Hot quarkonium spectral functions from QCD sum rules and MEM

P. Gubler and M. Oka, Prog. Theor. Phys. 124, 995 (2010).

P. Gubler, K. Morita and M. Oka, Phys. Rev. Lett. 107, 092003 (2011).

K. Suzuki, P. Gubler, K. Morita and M. Oka, Nucl. Phys. A897, 28 (2013).

Heavy quarks and quarkonia in thermal QCD @ ECT*, Villazzano, Italy 5. 4. 2013 Philipp Gubler (RIKEN, Nishina Center)

Collaborators:

M. Oka (Tokyo Tech), K. Morita (YITP), K. Suzuki (Tokyo Tech)

Contents

Introduction

- The method: QCD sum rules and the maximum entropy method
- Sum rule results for quarkonium + bottomonium
- Conclusions

Introduction: Quarkonia

General Motivation: Understanding the behavior of matter at high T.



QCD sum rules

M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B147, 385 (1979); B147, 448 (1979).

In QCD sum rules one considers the following correlator:



$$\Pi(q^2) = i \int d^4x e^{iqx} \langle 0|T\{\chi(x)\overline{\chi}(0)\}|0\rangle$$

In the region of $\Pi(q)$ dominated by large energy scales such as



it can be calculated by the operator product expansion (OPE):



$$\Pi(q^2) = i \int d^4x e^{iqx} \langle 0|T\{\chi(x)\overline{\chi}(0)\}|0\rangle$$

On the other hand, we consider the above correlator in the region of



where the optical theorem (unitarity) gives





After the Borel transormation: $G(M) \equiv \lim_{-q^2, n \to \infty} \lim_{(\frac{-q^2}{n} = M^2)} \frac{(-q^2)^{n+1}}{n!} \left(\frac{d}{dq^2}\right)^n \Pi(q^2)$

$$G(M) = \int_0^\infty ds e^{-\frac{s}{M^2}} \rho(s)$$

The basic problem to be solved



(but only incomplete and with error)

This is an ill-posed problem.

But, one may have additional information on $\rho(\omega)$, which can help to constrain the problem:

- Positivity:
- Asymptotic values:

$$egin{aligned} &
ho(\omega) \geq 0 \ &
ho(\omega=0),
ho(\omega=\infty) \end{aligned}$$



PG and M. Oka, Prog. Theor. Phys. 124, 995 (2010).

K. Ohtani, PG and M. Oka, Eur. Phys. J. A 47, 114 (2011).K. Ohtani, PG and M. Oka, Phys. Rev. D 87, 034027 (2013).

The quarkonium sum rules at finite T

The application of QCD sum rules has been developed in:

- A.I. Bochkarev and M.E. Shaposhnikov, Nucl. Phys. B 268, 220 (1986).
- T. Hatsuda, Y. Koike and S.H. Lee, Nucl. Phys. B 394, 221 (1993).

$$M(\nu) = \int_0^\infty e^{-\nu t} \rho(4m_c^2 t) dt \qquad (\nu \equiv \frac{M^2}{4m_c^2})$$

$$M(\nu) = A(\nu) \Big[1 + a(\nu)\alpha_s(\nu) + b(\nu) \frac{\langle \frac{\alpha_s}{\pi}G^2 \rangle_T}{m_c^4} + c(\nu) \frac{\langle \frac{\alpha_s}{\pi}G^2 \rangle_{T,2}}{m_c^4} + \frac{d(\nu) \langle \frac{g^3G^3 \rangle_T}{m_c^6}}{m_c^6} \Big]$$

depend on T

Compared to lattice:

- Continuum, infinite volume calculation: no cutoff or finite volume effects
- Same kernel and same number of data points at T=0 and T \neq 0

However:

- Effects of higher order terms of the OPE are difficult quantify



The T-dependence of the condensates

K. Morita and S.H. Lee, Phys. Rev. Lett. 100, 022301 (2008).



Considering the trace and the traceless part of the energy momentum tensor, one can show that in tht quenched approximation, the T-dependent parts of the gluon condensates by thermodynamic quantities such as energy density $\epsilon(T)$ and pressure p(T).

$$\langle \frac{\alpha_s}{\pi} G^2 \rangle_T = \langle \frac{\alpha_s}{\pi} G^2 \rangle_{\text{Vac.}} - \frac{8}{11} (\epsilon - 3p)$$

 $\langle \frac{\alpha_s}{\pi} G^2 \rangle_{T,2} = -\frac{\alpha_s(T)}{\pi} (\epsilon + p)$

The values of $\epsilon(T)$ and p(T) are obtained from quenched lattice calculations:

G. Boyd et al, Nucl. Phys. B 469, 419 (1996).

O. Kaczmarek et al, Phys. Rev. D 70, 074505 (2004).

A first test: mock data analysis



Charmonium at T=0



PG, K. Morita and M. Oka, Phys. Rev. Lett. 107, 092003 (2011).

 $\langle \frac{\alpha_s}{\pi} G^2 \rangle = 0.012 \pm 0.0036 \text{ GeV}^4$ $m_c = 1.277 \pm 0.026 \text{ GeV}$

Charmonium at finite T



PG, K. Morita and M. Oka, Phys. Rev. Lett. 107, 092003 (2011).

 $\langle \frac{\alpha_s}{\pi} G^2
angle = 0.012 \pm 0.0036 \text{ GeV}^4$ $m_c = 1.277 \pm 0.026 \text{ GeV}$

What is going on behind the scenes ?

The OPE data in the Vector channel at various T:



Comparison with lattice results

Imaginary time correlator ratio: $G(\tau,T)/G_{\rm rec}$



Lattice data are taken from: A. Jakovác, P. Petreczky, K. Petrov and A. Velytsky, Phys. Rev. D 75, 014506 (2007).

Bottomonium at finite T



K. Suzuki, PG, K. Morita and M. Oka, Nucl. Phys. A897, 28 (2013).

 $\langle \frac{\alpha_s}{\pi} G^2 \rangle = 0.012 \pm 0.0036 \text{ GeV}^4$ $m_b = 4.167 \pm 0.013 \text{ GeV}$

What about the excited states?



Extracting information on the excited states

However, we can at least investigate the behavior of the residue as a function of T.



Remaining issues

- Higher order gluon condensates?
 - Has to be checked! Lattice calculation is needed.
- Higher orders is α_s?
 - Are potentially large. Need some sort of resummation?
- Division between high- and low-energy contributions in OPE?
 - Is a problem at high T. Needs to be investigated carefully.

Conclusions

- We have shown that MEM can be applied to QCD sum rules
- The resolution of the method is limited, therefore it is generally difficult to obtain the peak-width
- We could observe the melting of the S-wave and P-wave quarkonia and estimated the corresponding melting temperatures

Backup slide

Estimation of the error of G(M)

$$G_{OPE}(M) = \frac{1}{4\pi^2} (1 + \frac{\alpha_s}{\pi}) + \left(2m\langle \bar{q}q \rangle + \frac{1}{12} \langle \frac{\alpha_s}{\pi} G^2 \rangle \right) \frac{1}{M^4} - \frac{112\pi}{81} \alpha_s \kappa \langle \bar{q}q \rangle^2 \frac{1}{M^6}$$

Gaussianly distributed values for the various parameters are randomly generated. The error is extracted from the resulting distribution of $G_{OPE}(M)$.

D.B. Leinweber, Annals Phys. 322, 1949 (1996).



PG, M. Oka, Prog. Theor. Phys. 124, 995 (2010).