Higher-order corrections to inclusive Higgs production

Claude Duhr

Zurich Phenomenology Workshop, 09/01/2013

Based on work in collaboration with C. Anastasiou, S. Buehler, F. Dulat, F. Herzog and B. Mistlberger

Introduction

- ATLAS and CMS have discovered a new boson likely to be the Higgs boson.
    - ➡ Question whether this is the SM Higgs boson is still open.

- Most important Higgs production mechanism at the LHC is gluon fusion.
    - ➡ Inclusive production cross section known to NNLO in QCD.

- Next goal: improve theoretical prediction by moving to N3LO!

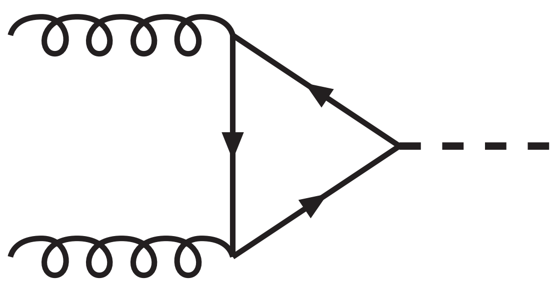
Outline

- The gluon fusion cross section at NNLO:
  - ➡ 'Reverse-unitarity' approach for inclusive phase space integrals.
- First steps towards N3LO:
  - ➡ The NNLO cross section to higher orders dimensional regularization.
  - ➡ The triple real emission contribution in the soft approximation.

The gluon fusion cross section

The Gluon fusion cross section

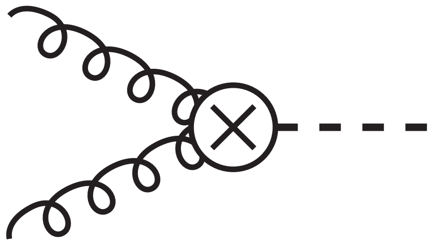
- The dominant Higgs production mechanism at the LHC is gluon fusion.
  - ➡ Loop-induced process.



- For a light Higgs boson, the top quark can be integrated out.

- As a result, we obtain a dimension five operator describing a tree-level coupling of the gluons to the Higgs boson:

$$\mathcal{L} = \mathcal{L}_{QCD,5} - \frac{1}{4v} C_1\, H\, G^a_{\mu\nu}\, G_a^{\mu\nu}$$



- In the rest of the talk, I will only concentrate on the effective theory.

The Gluon fusion cross section

- The gluon fusion cross section is given in perturbation theory by

$$\sigma(pp \to H + X) = z \sum_{ij} \left[f_i \otimes f_j \otimes (\hat{\sigma}_{ij}(x)/x) \right](z)$$

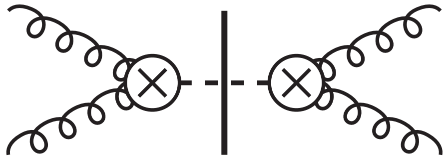
- The (partonic) cross section depends up to an overall scale only on the ratio

$$z = \frac{m^2}{s}$$

- The partonic cross section can be expanded into a perturbative series

$$\hat{\sigma}(z) = \hat{\sigma}^{LO}(z) + \alpha_s\, \hat{\sigma}^{NLO}(z) + \alpha_s^2\, \hat{\sigma}^{NNLO}(z) + \ldots$$

Higher-order computations

- LO: 

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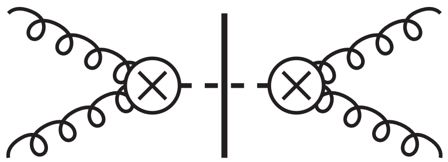
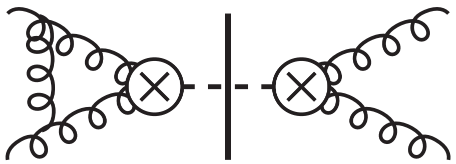
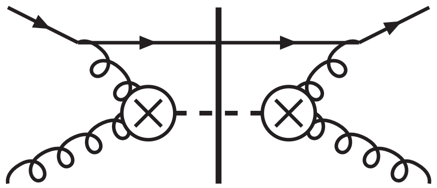
- LO: 
- NLO:  

