

# SCET predictions for the Higgs transverse-momentum distribution and jet-veto cross section


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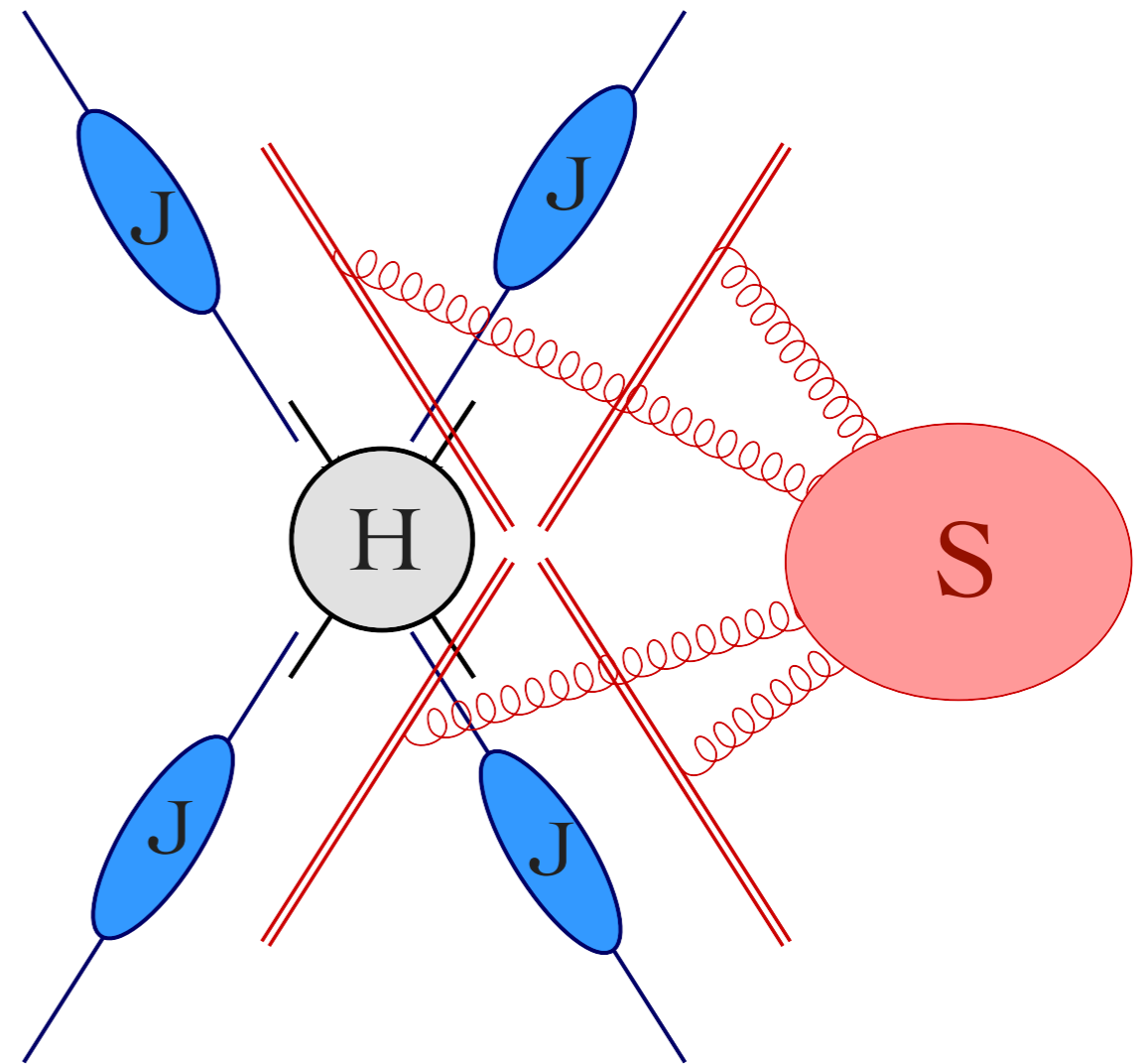
 **PRISMA Cluster of Excellence**  
Precision Physics, Fundamental Interactions and Structure of Matter

 **ERC Advanced Grant (EFT4LHC)**  
An Effective Field Theory Assault on the  
Zeptometer Scale: Exploring the Origins of  
Flavor and Electroweak Symmetry Breaking



# Introduction

- Hadron-collider processes are prime examples of **multi-scale problems** involving several hierarchical scales
- Due to **light-like nature** of these processes, scale separation cannot be performed using a conventional OPE
- Instead, any field-theory description of these processes must be intrinsically **non-local**



**QCD factorization theorems:**

$$d\sigma \sim H(\{s_{ij}\}, \mu) \prod_i J_i(M_i^2, \mu) \otimes S(\{\Lambda_{ij}^2\}, \mu)$$

operators containing **Wilson lines**

# Scale separation in Sudakov problems

Separation of **short-distance** and **long-distance contributions** is subtle:

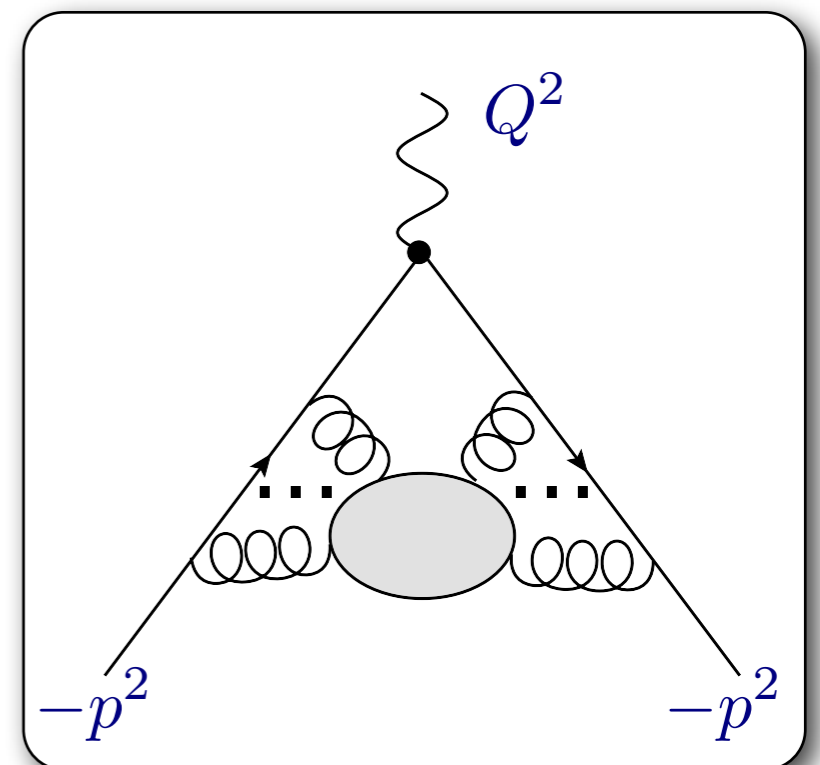
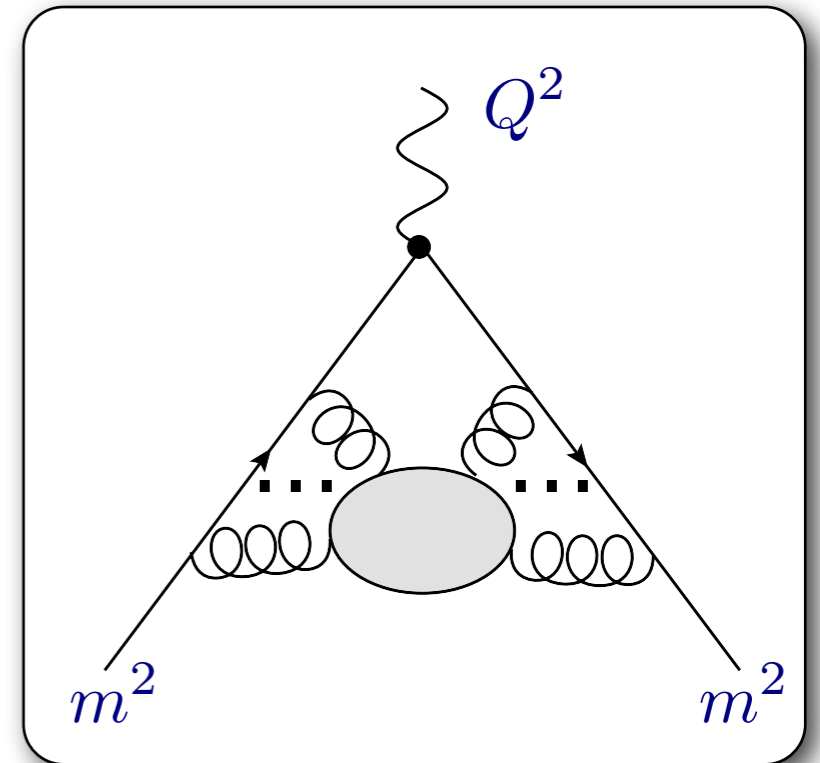
- usually, large logarithms in QFT arise from hierarchy between a long-distance (soft) scale  $m$  and a short-distance (hard) scale  $Q \gg m$ :

$$\ln \frac{Q^2}{m^2} = \ln \frac{Q^2}{\mu^2} + \ln \frac{\mu^2}{m^2}$$

- in Sudakov problems, dependence on the **hard scale  $Q$**  is affected by **long-distance physics**:

$$F(Q)|_{\text{on shell}} = 1 - \frac{C_F \alpha_s}{4\pi} \left( \ln^2 \frac{Q^2}{m^2} + \dots \right)$$

$$F(Q)|_{\text{off shell}} = 1 - \frac{C_F \alpha_s}{4\pi} \left( 2 \ln^2 \frac{Q^2}{-p^2} + \dots \right)$$



# Scale separation in Sudakov problems

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**Soft-collinear effective theory (SCET):** convenient framework to study Sudakov problems by describing collinear and soft particles by effective quark and gluon fields with well-defined interactions and power counting

- **factorization** of short- and long-distance contributions follows from structure of  $L_{\text{eff}}$
- **gauge invariance** implemented at Lagrangian level
- **operator definitions** of jet and soft functions
- **resummation** of Sudakov logarithms is accomplished by solving RG equations



Elegant method for (re-) deriving and using **factorization theorems** in collider and heavy-flavor physics, several times going beyond existing calculations

- in few cases, **new factorization theorems** have been established:  
**Higgs cross section with a jet veto is an important example!**



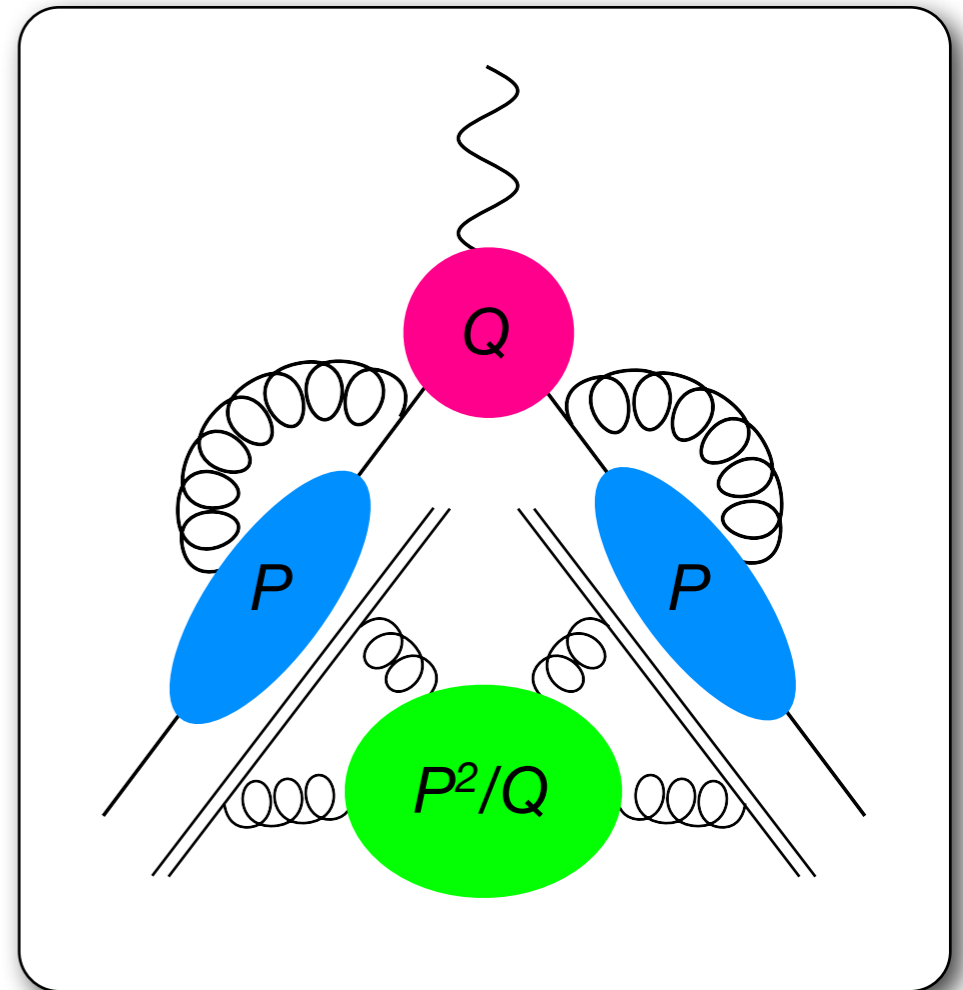
# SCET-I: Correlated scales

Sudakov problems with scale hierarchy  $Q \gg P$  really involve **three correlated scales**:

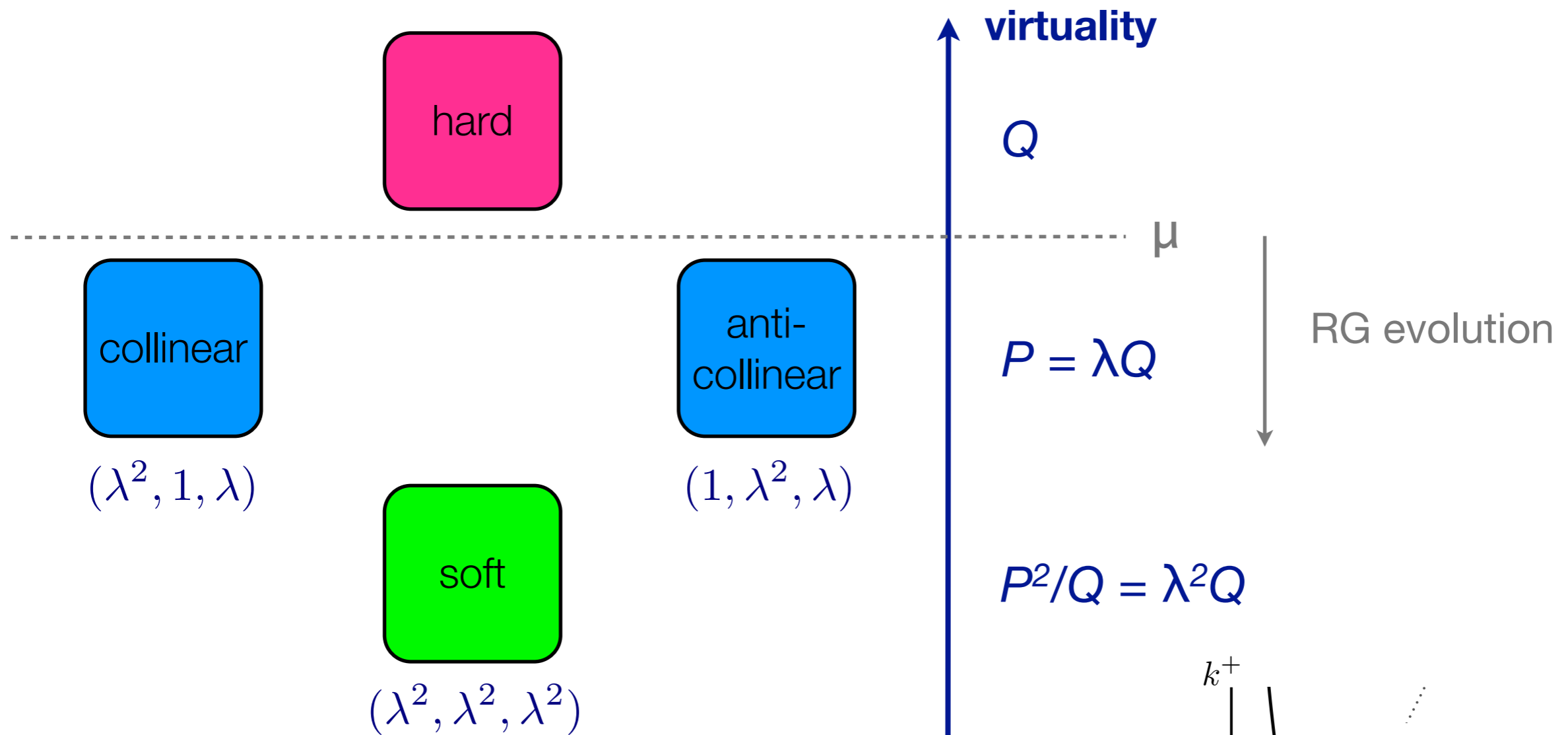
- hard scale  $Q$
- (anti-)collinear scale  $P$
- soft scale  $P^2/Q$

