MERGING H/W/Z + 0 AND 1 JET AT NLO

WITH NO MERGING SCALE

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NLO with multiple scales

- From the solution of the renormalization group equation for α_s

$$\alpha_s(\mu_{\rm R}^2) = \alpha_s(\mu_{\rm R}'^2) - b_0 \log \frac{\mu_{\rm R}^2}{\mu_{\rm R}'^2} \, \alpha_s^2(\mu_{\rm R}'^2) \ldots$$

- it follows that the formal structure of a NLO differential cross section is given by

$$\frac{d\sigma}{d\Phi} = \alpha_s^N(\mu_{\rm R}^2)\, B + \alpha_s^{N+1}(\mu_{\rm R}^2) \left[V + N\, b_0 \, \log \frac{\mu_{\rm R}^2}{Q^2}\, B \right] + \alpha_s^{N+1}(\mu_{\rm R}^2)\, R$$

- Should we choose to evaluate the N coupling constants in the Born term at different scales $\{\mu_i\}$, as in the matrix-element parton-shower merging algorithms, in order for NLO scale compensation to take place we must generalize to

$$\frac{d\sigma}{d\Phi} = \prod_{i=1}^{N} \alpha_s(\mu_i^2)\, B + \alpha_s^{N+1}(\mu_{\rm R}') \left[V + b_0 \sum_{i=1}^{N} \log \frac{\mu_i^2}{Q^2}\, B \right] + \alpha_s^{N+1}(\mu_{\rm R}'')\, R$$

  where the scales $\mu_{\rm R}'$ and $\mu_{\rm R}''$ are irrelevant from the point of view of scale compensation since

$$\alpha_s(\mu_{\rm R}) - \alpha_s(\mu_{\rm R}') = \mathcal{O}(\alpha_s^2)$$

- The renormalization scale $\mu_{\rm R}$ that appears in the virtual term can then be set to

$$\mu_{\rm R} = \left(\prod_{i=1}^{N} \mu_i \right)^{\frac{1}{N}}$$

MINLO+NLO

- ✓ MINLO: Multi-scale Improved NLO (Hamilton, Nason, Zanderighi, arXiv:1206.3572)
- ✓ The purpose of MINLO is to improve the NLO computation of inclusive quantities when regions of the phase space with widely different scales are approached.
- ✓ The MINLO procedure has been inspired by the CKKW method. It achieves its goals by:
  - recursively clustering all the coloured partons in the event using the k_T-clustering algorithm, in order to reconstruct the most likely branching history
  - at each of the vertexes of the branching history, it assigns a nodal scale q_i, equal to the relative transverse momentum at which the clustering has taken place

$$q_1 \leq q_2 \leq q_3 \leq \ldots \leq Q$$

    and use q_i as renormalization scale to compute the value of α_s at that vertex.
  - If the event is a real contribution, the first merging scale is called q_0

MINLO+NLO

- Assign an appropriate Sudakov form factor to all external and to all intermediate lines

$$\Delta_f(Q, q_T) = \exp\left\{-\int_{q_T^2}^{Q^2} \frac{dq^2}{q^2} \alpha_s(q^2) \left[A_1 \log\frac{Q^2}{q^2} + B_1\right]\right\} \qquad f = q, g$$

  It exponentiates large logarithms present in the fixed NLO cross section
- Its expansion is given by

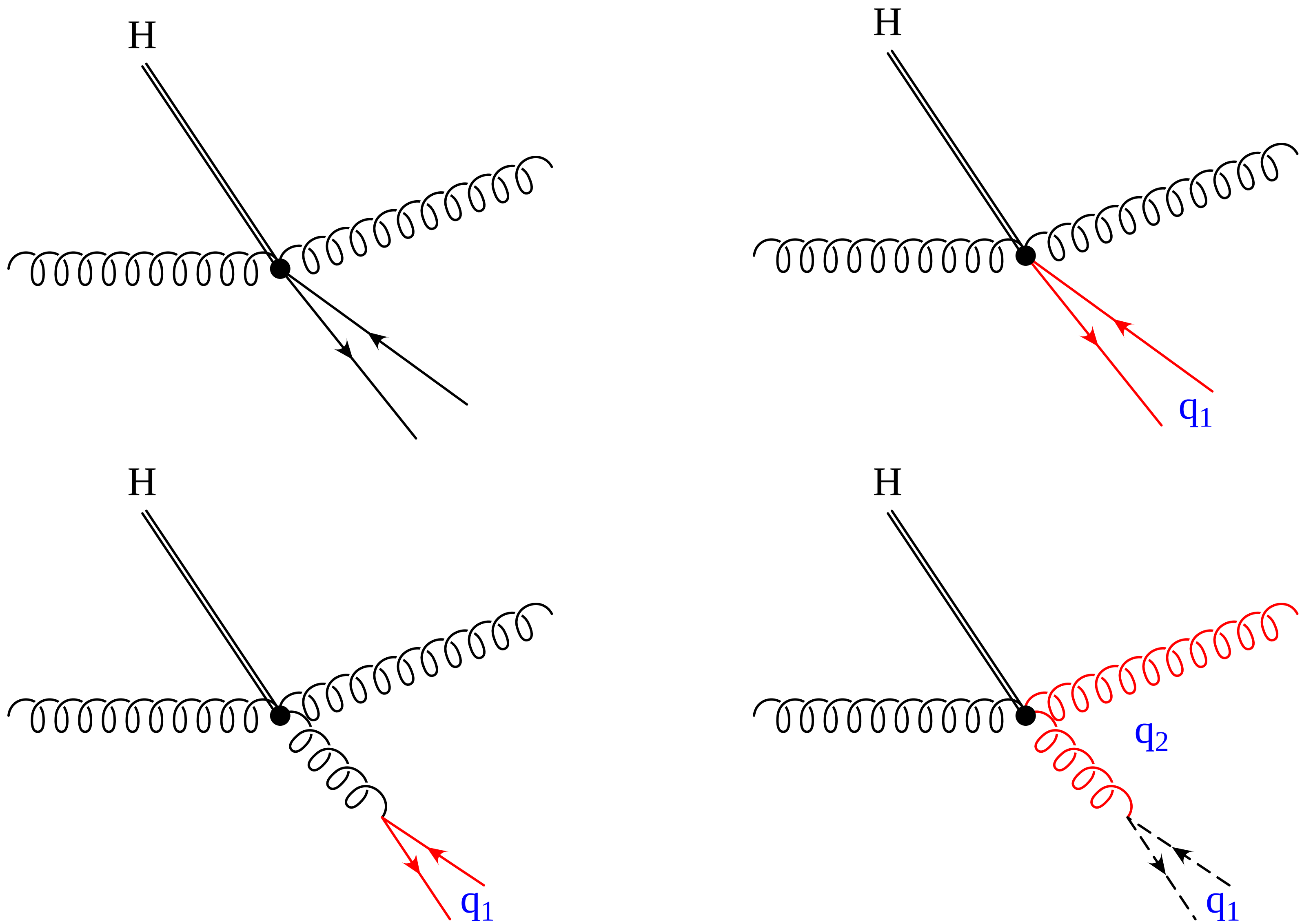
$$\Delta_f(Q, q_T) = 1 + \alpha_s \left[-\frac{1}{2} A_1 \log^2 \frac{q_T^2}{Q^2} + B_1 \log\frac{q_T^2}{Q^2}\right] + \mathcal{O}(\alpha_s^2)$$