Virtual

Real

Higher-order computations

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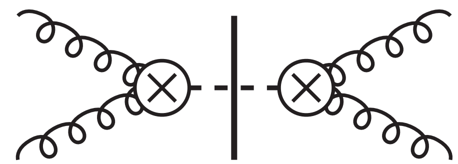

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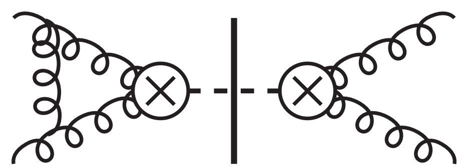

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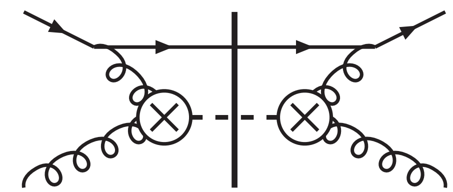

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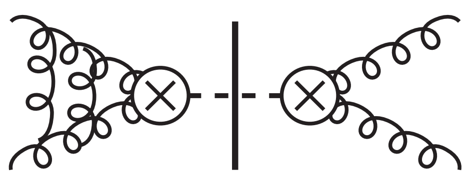
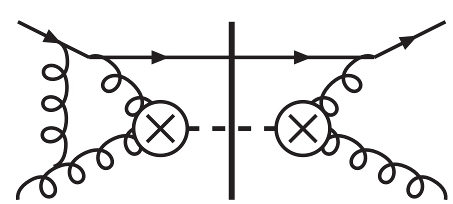


Double virtual



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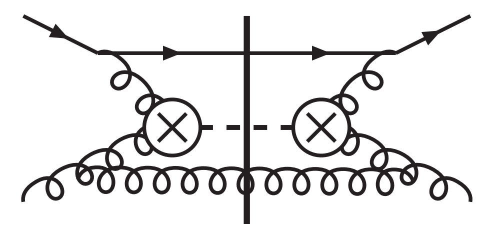

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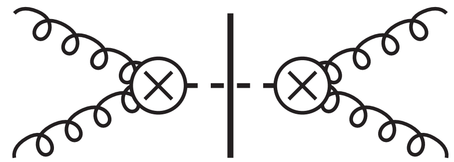

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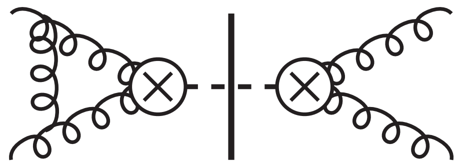

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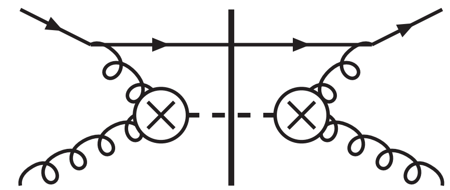

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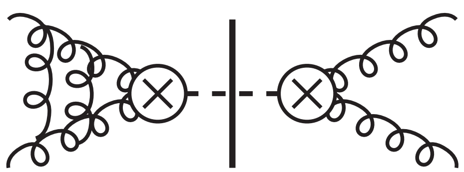

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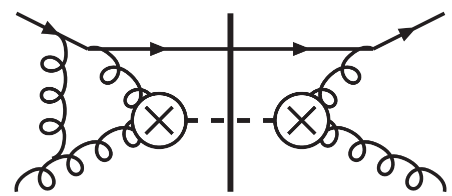

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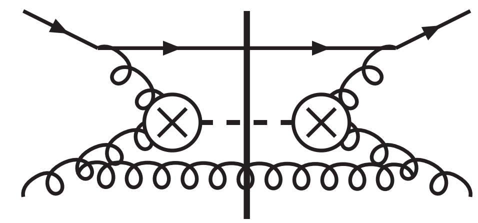
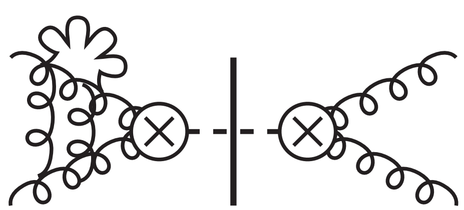
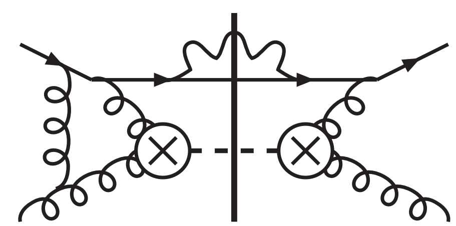