Region analysis of off-shell Sudakov form factor reveals that (with  $P^2 = -p^2$ ):

$$\ln^2 \frac{Q^2}{P^2} = \underbrace{\frac{1}{2} \ln^2 \frac{Q^2}{\mu^2}}_{\text{hard}} - \underbrace{\ln^2 \frac{P^2}{\mu^2}}_{\text{collinear}} + \underbrace{\frac{1}{2} \ln^2 \frac{P^4/Q^2}{\mu^2}}_{\text{soft}}$$

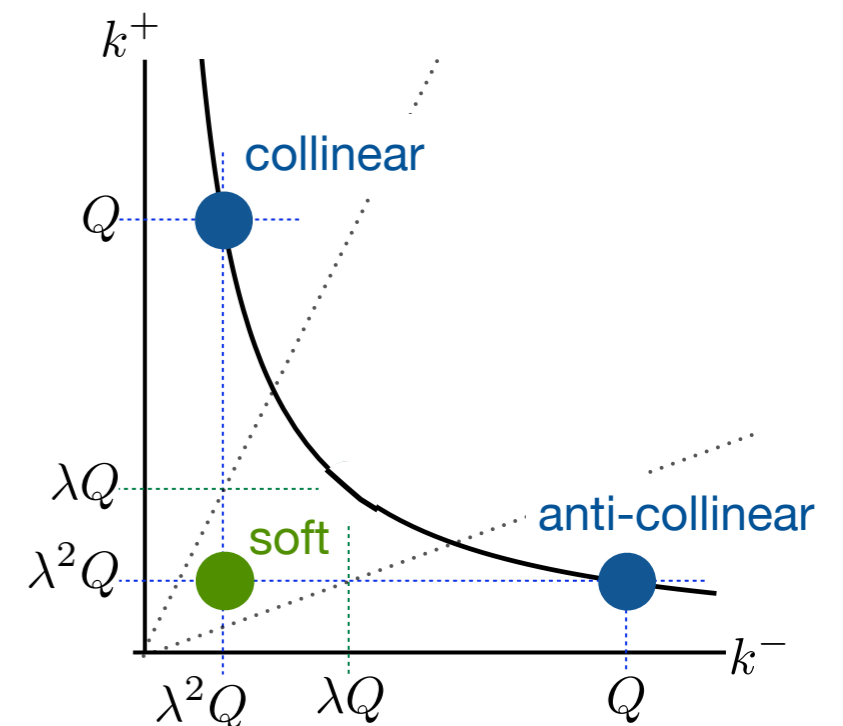


# SCET-I: Correlated scales

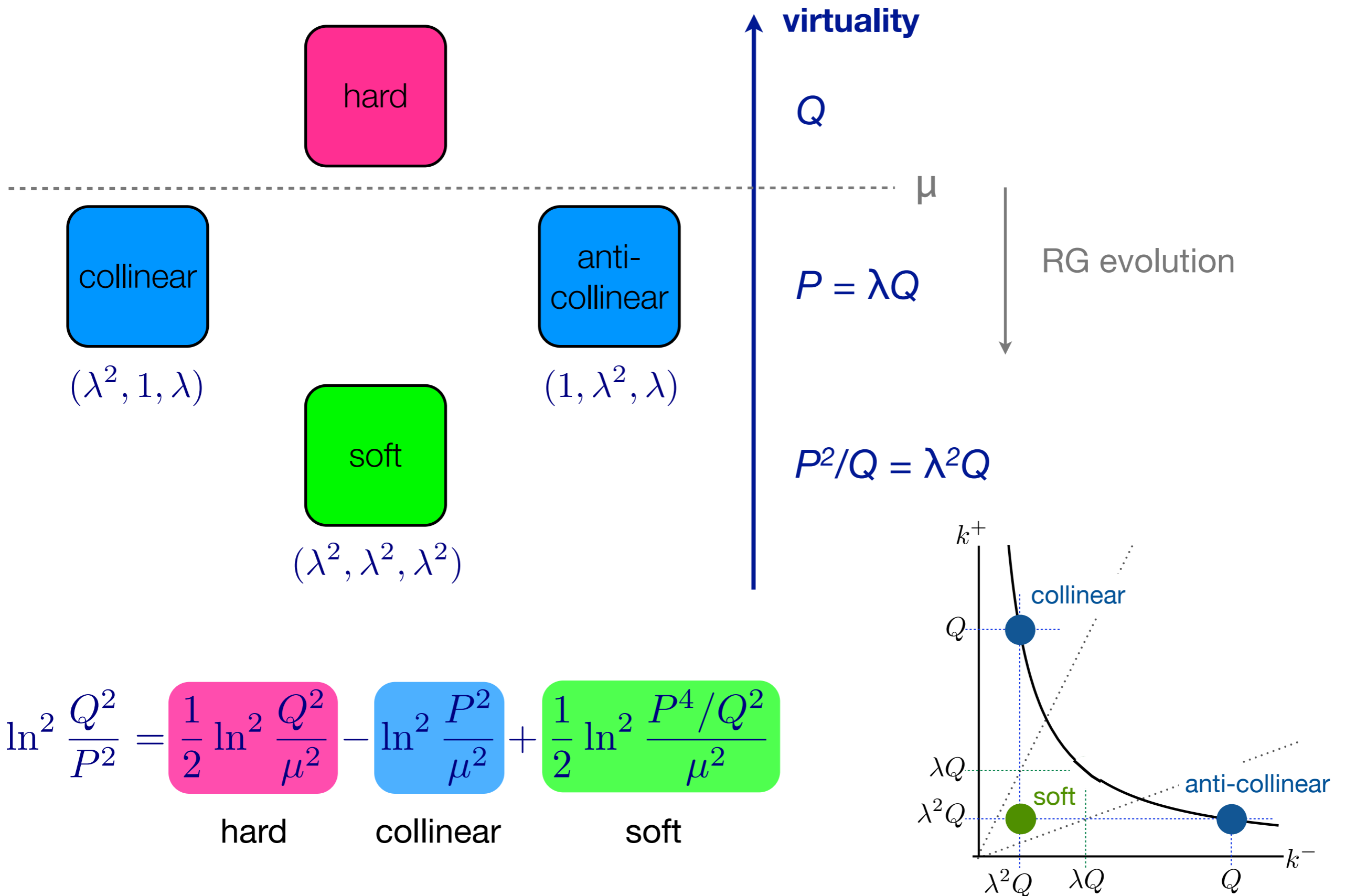


Generic **SCET-I** factorization theorem:

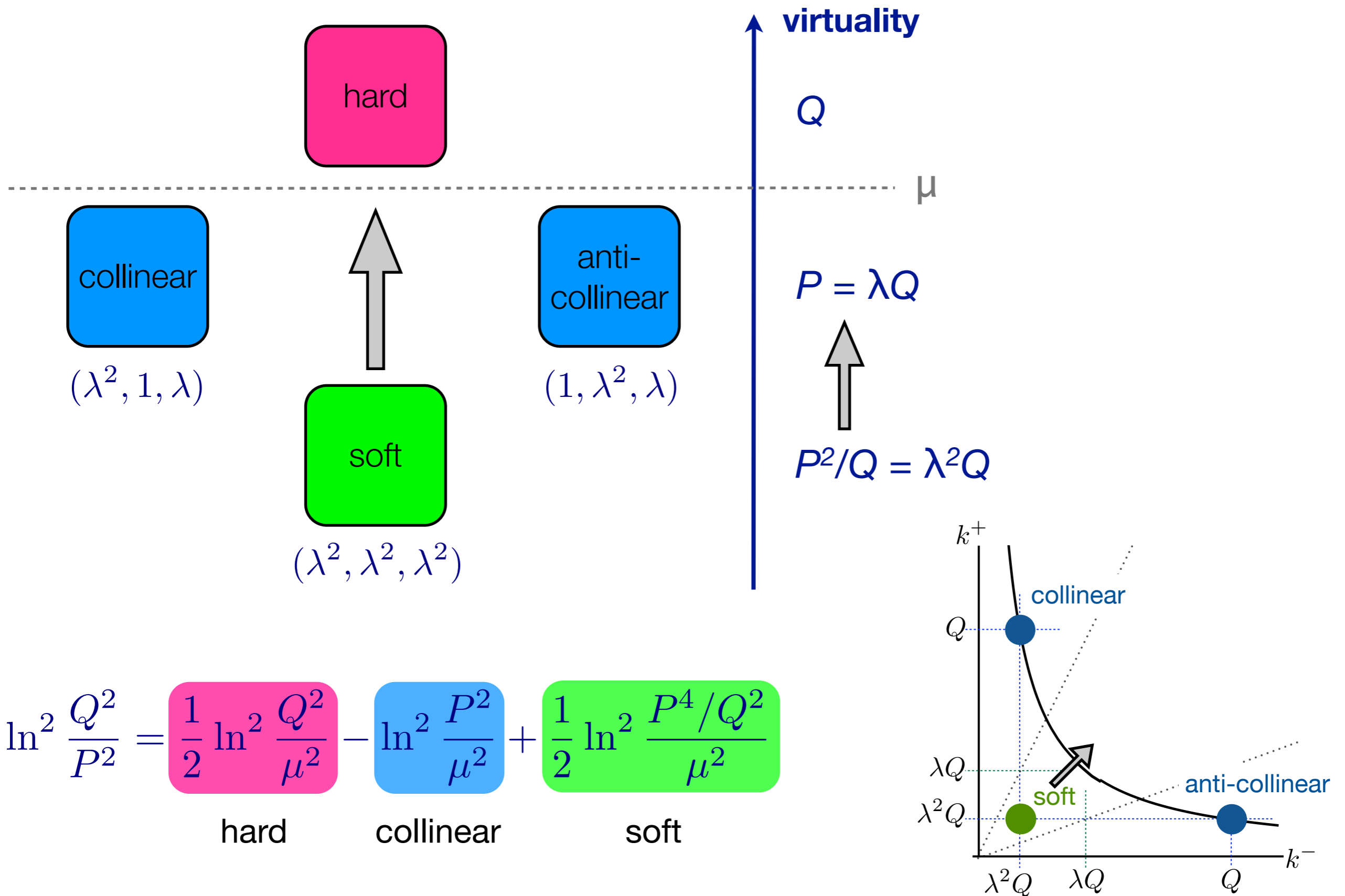
$$d\sigma = H J \otimes J \otimes S$$



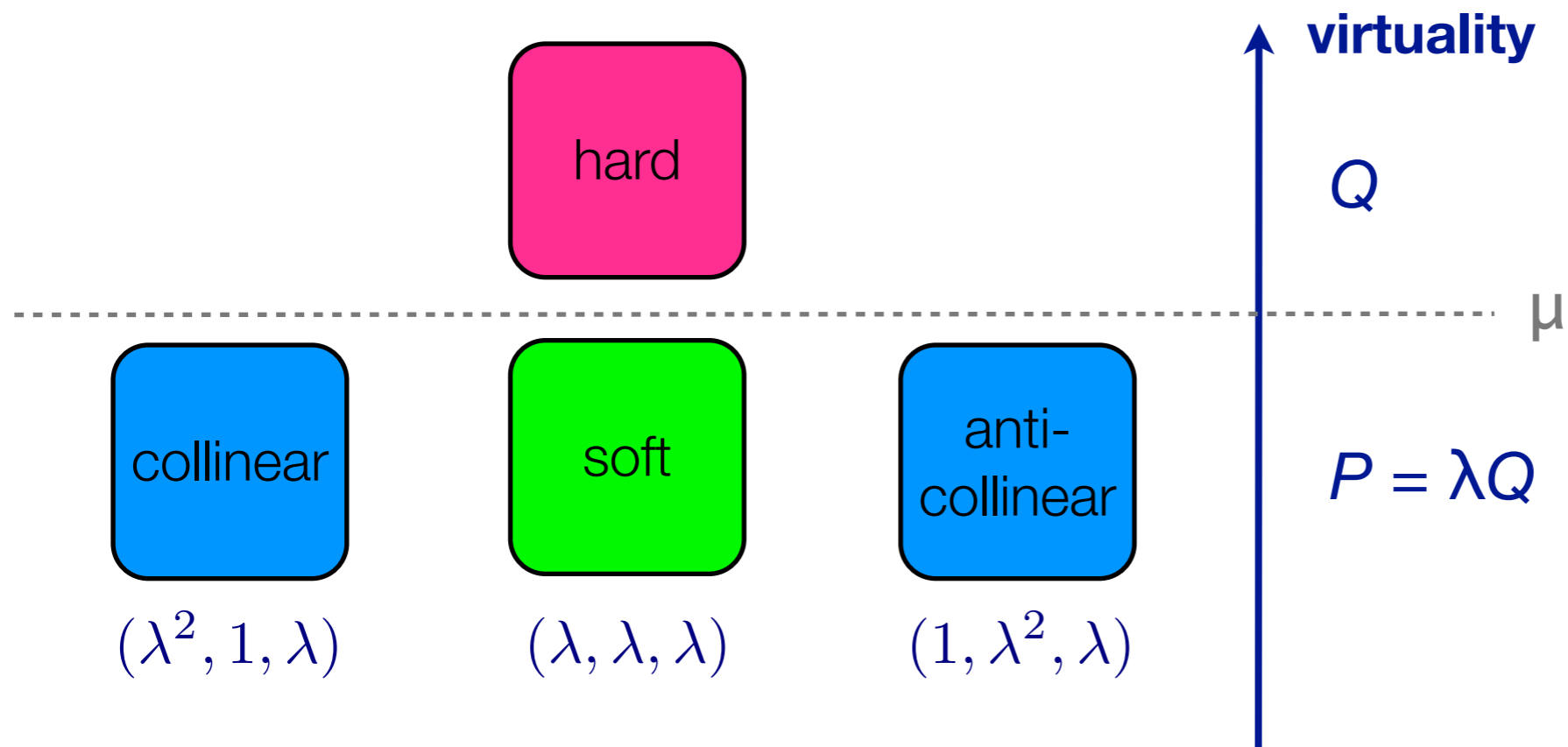
# SCET-I: Correlated scales



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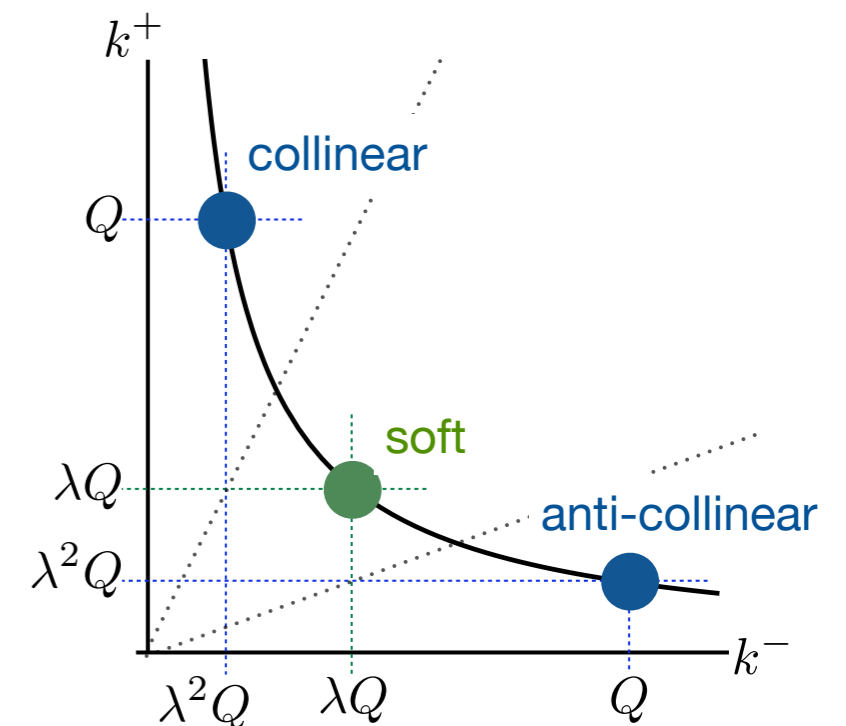
# SCET-II: Absence of the third (soft) scale



- For observables sensitive to transverse momentum, standard (ultra-)soft modes do not contribute
- How to maintain RG invariance?