- At NLO, a Sudakov form factor contributes with a term proportional to the Born

$$B\, \alpha_s \left[-\frac{1}{2} A_1 \log^2 \frac{q'^2}{q^2} + B_1 \log\frac{q'^2}{q^2}\right]$$

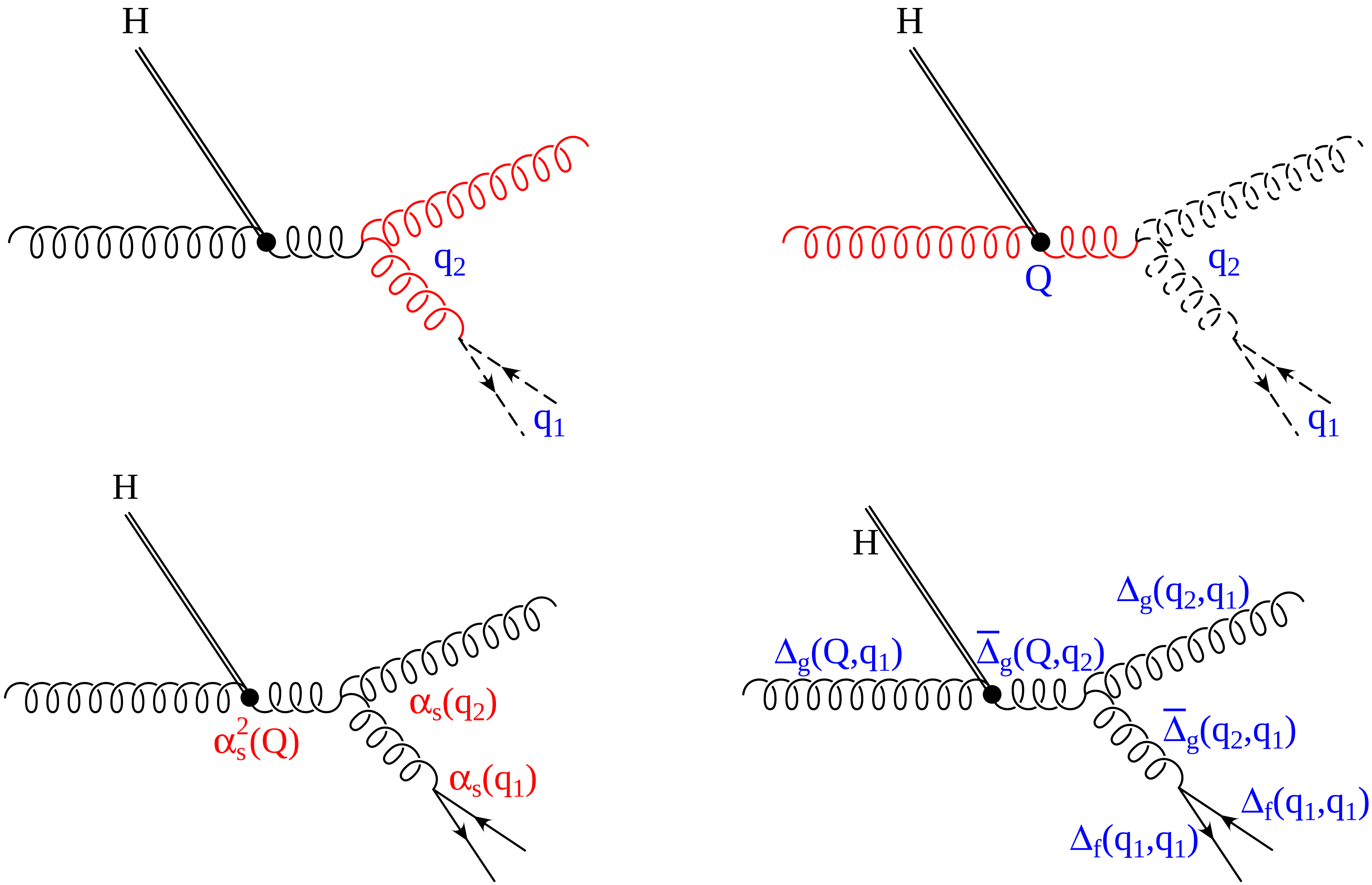
  that need to be subtracted from the exact NLO differential cross section, since the NLO differential cross already contains such logarithmic contributions
- set the factorization scale to q_1
- some degree of freedom is left for the value of the $(N+1)^{\text{th}}$ power of α_s in the real, in the virtual and in the expansion of the Sudakov form factor

MINLO+NLO



Not Feynman diagrams, but the most likely branching history: $q_1 \leq q_2$

MINLO+NLO



$$q_1 \leq q_2 \leq Q$$

$$\overline{\Delta}(q_i, q_j) \equiv \frac{\Delta(q_1, q_i)}{\Delta(q_1, q_j)}$$

MINLO+NLO

- ✓ The full result has formal NLO accuracy, therefore the scale variation around the central values is formally of NNLO order
- ✓ The accuracy and the smooth behaviour near the Sudakov regions is comparable to that of the corresponding tree-level calculation in the adopted CKKW scheme
- ✓ The procedure is simple and easily implemented for any NLO parton level generator, requiring only minor work on top of the NLO calculation available.

