Multi-jet production at NLO with NJET

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Outline

- Quantitative predictions need NLO accuracy
- Virtual corrections in massless QCD with NJET [https://bitbucket.org/njet/njet]
  - full colour one-loop amplitudes 2,3,4 and 5 jets
- NLO corrections to 3 and 4 jet production at the LHC
  - Validation with 4 jet production at 7 TeV
    [1st by BLACKHAT arXiv:1112.3940]
  - Full distributions for p_T and η at 8 TeV

Based on work with
Benedikt Biedermann, Peter Uwer and Valery Yundin
arXiv:1209.098, arXiv:1209.0100

NLO Ingredients

$$\sigma^{NLO} = \int_n \left(d\sigma^B + d\sigma^V\right) + \int_{n+1} d\sigma^R$$

schematically in pure QCD,

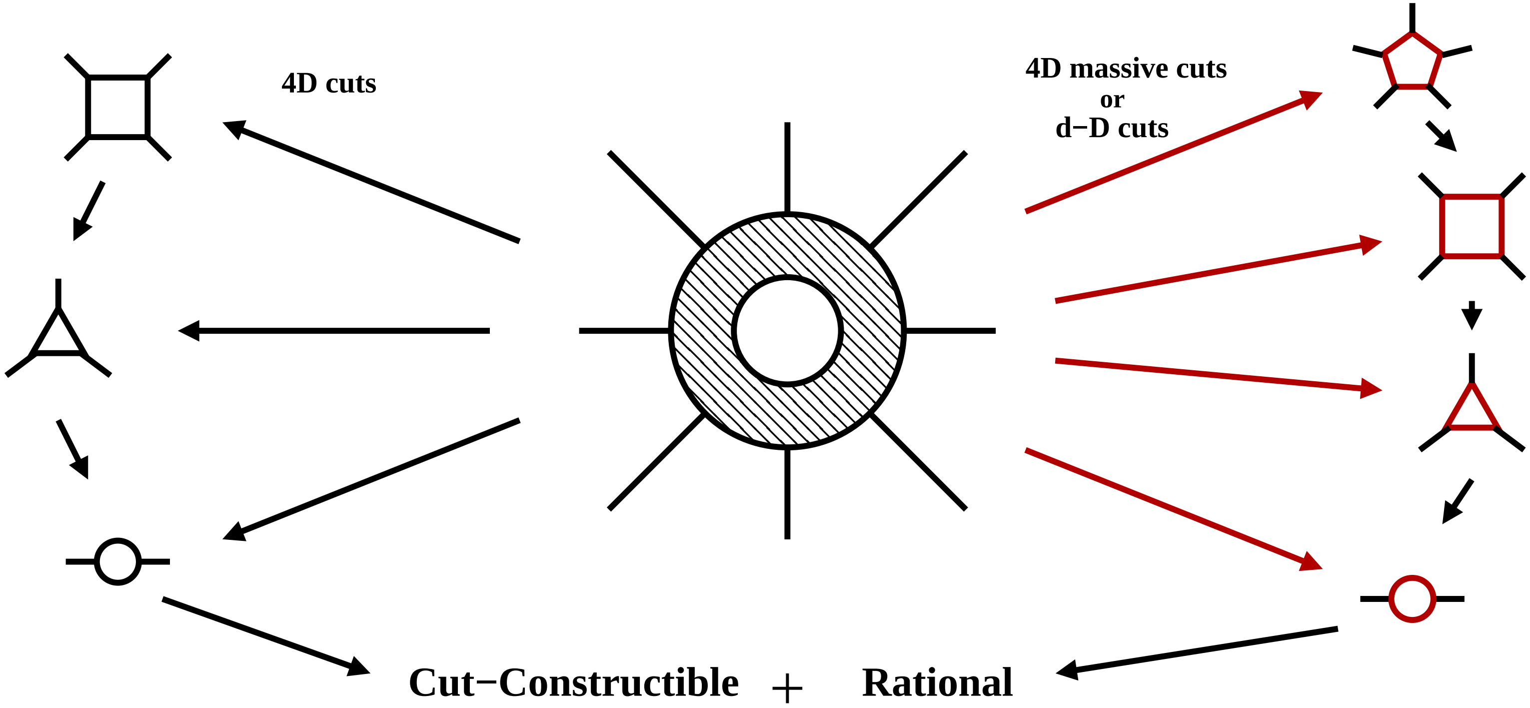
$$d\sigma^V \sim \sum_{\text{hel.}} \sum_{\text{col.}} \mathcal{A}^{\text{tree}\dagger} \mathcal{A}^{1-\text{loop}}$$
$$= \sum_{\text{hel.}} A_i^{\text{tree}\dagger} C_{ij}(N_c) A_j^{1-\text{loop}}$$

partial amplitudes

colour matrix

Generalized Unitarity

$$A^{1-\text{loop}} = \sum_k c_{4,k} I_{4,k} + \sum_k c_{3,k} I_{3,k} + \sum_k c_{2,k} I_{2,k} + R$$