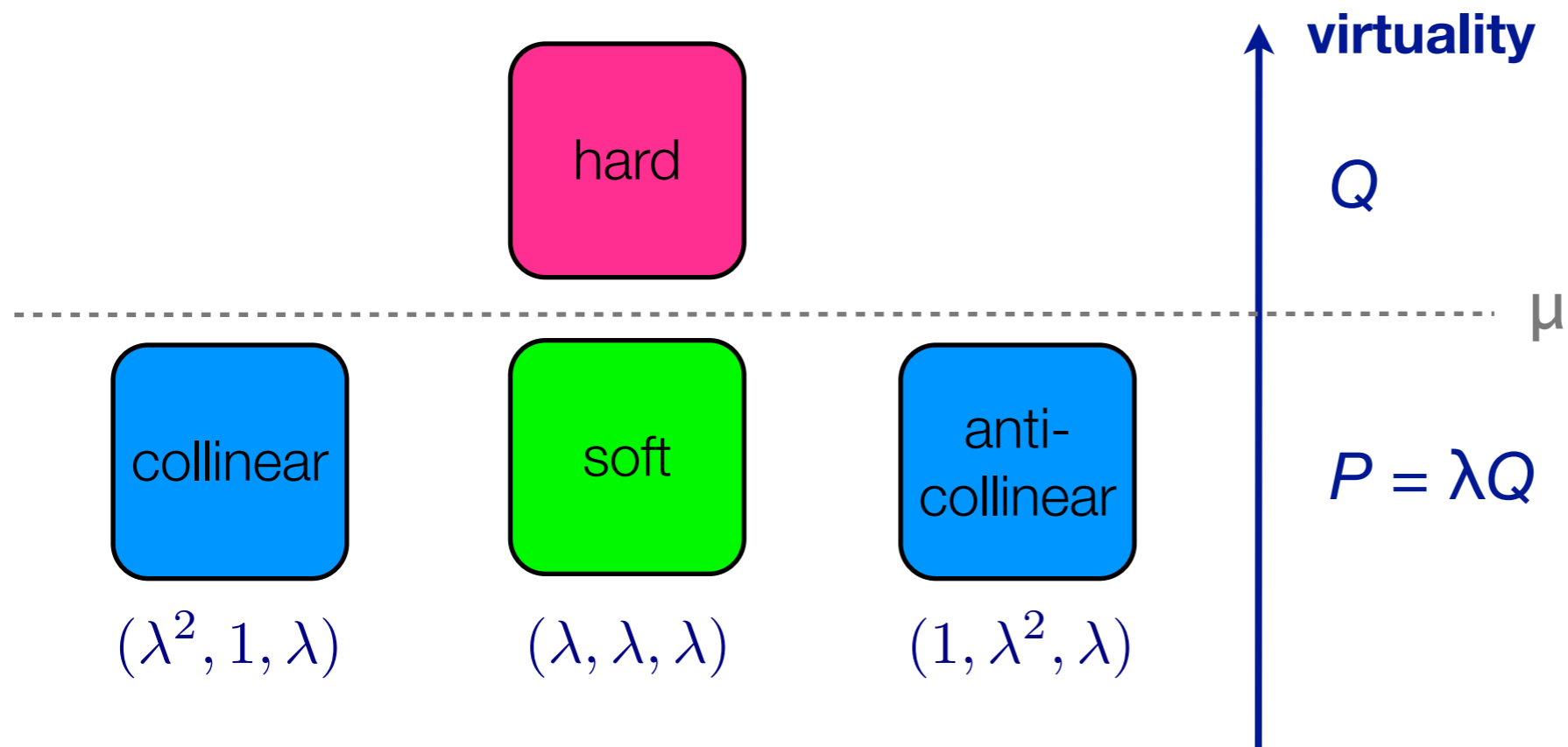
$$\frac{1}{2} \ln^2 \frac{Q^2}{P^2} = \frac{1}{2} \ln^2 \frac{Q^2}{\mu^2} - \frac{1}{2} \ln^2 \frac{P^2}{\mu^2} + ?$$

hard
collinear





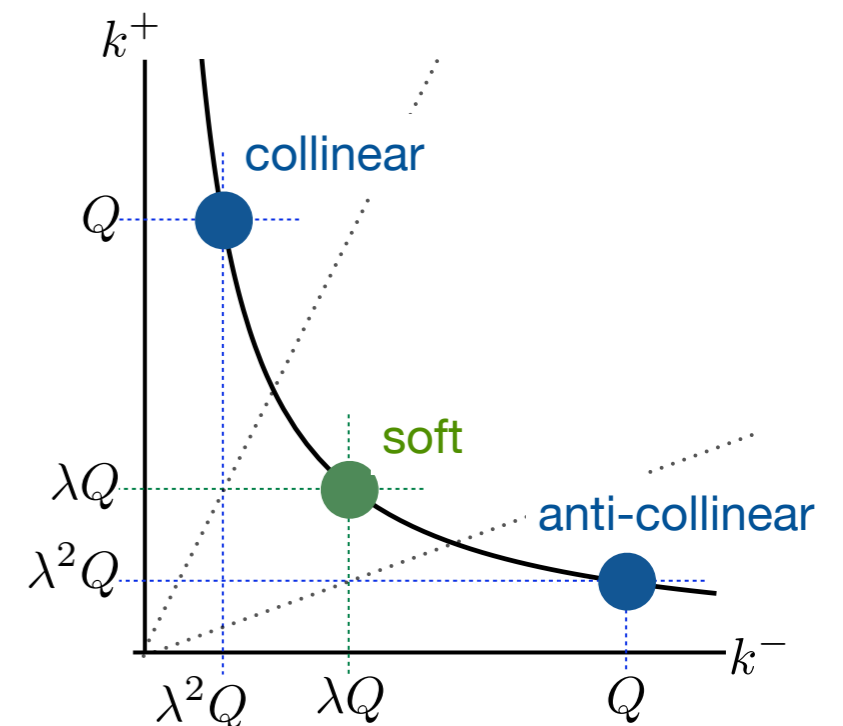
# SCET-II: Absence of the third (soft) scale



- Problem is cured by an anomaly of the effective theory SCET-II: **collinear factorization anomaly**

Becher, MN: 1007.4005

$$\frac{1}{2} \ln^2 \frac{Q^2}{P^2} = \underbrace{\frac{1}{2} \ln^2 \frac{Q^2}{\mu^2}}_{\text{hard}} - \underbrace{\frac{1}{2} \ln^2 \frac{P^2}{\mu^2}}_{\text{collinear}} - \underbrace{\ln \frac{P^2}{\mu^2}}_{\text{anomalous}} \underbrace{\ln \frac{Q^2}{P^2}}_{\text{anomalous}}$$



# SCET-II: Collinear factorization anomaly

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Generic phase-space integrals in SCET-II are **ill-defined** in dimensional regularization and require an **additional regulator**

- integrals such as  $\int_0^\infty dk_+/k_+$  can be avoided using an analytic regulator:

$$\int d^D k \delta(k^2) \theta(k^0) \rightarrow \int d^D k \delta(k^2) \theta(k^0) \left( \frac{\nu}{k_+} \right)^\alpha$$

Becher, Bell: 1112.3907

- **poles in  $1/\alpha$**  cancel when one adds the collinear, anti-collinear, and soft contributions, but an anomalous **dependence on the hard scale  $Q$**  remains
- a variant of the analytic regularization scheme is the **rapidity regularization scheme** proposed in [Chiu, Jain, Neill, Rothstein: 1202.0814](#)

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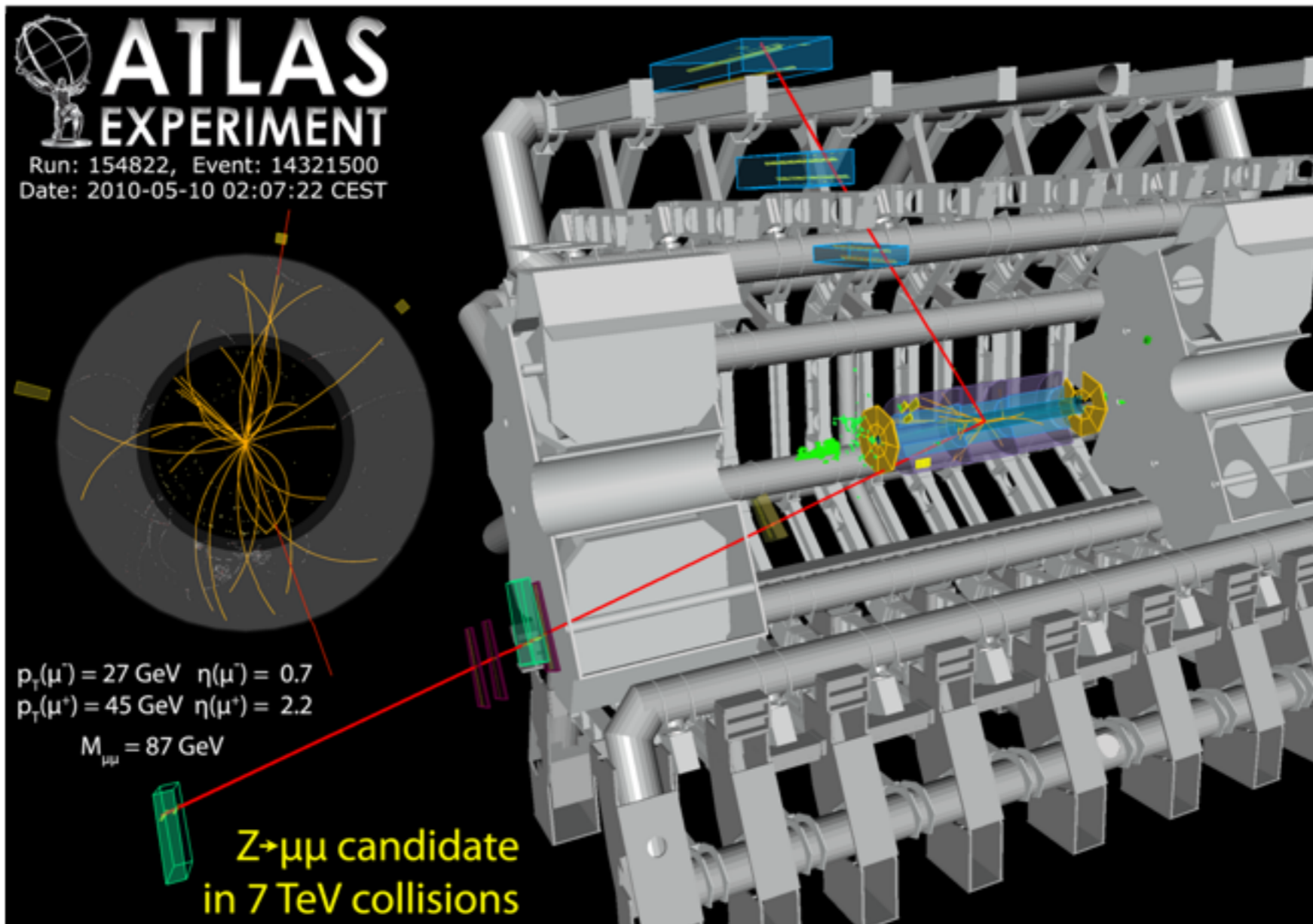
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In SCET-II, this phenomenon can be interpreted as an anomaly: the **breaking of a classical symmetry** of the effective Lagrangian by **quantum effects**

- as a result, the **functional dependence on  $Q$**  is highly constrained and can be derived from simple **differential equations** w.r.t. regulator



Application I: Transverse-momentum resummation for Z and Higgs production

# Drell-Yan production at small $q_T$

Drell-Yan production of Z, W or Higgs bosons at small transverse momentum ( $q_T \ll M$ ) is a classical two-scale process, for which the resummation of Sudakov logs  $\sim \alpha_s^n \ln^{2n}(M/q_T)$  is essential

- no reasonable fixed-order perturbative approximation can be obtained, even if  $q_T \gg \Lambda_{\text{QCD}}$