MINLO+POWHEG

- ✓ The (simplified) POWHEG cross section is given by

$$d\sigma = \overline{B}(\boldsymbol{\Phi}_n)\left\{\Delta\left(p_T^{min}\right) + \Delta\left(p_\mathrm{T}\right)\frac{R(\boldsymbol{\Phi}_{n+1})}{B(\boldsymbol{\Phi}_n)}\,d\Phi_r\right\}d\boldsymbol{\Phi}_n$$

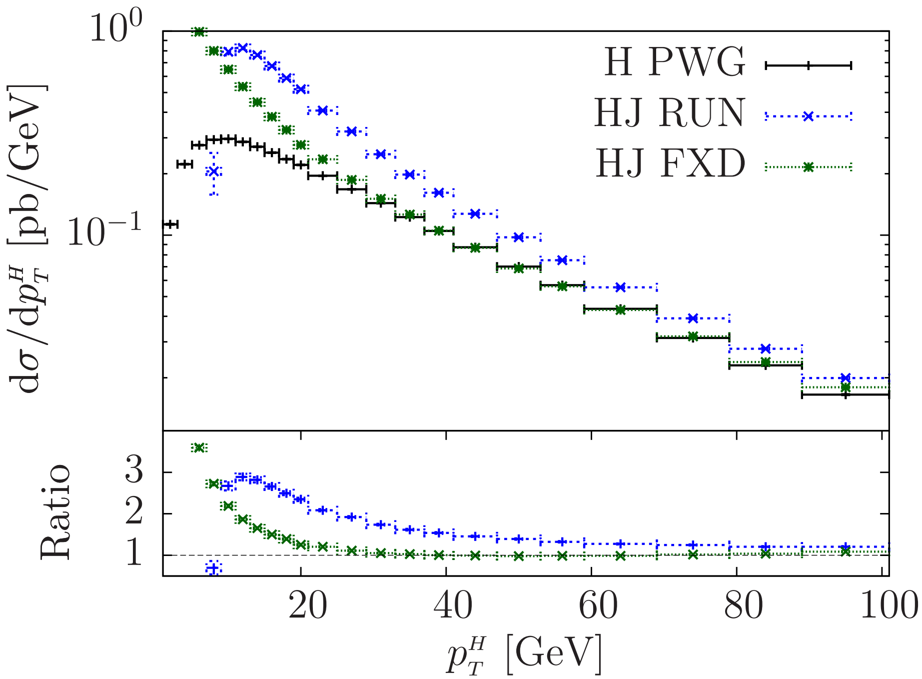
$$\overline{B}(\boldsymbol{\Phi}_n) = B(\boldsymbol{\Phi}_n) + V(\boldsymbol{\Phi}_n) + \int d\Phi_r R(\boldsymbol{\Phi}_n, \Phi_r)$$

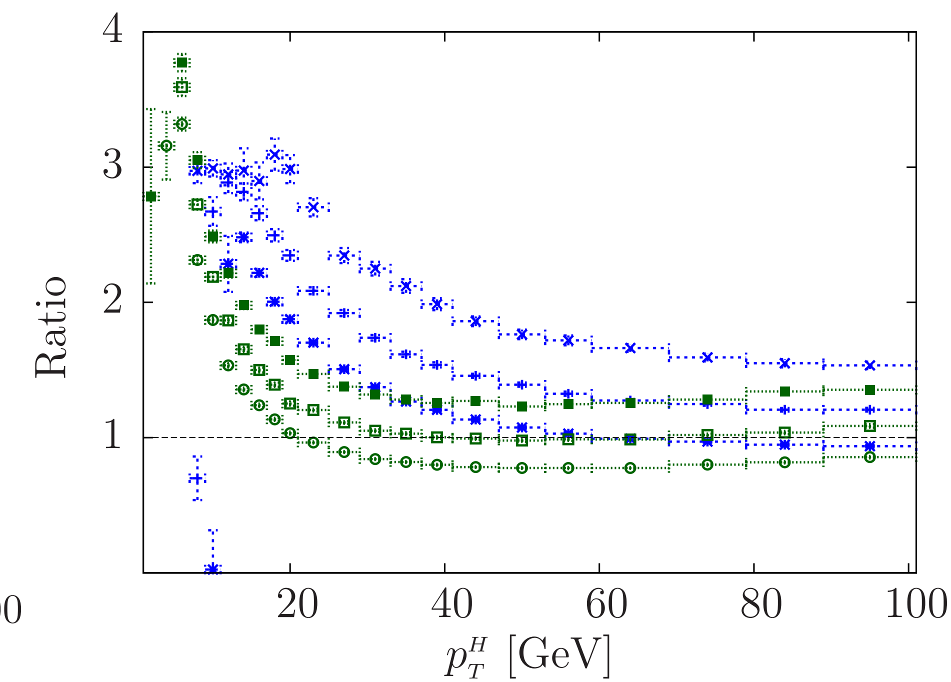
$$\Delta(p_\mathrm{T}) = \exp\left[-\int d\Phi'_r\,\frac{R(\boldsymbol{\Phi}_n, \Phi'_r)}{B(\boldsymbol{\Phi}_n)}\,\theta\left(p'_\mathrm{T} - p_\mathrm{T}\right)\right]$$

- ✓ The underlying Born kinematics is generated with a probability proportional to the NLO inclusive cross section (the $\overline{B}$ term), at a given point in the Born phase space $\boldsymbol{\Phi}_n$
- ✓ The radiation jet is already accompanied by its Sudakov form factor
- ✓ We can then improve the POWHEG formula by implementing MINLO on the inclusive $\overline{B}$ function
- ✓ The MINLO procedure has been implemented and made public in the POWHEG BOX and can be found at:

`svn://powhegbox.mib.infn.it/trunk/POWHEG-BOX/MINLO`

HJ NLO vs H POWHEG





- H PWG: the POWHEG BOX H generator interfaced with PYTHIA, $m_{\mathrm{H}} = 120$ GeV
- HJ RUN: $H + 1$ jet NLO calculation with $\mu_{\mathrm{R}} = \mu_{\mathrm{F}} = p_{\mathrm{T}}^{\mathrm{H}}$
- HJ FXD: $H + 1$ jet NLO calculation with $\mu_{\mathrm{R}} = \mu_{\mathrm{F}} = m_{\mathrm{H}}$
- Ratios over H PWG