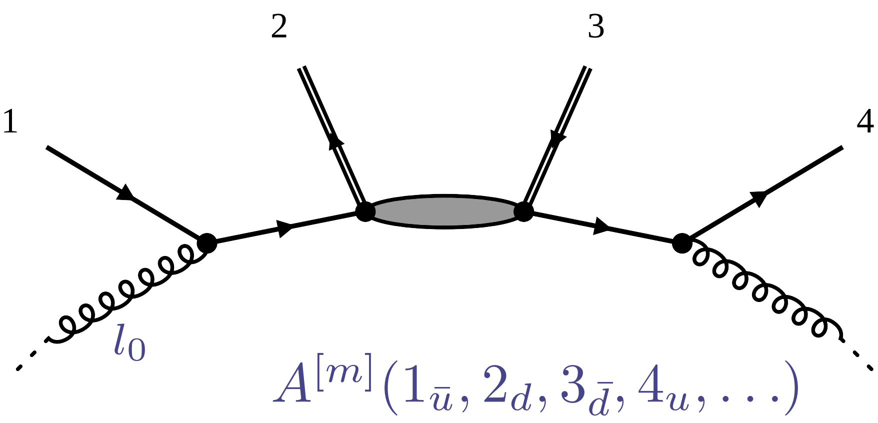
Bern, Dixon, Dunbar, Kosower, Morgan, Ossola, Papadopoulos, Pittau, Ellis, Giele, Kunszt, Melnikov, Forde, Anastasiou, Britto, Cachazo, Feng, Mastrolia, . . .

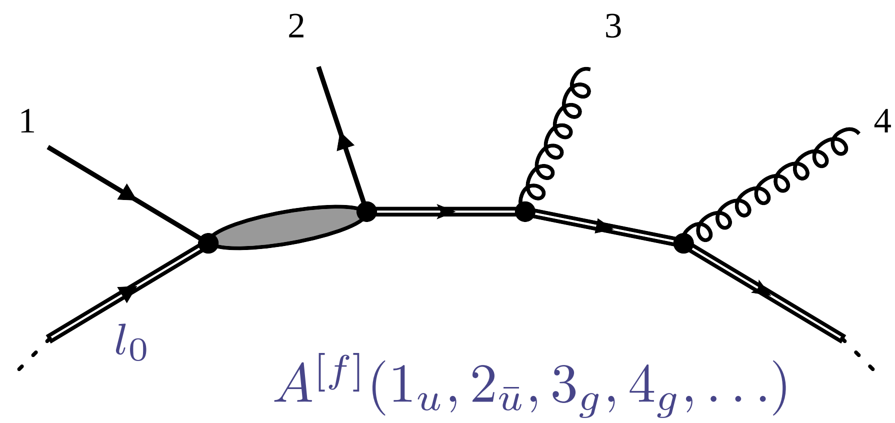
Multi-Gluon Primitive Amplitudes

- Unitarity : Re-cycle tree amplitudes into one-loop amplitudes.
  - gluonic radiation difficult with Feynman diagram approach
  - complex momenta allow us to work fully on-shell
- Basic primitive amplitudes - unique set of ordered propagator
- Berends-Giele recursion relations for tree-level input
- Rational terms from 4-D massive tree amplitudes (D-dimensional cuts)

  [SB (2009)]
- NGLUON Public C++ library for high multiplicity [SB, Biedermann, Uwer arXiv:1011.2900]
  - QD package for quadruple precision [Hida,Bailey,Li]
  - Scalar loop integrals from QCDLOOP/FF [Ellis, Zanderighi]