Double real

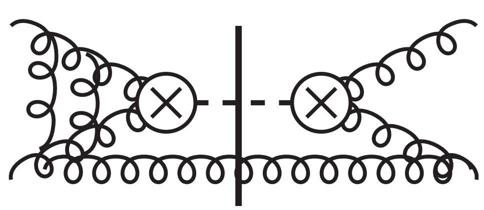
- N3LO:



Triple virtual



Real real virtual



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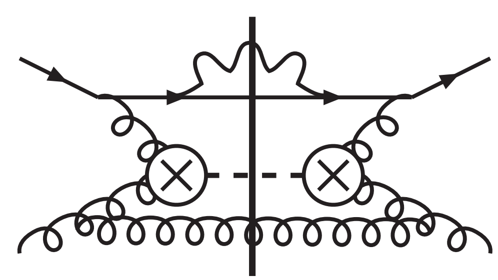
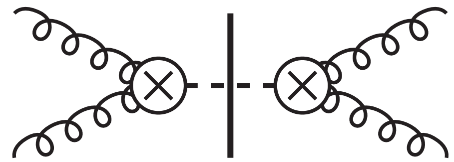
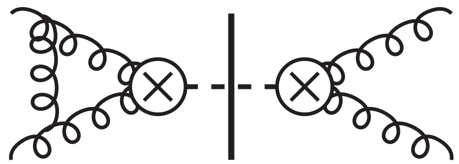


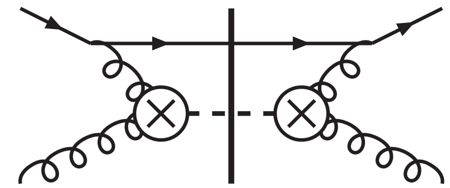
Triple real

Higher-order computations

- LO: 
- NLO:

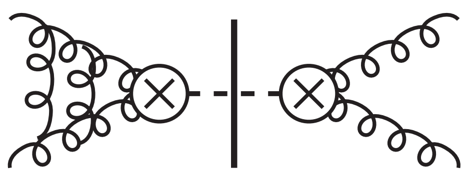


Virtual

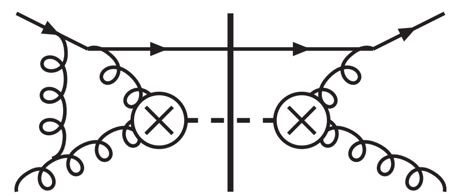


Real

- NNLO:



Double virtual



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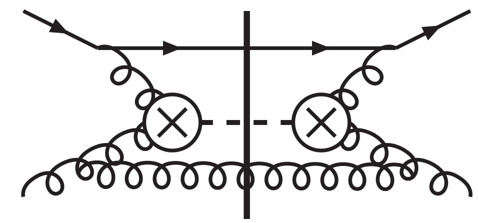
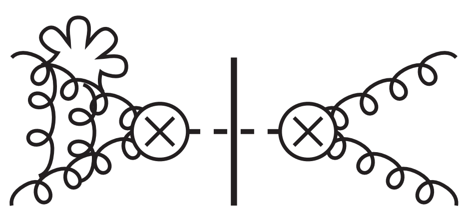
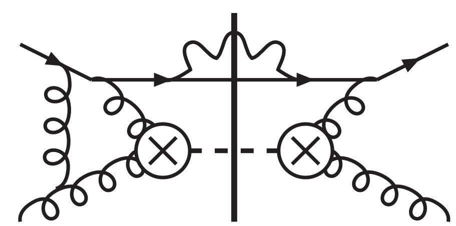


Double real

- N3LO:



Triple virtual



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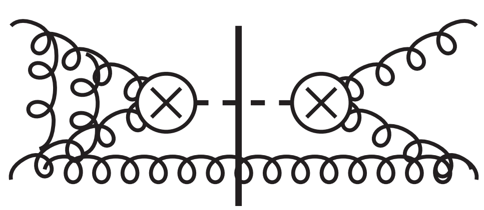

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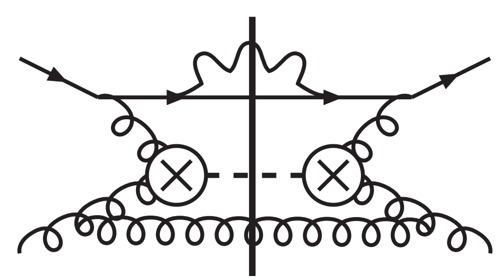
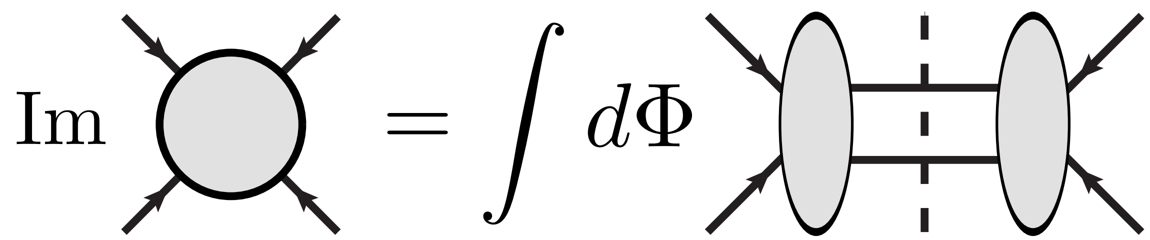


Triple real

- + convolution with PDFs. ➡ More on this later.

Unitarity

- Optical theorem:



  ➡ Discontinuities of loop amplitudes are phase space integrals.

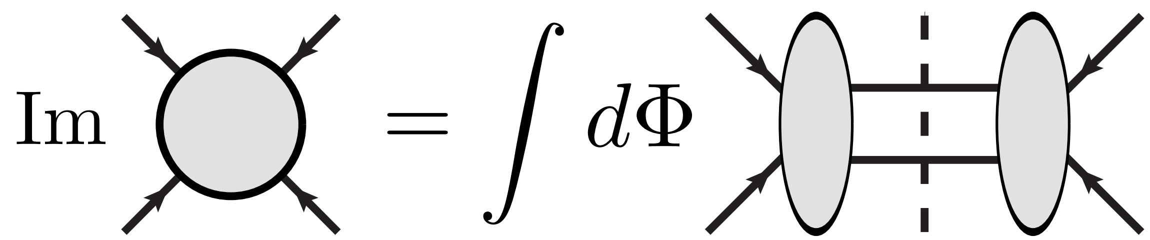
- Discontinuities of loop integrals are given by Cutkosky's rule:

$$\frac{1}{p^2 - m^2 + i\varepsilon} \to \delta_+(p^2 - m^2) = \delta(p^2 - m^2)\,\theta(p^0)$$

- These relations are at the heart to all the unitarity-based approaches to loop computations.

‘Reverse-unitarity’

- Optical theorem:



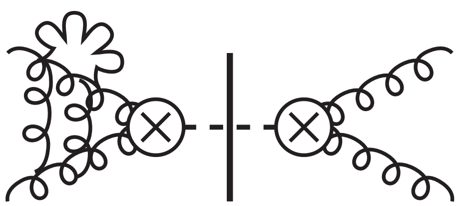
- We can read the optical theorem ‘backwards’ and write our inclusive phase space integrals as unitarity cuts of loop integrals.
  - ➡ Makes them accessible to all the technology developed for loop computations! [Anastasiou, Melnikov]
  - ➡ Integration-by-parts.
  - ➡ Master integrals.
  - ➡ Differential equations.