**Factorization theorem** obtained using the collinear anomaly: [Becher, MN: 1007.4005](#)

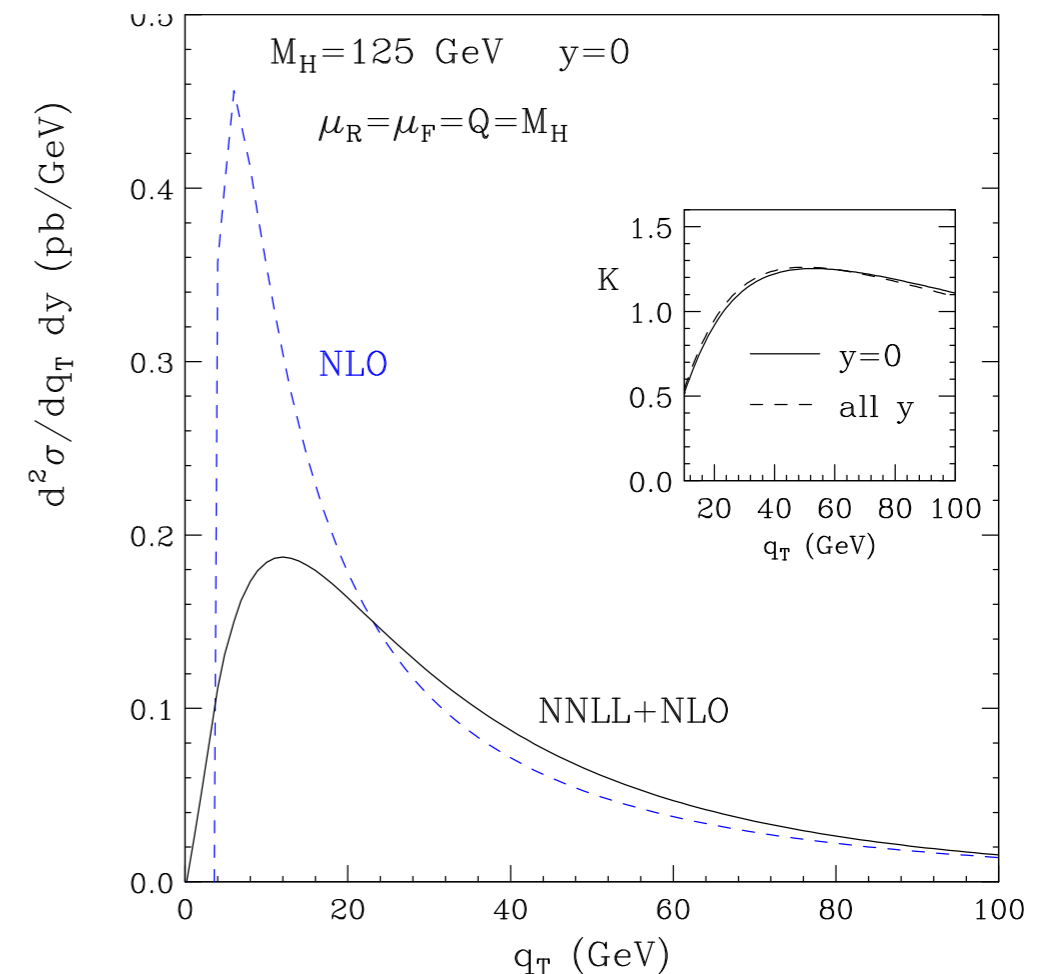
$$\frac{d\sigma}{dq_T} \sim H(M) \int d^2x_T e^{-iq_T \cdot x_T} [I(x_T) \otimes \phi] [I(x_T) \otimes \phi] (M^2 x_T^2)^{-F_{q\bar{q}}(x_T)}$$

beam functions

[Stewart, Tackmann, Waalewijn: 0910.0467](#)

anomalous M dependence is a **pure power** in  $x_T$  space

[Bozzi, Catani, de Florian, Grazzini: 0705.3887](#)





# Drell-Yan production at small $q_T$

In full detail:

$$\frac{d^3\sigma}{dM^2 dq_T^2 dy} = \frac{4\pi\alpha^2}{3N_c M^2 s} \sum_q e_q^2 \sum_{i=q,g} \sum_{j=\bar{q},g} \int_{\xi_1}^1 \frac{dz_1}{z_1} \int_{\xi_2}^1 \frac{dz_2}{z_2} \times \left[ C_{q\bar{q}\rightarrow ij}(z_1, z_2, q_T^2, M^2, \mu) \phi_{i/N_1}(\xi_1/z_1, \mu) \phi_{j/N_2}(\xi_2/z_2, \mu) + (q, i \leftrightarrow \bar{q}, j) \right]$$

with:

$$C_{q\bar{q}\rightarrow ij}(z_1, z_2, q_T^2, M^2, \mu) = |C_V(-M^2, \mu)|^2 \frac{1}{4\pi} \int d^2x_\perp e^{-iq_\perp \cdot x_\perp} \left( \frac{x_T^2 M^2}{4e^{-2\gamma_E}} \right)^{-F_{q\bar{q}}(x_T^2, \mu)} \times I_{q\leftarrow i}(z_1, x_T^2, \mu) I_{\bar{q}\leftarrow j}(z_2, x_T^2, \mu)$$

Result can be matched onto the standard **CSS resummation formula** (Collins, Soper, Sterman 1984)

Using the anomaly equations, we have derived the **last missing ingredient** (the three-loop coefficient  $A_3$ ) required for resummation at NNLL order

# Infrared protection at very small $q_T$

Becher, MN, Wilhelm: 1109.6027

A careful analysis reveals that the spectrum  $d\sigma/dq_T$  is **short-distance dominated** (but genuinely non-perturbative) all the way down to zero transverse momentum

The appropriate choice of  $\mu$  eliminating large logarithms from the Fourier integral is:

$$\mu \sim \max(q_T, q_*) \quad \text{with:} \quad q_* \approx M \exp\left(-\frac{2\pi}{(4C_{F/A} + \beta_0)\alpha_s(M)}\right)$$

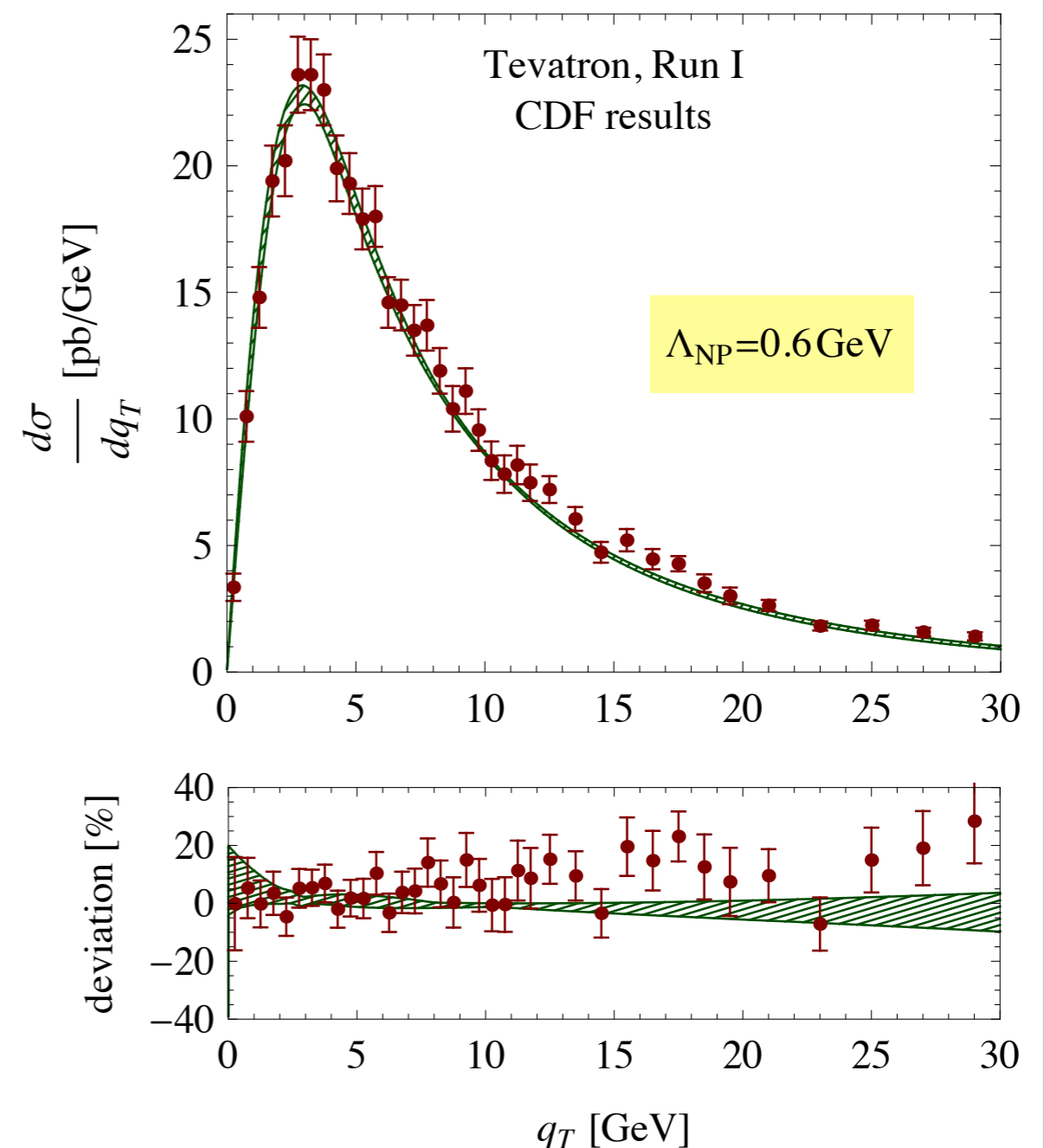
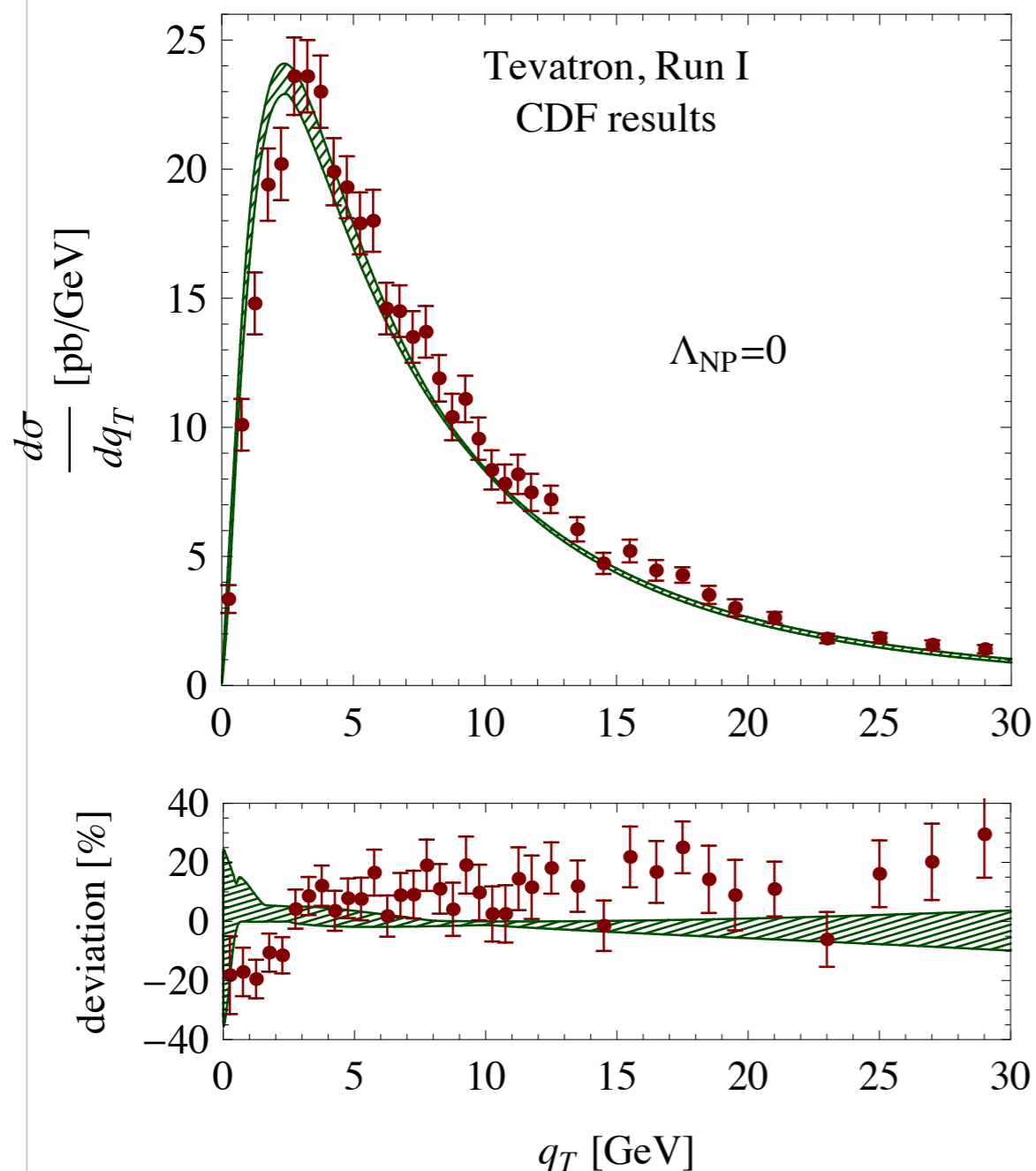
→ yields **1.9 GeV** for Z production, and **7.7 GeV** for Higgs production

Scale  $q_*$  controls the size of **long-distance hadronic corrections**, which can be noticeable for Z production but are very small for Higgs production

