

Higher-order corrections to inclusive Higgs production

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Based on work in collaboration with C. Anastasiou, S. Buehler, F. Dulat, F. Herzog and B. Mistlberger

Introduction

- ATLAS and CMS have discovered a new boson likely to be the Higgs boson.
	- ➡ Question whether this is the SM Higgs boson is still open.
- Most important Higgs production mechanism at the LHC is gluon fusion.
	- ➡ Inclusive production cross section known to NNLO in QCD.
- Next goal: improve theoretical prediction by moving to N3LO!

Outline

- The gluon fusion cross section at NNLO:
	- ➡ 'Reverse-unitarity' approach for inclusive phase space integrals.
- First steps towards N3LO:
	- ➡ The NNLO cross section to higher orders dimensional regularization.
	- \rightarrow The triple real emission contribution in the soft approximation.

The gluon fusion cross section

The Gluon fusion cross section

- The dominant Higgs production mechanism σ 000 at the LHC is gluon fusion.
	- ➡ Loop-induced process.
- For a light Higgs boson, the top quark can be integrated out.
- As a result, we obtain a dimension five operator describing a tree-level coupling of the gluons to the Higgs boson:

$$
\mathcal{L} = \mathcal{L}_{QCD,5} - \frac{1}{4v} C_1 H G^a_{\mu\nu} G^{\mu\nu}_a
$$

0 0 0

In the rest of the talk, I will only concentrate on the effective theory.

The Gluon fusion cross section

• The gluon fusion cross section is given in perturbation theory by

$$
\sigma(p p \to H + X) = z \sum_{ij} [f_i \otimes f_j \otimes (\hat{\sigma}_{ij}(x)/x)](z)
$$

• The (partonic) cross section depends up to an overall scale only on the ratio

$$
z=\frac{m^2}{s}
$$

The partonic cross section can be expanded into a perturbative series

$$
\hat{\sigma}(z) = \hat{\sigma}^{LO}(z) + \alpha_s \,\hat{\sigma}^{NLO}(z) + \alpha_s^2 \,\hat{\sigma}^{NNLO}(z) + \ldots
$$

• LO:

Unitarity

• Optical theorem:

$$
\operatorname{Im} \sum \int d\Phi \prod \frac{1}{\prod_{i=1}^{n} \prod_{j=1}^{n} \prod_{j=1}^{n}
$$

- ➡ Discontinuities of loop amplitudes are phase space integrals.
- Discontinuities of loop integrals are given by Cutkosky's rule:

$$
\frac{1}{p^2 - m^2 + i\varepsilon} \to \delta_+(p^2 - m^2) = \delta(p^2 - m^2) \,\theta(p^0)
$$

• These relations are at the heart to all the unitarity-based approaches to loop computations.

'Reverse-unitarity'

• Optical theorem:

= z
Z ${\rm Im} \left(\begin{array}{c} \end{array} \right) = \int d\Phi$

- We can read the optical theorem 'backwards' and write our inclusive phase space integrals as unitarity cuts of loop integrals.
	- \rightarrow Makes them accessible to all the technology developed for loop computations! [Anastasiou, Melnikov]
	- ➡ Integration-by-parts.
	- Master integrals.
	- ➡ Differential equations.

'Reverse-unitarity' @ NNLO

- At NNLO, the cross section can be reduced to 29 master integrals:
	- ➡ 5 double virtual integrals (~form factor). [Gonsalves; Kramer, Lampe; Gehrmann, Huber, Maître]

[Anastasiou, Melnikov]

- \rightarrow 6 real-virtual integrals.
- \rightarrow 18 double real integrals.

• The real-virtual and double real master integrals can be evaluated analytically using differential equations.

- [Anastasiou, Melnikov] • Evaluating all master integral up to $\mathcal{O}(\epsilon^0)$ yields the NNLO inclusive gluon fusion cross section.
- N.B.: The same master integrals contribute to any 2-to-1 inclusive cross section.

Towards N3LO

The N3LO cross section

- Figue virtual and the Reverse-unitarity technique can also be used to compute N3LO cross section.
	- \rightarrow There are many different building blocks.
	- \blacktriangleright We are not there yet...

