Parton Shower Unitarity and NLO Matching

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*Most work done by Stefan Prestel

Introduction

- Introduction
- Improving unitarity for CKKW(-L) $\rightarrow$ UMEPS
- Multi-jet merging to NLO $\rightarrow$ UNLOPS

General Philosophy

Keep the Parton Shower description intact as far as possible, but improve description for partonic configuration with hard, well separated partons using fixed-order matrix elements.

ME region typically defined by a *merging scale* cutoff, regularizing soft and collinear divergencies.

Fixed-Order Matrix Elements

- One mutiplicity at the time
- Fixed renormalization/factorization scale
- Beyond leading order we need Exclusive ME's

Assume that we have a ME generator that can give us samples (eg. in LHE files) of some Born-level configurations, and also samples with $+n$ extra partons ($n \leq N$).

For $n \leq M$ these may be calculated to NLO.

Parton Showers

- All-order resummation to (N)LL accuracy
- Process-independent (more or less)
- Exclusive final states with arbitrary multiplicies
- Prerequisite for any hadronization model
- Any Parton Shower will do
  (as long as it has on-shell intermediate states)
  (PYTHIA8)
- Parton Showers are unitary

The Unitary nature of Parton Showers

Start with a state from a Born-level ME

$$\frac{d\sigma_0^{inc}}{d\phi_0} \equiv F_0|\mathcal{M}_0|^2,$$

A parton shower will turn this into a +1-parton event with a probability

$$\frac{d\sigma_1^{first}}{d\phi_0} = F_0|\mathcal{M}_0|^2 \alpha_s \mathcal{P}_1 d\rho dz \Gamma_0(\rho_0, \rho).$$

Using a splitting function and a no-emission probability (the *first* or *hardest* splitting).

The PS does not only add a state with an extra parton, it also subtracts the total cross section for this to happen:

$$-\int F_0|\mathcal{M}_0|^2\alpha_s\mathcal{P}_1 d\rho dz\,\Gamma_0(\rho_0,\rho).$$

The *exclusive* zero-parton cross section that is left is

$$\begin{aligned}\frac{d\sigma_0^{excl}}{d\phi_0} &= F_0|\mathcal{M}_0|^2\left(1-\int_{\rho_c}\alpha_s\mathcal{P}_1 d\rho dz\,\Gamma_0(\rho_0,\rho)\right)\\ &= F_0|\mathcal{M}_0|^2\exp\left(-\int_{\rho_c}^{\rho_0}\alpha_s\mathcal{P}_1 d\rho dz\right)\\ &= F_0|\mathcal{M}_0|^2\Gamma_0(\rho_0,\rho_c)\end{aligned}$$

The PS then continues to turn the 1-parton state into a 2-parton state with cross section

$$\frac{d\sigma_2^{first}}{d\phi_0} = F_0|\mathcal{M}_0|^2\alpha_s\mathcal{P}_1 d\rho_1 dz_1 \Gamma_0(\rho_0,\rho_1)\alpha_s\mathcal{P}_2 d\rho_2 dz_2 \Gamma_1(\rho_1,\rho_2).$$

Again it adds the emission and subtracts the corresponding 1-parton state (integrated over the second emission) leaving the exclusive 1-jet cross-section

$$\frac{d\sigma_1^{excl}}{d\phi_0} = F_0|\mathcal{M}_0|^2\alpha_s\mathcal{P}_1 d\rho_1 dz_1 \Gamma_0(\rho_0,\rho_1)\Gamma_1(\rho_1,\rho_c).$$

And so on with a third parton, etc.