Multi-Fermion Primitive Amplitudes

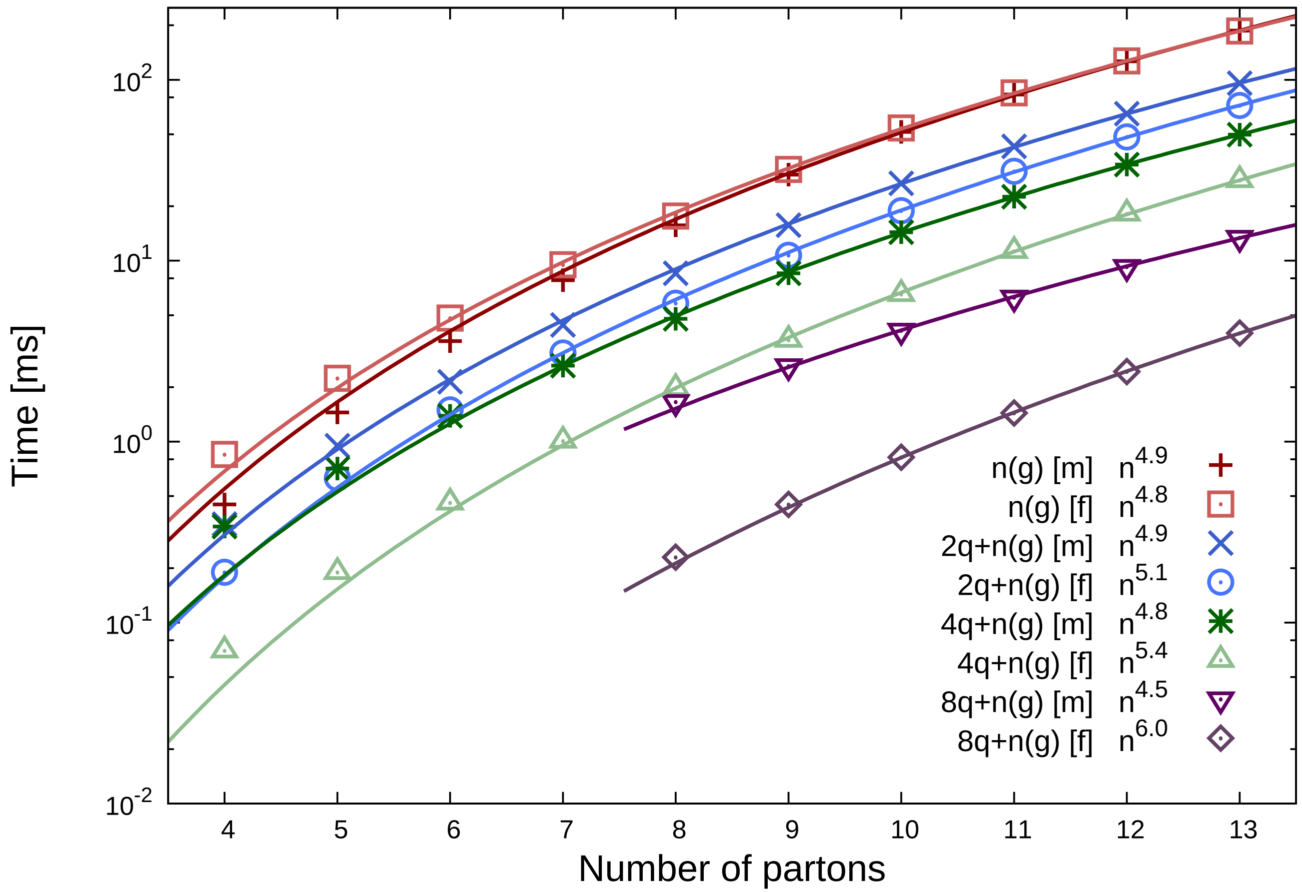




- NGLUON extended to treat arbitrary multi-fermion primitives
- Two classes :
  - Those with mixed fermion and gluon propagators (l_0 =gluon)
  - Those with internal fermion loops (l_0 =quark)

Polynomial Scaling

Primitive amplitudes have polynomial growth of computation time with number of legs



Colour sums : from primitives to partials

- Using standard colour decomposition

$$\mathcal{A}_n(p_i) = \sum_c T_c(\{a_i\}) A_{n;c}(p_1, \ldots, p_n)$$

$T_c(\{a_i\})$ - colour basis in terms of T^a, $\qquad A_{n;c}$ - partial amplitudes

- Given explicit form of $T_c(\{a_i\})$ we can compute the matrix $\mathcal{C}_{cc'}$

$$d\sigma^{\mathrm{v}} \sim \sum_{\mathrm{hel.}} A_{n;c}^{\mathrm{tree}\dagger} \mathcal{C}_{cc'} A_{n;c'}^{1-\mathrm{loop}}$$

- Partial amplitudes computed from linear combinations of primitives

$$A_{n;c}^{1-\mathrm{loop}} \sim \sum_k c_k^{[m]}(N_c) A_n^{[m]} + N_f c_k^{[f]}(N_c) A_n^{[f]}$$

- Known all multiplicity examples: [Del Duca, Dixon, Maltoni (1999)]
    - gluons - $(n-1)!/2 \times (n-2)!$
    - $q\bar{q}$ + gluons - $(n-1)! \times (n-2)!$
- Closed form for multi-quark processes unknown

Automating primitive decompositions

- Feynman diagram matching procedure [Ellis,Giele,Kunszt,Melnikov,Zanderighi] [Ita,Ozeren]
- General implementation : `qgraf+FORM+Mathematica` [arXiv:1209.0100]

$$(\text{primitive})_i = \sum_j M_{ij} \text{kinematic}(\text{diagram}_j), \qquad M_{ij} = \begin{cases} +1 & \text{even order} \\ -1 & \text{odd order} \\ 0 & \text{no order} \end{cases}$$

linear system $P_i = M_{ij} K_i$ can be solved to find

$$\mathcal{A}^{1-\text{loop}} = \sum_j \text{colour}(\text{diagram}_j) \sum_{i'} B_{ji'} (\text{primitive})_{i'}$$

as a function of independent primitives.
M_{ij} contains diagram symmetries incl. Furry's theorem.

- Tested up to 8 external legs

Automating primitive decompositions

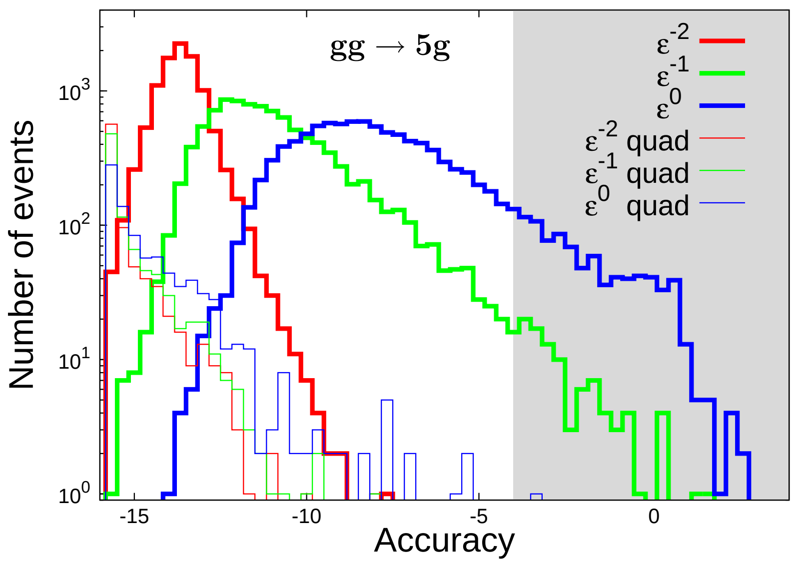
Numbers primitive amplitudes in $pp \to 2, 3, 4$ and 5 jets:

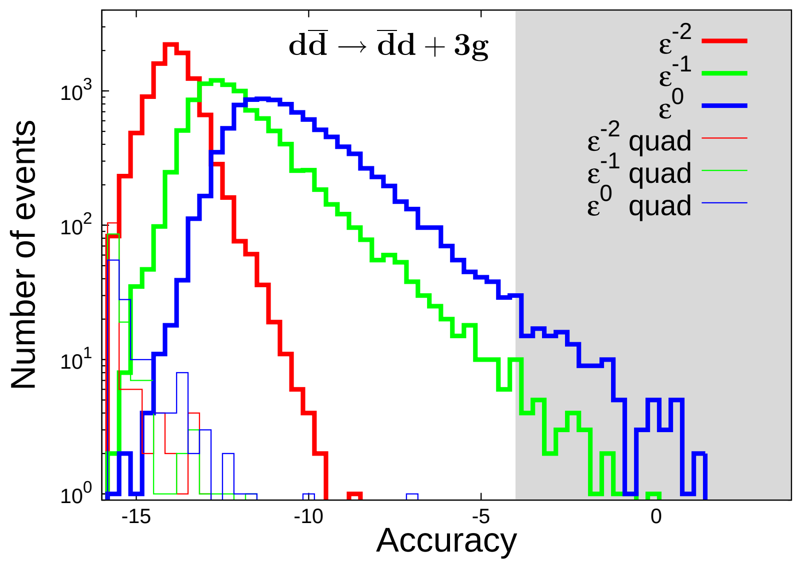
process	tree	mixed	fermion loop	process	tree	mixed	fermion loop
0q4g	2	3	3	0q5g	6	12	12
2q2g	2	6	1	2q3g	6	24	6
4q0g	1	4	1	4q1g	3	16	3
process	tree	mixed	fermion loop	process	tree	mixed	fermion loop
0q6g	24	60	60	0q7g	120	360	360
2q4g	24	120	33	2q5g	120	720	230
4q2g	12	80	13	4q3g	60	480	75
6q0g	4	32	4	6q1g	20	192	20

Accuracy tests

Estimate precision with momentum scaling test - two amplitude evaluations

$$\text{Accuracy} = \log_{10}\left(\frac{2(A_{\text{NJET}} - A^{\text{scaled}}_{\text{NJET}})}{A_{\text{NJET}} - A^{\text{scaled}}_{\text{NJET}}}\right)$$





- thick lines - single double precision
- thin lines - quadruple precision for points with accuracy < -4

Since this is a statistical test, best to play safe by a couple of digits.

Evaluation times

- Full col. and hel. sums including accuracy test [clang, intel core i7 3.33GHz]

process	T_{sd}[s]	$T_{4\text{ digits}}$[s]	(% fixed)
4g	0.030	0.030	(0.00)
2u2g	0.032	0.032	(0.00)
2u2d	0.011	0.011	(0.00)
4u	0.022	0.022	(0.00)

process	T_{sd}[s]	$T_{4\text{ digits}}$[s]	(% fixed)
5g	0.22	0.22	(0.22)
2u3g	0.34	0.35	(0.06)
2u2d1g	0.11	0.11	(0.00)
4u1g	0.22	0.22	(0.03)

process	T_{sd}[s]	$T_{4\text{ digits}}$[s]	(% fixed)
6g	6.19	6.81	(1.37)
2u4g	7.19	7.40	(0.38)
2u2d2g	2.05	2.06	(0.08)
4u2g	4.08	4.15	(0.21)
2u2d2s	0.38	0.38	(0.00)
2u4d	0.74	0.74	(0.00)
6u	2.16	2.17	(0.02)

process	T_{sd}[s]	$T_{4\text{ digits}}$[s]	(% fixed)
7g	171.3	276.7	(8.63)
2u5g	195.1	241.2	(3.25)
2u2d3g	45.7	48.8	(0.88)
4u3g	92.5	101.5	(1.29)
2u2d2s1g	7.9	8.1	(0.23)
2u4d1g	15.8	16.2	(0.29)
6u1g	47.1	48.6	(0.41)

De-symmetrizing Colour Sums

Make use of Bose symmetry in the final state phase space:

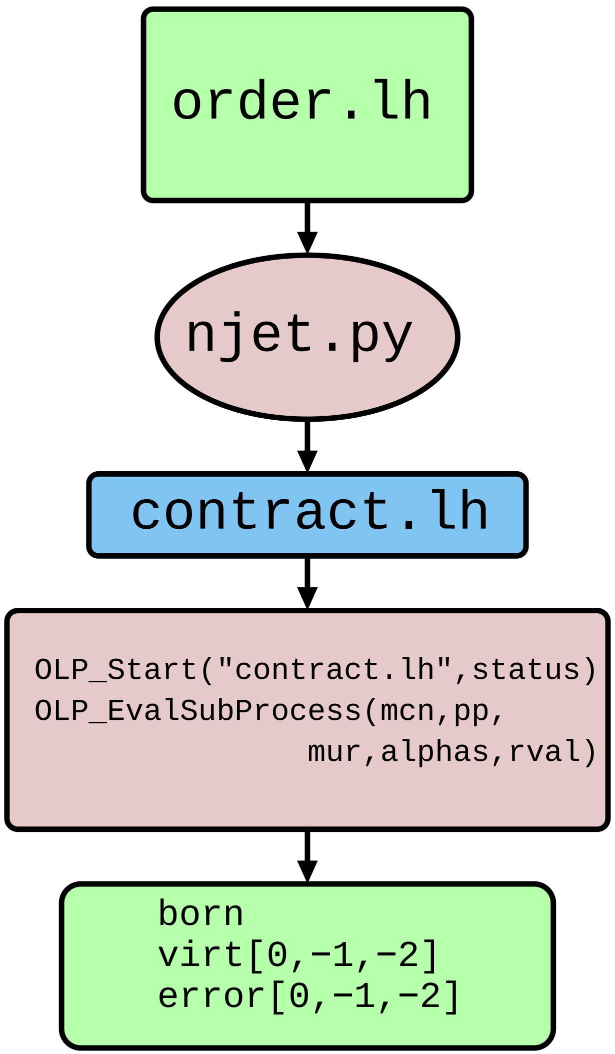
$$\sigma^{V}_{gg\to n(g)} = \int d\mathrm{LIPS}_n \sum_{c=1}^{(n-2)!} \sum_{c'=1}^{(n-1)!/2} A^{\mathrm{tree}\dagger}_{n;c} \cdot \mathcal{C}_{cc'} \cdot A^{1-\mathrm{loop}}_{n;c'}$$

