

$$\int_{\mathrm{d}\Phi_m} \left(\mathrm{d}\hat{\sigma}^{VV}_{NNLO} - \mathrm{d}\hat{\sigma}^{U}_{NNLO} \right)$$

- renormalized $\mathrm{d}\hat{\sigma}^{VV}_{NNLO}$ contains explicit infrared ϵ-poles
- $\mathrm{d}\hat{\sigma}^{U}_{NNLO}$ is made up of integrated subtraction terms from the double-real radiation and real-virtual radiation
- initial-state collinear singularities absorbed by the mass factorization counterterm $\mathrm{d}\hat{\sigma}^{MF,2}_{NNLO}$

$$\mathrm{d}\sigma^{U}_{NNLO} = -\int_2 \mathrm{d}\hat{\sigma}^{S}_{NNLO} - \int_1 \mathrm{d}\hat{\sigma}^{VS}_{NNLO} - \mathrm{d}\hat{\sigma}^{MF,2}_{NNLO}$$

- to show explicit pole cancellation at NNLO recast integrated subtraction terms and mass factorization contribution in a form of a convolution integral evaluated analytically

	gg	qg	qq
$\Gamma^1 \otimes \Gamma^1$	✓[1]	✓	✓
$\mathcal{X}^0_3 \otimes \Gamma^1$	✓[1]	✓	✓
$\mathcal{X}^0_3 \otimes \mathcal{X}^0_3$	✓[1]	✓	✓

[1] [A. Gehrmann-De Ridder, T. Gehrmann, N. Glover, JP]

DOUBLE-VIRTUAL CONTRIBUTION

- new structures arise made from the integrated antennae building blocks

$$\mathbb{X}_3^0(\bar{1}_g, \bar{2}_g, i_g, j_g; z_1, z_2) = \mathcal{F}_3^0(s_{\bar{1}\bar{2}}, z_1, z_2) + \frac{1}{2}\mathcal{F}_3^0(s_{\bar{2}i}, z_1, z_2) + \frac{1}{3}\mathcal{F}_3^0(s_{ij}, z_1, z_2) + \frac{1}{2}\mathcal{F}_3^0(s_{j\bar{1}}, z_1, z_2)$$

- integrated antennae string with the mass factorization contribution is in direct connection with the I_1 operator of Catani at NLO

$$-2\boldsymbol{I}^{(1)}(\epsilon; \bar{1}_g, \bar{2}_g, i_g, j_g; z_1, z_2) = \mathbb{X}_3^0(\bar{1}_g, \bar{2}_g, i_g, j_g; z_1, z_2) - \Gamma^{(1)}_{gg;gg}(z_1, z_2)$$

- similarly at NNLO the integrated antennae convolution integrals together with the mass factorization contribution yield in the double-virtual channel

$$-2\boldsymbol{I}^{(1)}(\epsilon; \bar{1}_g, \bar{2}_g, i_g, j_g; z_1, z_2)^2 = \left(\mathbb{X}_3^0 - \Gamma^{(1)}_{gg;gg}\right) \otimes \left(\mathbb{X}_3^0 - \Gamma^{(1)}_{gg;gg}\right)(\bar{1}_g, \bar{2}_g, i_g, j_g; z_1, z_2)$$

DOUBLE-VIRTUAL CONTRIBUTION

- double virtual antennae subtraction term $\mathrm{d}\sigma^{U}_{NNLO}$ written compactly rederives the predicted Catani pole structure of the two-loop contribution in the antennae language
- local (pointwise) analytic cancellation of all infrared explicit ϵ-poles when combined with two-loop matrix elements

$$\mathcal{P}oles\left(\mathrm{d}\hat{\sigma}^{VV}_{NNLO} - \mathrm{d}\hat{\sigma}^{U}_{NNLO}\right) = 0 \qquad \text{(gluons only)}$$

NUMERICAL SETUP

[A. Gehrmann-De Ridder, T. Gehrmann, N. Glover, JP] (in preparation)

- jets identified with the anti-k_T jet algorithm with resolution parameter $R = 0.7$
- jets accepted at rapidities $|y| < 4.4$
- leading jet with transverse momentum $p_t > 80$ GeV
- subsequent jets required to have at least $p_t > 60$ GeV
- MSTW2008nnlo PDF
- dynamical factorization and renormalization scales equal to the leading jet p_T ($\mu_R = \mu_F = \mu = p_T$)