CKKW(-L)

We can now use full tree-level matrix elements instead, by multiplying them with appropriate no-emission probabilities, thus making them exclusive:

- $F_0|\mathcal{M}_0|^2\Gamma_0(\rho_0, \rho_{\rm MS}) \rightarrow F_0|\mathcal{M}_0|^2\Gamma_0(\rho_0, \rho_{\rm MS})$
- $F_0|\mathcal{M}_0|^2\alpha_s\mathcal{P}_1 d\rho_1 dz_1 \Gamma_0(\rho_0, \rho_1)\Gamma_{(}\rho_1, \rho_{\rm MS})$

  $\rightarrow F_0|\mathcal{M}_1|^2 d\rho_1 dz_1 \Gamma_0(\rho_0, \rho_1)\Gamma_{(}\rho_1, \rho_{\rm MS})$
- $F_0|\mathcal{M}_0|^2\alpha_s\mathcal{P}_1 d\rho_1 dz_1 \Gamma_0(\rho_0, \rho_1)\alpha_s\mathcal{P}_2 d\rho_2 dz_2 \Gamma_1(\rho_1, \rho_2)$

  $\rightarrow F_0|\mathcal{M}_2|^2 d\rho_1 dz_1 \Gamma_0(\rho_0, \rho_1) d\rho_2 dz_2 \Gamma_1(\rho_1, \rho_2)$

Where $\rho_{\rm MS}$ is some merging scale (defined in the PS evolution variable). ρ_i and z_i are (PS) reconstructed splittings.

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- $F_0|\mathcal{M}_0|^2\alpha_s\mathcal{P}_1 d\rho_1 dz_1 \Gamma_0(\rho_0, \rho_1)\Gamma_{(}\rho_1, \rho_{\text{MS}})$

  $\to F_0|\mathcal{M}_1|^2 d\rho_1 dz_1 \Gamma_0(\rho_0, \rho_1)\Gamma_{(}\rho_1, \rho_{\text{MS}})$
- $F_0|\mathcal{M}_0|^2\alpha_s\mathcal{P}_1 d\rho_1 dz_1 \Gamma_0(\rho_0, \rho_1)\alpha_s\mathcal{P}_2 d\rho_2 dz_2 \Gamma_1(\rho_1, \rho_2)$

  $\to F_0|\mathcal{M}_2|^2 d\rho_1 dz_1 \Gamma_0(\rho_0, \rho_1) d\rho_2 dz_2 \Gamma_1(\rho_1, \rho_2)$

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- $F_0|\mathcal{M}_0|^2\alpha_s\mathcal{P}_1 d\rho_1 dz_1 \Gamma_0(\rho_0, \rho_1)\Gamma_{(}\rho_1, \rho_{\text{MS}})$

  $\to F_0|\mathcal{M}_1|^2 d\rho_1 dz_1 \Gamma_0(\rho_0, \rho_1)\Gamma_{(}\rho_1, \rho_{\text{MS}})$
- $F_0|\mathcal{M}_0|^2\alpha_s\mathcal{P}_1 d\rho_1 dz_1 \Gamma_0(\rho_0, \rho_1)\alpha_s\mathcal{P}_2 d\rho_2 dz_2 \Gamma_1(\rho_1, \rho_2)$

  $\to F_0|\mathcal{M}_2|^2 d\rho_1 dz_1 \Gamma_0(\rho_0, \rho_1) d\rho_2 dz_2 \Gamma_1(\rho_1, \rho_2)$

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  $\to F_0|\mathcal{M}_2|^2 d\rho_1 dz_1\Gamma_0(\rho_0, \rho_1) d\rho_2 dz_2\Gamma_1(\rho_1, \rho_2)$

Where ρ_{MS} is some merging scale (defined in the PS evolution variable). ρ_i and z_i are (PS) reconstructed splittings.

