The Polyakov loop correlator and the cyclic Wilson loop in perturbation theory and EFTs

Jacopo Ghiglieri, McGill University
in collaboration with M. Berwein, N. Brambilla, P. Petreczky and A. Vairo
ECT*, April 4th 2013

Outline

- Introduction to the thermodynamical free energies for QQbar pairs
- The Polyakov loop correlator in perturbation theory and EFT
- The Cyclic Wilson loop and its renormalization

Brambilla JG Petreczky Vairo **PRD82** (2012)
Berwein Brambilla JG Vairo **JHEP1303** (2013)

Thermodynamical free energies

The Polyakov loop

- Polyakov loop in a color representation R

$$L = P\exp\left(ig\int_0^\beta d\tau A^0(\tau,\mathbf{x})\right) \qquad \langle L_R\rangle \equiv \langle\tilde{\mathrm{Tr}} L_R\rangle, \quad \tilde{\mathrm{Tr}} \equiv \frac{\mathrm{Tr}}{d(R)}$$

- Thermodynamic relation to the free energy of a (infinitely) heavy quark

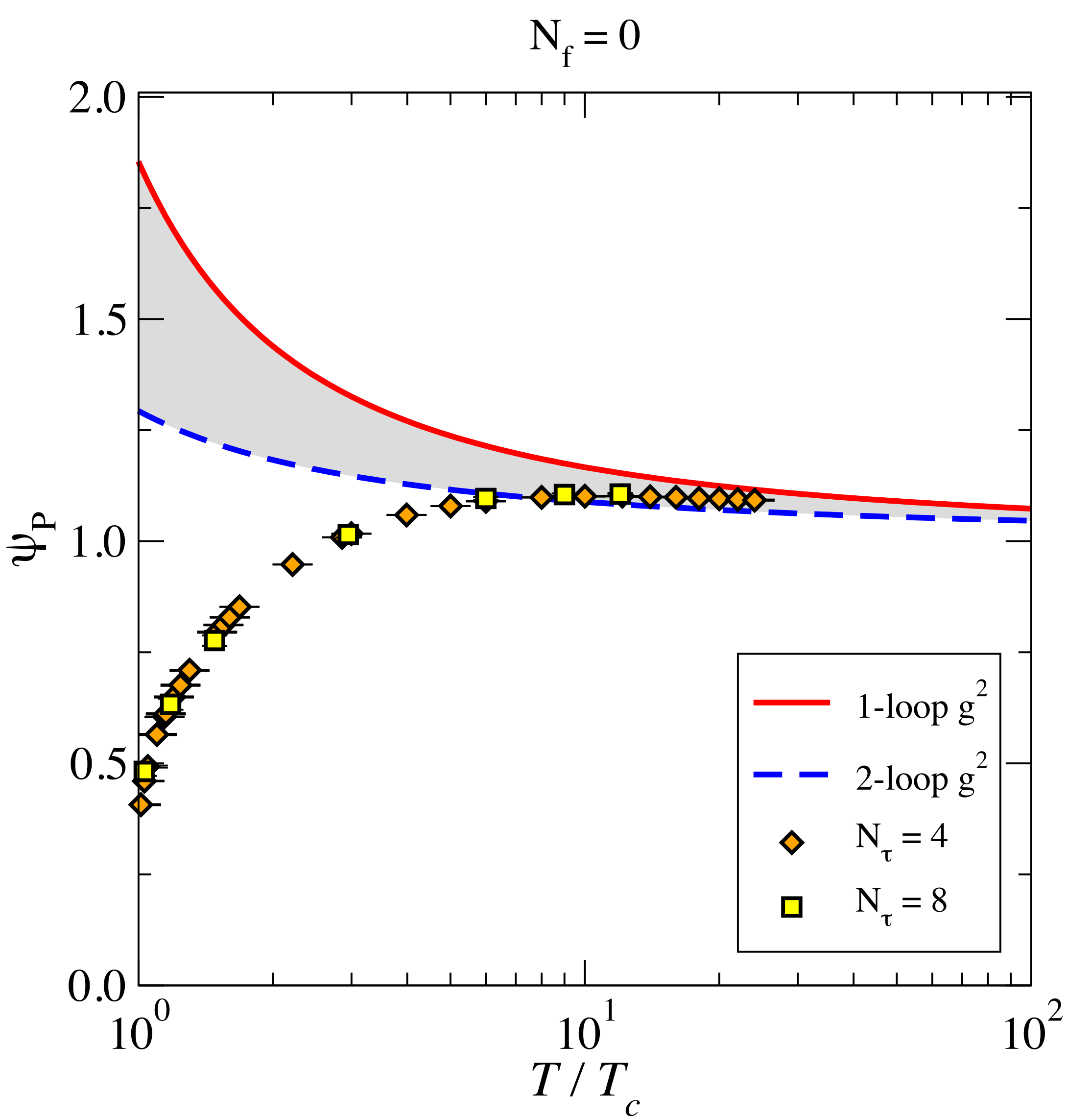
$$\langle L_F\rangle = e^{-F_Q/T}$$

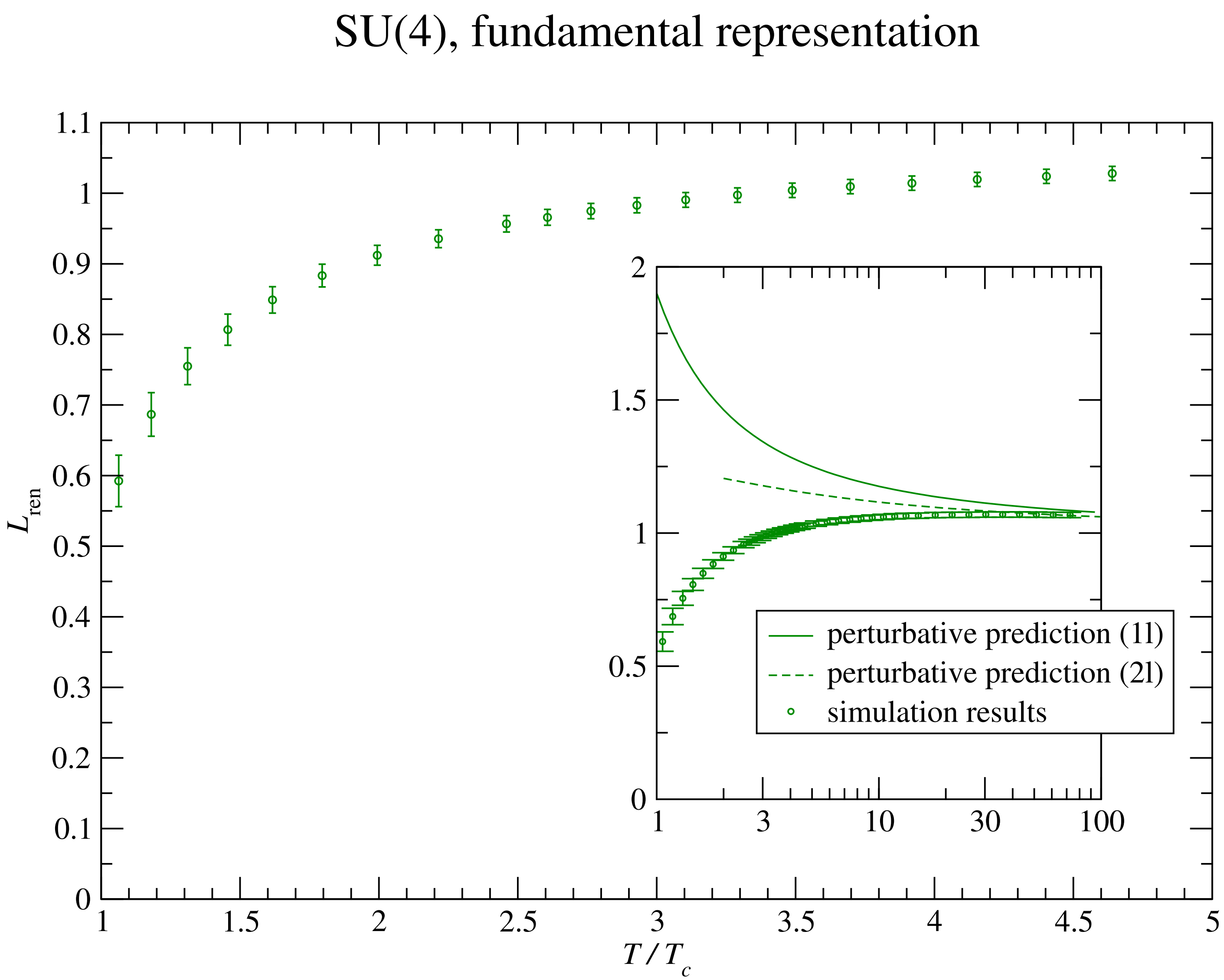
McLerran Svetitsky **PRD24** 1981

- Order parameter for the deconfinement phase transition.