# Z-boson production at Tevatron

Becher, MN, Wilhelm: 1109.6027

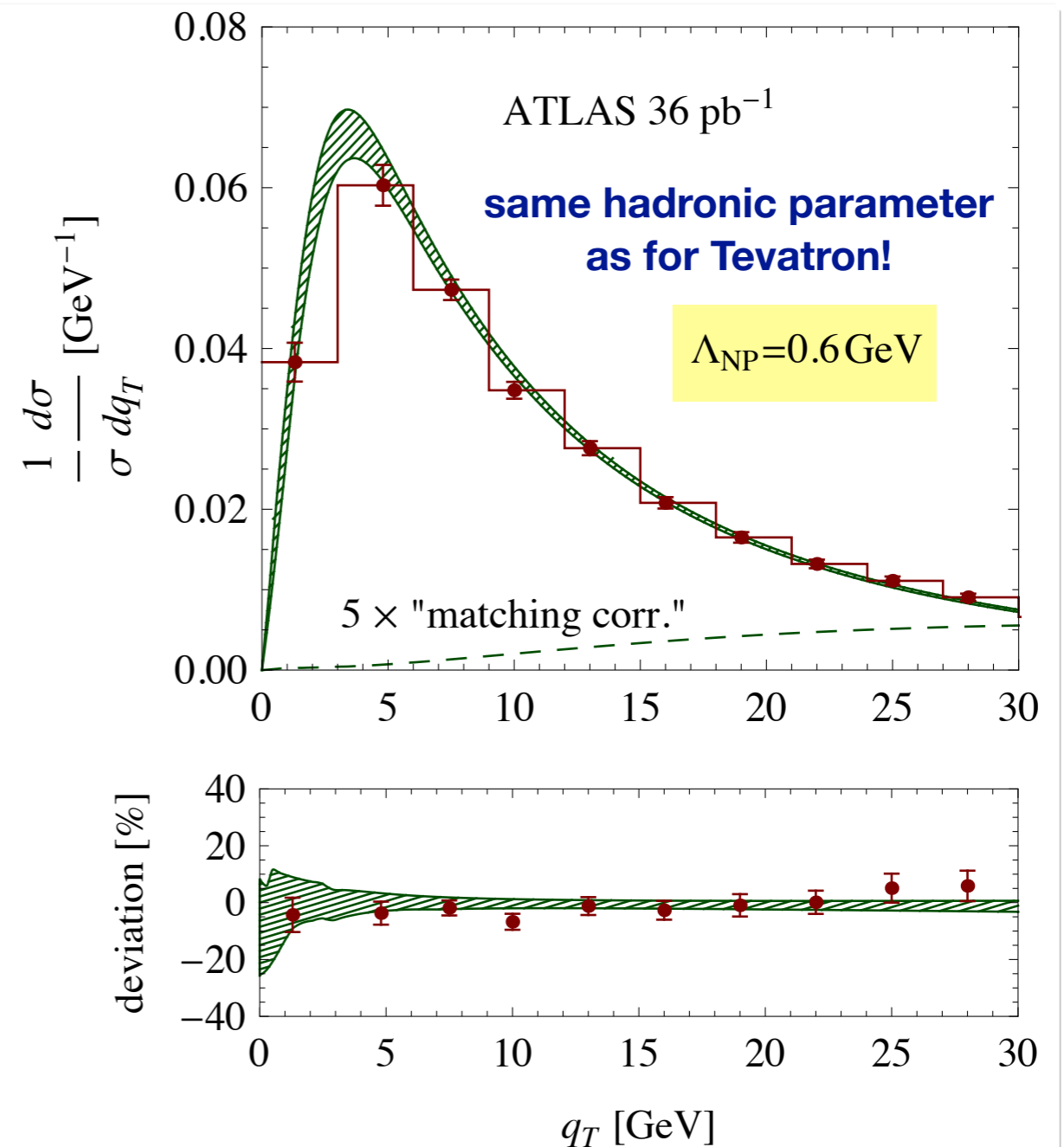
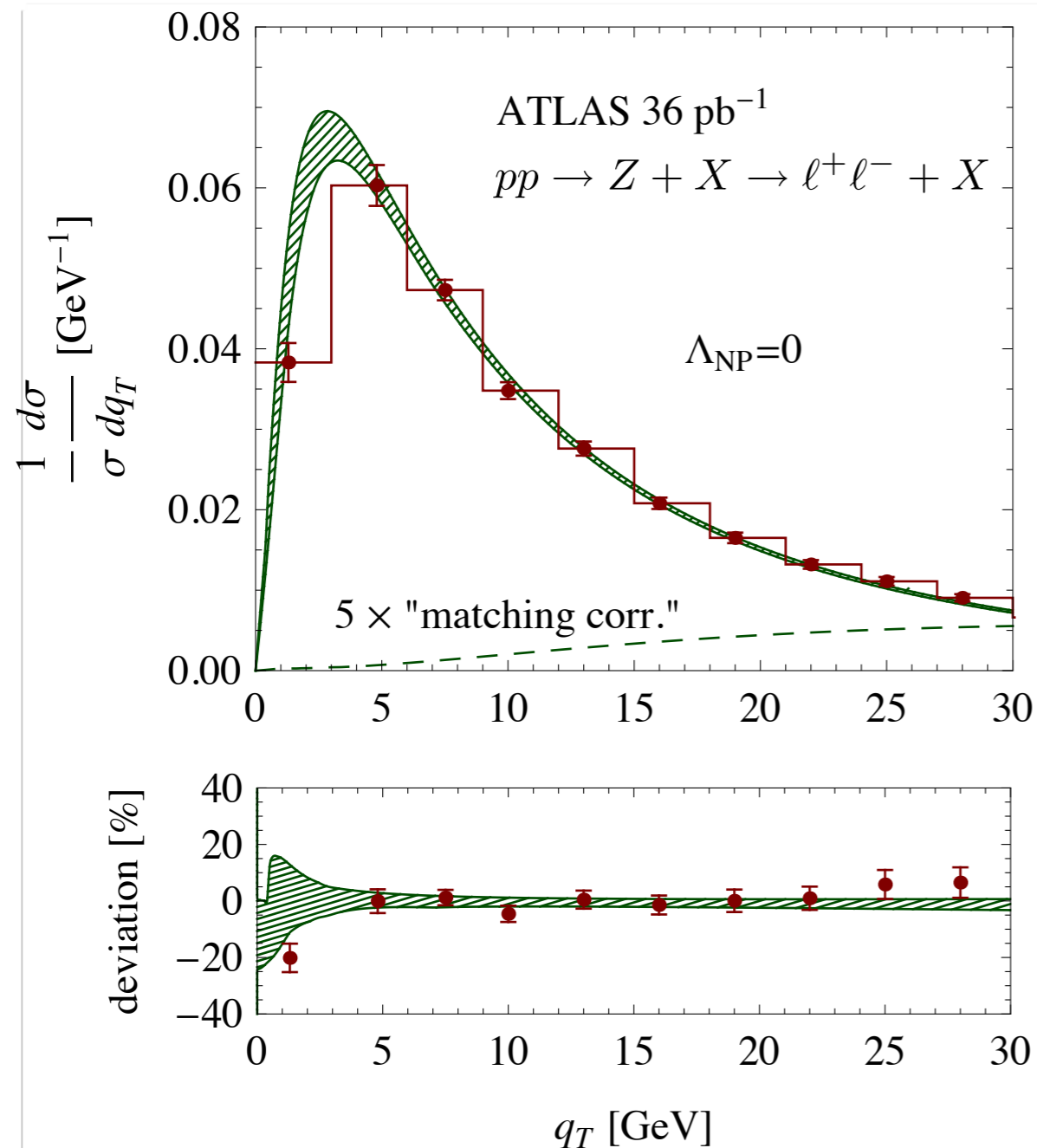
- First complete calculation of Z-boson and Higgs production at NNLL+NLO
- Extension to NNLL+NNLO is technically possible (work in progress)



# Z-boson production at LHC

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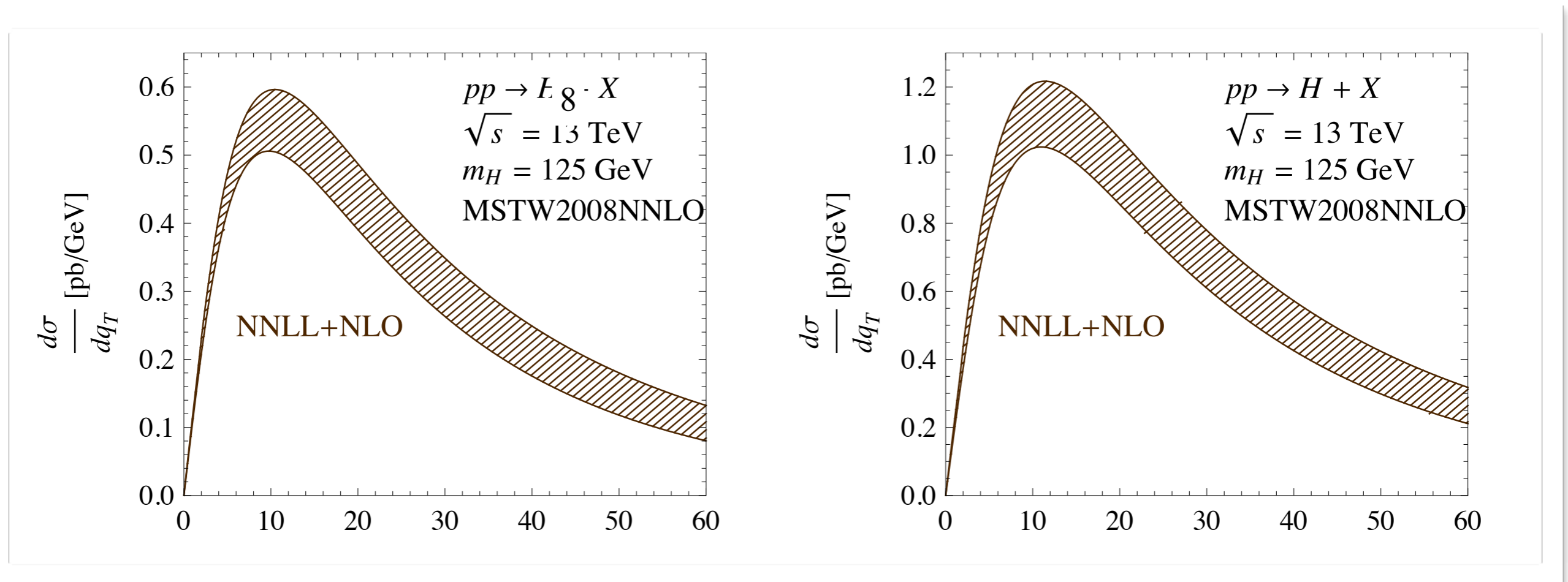
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# Higgs-boson production at LHC

Becher, MN, Wilhelm: 1212.2621

- Higgs  $q_T$  spectrum is predicted using same formalism, only that long-distance hadronic corrections are much smaller in this case
- Eagerly awaiting data ...



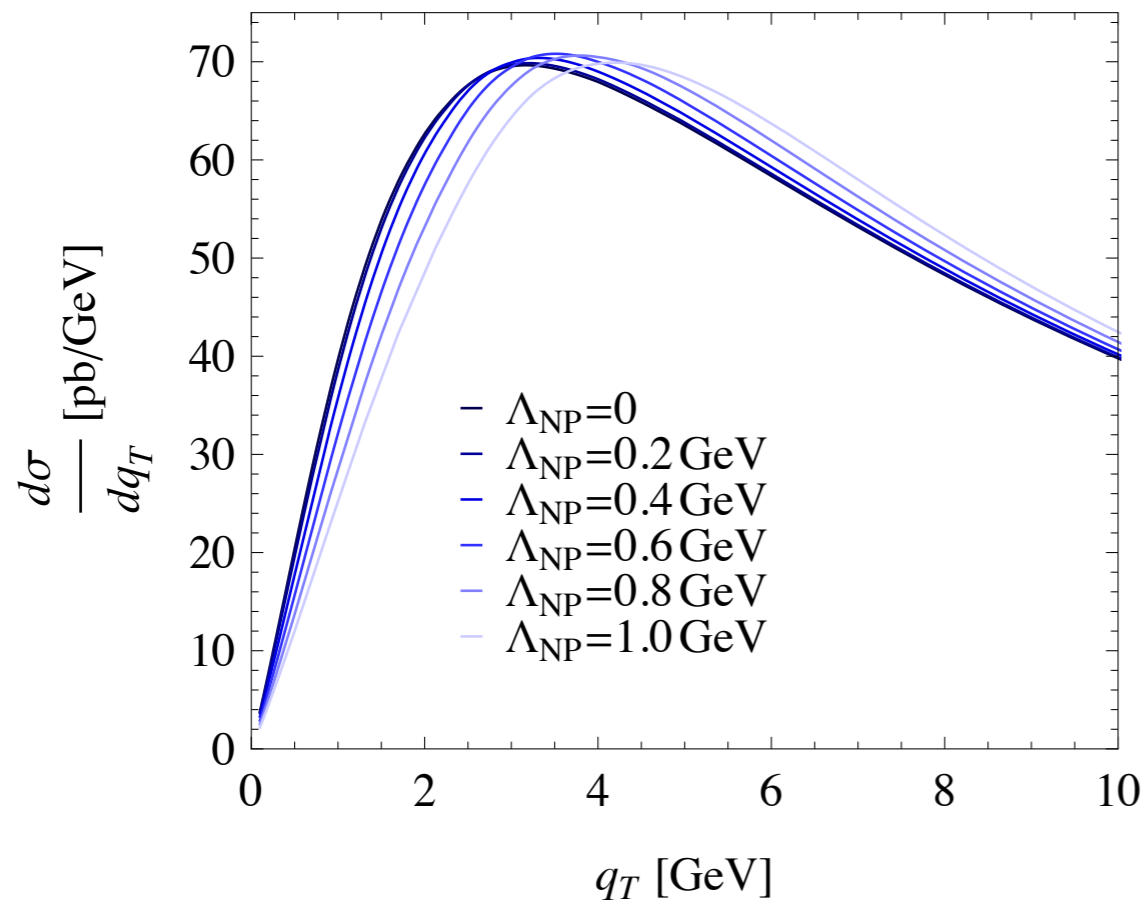
→ public code CuTe available at: <http://cute.hepforge.org>



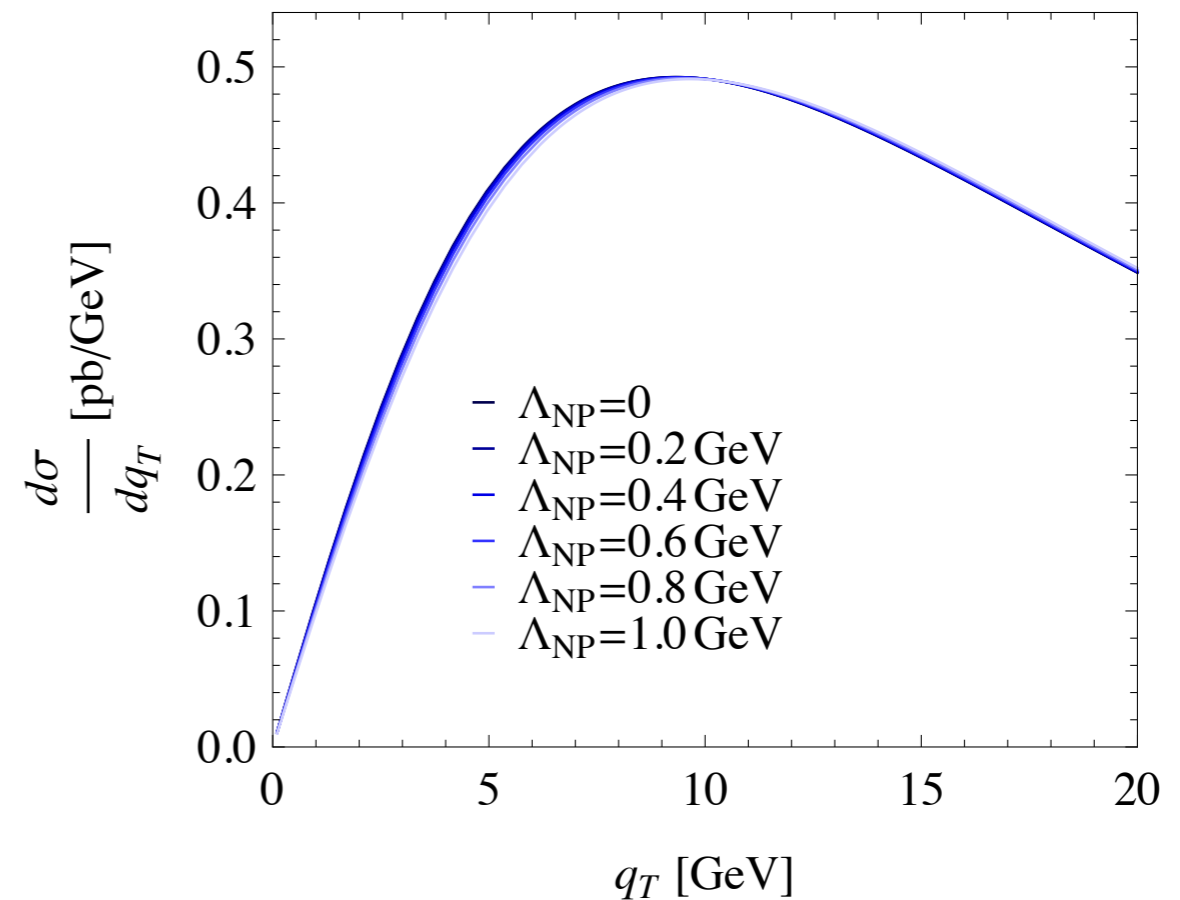
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hadronic corrections: Z-boson case