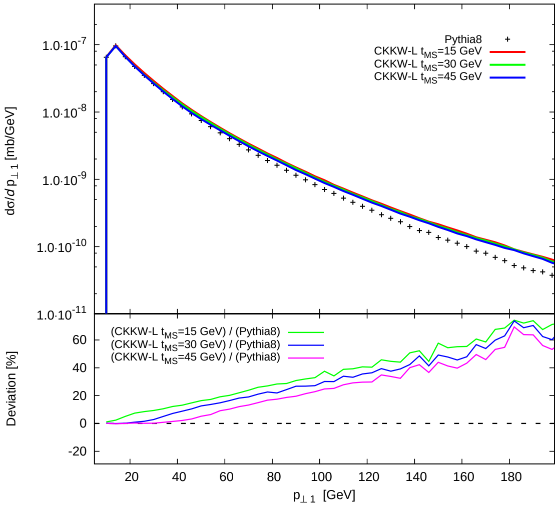
We let eg. MadEvent generate 0-, 1-, and 2-jet samples. We make the 0- and 1-jet samples exclusive and the 2-jet sample *hardest* inclusive by reweighting with no-emission probabilities. We can now add a normal PS below ρ_{MS} (or below ρ_2 in the 2-jet case), and add all samples together avoiding all double-counting.

However, what we add is no longer what we subtract.

- We add the full tree-level ME
- We subtract the PS-approximation

This will give us a dependence of the inclusive cross section on the merging scale.

W+jets


Pythia8
CKKW-L t_{MS}=15 GeV
CKKW-L t_{MS}=30 GeV
CKKW-L t_{MS}=45 GeV
1.0·10^-7
1.0·10^-8
1.0·10^-9
1.0·10^-10
1.0·10^-11
dσ/d p⊥1 [mb/GeV]
(CKKW-L t_{MS}=15 GeV) / (Pythia8)
(CKKW-L t_{MS}=30 GeV) / (Pythia8)
(CKKW-L t_{MS}=45 GeV) / (Pythia8)
Deviation [%]
60
40
20
0
-20
20 40 60 80 100 120 140 160 180
p⊥1 [GeV]

Even far above the merging scales we have a 5-10% merging scale dependence.

No problem for a tree-level calculation, as the scale uncertainties are larger.

But if we want to use this procedure as a starting point for an NLO matching we need to worry.

UMEPS

Instead of making the tree-level ME-samples exclusive, make all of them *hardest* inclusive:

- $F_0|\mathcal{M}_0|^2$
  $-\int F_0|\mathcal{M}_1|^2 d\rho_1 dz_1 \Gamma_0(\rho_0, \rho_1)$
- $F_0|\mathcal{M}_1|^2 d\rho_1 dz_1 \Gamma_0(\rho_0, \rho_1)$
  $-d\rho_1 dz_1 \Gamma_0(\rho_0, \rho_1) \int F_0|\mathcal{M}_2|^2 d\rho_2 dz_2 \Gamma_1(\rho_1, \rho_2)$
- $F_0|\mathcal{M}_2|^2 d\rho_1 dz_1 \Gamma_0(\rho_0, \rho_1) d\rho_2 dz_2 \Gamma_1(\rho_1, \rho_2)$

For each extra parton we add the reweighted ME sample but we also subtract the integrated version from the parton multiplicity below making them exclusive.

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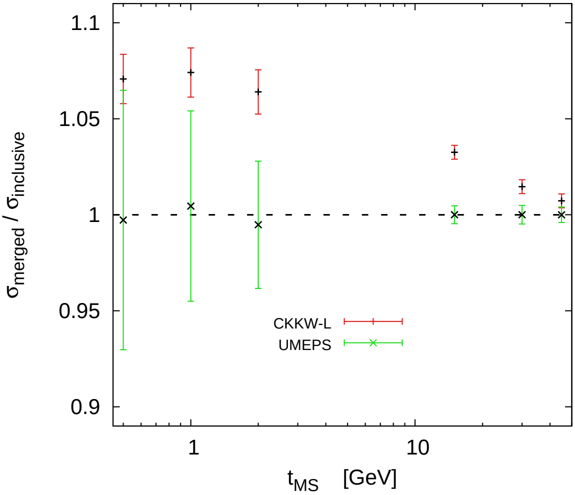
- $F_0|\mathcal{M}_0|^2$
  $-\int F_0|\mathcal{M}_1|^2 d\rho_1 dz_1 \Gamma_0(\rho_0,\rho_1)$
- $F_0|\mathcal{M}_1|^2 d\rho_1 dz_1 \Gamma_0(\rho_0,\rho_1)$
  $-d\rho_1 dz_1 \Gamma_0(\rho_0,\rho_1)\int F_0|\mathcal{M}_2|^2 d\rho_2 dz_2 \Gamma_1(\rho_1,\rho_2)$
- $F_0|\mathcal{M}_2|^2 d\rho_1 dz_1 \Gamma_0(\rho_0,\rho_1) d\rho_2 dz_2 \Gamma_1(\rho_1,\rho_2)$