- Extensively measured on the lattice

The Polyakov loop

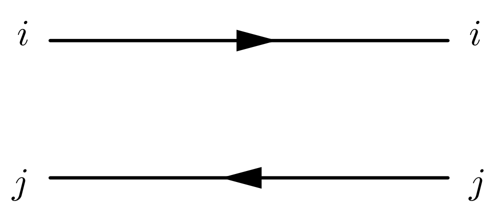




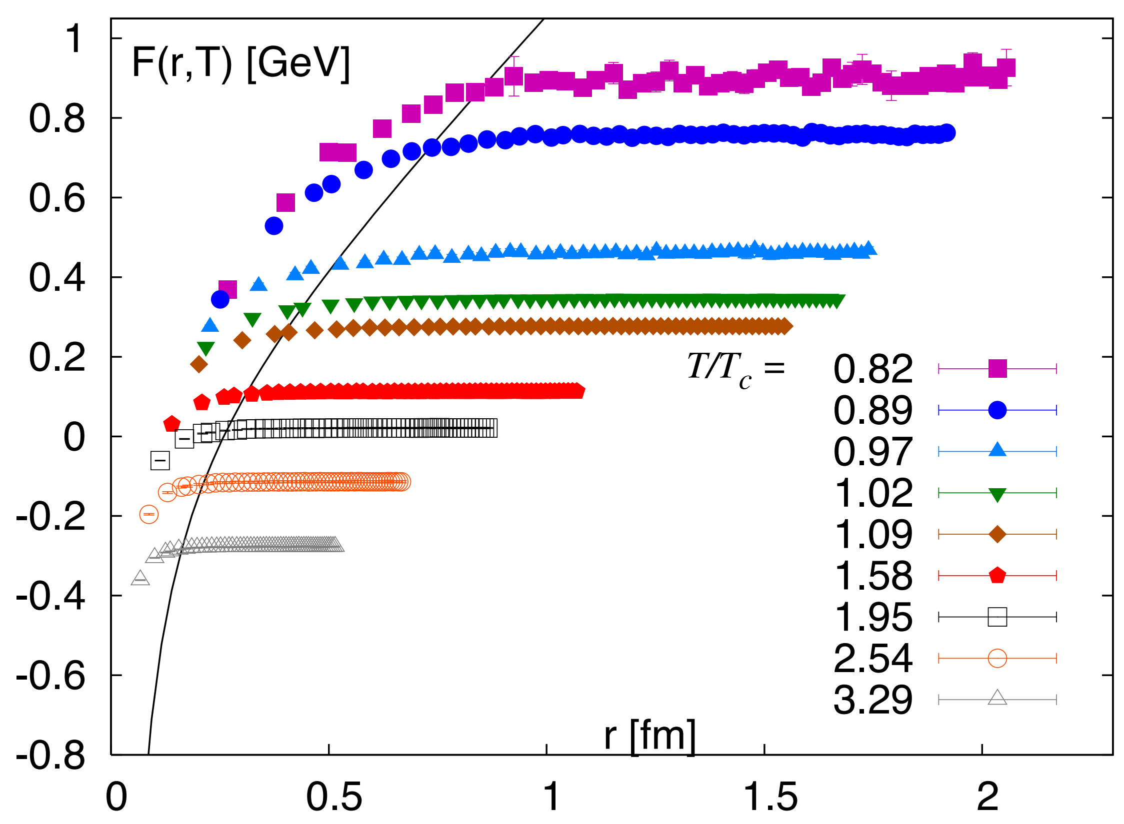
- Lattice: Gupta Hubner Kaczmarek **PRD77** (2008) (left), Mykkanen Panero Rummukainen **JHEP1205** (2012) (right, figure)
  pQCD: Burnier Laine Vepsalainen **JHEP1011** (2009, left fig.), Brambilla Petreczky JG Vairo **PRD82** (2010)