- Error bands obtained varying μ_{R} and μ_{F} by a factor of 2 above and below their common central value, with the constraint $1/2 \le \mu_{\mathrm{R}}/\mu_{\mathrm{F}} \le 2$
- Bands do not overlap at $p_{\mathrm{T}}^{\mathrm{H}} \lesssim 30$ GeV
- NLO shapes differ from LL resummed result (POWHEG) at small $p_{\mathrm{T}}^{\mathrm{H}}$

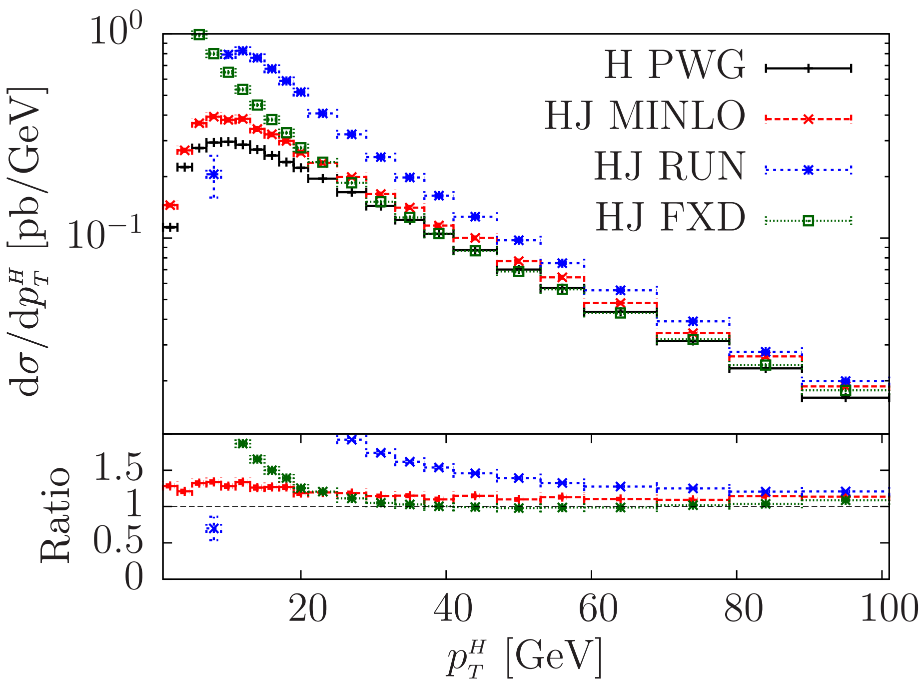
MINLO HJ

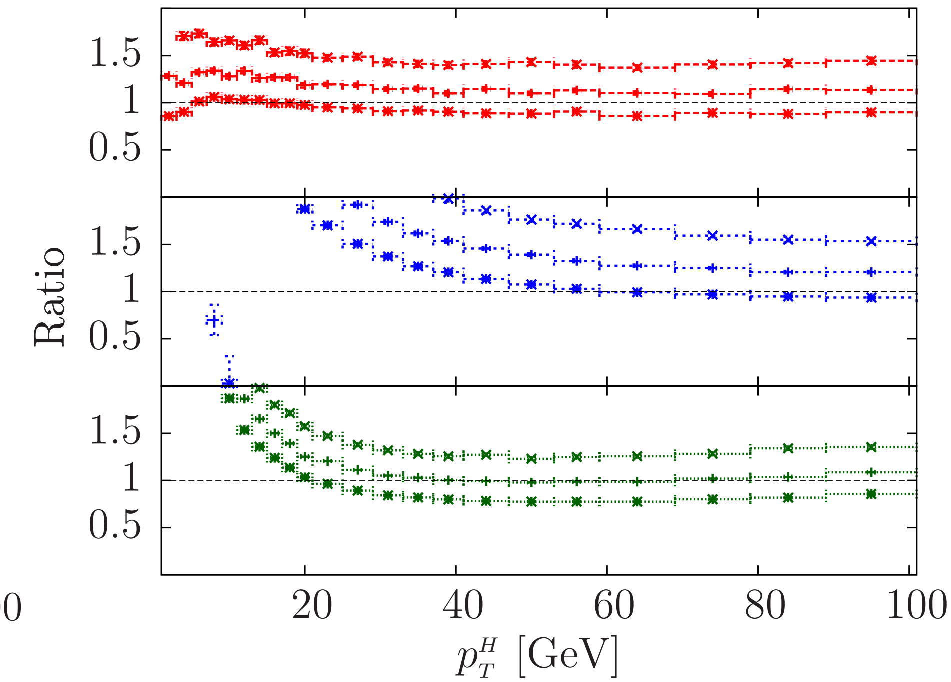
- In the standard POWHEG implementation of $H + 1$ jet production, the NLO computation of the HJ cross section suffers from large uncertainties due to scale choices, and, furthermore, does not have a good match with the H production POWHEG generator, at small transverse momenta.

- This problem is easily tracked back to the fact that the HJ NLO calculation does not attempt to resum large logarithms of the jet transverse momentum, not even at the LO level.

- We then apply the MINLO procedure when computing the $\bar{B}$ function for the generation of the underlying Born kinematics.

MINLO HJ





- H PWG: the (showered) H POWHEG BOX result
- RUN and FXD need a generation cut (or Born suppression factor) at small p_T^H. The MINLO result is instead finite (up to a cut-off $\approx \Lambda_{QCD}$)
- The MINLO result at small p_T^H is almost NLO accurate. More details later on.
- We get a result that is sensible also at low p_T^H, rather than divergent.

MINLO+POWHEG

- ✓ The MINLO approach improves the POWHEG implementations involving associated jet production, in the singular phase-space region.
- ✓ It provides a better match with the corresponding lower-multiplicity process.
  For example, $H+2$ jets matches $H+1$ jet when approaching the one-jet region, and $H+1$ jet matches H when approaching the zero-jet region. See arXiv:1206.3572 for more details.
- ✓ It turned out that it eases considerably the construction of matched samples with different jet multiplicities.