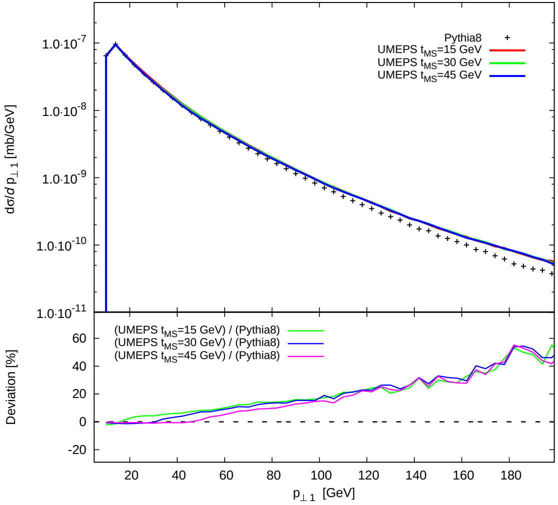
For each extra parton we add the reweighted ME sample but we also subtract the integrated version from the parton multiplicity below making them exclusive.

We can still add a normal PS below ρ_{MS} (or below ρ_2 in the 2-jet case), to avoid all double-counting.

But the procedure is now (almost) completely unitary.

Lönnblad & Prestel arxiv:1211.4827 [hep-ph]


1.1
1.05
1
0.95
0.9
σ_{merged} / $\sigma_{inclusive}$
1
10
t_{MS} [GeV]
CKKW-L
UMEPS


Pythia8
UMEPS t_{MS}=15 GeV
UMEPS t_{MS}=30 GeV
UMEPS t_{MS}=45 GeV
1.0·10^-7
1.0·10^-8
1.0·10^-9
1.0·10^-10
1.0·10^-11
dσ/d $p_{\perp 1}$ [mb/GeV]
(UMEPS t_{MS}=15 GeV) / (Pythia8)
(UMEPS t_{MS}=30 GeV) / (Pythia8)
(UMEPS t_{MS}=45 GeV) / (Pythia8)
60
40
20
0
-20
Deviation [%]
20
40
60
80
100
120
140
160
180
$p_{\perp 1}$ [GeV]

Caveats

We can use any merging scale definition - no need for truncated showers. We still need vetoed showers, but only the first shower emission need to be vetoed.

Only states where the n hardest partons according to the PS are above the merging scale, will be ME-correct.

When reclustered, an n-parton state above the merging scale may result in a $n-1$-parton state below the merging scale. Rather than subtracting this from the exclusive $n-1$ parton sample, it is instead reclustered again and subtracted from the $n-2$ sample.

Negative weights

For small merging scales, the 0-jet exclusive cross section is very small, and the the 0-jet inclusive sample is almost completely cancelled by reclustered 1-jet events (with negative weights).

Not a problem in principle, but statistics is an issue.

It would be nice if we could bias our ME-generator to generate LHE-files with suitable weights.

UNLOPS†

We can now go on to add also merge multi-jet NLO calculations.

- From the NL^3 NLO-merging we know how to expand out the no-emission probabilities in orders of α_s, and subtract any given order.
- We also know how to expand out PDF-ratios with running factorization scales used in the PS to any given order.
- Likewise, the running of α_s in the PS can be trivially expanded.
- If we want we can multiply the UMEPS samples with a K-factor - again, trivially expanded.