‘Reverse-unitarity’ @ NNLO

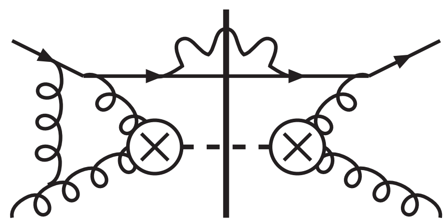
- At NNLO, the cross section can be reduced to 29 master integrals:
  - ➡ 5 double virtual integrals (~form factor). [Gonsalves; Kramer, Lampe; Gehrmann, Huber, Maître]
  - ➡ 6 real-virtual integrals.
  - ➡ 18 double real integrals. [Anastasiou, Melnikov]
- The real-virtual and double real master integrals can be evaluated analytically using differential equations.
- Evaluating all master integral up to $\mathcal{O}(\epsilon^0)$ yields the NNLO inclusive gluon fusion cross section. [Anastasiou, Melnikov]
- N.B.: The same master integrals contribute to any 2-to-1 inclusive cross section.

Towards N3LO

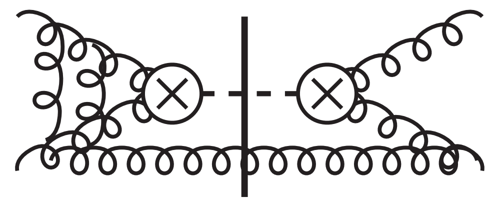
The N3LO cross section



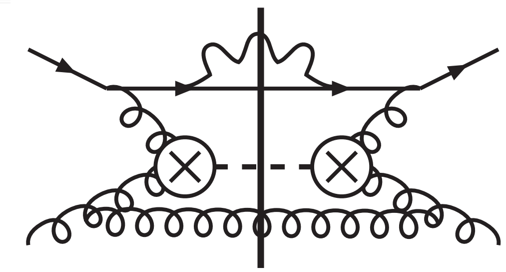
Triple virtual



Real real virtual



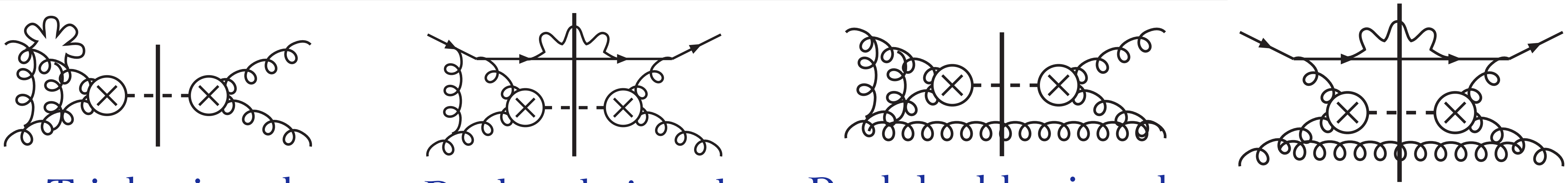
Real double virtual



Triple real

- Reverse-unitarity technique can also be used to compute N3LO cross section.
  - ➡ There are many different building blocks.
  - ➡ We are not there yet...

The N3LO cross section



Triple virtual | Real real virtual | Real double virtual | Triple real

- Reverse-unitarity technique can also be used to compute N3LO cross section.
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- Purely virtual contributions at N3LO are known.
  - ➡ 3-loop QCD form factor known. [Baikov, Chetyrkin, Smirnov, Smirnov, Steinhauser; Gehrmann, Glover, Huber, Ikizlerli, Studerus]
  - ➡ 1 & 2-loop QCD form factors known to all orders in dimensional regularization. [Gonsalves; Kramer, Lampe; Gehrmann, Huber, Maître]

The N3LO cross section

- Initial-state collinear divergences must be absorbed into PDFs.
  - ➡ Achieved by introducing a counterterm:

$$\delta\hat{\sigma}^{(3)} \sim \frac{1}{\epsilon}C_1 \times \hat{\sigma}^{(2)} + \left(\frac{1}{\epsilon^2}C_2 + \frac{1}{\epsilon}C_3\right) \times \hat{\sigma}^{(1)} + \left(\frac{1}{\epsilon^3}C_4 + \frac{1}{\epsilon^2}C_5 + \frac{1}{\epsilon}C_6\right) \times \hat{\sigma}^{(0)}$$

- Counterterms are related to splitting functions.
  - ➡ Three-loop splitting functions are known.