The N3LO cross section

- Figure 1 Reverse-unitarity technique can also be used to compute N3LO cross section.
	- \rightarrow There are many different building blocks.
	- \blacktriangleright We are not there yet...
- Purely virtual contributions at N3LO are known. ➡ 3-loop QCD form factor known. [Baikov, Chetyrkin, Smirnov, Smirnov, Steinhauser; Gehrmann, Glover, Huber, Ikizlerli, Studerus] ➡ 1 & 2-loop QCD form factors known to all orders in dimensional regularization. [Gonsalves; Kramer, Lampe; Gehrmann, Huber, Maître]

The N3LO cross section bation theory, the individual pieces contributing to a given loop order are divergent. Final state infrared (IR) divergences can interestions, the real and virtual corrections, in the real and virtual co whereas ultraviolet (UV) and initial-state initial-state IR divergences have to be dealth with by replacing th
In the divergences have to be dealth by replacing the divergence of the divergence of the state of the diverge

the bare coupling and PDF's by the boundary renormalized counterparts, which respects the in-BDF \sim

• Initial-state collinear divergences must be absorbed into PDFs. α Initial-state collinear divergences must be absorbed into $\mathbf{p}_{\mathbf{D}}$ denotes the '-th order correction to the '-th order correction' points section, the N3LO UV/PDF counterterms section, the N3LO UV/PDF counterterms of the N3LO UV/PDF counterterms of the N3LO UV/PDF counterterms of the N3LO

 \bullet Counterterms are related to splitting functions. [Moch, → Achieved by introducing a counterterm: [Moch, Vermaseren, Vogt] $\delta \hat{\sigma}^{(3)} \sim$ 1 ϵ $C_1 \times \hat{\sigma}^{(2)}$ + $\left(\begin{array}{c} 1 \end{array} \right)$ $\frac{1}{\epsilon^2}C_2 +$ 1 ϵ C_{3})
) $\times \hat{\sigma}^{(1)} +$ $\left(\begin{array}{c} 1 \end{array} \right)$ $\frac{1}{\epsilon^3}C_4 +$ 1 $\frac{1}{\epsilon^2}C_5 +$ 1 ϵ C_{6})
) $\times \hat{\sigma}^{(0)}$, (2.7) Γ Three loop splitting functions are known $Vermaseren, Vogt$ section from the the section of ϵ and ➡ Three-loop splitting functions are known.

The aim of this paper is to provide the NNLO master is to provide the NNLO master integrals which are required

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- $\frac{1}{2}$ requires the knowledge of the NNI Ω gross to one • Pole requires the knowledge of the NNLO cross to one order inglier in the differential regulator. [Pak Rogal corrections] order higher in the dimensional regulator.
	- \rightarrow It is enough to solve the differential equations Steinhauser; for the master integrals to one order higher. [Pak, Rogal, Steinhauser; Anastasiou, Buehler, CD, Herzog]

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- \rightarrow It is enough to solve the differential equations Steinhauser; for the master integrals to one order higher. Anastasiou, Buehler, [Pak, Rogal, Steinhauser; CD, Herzog] • Convolution with splitting functions was recently
performed. [Höschele, Hoff, Pak, S [Höschele, Hoff, Pak, Steinhauser, Ueda]

Soft limit @ NNLO

- Solving the differential equation for the master integrals requires the knowledge of an initial condition.
- **•** Two natural candidates:
	- $\Rightarrow z = \frac{m}{2} \rightarrow 0$ massless limit changes singularity structure. m^2 *s* $\rightarrow 0$ $z =$ m^2 *s* $z = \frac{m}{s} \rightarrow 1$ soft limit (threshold).
- The NNLO master integrals can be evaluated to all orders in dimensional regularization in the soft limit.

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- The NNLO master integrals can be evaluated to all orders in dimensional regularization in the soft limit.
	- Surprise: 17 out of 18 double real master integrals are proportional to the phase space volume in the soft limit!

Soft limit @ NNLO Soft limit @ NNLO

 $\mathbf{X}_i^S(z,\epsilon) = \mathbf{S}_i(z,\epsilon) \, \mathbf{X}_1^S(z,\epsilon) \, .$

 $S_{3/2}$

^S4(z, !) = ^S5(z, !) = [−]² (1 [−] ²!) (3 [−] ⁴!) (1 [−] ⁴!)

 $S_{\rm eff}$

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Soft limit @ NNLO $\overline{}$ $\lim_{x \to 0}$ MNLO Ω Ω \mathbb{P} Ω orders in Ω POIT HUIL CASSESS

 $\mathbf{X}_i^S(z,\epsilon) = \mathbf{S}_i(z,\epsilon) \, \mathbf{X}_1^S(z,\epsilon) \qquad \qquad \mathbf{X}_1^S(z,\epsilon) = (1-z)^{3-4\epsilon} \frac{\mathbf{1} \, \mathbf{1} \, \mathbf{1} \, \mathbf{1} }{\Gamma(z)}$ $_{3-4\epsilon}\frac{\Gamma(2-2\epsilon)^2}{}$ $\frac{1}{\Gamma(4-4\epsilon)}$

the constant of proportionality being a rational function of $\mathcal{L}_\mathcal{P}$ and $\mathcal{L}_\mathcal{P}$

S3(z, 2)
S3(z, 2) = S9(z, 2) = S10(z, 3) = S10(z, 3)
S10(z, 3) = S10(z, 3)

^S4(z, !) = ^S5(z, !) = [−]² (1 [−] ²!) (3 [−] ⁴!) (1 [−] ⁴!)