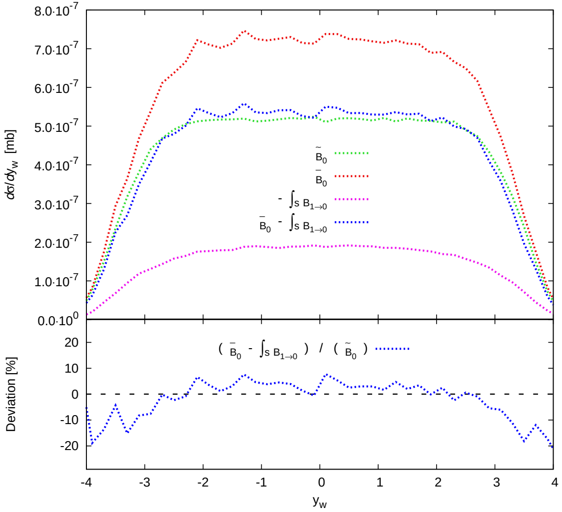
† Lönnblad & Prestel arxiv:1211.7278 [hep-ph]

For each exclusive UMEPS multiplicity we can subtract the α_s^n and α_s^{n+1} terms and instead add a sample generated according to the *exclusive* NLO cross section.

An exclusive NLO sample can be obtained by slightly hacking POWHEG.

But it can also be obtain by turning off the Sudakov-generated emission in POWHEG (giving $\bar{B}_n$) and subtracting the integrated +1-parton tree-level ME with a ρ_{MS}-cut.

But the $1 \to 0$ phase space mapping is different in PYTHIA8 (one fixed x) and POWHEG (fixed y_W)


8.0·10^-7
7.0·10^-7
6.0·10^-7
5.0·10^-7
4.0·10^-7
3.0·10^-7
2.0·10^-7
1.0·10^-7
0.0·10^0
dσ/dy_W [mb]
B̃_0
B̄_0
- ∫_S B_1→0
B̄_0 - ∫_S B_1→0
(B̄_0 - ∫_S B_1→0) / (B̃_0)
Deviation [%]
20
10
0
-10
-20
-4
-3
-2
-1
0
1
2
3
4
y_W

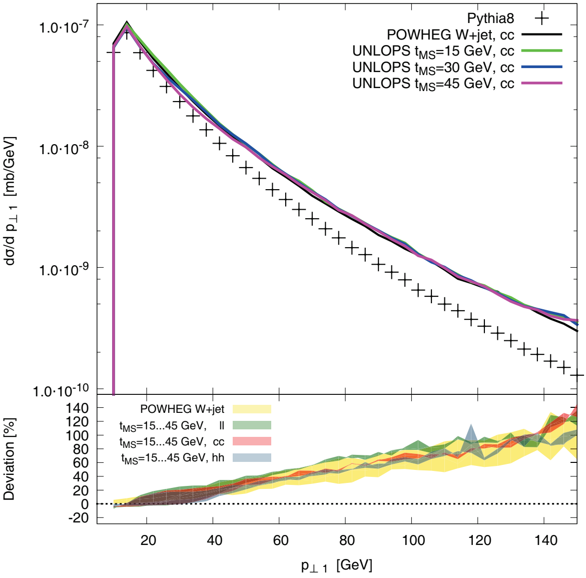
But we also need to subtract what we add.

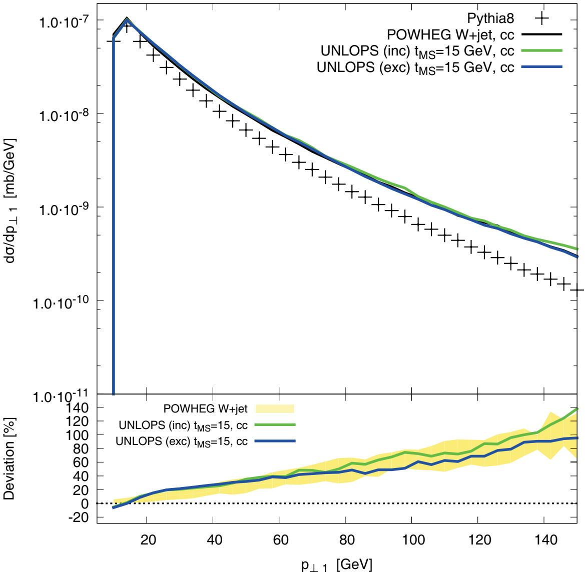
We take the exclusive NLO sample minus the α_s-terms we subtracted from UMEPS reweighted tree-level ME, integrate them over the last emission and subtract them from the multiplicity below.