Merging samples

✓ Several codes provide NLO + Parton Shower results for the production of color-neutral object ($H/W/Z$, that we call B) plus 0, 1 and 2 jets

✓ Events produced with these Monte Carlo programs overlap in their population of the phase space, but the relative accuracies of each one in the various regions is complementary:

- the `B` generator
  - NLO accurate for inclusive boson distribution
  - LO accurate in the description of the hard radiation
- the `BJ` generator
  - NLO accurate for boson plus one jet distributions
  - LO accurate in the description of two jets
- ...

✓ Merging the `B`, `BJ`,... simulations means having an "output" that

- has NLO accuracy for inclusive boson distributions
- has NLO accuracy for boson plus one jet distributions
- ...

NEW MINLO+POWHEG

In a recent paper (Hamilton, Nason, Zanderighi, C.O. arXiv:1212.4504), we have investigated the accuracy of the BJ+MINLO results. We have found that:

- ✓ The inclusive boson observables are described by the BJ+MINLO programs at relative order α_s with respect to the Born cross section. However, they do not reach NLO accuracy, since they also include ambiguous contributions of relative order $\alpha_s^{1.5}$, rather than α_s^2.
- ✓ It is possible to modify the BJ+MINLO procedure in a very simple way in such a way that to reach NLO accuracy for inclusive observables.
- ✓ NNLO + Parton Shower accuracy on the inclusive boson distribution can be reached.
- ✓ We can then produce a sample of “merged” events without actually merging different samples, i.e. no merging scale is needed.

NEW MINLO+POWHEG

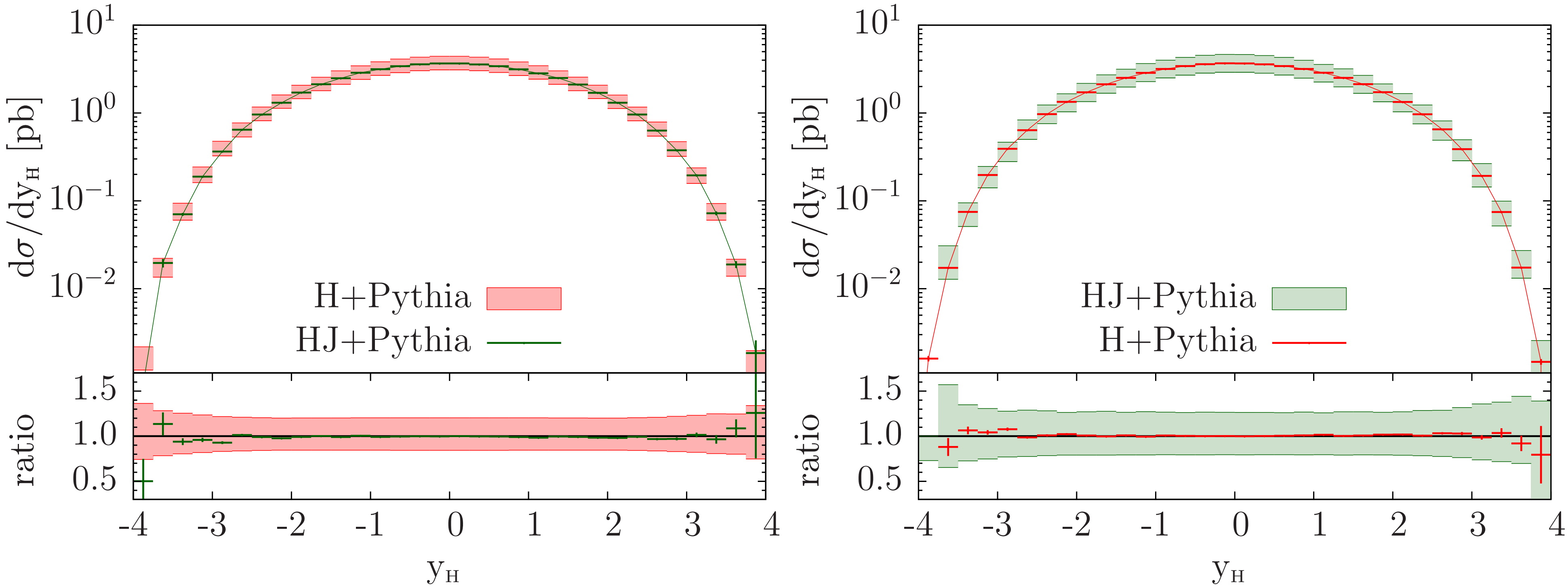
The modifications are very simple:

- ✓ In the Sudakov form factor we have to include the A_2 and B_2 terms

$$\Delta(Q, q_{\rm T}) = \exp\left\{-\int_{q_{\rm T}^2}^{Q^2} \frac{dq^2}{q^2}\left[\left(A_1\alpha_s(q^2) + A_2\,\alpha_s^2(q^2)\right)\log\frac{Q^2}{q^2} + \left(B_1\alpha_s(q^2) + B_2\,\alpha_s^2(q^2)\right)\right]\right\}$$