In this normalization the results for the soft limits of the master integrals read

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space volume,

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Soft limit @ NNLO $\overline{}$ $\lim_{x \to 0}$ MNLO Ω Ω \mathbb{P} Ω orders in Ω POIT HUIL CASSESS S_{Ω} the limit ω NNL Ω $\sum_{i=1}^n$ \mathcal{L} \mathcal{L} \mathcal{L} \mathcal{L} \mathcal{L} \mathcal{L} \mathcal{L} \mathcal{L}

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\n[Anastasiou, Buchler, CD, Herzog]

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imply that the hypergeometric function cannot be reduced to S gamma functions!
Coefficients of the soft phase space volume are reminiscent of eometric function cannot be reduc ⁴ , (4.41) ^S12(z, !) = ¹ $\frac{1}{2}$ $\frac{1}{2}$ ● Standard conjectures about the structure of multiple zeta values imply that the hypergeometric function cannot be reduced to gamma functions!
	- Coefficients of the soft phase space volume are remin

	BP identities. !
3(1 − z) − z) − z) − z
2(1 − z) − z) − z) − z) − z) − z) − z ^S4(z, !) = ^S5(z, !) = [−]² (1 [−] ²!) (3 [−] ⁴!) (1 [−] ⁴!) :
3(1 ≤ z) = z) = z) = z $\frac{4}{3}$ e soft pl nues. ² , (4.42) iniscent of ^S13(z, !) = [−] ³ [−] ⁴! $\overline{}$ ² , (4.42) $\mathbf{1}$, 2 − 2
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(2 − 2 − 2) , (4.43) and • Coefficients of the soft phase space volume are reminiscent of IBP identities.

Soft limit @ NNLO

- Explanation: In the soft limit the number of real emission master integrals drops dramatically!
- Combine IBP identities with threshold expansion, and solve IBP identities only in this limit.
	- ➡ Technically speaking: expansion by regions, but only keep the leading term in the soft region.

Soft limit @ NNLO

- Explanation: In the soft limit the number of real emission master integrals drops dramatically!
- Combine IBP identities with threshold expansion, and solve IBP identities only in this limit.
	- ➡ Technically speaking: expansion by regions, but only keep the leading term in the soft region.
- If we rerun the IBP reduction taking into account the threshold expansion, all real emission phase space integral reduce to only two master integrals.
	- ➡ The coefficients coming out of the IBP reduction are those shown in the previous slide!
- Extremely useful at N3LO, where the number of real emission master integrals is (expected to be) quite large.

Triple real soft emission

- Next step towards N3LO: The cross section in the soft limit.
	- ➡ Physically important (threshold).
	- ➡ Initial condition for differential equations away from threshold.
- We started by investigating the soft limit of the triple real phase space integrals.
- IBP reduction reveals 9 different master integrals in the soft
limit. [Anastasiou, Dulat, CD, Mistlberger]

Triple real soft emission

Triple real soft emission \blacksquare 2 + 1; 1 + \blacksquare

• The master integrals can be computed analytically. egrals can be com $\frac{1}{2}$ ate " analytic $\overline{11}$ \cdot any

 $\frac{1}{2}$ order to proceed, we first apply the identity the identity of $\frac{1}{2}$

 \rightarrow There is an algorithmic way to express the soft master integrals as angular integrals, for which a Mellin-Barnes (MB) representation can be obtained. (TID) (TID) first, legislation can be obtained. [Van Neerven, Somogyi]

(11.2)

$$
\mathcal{F}_8 = \frac{\Gamma(6-6\epsilon)}{64\bar{z}^6\Gamma(1-\epsilon)^4\Gamma(-6\epsilon)} \int_{-i\infty}^{+i\infty} \frac{dz_2 dz_3 dz_4}{(2\pi i)^3} \Gamma(-z_2) \Gamma(-z_3) \Gamma(-z_4)
$$

× $\Gamma(z_3+1) \Gamma(z_2-2\epsilon) \Gamma(-z_2-z_4) \Gamma(z_2+z_4+1) \Gamma(-\epsilon-z_3) \Gamma(z_3-\epsilon)$
× $\frac{\Gamma(-2\epsilon+z_2-z_3) \Gamma(-\epsilon-z_4) \Gamma(z_4-\epsilon)}{\Gamma(-2\epsilon+z_2+1) \Gamma(-2\epsilon-z_3-z_4)}$
[Anastasiou, Dulat, CD, Mistlberger]

integral. We proceed in the standard way and resolve singularities in !. At the end of

this procedure, we have a collection of MB integrals of dimensionality at most three with

integration contours that are straight vertical lines. These integrals can then be safely

expanded in the integration sign. In the integration sign. In the following we discuss the following we discuss
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Triple real soft emission \blacksquare 2 + 1; 1 + \blacksquare