We are still unitary:

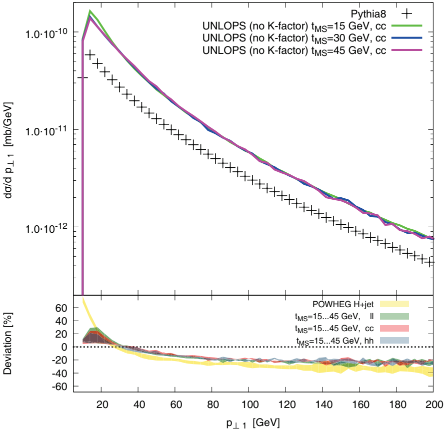
- The inclusive total cross section will be given by the NLO calculation.
- The inclusive 1-parton cross section will be given by the corresponding NLO calculation
- ...

NNLO is also possible in this framework.

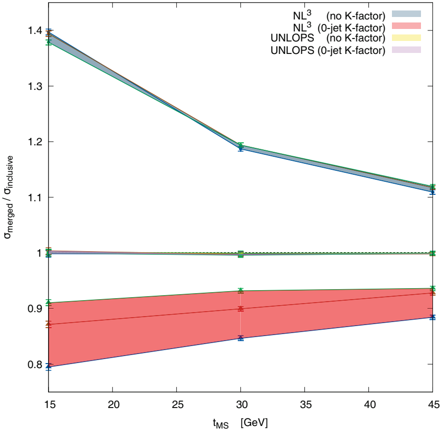

Pythia8
POWHEG W+jet, cc
UNLOPS t_{MS}=15 GeV, cc
UNLOPS t_{MS}=30 GeV, cc
UNLOPS t_{MS}=45 GeV, cc
$d\sigma/d\,p_{\perp\,1}$ [mb/GeV]
$1.0\cdot10^{-7}$
$1.0\cdot10^{-8}$
$1.0\cdot10^{-9}$
$1.0\cdot10^{-10}$
POWHEG W+jet
t_{MS}=15...45 GeV, ll
t_{MS}=15...45 GeV, cc
t_{MS}=15...45 GeV, hh
Deviation [%]
140
120
100
80
60
40
20
0
-20
20
40
60
80
100
120
140
$p_{\perp\,1}$ [GeV]


Pythia8
POWHEG W+jet, cc
UNLOPS (inc) t_MS=15 GeV, cc
UNLOPS (exc) t_MS=15 GeV, cc
1.0·10^-7
1.0·10^-8
1.0·10^-9
1.0·10^-10
1.0·10^-11
dσ/dp⊥ 1 [mb/GeV]
POWHEG W+jet
UNLOPS (inc) t_MS=15, cc
UNLOPS (exc) t_MS=15, cc
140
120
100
80
60
40
20
0
-20
Deviation [%]
20
40
60
80
100
120
140
p⊥ 1 [GeV]

Higgs production



Use of K-factors



Multi-jet NLO merging with parton showers is a solved problem. Several algorithms exists.

UNLOPS (and UMEPS) has a couple of attractive features:

- Low jet-multiplicity cross section explicitly preserved without merging scale dependence.
- Merging scale can be taken arbitrarily low (in principle down to the shower cutoff).
- Extension to NNLO is straight forward (trivial for the lowest multiplicity).

Still, there are downsides:

- Need full exclusive *n*-parton states calculated to (N)NLO. Resolution scale must be defined similar to the PS evolution scale.
- Need biased ME event samples to get reasonable statistics for low merging scales.
- For exclusive observables, resummation of higher orders is never better than what the PS gives.

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