[Moch, Vermaseren, Vogt]

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- Pole requires the knowledge of the NNLO cross to one order higher in the dimensional regulator.
  - ➡ It is enough to solve the differential equations for the master integrals to one order higher.

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- Pole requires the knowledge of the NNLO cross to one order higher in the dimensional regulator.
  - ➡ It is enough to solve the differential equations for the master integrals to one order higher. [Pak, Rogal, Steinhauser; Anastasiou, Buehler, CD, Herzog]
- Convolution with splitting functions was recently performed. [Höschele, Hoff, Pak, Steinhauser, Ueda]

Soft limit @ NNLO

- Solving the differential equation for the master integrals requires the knowledge of an initial condition.
- Two natural candidates:
  - ➡ $z = \frac{m^2}{s} \to 0$ massless limit - changes singularity structure.
  - ➡ $z = \frac{m^2}{s} \to 1$ soft limit (threshold).
- The NNLO master integrals can be evaluated to all orders in dimensional regularization in the soft limit.

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  - ➡ $z = \frac{m^2}{s} \to 1$ soft limit (threshold).
- The NNLO master integrals can be evaluated to all orders in dimensional regularization in the soft limit.
- Surprise: 17 out of 18 double real master integrals are proportional to the phase space volume in the soft limit!

Soft limit @ NNLO

$$\mathbf{X}_i^S(z,\epsilon) = \mathbf{S}_i(z,\epsilon)\,\mathbf{X}_1^S(z,\epsilon)$$

Soft limit @ NNLO

$$\mathbf{X}_i^S(z,\epsilon) = \mathbf{S}_i(z,\epsilon)\,\mathbf{X}_1^S(z,\epsilon) \qquad \mathbf{X}_1^S(z,\epsilon) = (1-z)^{3-4\epsilon}\frac{\Gamma(2-2\epsilon)^2}{\Gamma(4-4\epsilon)}$$

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[Anastasiou, Buehler, CD, Herzog]

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[Anastasiou, Buehler, CD, Herzog]

- Standard conjectures about the structure of multiple zeta values imply that the hypergeometric function cannot be reduced to gamma functions!
- Coefficients of the soft phase space volume are reminiscent of IBP identities.

Soft limit @ NNLO

- Explanation: In the soft limit the number of real emission master integrals drops dramatically!
- Combine IBP identities with threshold expansion, and solve IBP identities only in this limit.
  - ➡ Technically speaking: expansion by regions, but only keep the leading term in the soft region.