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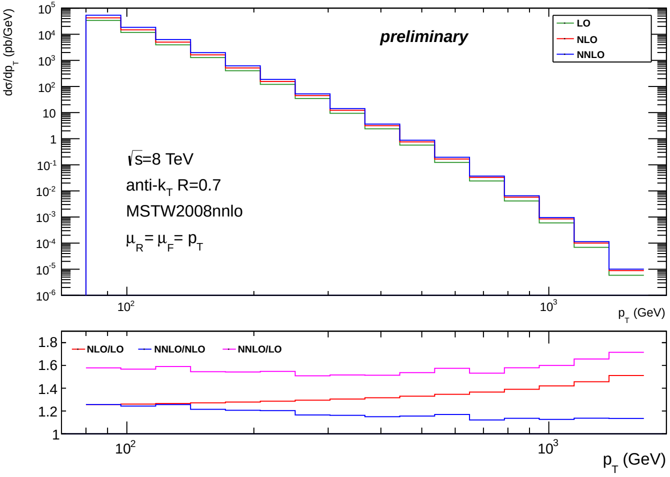
Integrated cross section results (gluons only channel - preliminary)

	$\sigma^{8TeV-LO}_{incl.jet}(pb)$	$\sigma^{8TeV-NLO}_{incl.jet}(pb)$	$\sigma^{8TeV-NNLO}_{incl.jet}(pb)$
$\mu = 0.5p_t$	$(12.586 \pm 0.001) \times 10^5$	$(11.299 \pm 0.001) \times 10^5$	$(15.33 \pm 0.03) \times 10^5$
$\mu = p_t$	$(9.6495 \pm 0.001) \times 10^5$	$(12.152 \pm 0.001) \times 10^5$	$(15.20 \pm 0.02) \times 10^5$
$\mu = 2.0p_t$	$(7.5316 \pm 0.001) \times 10^5$	$(11.824 \pm 0.001) \times 10^5$	$(15.21 \pm 0.01) \times 10^5$

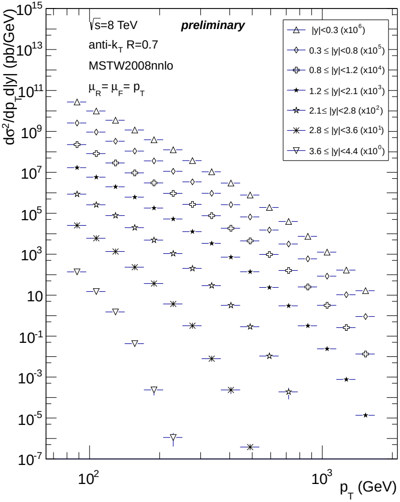
- NNLO result increased by about 25% with respect to the NLO cross section
- flat scale dependence at NNLO

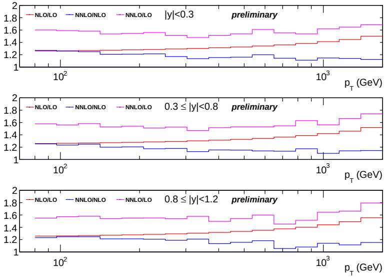
- NNLO QCD corrections to inclusive jet p_T distribution (gluons only)
  [A. Gehrmann-De Ridder, T. Gehrmann, N. Glover, JP] (in preparation)



- NNLO effect stabilizes the NLO k-factor growth with p_T

- double differential inclusive jet p_T distribution at NNLO
  [A. Gehrmann-De Ridder, T. Gehrmann, N. Glover, JP] (in preparation)

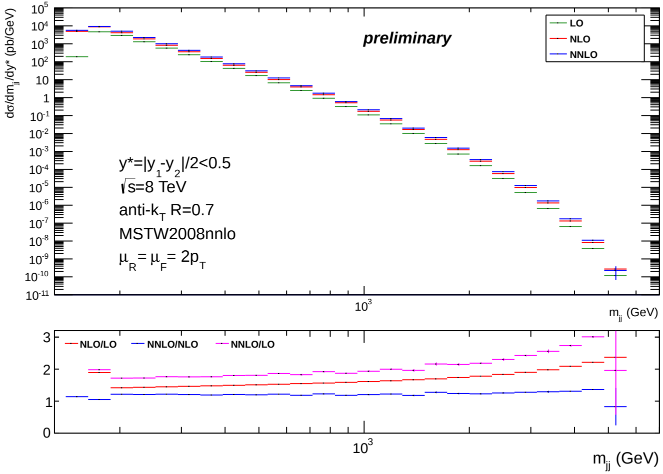




double differential k-factors

- NNLO result varies between 25% to 12% with respect to the NLO cross section
- similar behaviour between the rapidity slices

- NNLO QCD corrections to dijet mass distribution (gluons only)
  [A. Gehrmann-De Ridder, T. Gehrmann, N. Glover, JP] (in preparation)



Conclusions

- antenna subtraction method generalised for the calculation of NNLO QCD corrections for exclusive collider observables with partons in the initial-state
- explicit ϵ-poles in the matrix elements are analytically cancelled by the ϵ-poles in the subtraction terms
- non-trivial check of analytic cancellation of infrared singularities between double-real, real-virtual and double-virtual corrections
- proof-of principle implementation of the $gg \to gg$ contribution to $pp \to 2j$ at NNLO in the new NNLOJET parton-level generator

Future work:

- go beyond gluons only leading colour approximation
- include remaining channels
  - 4g2q processes
  - 2g4q processes
  - 6q processes