$$= \frac{(n-2)!}{2} \int d\mathrm{LIPS}_n \sum_{c=1}^{(n-2)!} \sum_{c'=1}^{n-1} A^{\mathrm{tree}\dagger}_{n;c} \cdot \mathcal{C}^{\mathrm{dsym}}_{cc'} \cdot A^{1-\mathrm{loop,dsym}}_{n;c'}$$

	$gg \to 2g$	$gg \to 3g$	$gg \to 4g$	$gg \to 5g$
standard sum	0.03	0.22	6.19	171.31
de-symmetrized	0.03	0.07	0.57	3.07

Binoth Accord Interface

[Binoth et al. arXiv:1001.1307]



write order file

process with python script

create contract file, check options etc.

link to `libnjet`:
`Start` called once to initialize
`EvalSubProcess` to call amplitude

Returned values are Born, $1/\epsilon^2$, $1/\epsilon$, finite
(optionally also error estimates)

Binoth Accord Interface

order file

```
# OLE_order for 5jet production



MatrixElementSquareType  CHsummed
CorrectionType           QCD
IRregularisation         CDR
AlphasPower              5
# process list
21 21 -> 21 21 21 21 21
1 -1 -> 21 21 21 21 21
1 -1 -> 21 -2 2 21 21
1 -1 -> 21 -1 1 21 21
1 -1 -> 21 -2 2 -3 3
1 -1 -> 21 -2 2 -2 2
1 -1 -> 21 -1 1 -1 1
```

contract file

```
# OLE_order for 5jet production
# Generated file. Do not edit by hand.
# Signed by NJet 3900867518.
# 12 1 1e-05 0.01 0 1 1 1 1 0 3 5
MatrixElementSquareType CHsummed | OK
CorrectionType QCD | OK
IRregularisation CDR | OK
AlphasPower 5 | OK
# process list
21 21 -> 21 21 21 21 21 | 1 1 # 70 120 4 64 0 (-2 -1 3 4 5 6 7)
1 -1 -> 21 21 21 21 21 | 1 2 # 71 120 4 9 0 (-1 -2 3 4 5 6 7)
1 -1 -> 21 -2 2 21 21 | 1 3 # 72 6 4 9 0 (-1 -2 4 5 3 6 7)
1 -1 -> 21 -1 1 21 21 | 1 4 # 73 6 4 9 0 (-1 -2 4 5 3 6 7)
1 -1 -> 21 -2 2 -3 3 | 1 5 # 74 1 4 9 0 (-1 -2 4 5 6 7 3)
1 -1 -> 21 -2 2 -2 2 | 1 6 # 75 4 4 9 0 (-1 -2 4 5 6 7 3)
1 -1 -> 21 -1 1 -1 1 | 1 7 # 76 4 4 9 0 (-1 -2 4 5 6 7 3)
```

Additional BLHA options

- `NJetReturnAccuracy` – return accuracy estimates
- `NJetSwitchAcc` – control accuracy threshold default $= 10^{-5}$
- `NJetType` – loop(default), loopds, tree
- `NJetNf` – number of light flavours (default $= 5$)
- `NJetNc` – number of colours (default $= 3$)

Multi-jet production at NLO

Recent progress in Fixed NLO corrections:

1-loop amplitudes: $2 \to 2$ [Ellis, Sexton (1985)],[Kunszt, Signer, Trocsanyi (1994)]
$2 \to 3$ [Bern, Dixon, Kosower (1993,1994)],[Kunszt, Signer, Trocsanyi (1994)]

- $pp \to 2$ jets
  [Kunszt, Soper (1992)]
  [Giele, Glover, Kosower (1993)]
- $pp \to 3$ jets
  [only gluons Trocsanyi (2003)]
  [only gluons Giele, Kilgore (1997)]
  [Nagy NLOJET++ (2003)]
- $pp \to 4$ jets
  [Bern et al. BLACKHAT (2012)]
  [SB, Biedermann, Uwer, Yundin (2013)]

Large amounts of real radiation still make the $2 \to 4$ process challenging
On-shell approach particularly effective for the pure gluon channels

Multi-jet production at the LHC : setup

- Interface NJet to Sherpa via the BLHA
  - Amegic++ tree level matrix elements [Krauss, Kuhn, Soff (2002)]
  - Automated Catani-Seymour subtraction scheme [Gleisberg, Krauss (2008)]
  - FastJet [Caccari, Salam, Soyez]
- ATLAS multi-jet cuts [arXiv:1107.2092]