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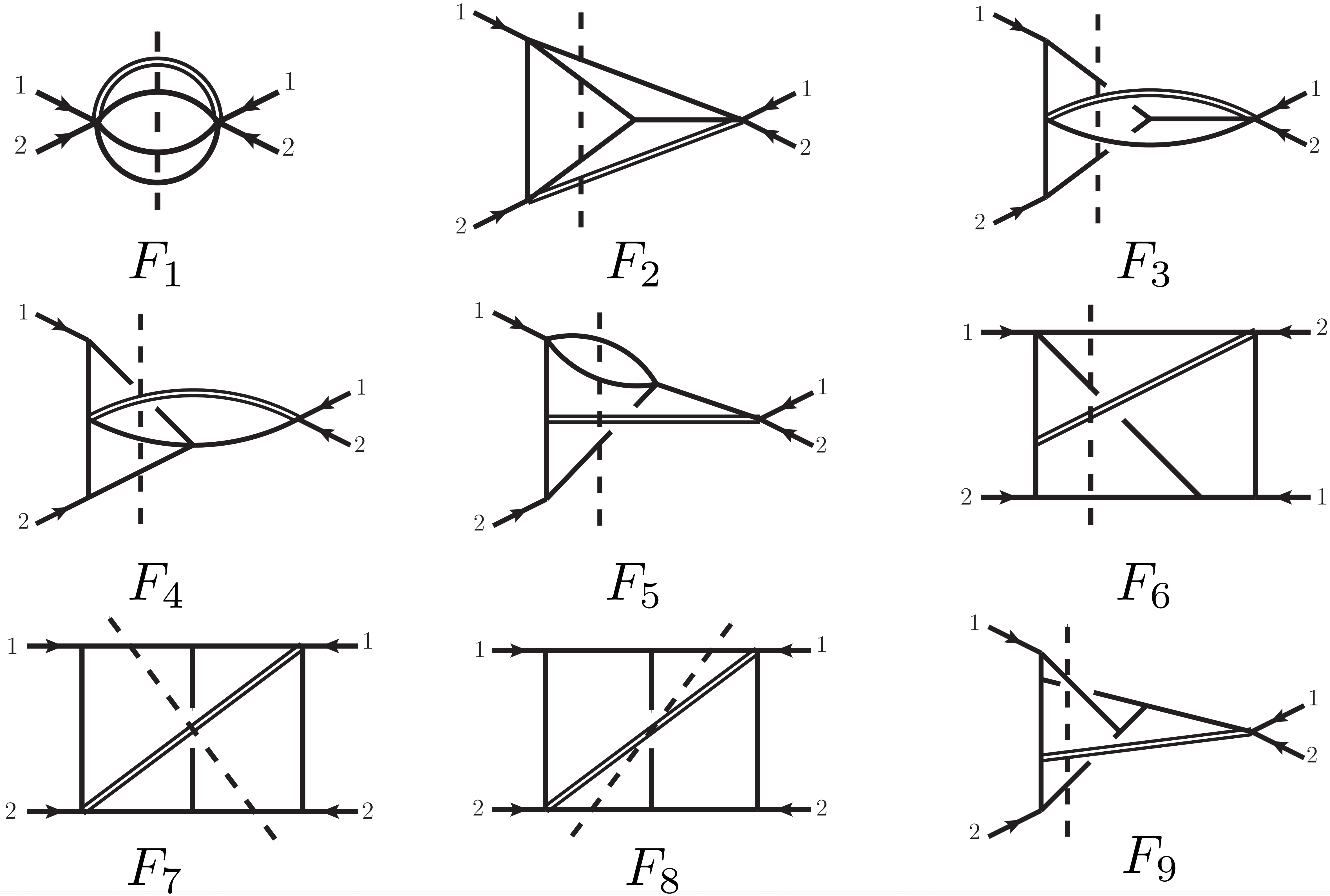
- Combine IBP identities with threshold expansion, and solve IBP identities only in this limit.
  - ➡ Technically speaking: expansion by regions, but only keep the leading term in the soft region.
- If we rerun the IBP reduction taking into account the threshold expansion, all real emission phase space integral reduce to only two master integrals.
  - ➡ The coefficients coming out of the IBP reduction are those shown in the previous slide!
- Extremely useful at N3LO, where the number of real emission master integrals is (expected to be) quite large.

Triple real soft emission

- Next step towards N3LO: The cross section in the soft limit.
  - ➡ Physically important (threshold).
  - ➡ Initial condition for differential equations away from threshold.
- We started by investigating the soft limit of the triple real phase space integrals.
- IBP reduction reveals 9 different master integrals in the soft limit.

[Anastasiou, Dulat, CD, Mistlberger]

Triple real soft emission



Triple real soft emission

- The master integrals can be computed analytically.
  - ➡ There is an algorithmic way to express the soft master integrals as angular integrals, for which a Mellin-Barnes (MB) representation can be obtained. [Van Neerven, Somogyi]

$$
\begin{aligned}
\mathcal{F}_8 \, &= \frac{\Gamma(6-6\epsilon)}{64\bar{z}^6\Gamma(1-\epsilon)^4\Gamma(-6\epsilon)} \int_{-i\infty}^{+i\infty} \frac{dz_2 dz_3 dz_4}{(2\pi i)^3} \Gamma\left(-z_2\right)\Gamma\left(-z_3\right)\Gamma\left(-z_4\right) \\
&\times \Gamma\left(z_3+1\right)\Gamma\left(z_2-2\epsilon\right)\Gamma\left(-z_2-z_4\right)\Gamma\left(z_2+z_4+1\right)\Gamma\left(-\epsilon-z_3\right)\Gamma\left(z_3-\epsilon\right) \\
&\times \frac{\Gamma\left(-2\epsilon+z_2-z_3\right)\Gamma\left(-\epsilon-z_4\right)\Gamma\left(z_4-\epsilon\right)}{\Gamma\left(-2\epsilon+z_2+1\right)\Gamma\left(-2\epsilon-z_3-z_4\right)} \, .
\end{aligned}
$$