The Polyakov loop correlator

- Correlator of two Polyakov loops: (difference in) free energy of a quark-antiquark pair
  $P_c \equiv \langle \mathrm{Tr}\, L(\mathbf{x})\, \mathrm{Tr}\, L^\dagger(\mathbf{0}) \rangle$



  Gauge independent and well defined, but probes the octet sector as well



Petreczky 1001.5284

- Perturbation theory at short distances / EFT analysis
  Brambilla JG Petreczky Vairo **PRD82** (2010)

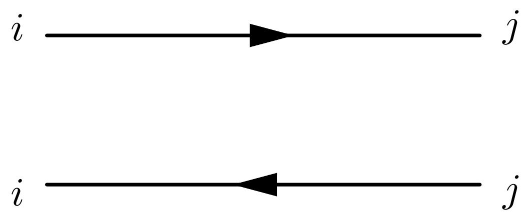
- Intermediate distances $r \sim 1/m_D$ Nadkarni **PRD33** (1986)

- Large distances $r \gg 1/m_D$
  Braaten Nieto **PRL74** (1995)

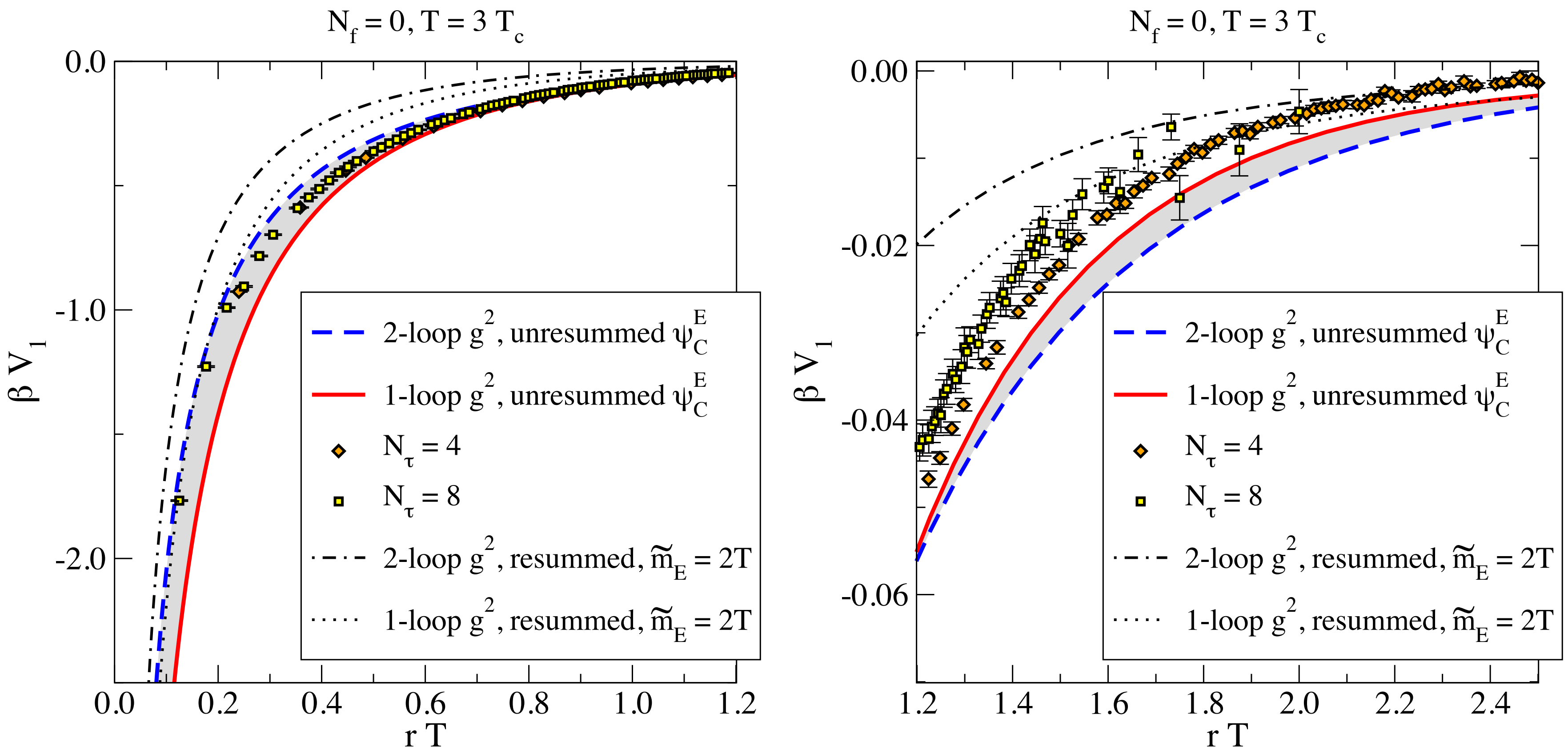
The singlet free energy

- Defined as