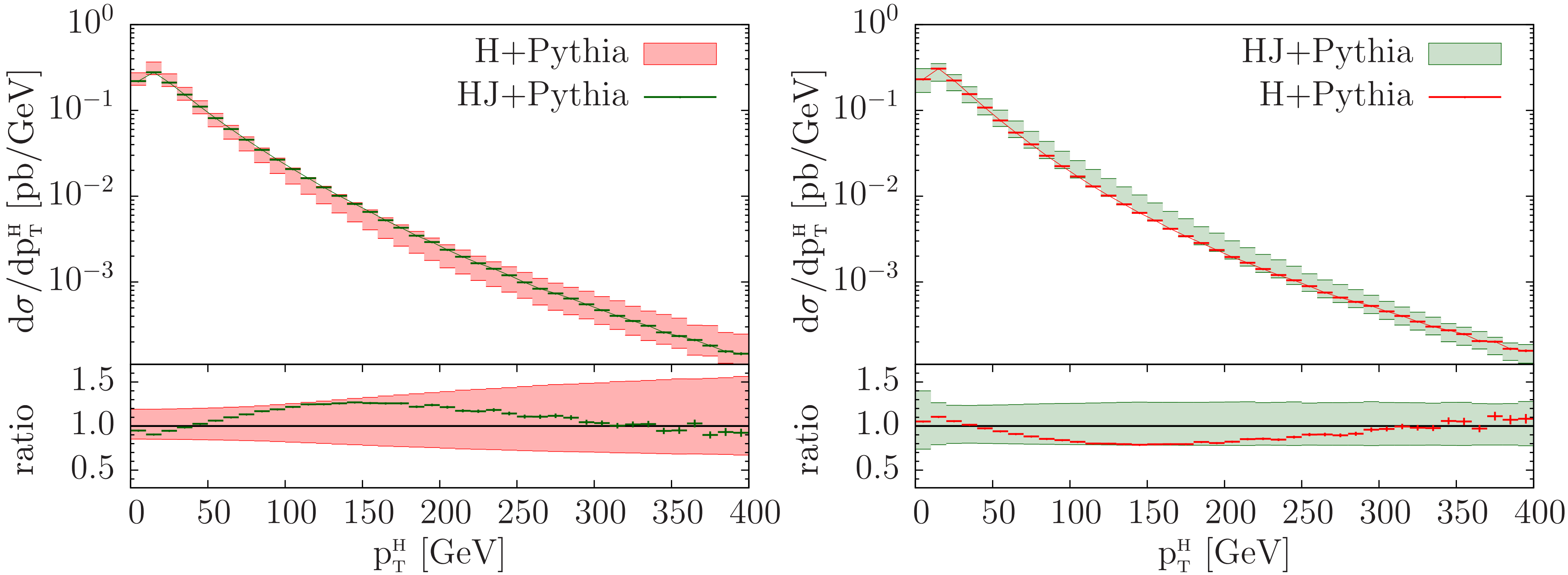
- ✓ $q_{\rm T}$ is the transverse momentum of the produced boson.
- ✓ The value of the $(N+1)^{\rm th}$ power of α_s in the real, in the virtual and in the expansion of the Sudakov form factor has to be computed using $q_{\rm T}$ as renormalization scale.
- ✓ The factorization scale has to be set to $q_{\rm T}$.

HJ-MINLO-NEW



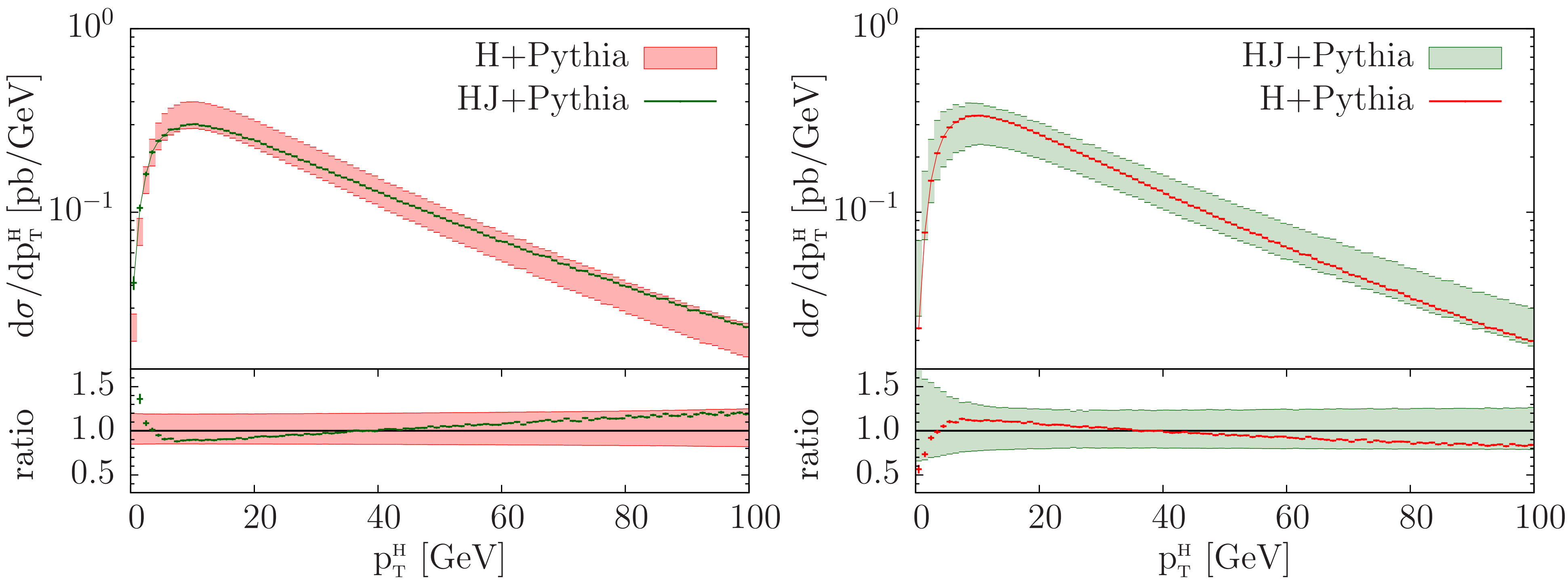
- $m_{\rm H} = 125$ GeV, LHC @ 8 TeV, `hfact` = $m_{\rm H}/1.2$
- envelope of the scale-variation bands obtained by varying the scale factor parameters by a factor of 2

HJ-MINLO-NEW



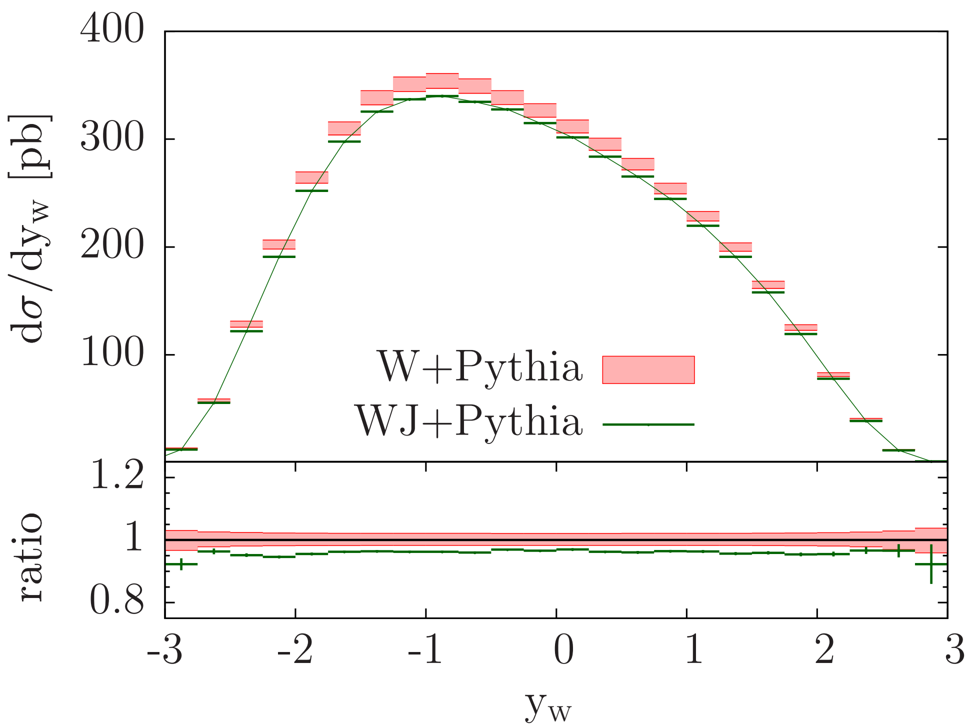
- central values of the H and HJ-MINLO generator in very good agreement
- the HJ-MINLO generator has a smaller scale-variation band: the HJ-MINLO generator achieves NLO accuracy for one-jet inclusive distributions, while the H generator is only tree-level accurate.