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(11.3)

$$
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× $\Gamma(z_3 + 1) \Gamma(z_2 - 2\epsilon) \Gamma(-z_2 - z_4) \Gamma(z_2 + z_4 + 1) \Gamma(-\epsilon - z_3) \Gamma(z_3 - \epsilon)$
× $\frac{\Gamma(-2\epsilon + z_2 - z_3) \Gamma(-\epsilon - z_4) \Gamma(z_4 - \epsilon)}{\Gamma(-2\epsilon + z_2 + 1) \Gamma(-2\epsilon - z_3 - z_4)}.$
[Anastasiou, Dulat, CD, Mistlberger]

 $I = I$ • An MB representation is very useful, but need to compute them analytically...

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expanded in the integration sign. In the integration sign. In the following we discuss the following we discuss
In the following we discuss the following we discuss the computation of the following we discuss the following

Triple real soft emission $\begin{array}{ccc} \n\frac{1}{2} & \frac{1}{2} & \frac{$ $\mathcal{L}(\mathcal{$ × ple real soft emission Γ SUIT CHIISSIUII

 \bullet In some cases the MB integrations can be done in closed form, or by closing contours and summing up residues. some cases the two integrations can be done in closed form, by closing contours and summing u

$$
\mathcal{F}_2 = \frac{\Gamma(6-6\epsilon)\Gamma(1-2\epsilon)^2}{8\bar{z}^3\epsilon\,\Gamma(3-6\epsilon)\Gamma(2-2\epsilon)^2} \, {}_3F_2(1,1,1-\epsilon;2-2\epsilon,2-2\epsilon;1)
$$

● In other cases the evaluation of the MB integrals is more 'tricky'... " the eval diudli
... \overline{m} \overline{a} or \overline{a} e MB inte Leg dis is. \overline{m} $\frac{3}{5}$ $\frac{1}{1}$ μ behave the transmitted all the two integrals is more

$$
\mathcal{F}_8 = \frac{1}{(1-z)^6} \left[-\frac{15}{16\epsilon^5} + \frac{411}{32\epsilon^4} + \frac{1}{\epsilon^3} \left(\frac{5\pi^2}{8} - \frac{2025}{32} \right) + \frac{1}{\epsilon^2} \left(\frac{75}{2}\zeta_3 + \frac{2295}{16} - \frac{137\pi^2}{16} \right) \right.
$$

+ $\frac{1}{\epsilon} \left(-\frac{2055}{4}\zeta_3 - \frac{1215}{8} + \frac{675\pi^2}{16} + \frac{37\pi^4}{16} \right)$
+ $\frac{10125}{4}\zeta_3 + \frac{35\pi^2}{4}\zeta_3 + 810\zeta_5 + \frac{243}{4} - \frac{765\pi^2}{8} - \frac{5069\pi^4}{160}$
+ $\epsilon \left(-\frac{11475}{2}\zeta_3 - \frac{959\pi^2}{8}\zeta_3 + \frac{735}{4}\zeta_3^2 - 11097\zeta_5 + \frac{405\pi^2}{4} + \frac{4995\pi^4}{32} + \frac{865\pi^6}{252} \right) + \mathcal{O}(\epsilon^2) \bigg].$
[Anastasiou, Dulat, CD, Mistlberger]

.

Triple real soft emission

• We were not able to find all-order results for all the master integral, but we have obtained a Laurent series up to weight 6 for all 9 masters.

• We have reduced the tree-level amplitude $H + 5g$ in the soft limit to a combination of the 9 master integrals.

• New non-trivial building block needed for the full N3LO evaluation of the gluon fusion cross section!

Conclusion

- The inclusive gluon fusion cross section might get within reach in the next few years!
	- \rightarrow Triple virtuals.

[Baikov, Chetyrkin, Smirnov, Smirnov, Steinhauser; Gehrmann, Glover, Huber, Ikizlerli, Studerus]

- [Pak, Rogal, Steinhauser; → NNLO cross section to higher orders. Anastasiou, Buehler, CD, Herzog]
- ➡ Convolution with splitting functions.

[Höschele, Hoff, Pak, Steinhauser, Ueda]

➡ Triple real soft emission.

[Anastasiou, Dulat, CD, Mistlberger]

Next goals:

- \rightarrow Full cross section at N3LO in the soft limit.
- \rightarrow Use soft limit as initial condition for general kinematics.