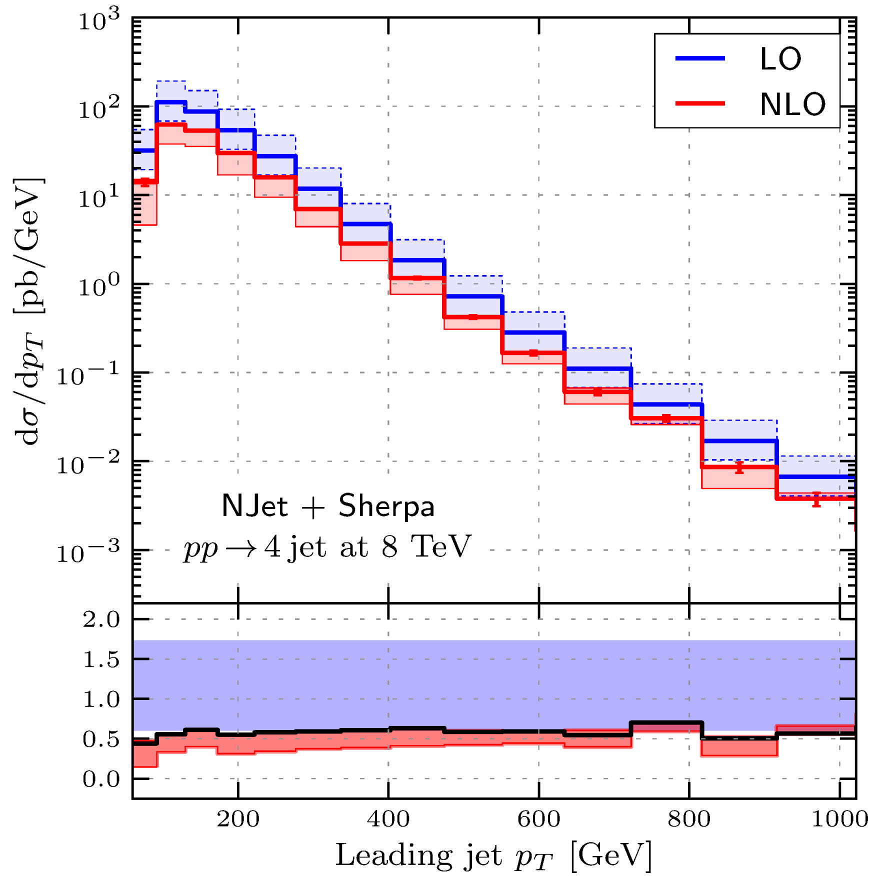
leading jet $p_T > 80$ GeV, $p_T > 60$ GeV,
$|\eta| < 2.8$, anti-kt $R = 0.4$, MSTW2008 PDFs
$\mu_R = \mu_F = \hat{H}_T/2$ (scale variations $\times 2$ – $1/2$)

- Full agreement with $\sqrt{s} = 7$ TeV results from BlackHat+Sherpa

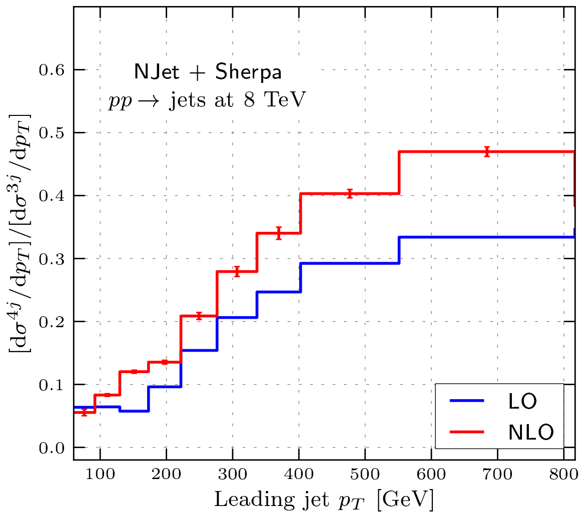
	3 jets	4 jets
σ_{LO}	$126.65(0.05)^{+66.56}_{-40.40}$ nb	$14.36(0.01)^{+10.38}_{-5.6}$ nb
σ_{NLO}	$72.57(0.16)^{+2.71}_{-28.08}$ nb	$8.15(0.09)^{+0.0}_{-3.24}$ nb
$\sigma_{8\text{ TeV}}/\sigma_{7\text{ TeV}}$	$\sim 35\%$	$\sim 46\%$

New results at
$\sqrt{s} = 8$ TeV
[arXiv:1209.098]