[Anastasiou, Dulat, CD, Mistlberger]

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$$\times \Gamma(z_3+1)\Gamma(z_2-2\epsilon)\Gamma(-z_2-z_4)\Gamma(z_2+z_4+1)\Gamma(-\epsilon-z_3)\Gamma(z_3-\epsilon)$$
$$\times \frac{\Gamma(-2\epsilon+z_2-z_3)\Gamma(-\epsilon-z_4)\Gamma(z_4-\epsilon)}{\Gamma(-2\epsilon+z_2+1)\Gamma(-2\epsilon-z_3-z_4)} .$$

[Anastasiou, Dulat, CD, Mistlberger]

- An MB representation is very useful, but need to compute them analytically...

Triple real soft emission

- In some cases the MB integrations can be done in closed form, or by closing contours and summing up residues.

$$\mathcal{F}_2 = \frac{\Gamma(6-6\epsilon)\Gamma(1-2\epsilon)^2}{8\bar{z}^3\epsilon\,\Gamma(3-6\epsilon)\Gamma(2-2\epsilon)^2}\,{}_3F_2(1,1,1-\epsilon;2-2\epsilon,2-2\epsilon;1)$$

- In other cases the evaluation of the MB integrals is more 'tricky'...

$$\begin{aligned}\mathcal{F}_8 = \frac{1}{(1-z)^6}\Bigg[&-\frac{15}{16\epsilon^5}+\frac{411}{32\epsilon^4}+\frac{1}{\epsilon^3}\left(\frac{5\pi^2}{8}-\frac{2025}{32}\right)+\frac{1}{\epsilon^2}\left(\frac{75}{2}\zeta_3+\frac{2295}{16}-\frac{137\pi^2}{16}\right)\\
&+\frac{1}{\epsilon}\left(-\frac{2055}{4}\zeta_3-\frac{1215}{8}+\frac{675\pi^2}{16}+\frac{37\pi^4}{16}\right)\\
&+\frac{10125}{4}\zeta_3+\frac{35\pi^2}{4}\zeta_3+810\zeta_5+\frac{243}{4}-\frac{765\pi^2}{8}-\frac{5069\pi^4}{160}\\
&+\epsilon\left(-\frac{11475}{2}\zeta_3-\frac{959\pi^2}{8}\zeta_3+\frac{735}{4}\zeta_3^2-11097\zeta_5+\frac{405\pi^2}{4}+\frac{4995\pi^4}{32}+\frac{865\pi^6}{252}\right)+\mathcal{O}(\epsilon^2)\Bigg].\end{aligned}$$

[Anastasiou, Dulat, CD, Mistlberger]

Triple real soft emission

- We were not able to find all-order results for all the master integral, but we have obtained a Laurent series up to weight 6 for all 9 masters.

- We have reduced the tree-level amplitude H + 5g in the soft limit to a combination of the 9 master integrals.

- New non-trivial building block needed for the full N3LO evaluation of the gluon fusion cross section!

Conclusion

- The inclusive gluon fusion cross section might get within reach in the next few years!
  - ➡ Triple virtuals. [Baikov, Chetyrkin, Smirnov, Smirnov, Steinhauser; Gehrmann, Glover, Huber, Ikizlerli, Studerus]
  - ➡ NNLO cross section to higher orders. [Pak, Rogal, Steinhauser; Anastasiou, Buehler, CD, Herzog]
  - ➡ Convolution with splitting functions. [Höschele, Hoff, Pak, Steinhauser, Ueda]
  - ➡ Triple real soft emission. [Anastasiou, Dulat, CD, Mistlberger]

- Next goals:
  - ➡ Full cross section at N3LO in the soft limit.
  - ➡ Use soft limit as initial condition for general kinematics.