hadronic corrections: Higgs case

→ public code CuTe available at: <http://cute.hepforge.org>



**Application II:  
Higgs production with a jet veto**

# Higgs production with a jet veto

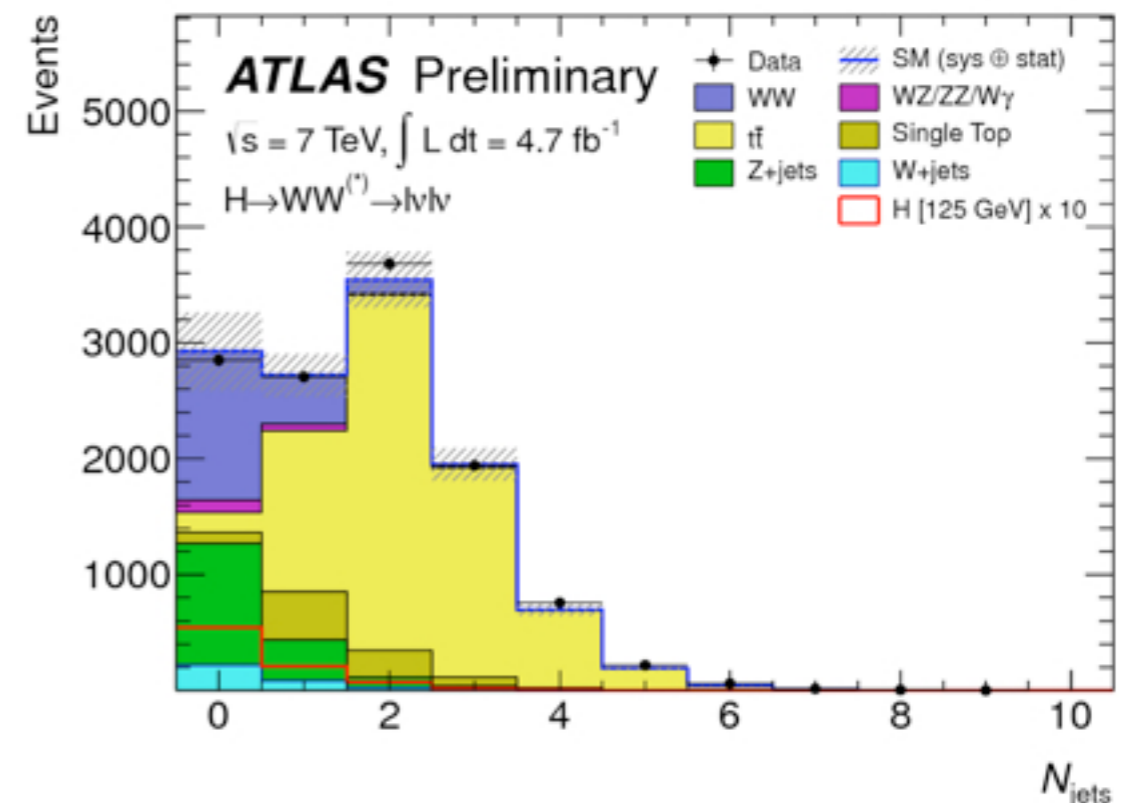
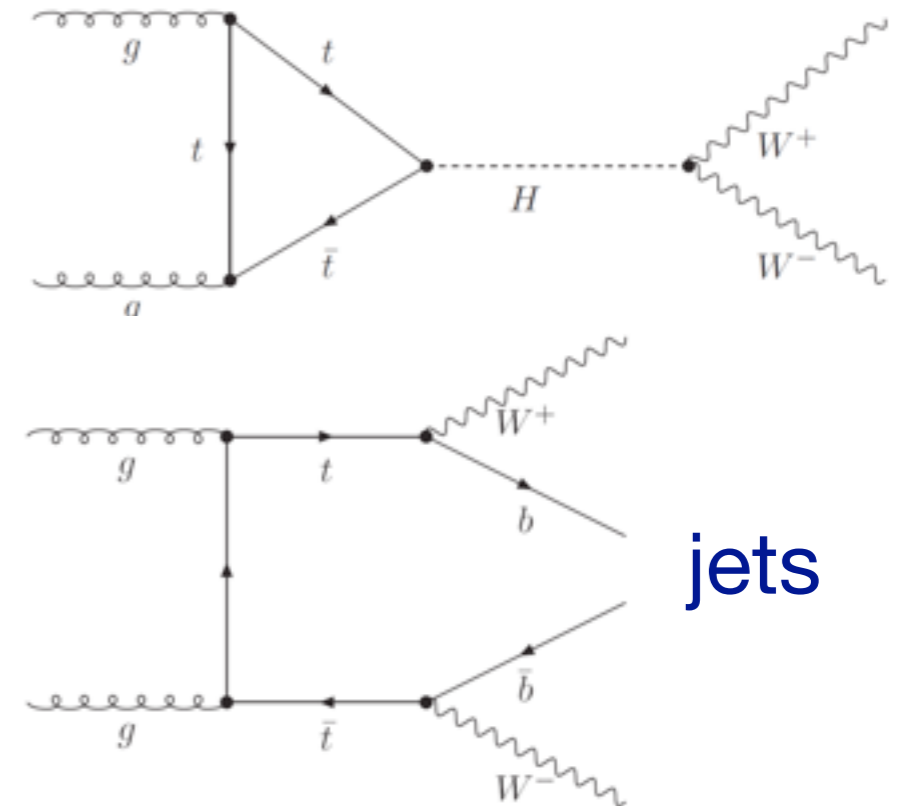
Searches for Higgs boson require stringent cuts to suppress background events

Since backgrounds are very different when the Higgs is produced in association with jets, the searches are performed in **jet bins**

- require precise predictions for  $H+n$  jets, in particular for the 0-jet bin, i.e., the cross section with a **jet veto**:

$$p_T^{\text{jet}} < p_T^{\text{veto}} \approx 15 - 30 \text{ GeV}$$

Until very recently, no resummed results for the cross section defined with a jet veto were available beyond LL order (parton shower)



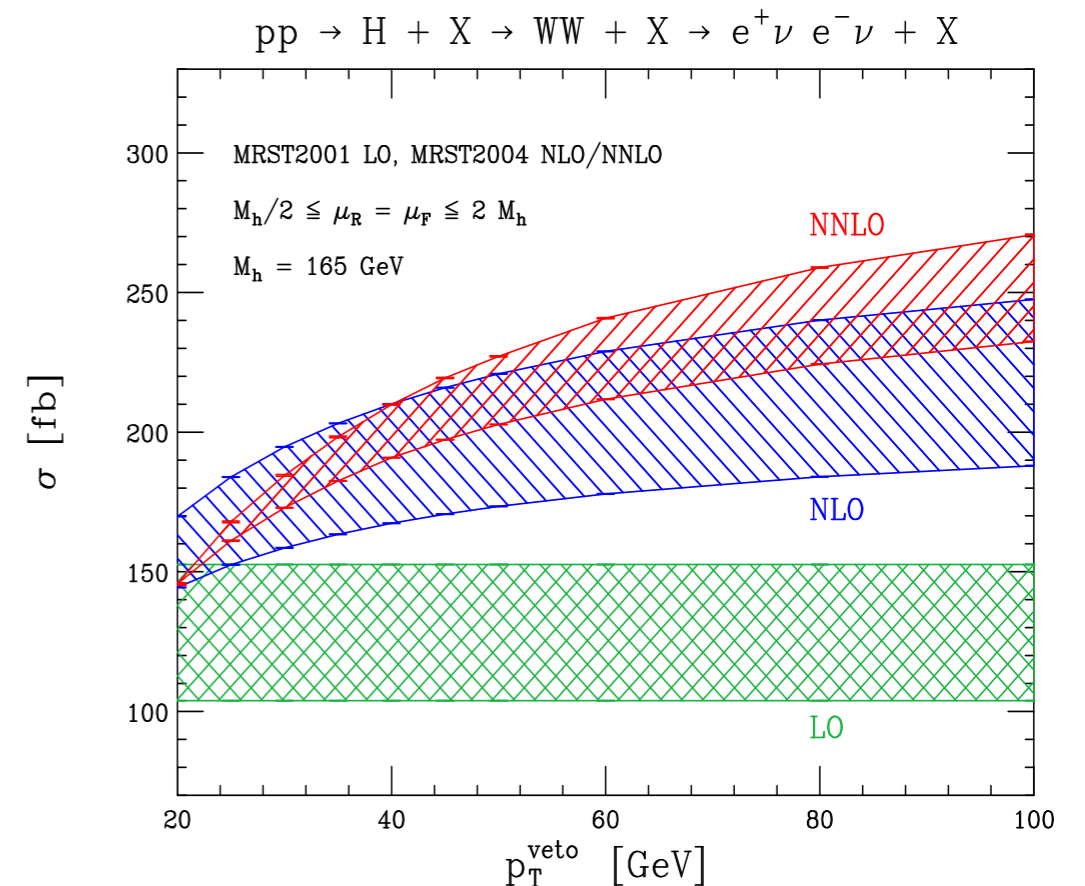
# Higgs production with a jet veto

Fixed-order predictions naively suggest that the cut rate has smaller uncertainties than the total cross section

Effect is due to an **accidental cancellation** of large corrections from two sources:

- large **positive** corrections to total cross sections from analytic continuation of scalar form factor to time-like region  
Ahrens, Becher, MN, Yang (2008)
- large **negative** corrections from Sudakov logarithms  $\alpha_s^n \ln^{2n}(m_H/p_T^{\text{veto}})$

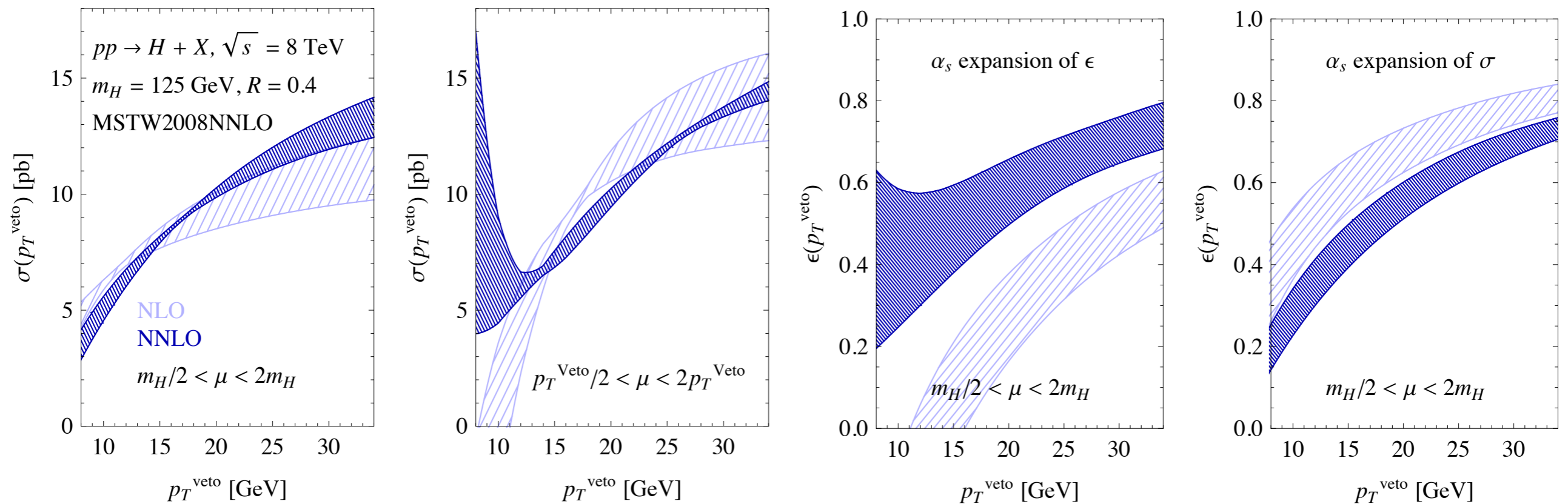
True perturbative uncertainty is most likely significantly larger  
Stewart, Tackmann, Waalewijn (2010)  
Stewart, Tackmann (2011)



Anastasiou, Dissertori, Stöckli (2007)

# Higgs production with a jet veto

Updated fixed-order predictions for different schemes and scale choices:



$\Rightarrow$  bands do not reflect true uncertainties!

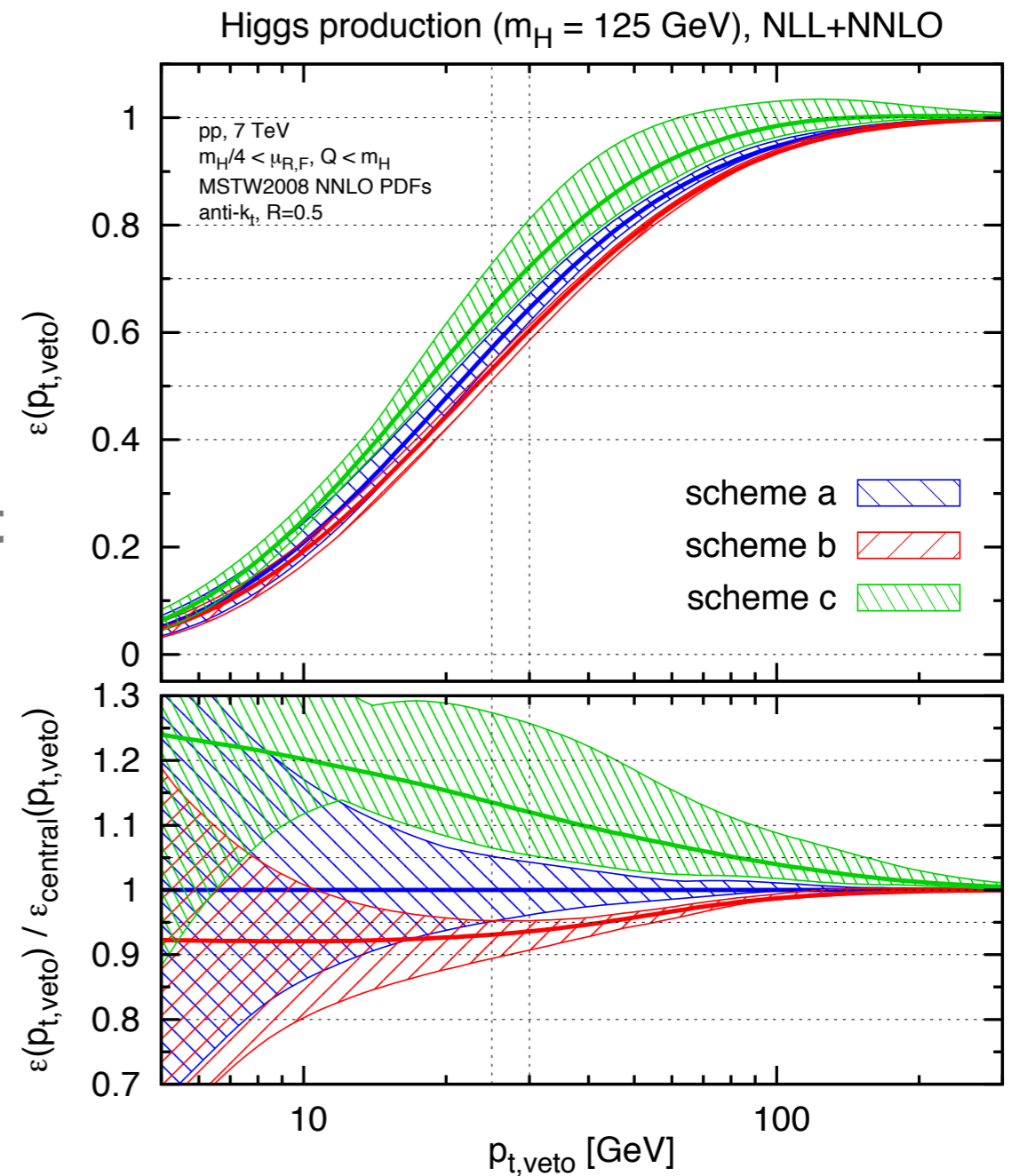


# Resummation at NLL order

Recently, it has been shown that the jet veto can be resummed at **NLL order** using the numerical resummation code **CAESAR**

Banfi, Salam, Zanderighi: 1203.5773

- NLL+NNLO calculation still suffers from significant perturbative uncertainties and scheme dependences; hence calculate **cut efficiency** instead of cross section



# Resummation at NNLL order and beyond

Recently, it has been shown that the jet veto can be resummed at **NLL order** using the numerical resummation code **CAESAR**