Multi-jet distributions

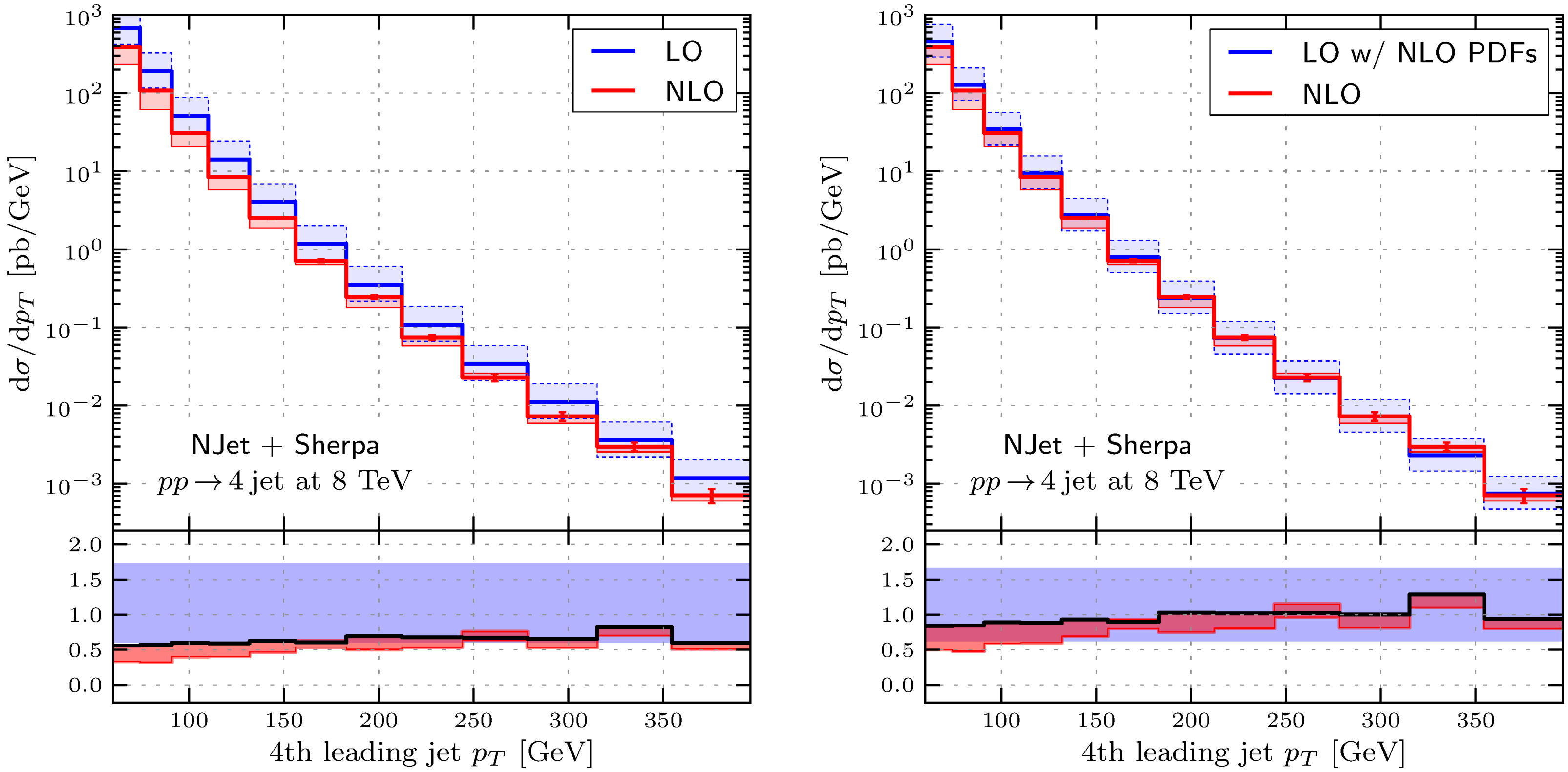


- NLO corrections $\sim 50\%$
- Reduced scale dependence
- K-factor very flat
- Central scale near top of variation band – $\hat{H}_T/2$ too high?



3/4 jet ratio increases at NLO
(except at low p_T)

Multi-jet distributions



- Large corrections from NLO α_s
  (NLO) / (Born w/ NLO pdfs) $\sim 0.2 - 0.25$
- low p_T region expected to receive large corrections from soft logs, c.f. di-jets w/ PS matching
  [POWHEG Alioli et al. (2011)]
  [MC@NLO Hoeche, Schoenherr (2012)]