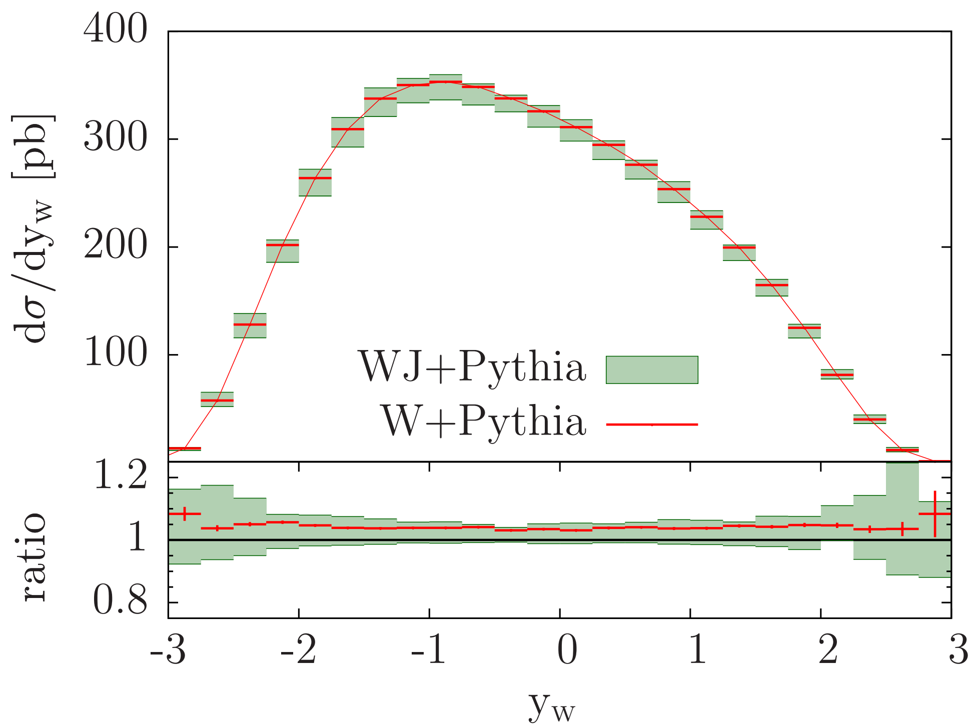
HJ-MINLO-NEW



- the scale uncertainty band of HJ-MINLO widens at small transverse momentum
  - approaching of the strong coupling regime
  - for $p_T^H < m_H$, the H result does not show a realistic scale uncertainty (S-type events)
- difference in shape in the very small transverse-momentum region, due to different NNLL and non-singular contributions in the two Sudakov form factors.

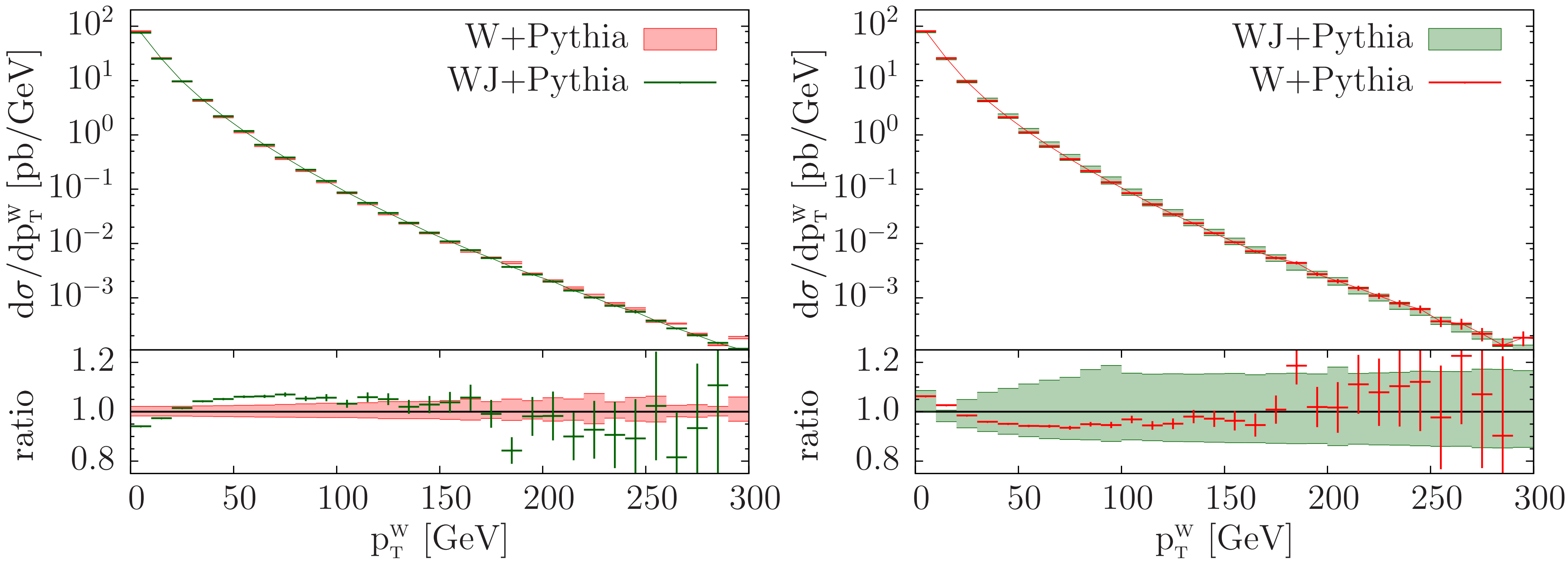
WJ-MINLO-NEW





- W^-, Tevatron @ 1.96 TeV. Symmetric error bands: $K_R = K_F = \{1/2, 1, 2\}$
- no shape difference. WJ+MINLO central value is about 5% below the W one. The WJ band is slightly larger than the W one for central rapidities, widening towards larger rapidities.