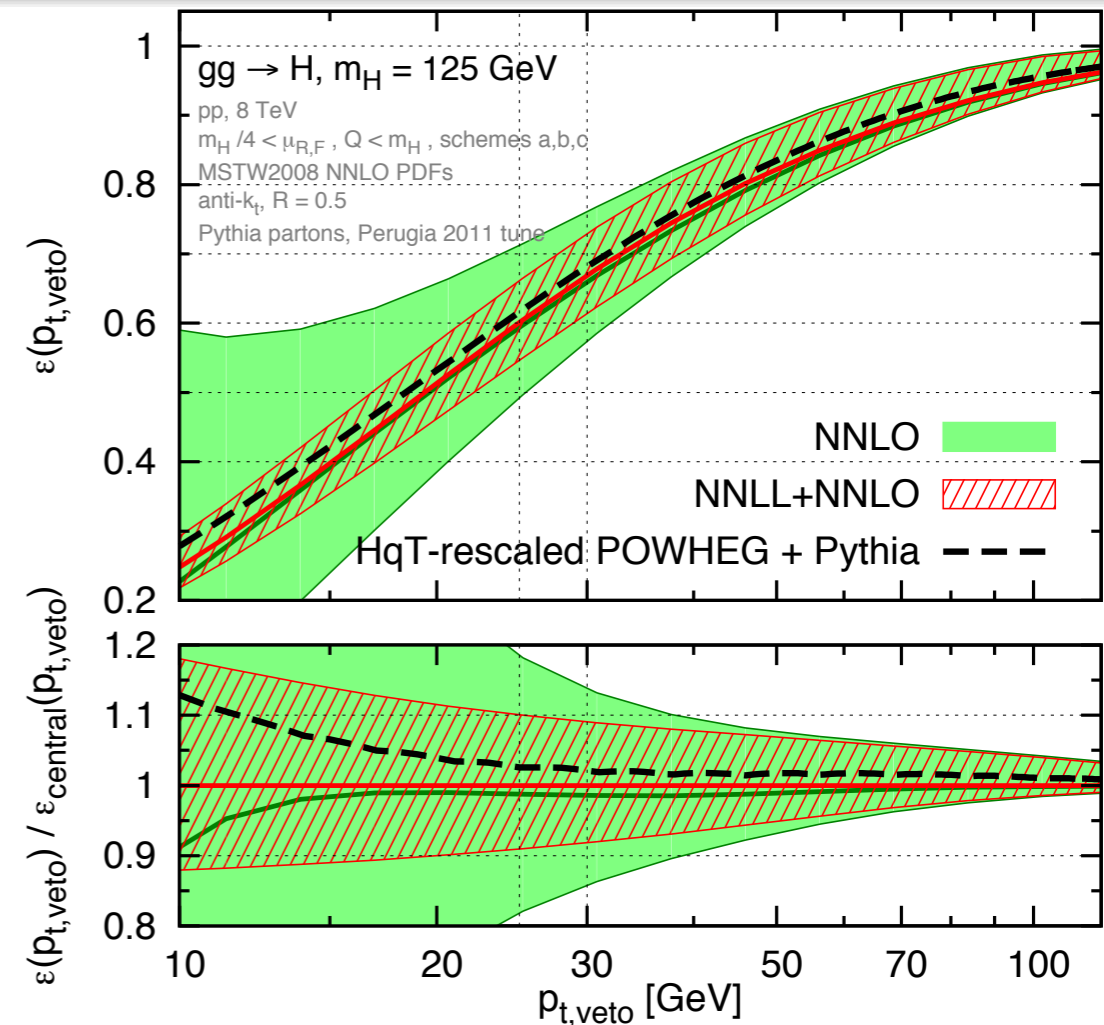
Banfi, Salam, Zanderighi: 1203.5773

- NLL+NNLO calculation still suffers from significant perturbative uncertainties and scheme dependences; hence calculate **cut efficiency** instead of cross section

Soon after, the resummation was extended to **NNLL order** (matched to NNLO)

Banfi, Monni, Salam, Zanderighi: 1206.4998

- uncertainties are significantly reduced



**Meanwhile, we have shown that the jet-veto cross factorizes to all orders and can be resummed using SCET !**

Becher, MN: 1205.3806

# Inclusive jet clustering algorithm

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Distance measure:

$$d_{ij} = \min(p_{Ti}^n, p_{Tj}^n) \frac{\sqrt{\Delta y_{ij}^2 + \Delta \phi_{ij}^2}}{R}$$
$$d_{iB} = p_{Ti}^n$$

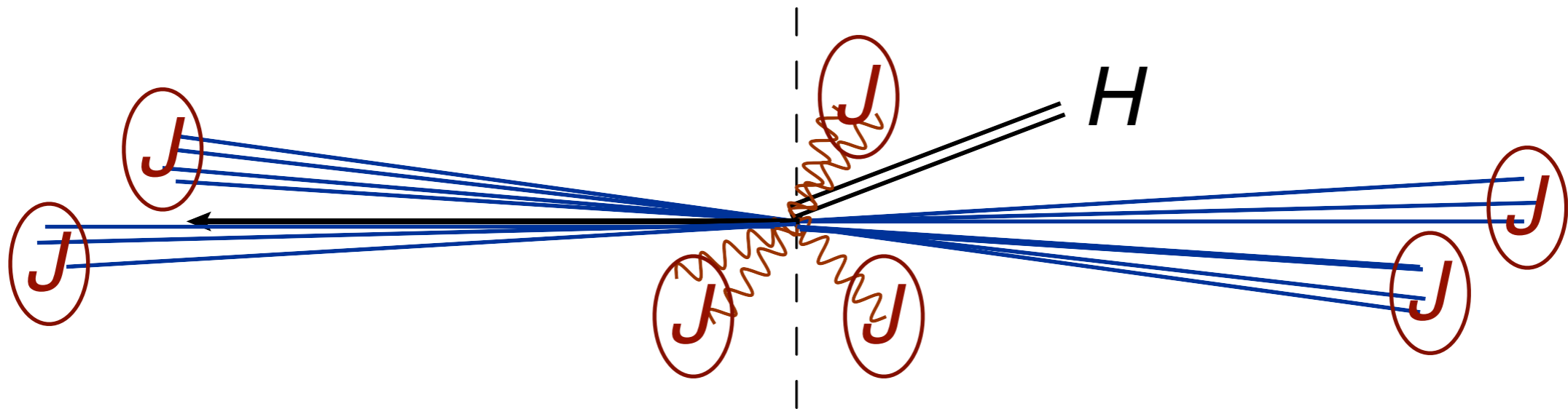
n=1:  $k_T$   
n=0: C/A  
n=-1: anti- $k_T$

Find the smallest of all  $d_{ij}$ ,  $d_{iB}$ . If it is a  $d_{ij}$ , combine particles  $i$  and  $j$  into one particle. If it is a  $d_{iB}$ , call particle  $i$  a jet and remove it from the list. Repeat until all particles are clustered in jets

Since two **different SCET modes have a large rapidity gap**, the jet algorithm clusters soft particles with soft ones and collinear particles with collinear ones, except in corners of phase space (power-suppressed effects)

→ **jet veto can be applied separately in each sector of SCET**  
(simple factorization theorem)

# All-order factorization theorem



Based on SCET analysis, propose first **all-order factorization formula** for the cross section with a jet veto with  $\mathbf{R}=\mathbf{O}(1)$ :

anomalous  $m_H$  dependence is a **pure power** in  $p_T$  space

$$\frac{d\sigma(p_T^{\text{veto}})}{dy} = \sigma_0(\mu) C_t^2(m_t^2, \mu) |C_S(-m_H^2, \mu)|^2 \left(\frac{m_H}{p_T^{\text{veto}}}\right)^{-2F_{gg}(p_T^{\text{veto}}, \mu)} \times \sum_{i,j} \int_{\xi_1}^1 \frac{dz_1}{z_1} \int_{\xi_2}^1 \frac{dz_2}{z_2} I_{g \leftarrow i}(z_1, p_T^{\text{veto}}, \mu) I_{g \leftarrow j}(z_2, p_T^{\text{veto}}, \mu) \phi_{i/P}(\xi_1/z_1, \mu) \phi_{j/P}(\xi_2/z_2, \mu)$$

Becher, MN: 1205.3806

Note close structural similarity with  $q_T$  resummation formula!

# All-order factorization theorem

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Ingredients required for **resummation at NNLL order**:

- $C_t$  and  $C_S$  at two-loop order in RG-improved perturbation theory (known to three loops)
- one-loop collinear kernel functions:

$$I_{g \leftarrow i}(z, p_T^{\text{veto}}, \mu) = \delta(1-z) \delta_{gi} \left[ 1 + a_s \left( \Gamma_0^A \frac{L_\perp^2}{4} - \gamma_0^g L_\perp \right) \right] + a_s \left[ -\mathcal{P}_{g \leftarrow i}^{(1)}(z) \frac{L_\perp}{2} + \mathcal{R}_{g \leftarrow i}(z) \right]$$

$$\mathcal{R}_{g \leftarrow g}(z) = -C_A \frac{\pi^2}{6} \delta(1-z), \quad \mathcal{R}_{g \leftarrow q}(z) = 2C_F z$$

$$L_\perp = 2 \ln \frac{\mu}{p_T^{\text{veto}}}$$

- two-loop anomaly coefficient:

$$F_{gg}(p_T^{\text{veto}}, \mu) = a_s \left( \Gamma_0^A L_\perp + d_1^{\text{veto}} \right) + a_s^2 \left( \Gamma_0^A \beta_0 \frac{L_\perp^2}{2} + \Gamma_1^A L_\perp + d_2^{\text{veto}} \right)$$

↑  
vanishes!

↑  
only dependence on **jet radius parameter R** at NNLL order

Becher, MN: 1205.3806

# All-order factorization theorem

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Determination of anomaly coefficient  $d_2^{\text{veto}}$ :

- matching our formula with NNLL result of BMSZ [Banfi, Monni, Salam, Zanderighi: 1206.4998](#) yields:

$$d_2^{\text{veto}} = d_2^g - 32C_A f(R)$$



from: [Becher, MN: 1007.4005](#)

with:

$$f(R) = - (1.0963 C_A + 0.1768 T_F n_f) \ln R + (0.6072 C_A - 0.0308 T_F n_f) \\ - (0.5585 C_A - 0.0221 T_F n_f) R^2 + (0.0399 C_A - 0.0004 T_F n_f) R^4 + \dots$$

- originally we had an extra constant term in the above relation, which appeared because we had incorrectly assumed that the BMSZ formula still holds in the limit  $R \rightarrow \infty$

We have now re-derived the expression for  $d_2^{\text{veto}}$  from a **two-loop calculation in SCET**, finding complete agreement with the BMSZ formula !

Thomas Becher, MN, Lorena Rothen (to appear)

# Matching to fixed-order results

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Study two different matching schemes:

- perform matching in naive way (scheme A)
- factor out hard function  $H$  times anomaly term (scheme B)

Since the hard function  $H$  contains the large corrections affecting the total cross section (time-like scalar form factor), scheme B is better motivated than scheme A

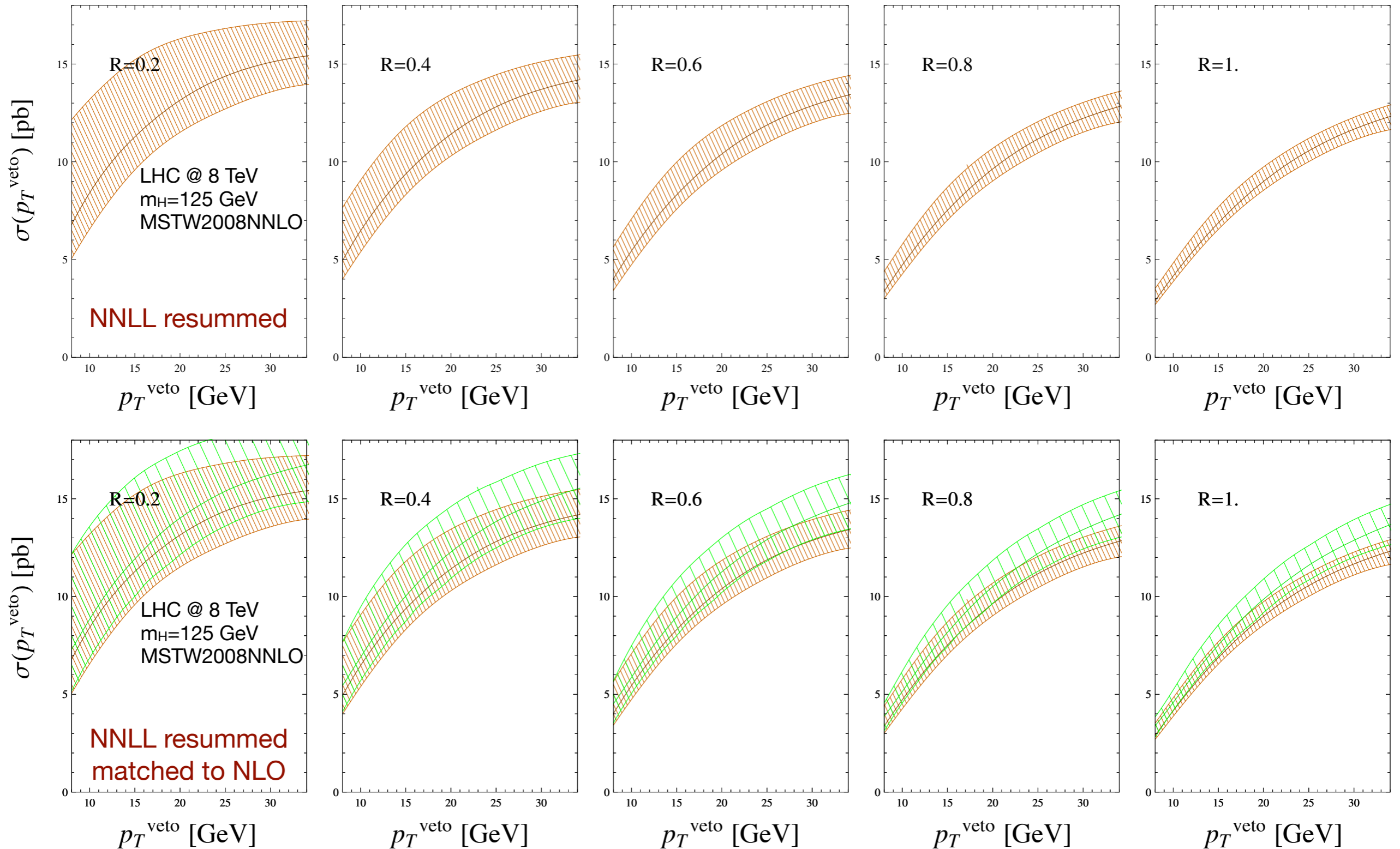
$$\sigma(p_T^{\text{veto}}) \sim H(m_H) [I(p_T^{\text{veto}}) \otimes \phi] [I(p_T^{\text{veto}}) \otimes \phi] \left( \frac{m_H^2}{(p_T^{\text{veto}})^2} \right)^{-F_{gg}^{\text{veto}}(p_T^{\text{veto}})}$$