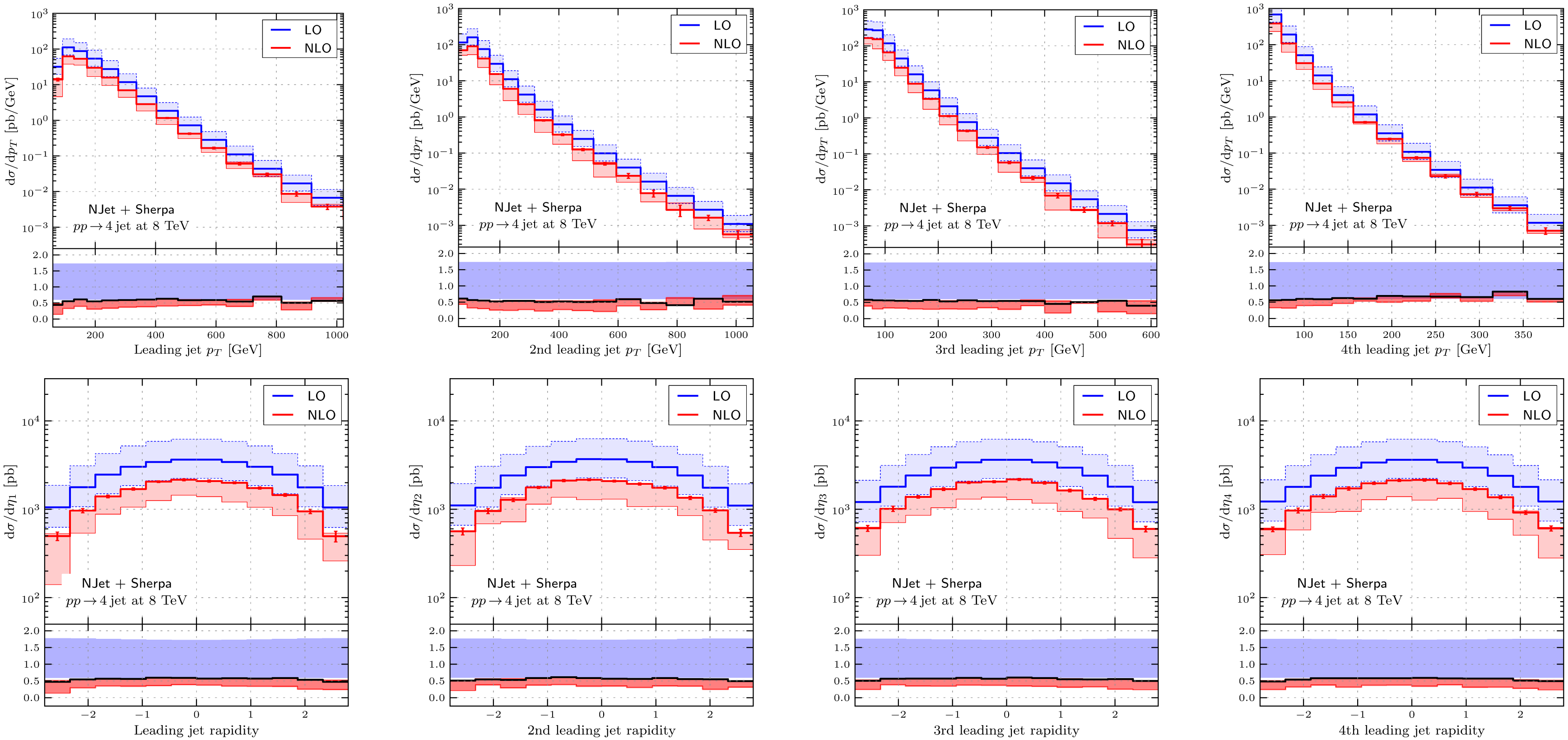
Outlook

- NJET : numerical evaluation of one-loop amplitudes in massless QCD.

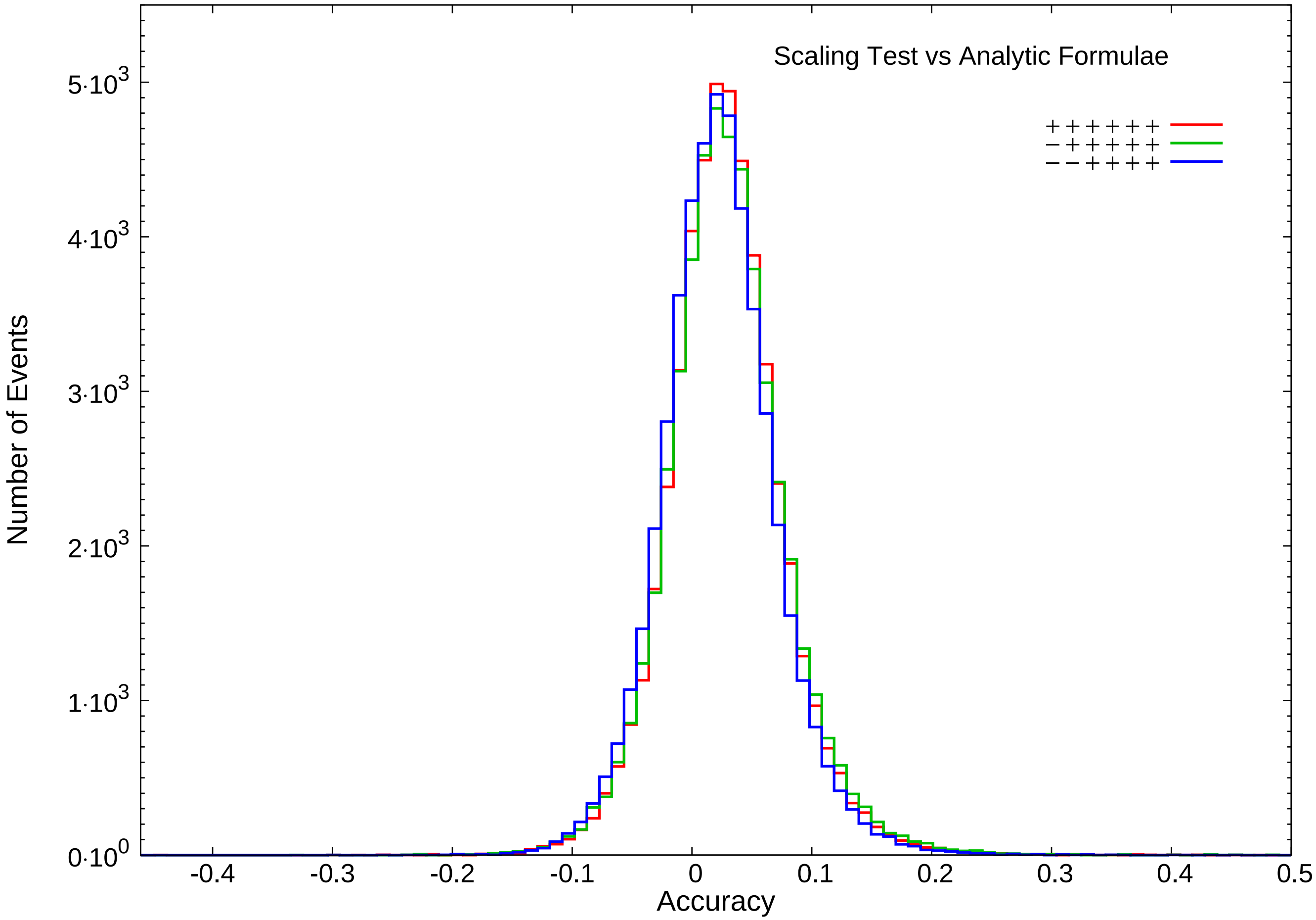
  `https://bitbucket.org/njet/njet`

- General construction for primitive and partial amplitudes.
- Fast and reliable results.
- Full colour for ≤ 5 jets.
  For more jets leading colour approx. may be necessary.
- Easy to use BLHA interface
- Phenomenological studies for 3 and 4 jet production @ LHC with NJET and SHERPA
- Real radiation for more jets will be very challenging

NLO distributions for $pp \to 4$ jets



Momentum Scaling Accuracy Tests



$$\text{Accuracy} = \log\left(\frac{A_{\text{NJET}} - A_{\text{analytic}}}{A_{\text{analytic}}}\right) - \log\left(\frac{2(A_{\text{NJET}} - A_{\text{NJET}}^{\text{scaled}})}{A_{\text{NJET}} + A_{\text{NJET}}^{\text{scaled}}}\right)$$