WJ-MINLO-NEW



- noticeable shape differences between the W and WJ+MINLO distribution, especially at low p_T^W: the WJ+MINLO Sudakov form factor peaks at a lower value of p_T^W.
- this distribution is described only at LO by the W generator, while the WJ+MINLO description is NLO accurate.
- the error band in WJ+MINLO is of an acceptable size at large transverse momenta, while it seems to be excessively small in the very low transverse momentum region.

NNLO+PS

✓ By reweighting the new BJ-MINLO generators with B production at NNLO, we get a NNLO calculation matched to a parton shower simulation, i.e. a NNLO+PS generator.

✓ HJ-MINLO differential cross section: $(d\sigma/dy)_{\mathtt{HJ}}$

- $\mathcal{O}(\alpha_s^3)$ accuracy for inclusive distributions
- $\mathcal{O}(\alpha_s^4)$ accuracy for all distributions involving at least one jet

✓ reweighting the HJ-MINLO output by $R(y)$

$$R(y) \equiv \frac{\left(\frac{d\sigma}{dy}\right)_{\mathrm{NNLO}}}{\left(\frac{d\sigma}{dy}\right)_{\mathtt{HJ}}} = \frac{c_2\alpha_s^2 + c_3\alpha_s^3 + c_4\alpha_s^4}{c_2\alpha_s^2 + c_3\alpha_s^3 + d_4\alpha_s^4} \approx 1 + \frac{c_4 - d_4}{c_2}\alpha_s^2 + \mathcal{O}(\alpha_s^3).$$

we achieve full NNLO accuracy for our generator. In fact the reweighting factor is such that

* the inclusive distributions are reweighted to achieve α_s^4 accuracy
* it does not spoil the α_s^4 accuracy of the HJ-MINLO generator in the one-jet region.

✓ Variants of this scheme are also possible (see arXiv:1212.4504)

Conclusions and outlooks

- ✓ With simple changes applied to MINLO, we could produce results for the production of a heavy, color-neutral system + 0 and 1 jet, accurate at NLO+PS for 0 and 1 jet distributions.
- ✓ We have tested our method in the framework of $H/W/Z$ production and we have found that the method performs remarkably well.
- ✓ Using the `BJ`-MINLO generators, it is actually possible to construct a NNLO+PS generator, by a simply reweighting procedure.
- ✓ This procedure could probably be generalized to higher jet multiplicities.
  More studies need to be done.

Backup slides

- (0) accuracy = accuracy of B inclusive cross section, integrated over its transverse momentum
  (1) accuracy = accuracy of the $B+1$ jet inclusive cross section
- The NNLL formula for the Higgs boson q_{T} distribution at fixed rapidity y is

$$\frac{d\sigma}{dy dq_{\mathrm{T}}^2} = \sigma_0 \frac{d}{dq_{\mathrm{T}}^2}\left\{ \left[C_{ga} \otimes f_{a/A}\right](x_A, q_{\mathrm{T}}) \times \left[C_{gb} \otimes f_{b/B}\right](x_B, q_{\mathrm{T}}) \exp \mathcal{S}(Q, q_{\mathrm{T}}) \right\} + R_f$$

- Its integral in dq_{T}^2 is given by

$$\frac{d\sigma}{dy} = \sigma_0 \left[C_{ga} \otimes f_{a/A}\right](x_A, Q) \times \left[C_{gb} \otimes f_{b/B}\right](x_B, Q) + \int dq_{\mathrm{T}}^2 R_f$$

  - In order to reach $\mathrm{NLO}^{(0)}$ accuracy, the C_{ij} functions should be accurate at order α_s and R_f should be $\mathrm{LO}^{(1)}$ accurate.
  - It is independent of the particular form of the Sudakov form factor

Backup slides

- if we take the derivative in q_T^2 and discard terms of higher order in α_s, the NLO$^{(0)}$ accuracy is maintained. In fact, after the derivative is taken, we get terms of the following form

$$\frac{d\sigma}{dy dq_T^2} \div \sigma_0 \frac{1}{q_T^2}\left[\alpha_s,\, \alpha_s^2,\, \alpha_s^3,\, \alpha_s^4,\, \alpha_s L,\, \alpha_s^2 L,\, \alpha_s^3 L,\, \alpha_s^4 L\right] \exp \mathcal{S}\left(Q, q_T\right), \qquad L = \log \frac{Q^2}{q_T^2}$$

- Using

$$\int_{\Lambda^2}^{Q^2} \frac{dq_T^2}{q_T^2}\left(\log\frac{Q^2}{q_T^2}\right)^m \alpha_s^n\left(q_T^2\right) \exp \mathcal{S}\left(Q, q_T\right) \approx \left[\alpha_s\left(Q^2\right)\right]^{n-\frac{m+1}{2}}$$

we can drop all terms at order α_s^3 and higher in the square brackets without spoiling the NLO$^{(0)}$ accuracy.

- Dropping these terms, we get essentially the full singular part of the BJ+MINLO formula, except that the original MINLO formula does not have the B_2 term in $\mathcal{S}$.
- If we dropped the B_2 term in the Sudakov we would miss, in the square brackets, the term

$$\sigma_0 \frac{1}{q_T^2} \alpha_s^2(q_T^2)\, B_2 \exp \mathcal{S}\left(Q, q_T\right)$$

so that, the old MINLO formula violates the NLO$^{(0)}$ accuracy by a term that, upon integration is of order of $\alpha_s^{2-\frac{1}{2}} = \alpha_s^{1.5}$