Caveat: numerical results are preliminary

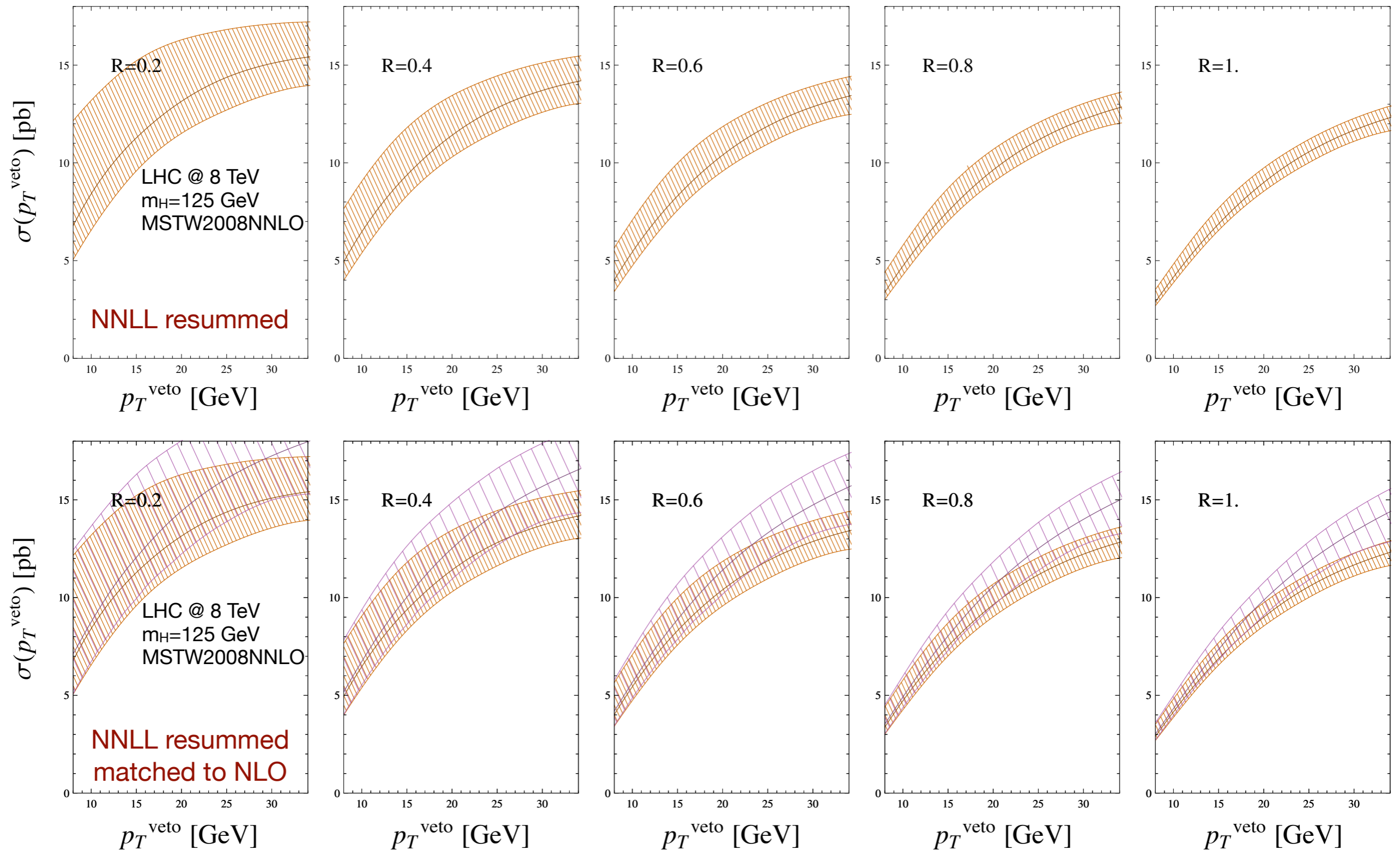
Thomas Becher, MN, Lorena Rothen (to appear)



# NNLL+NLO results (scheme A)



# NNLL+NLO results (scheme B)



# NNLL+NLO results

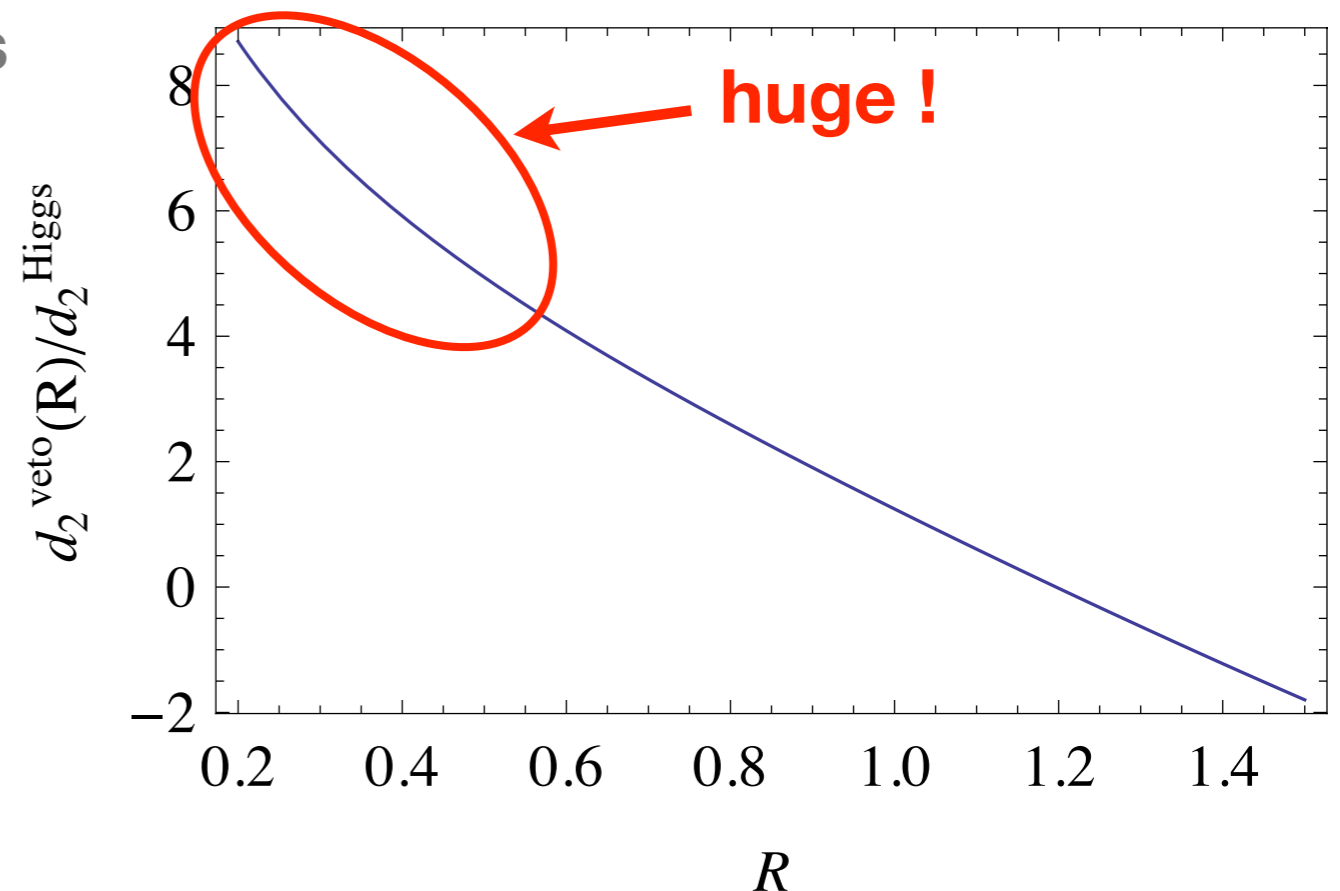
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## General observations:

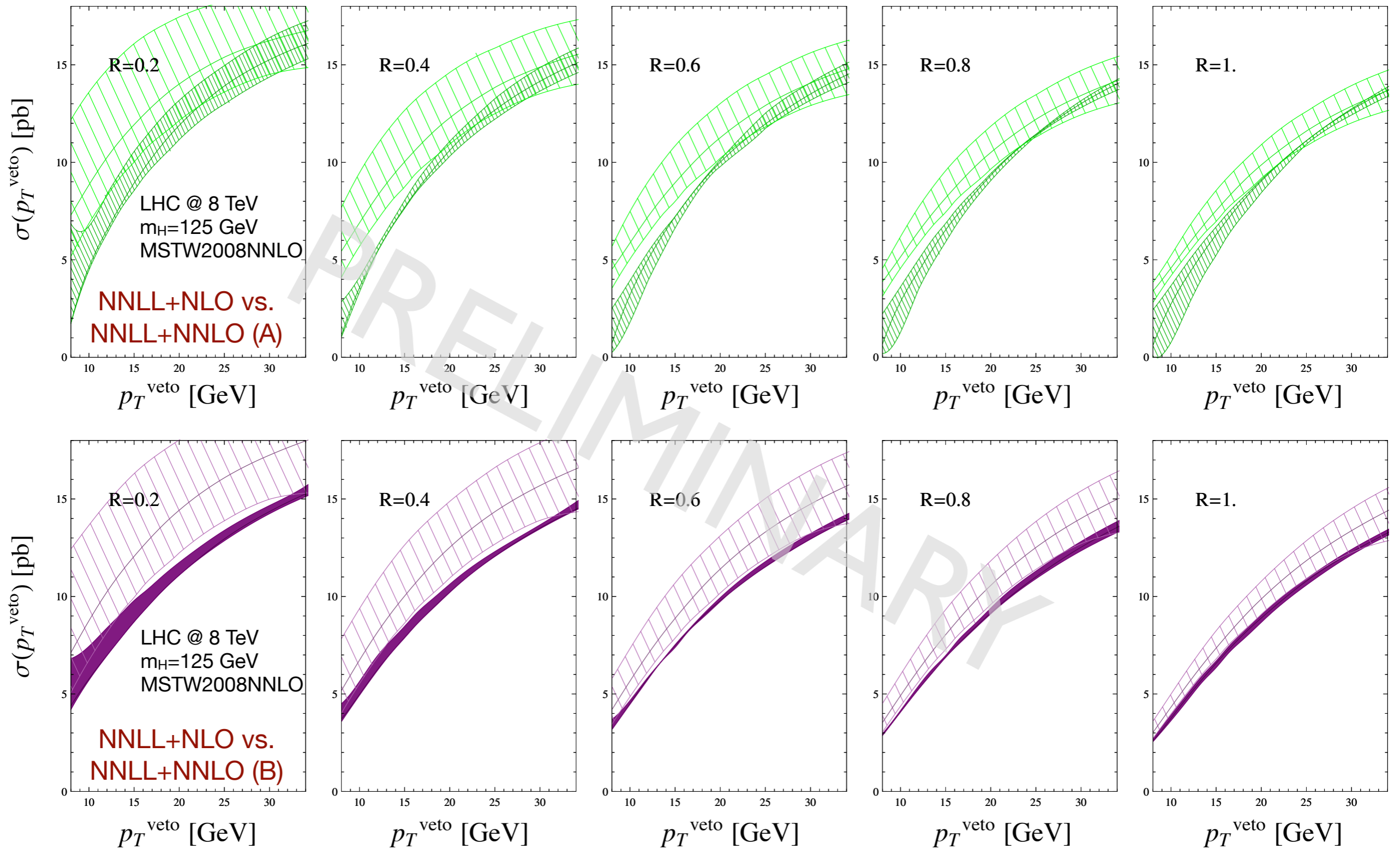
- matching corrections  $\sim (p_T^{\text{veto}}/m_H)^2$  small for low  $p_T^{\text{veto}}$  values and increase for larger ones
- scale variations under control for not too small jet radii ( $R > 0.6$ )
- larger scale dependence for smaller radii results from strong  $R$ -dependence of the two-loop anomaly coefficient  $d_2^{\text{veto}}$
- in this region, **clustering logarithms**  $\sim \alpha_s^{n+1} \ln^n R$  should be resummed

To further reduce the scale variations at small  $R$ , one should extend the analysis to **NNLL+NNLO** order (in progress)

- study NNLO matching as one important part of such an analysis



# NNLL+NNLO results



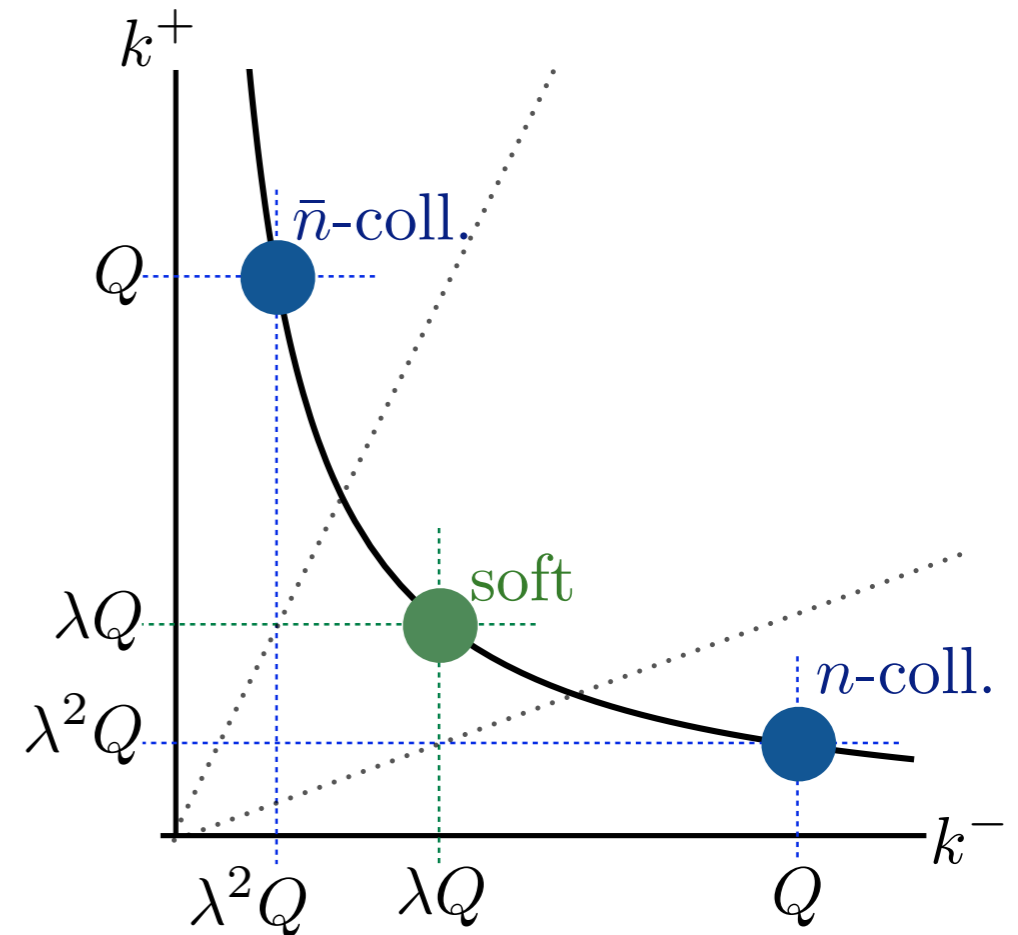
# Comparison to other work

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- NNLL formula of **Banfi, Monni, Salam, Zanderighi: 1206.4998** is fully consistent with our all-order factorization theorem; both general structure and explicit computations of ingredients agree
- **Tackmann, Walsh, Zuberi: 1206.4312 (TWZ)** use a different formalism (“rapidity renormalization group”) but obtain the same factorization formula
- however, they argued that the factorization formula is of little use, since it only holds at **parametrically small  $R \ll 1$** , where clustering logs must be resummed
- TWZ claim that **soft-collinear mixing terms** spoil factorization starting at NNLL order (later modified to “beyond NNLL order”)

# Soft-collinear mixing terms ?

- In dimensional regularization, soft and collinear contributions are integrated over full phase space
- Avoid double counting by **multi-pole expanding** the integrands, or by performing “**zero-bin**” **subtractions** of overlap contributions



We find that soft-collinear mixing contributions obtained without performing the multi-pole expansion **precisely cancel** against these zero-bin subtraction terms !

Have confirmed this by explicit two-loop calculations



# Conclusions

SCET provides efficient tools for addressing difficult collider-physics problems: systematic factorization and resummation

Many applications exist for Drell-Yan processes (production of Z, W, H bosons) and top-quark pair production

In several cases, SCET methods have pushed the limits of what has been accomplished using traditional techniques

Collinear anomaly is an important ingredient to factorization analyses for observables sensitive to transverse momenta

Have developed a consistent framework for  $q_T$  resummation and jet-veto cross sections for  $pp \rightarrow (\text{colorless bosons}) + 0 \text{ jets}$