$\langle \mathrm{Tr}\, L(\mathbf{x}) L^\dagger(\mathbf{0}) \rangle \qquad L = P \exp\left(ig \int_0^\beta d\tau A^0(\tau, \mathbf{x}) \right)$



Gauge dependent, Coulomb gauge popular



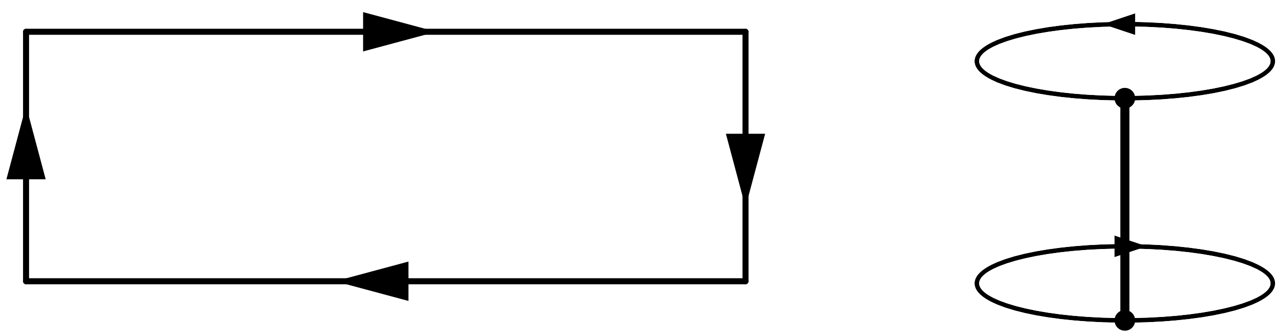
Perturbative: Burnier Laine Vepsäläinen **JHEP1001** (2010)

Lattice: Kaczmarek Karsch Petreczky Zantow **PLB243** (2002)

The cyclic Wilson loop

- A gauge invariant completion of the singlet free energy

$$W_c \equiv \frac{1}{N_c}\langle \mathrm{Tr}\, U(\tau=0;\mathbf{0},\mathbf{r})L(\mathbf{r})U^\dagger(\tau=0;\mathbf{0},\mathbf{r})L^\dagger(\mathbf{0})\rangle$$



- It corresponds to two Polyakov lines connected by an adjoint spacelike Wilson line

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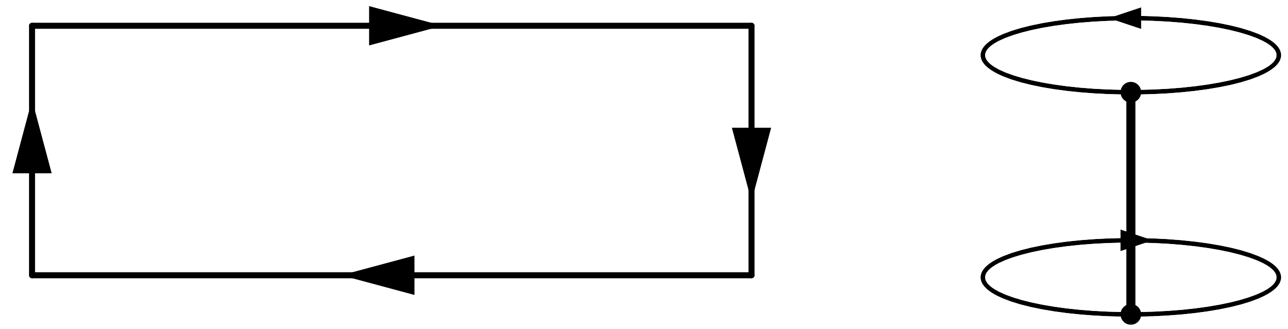


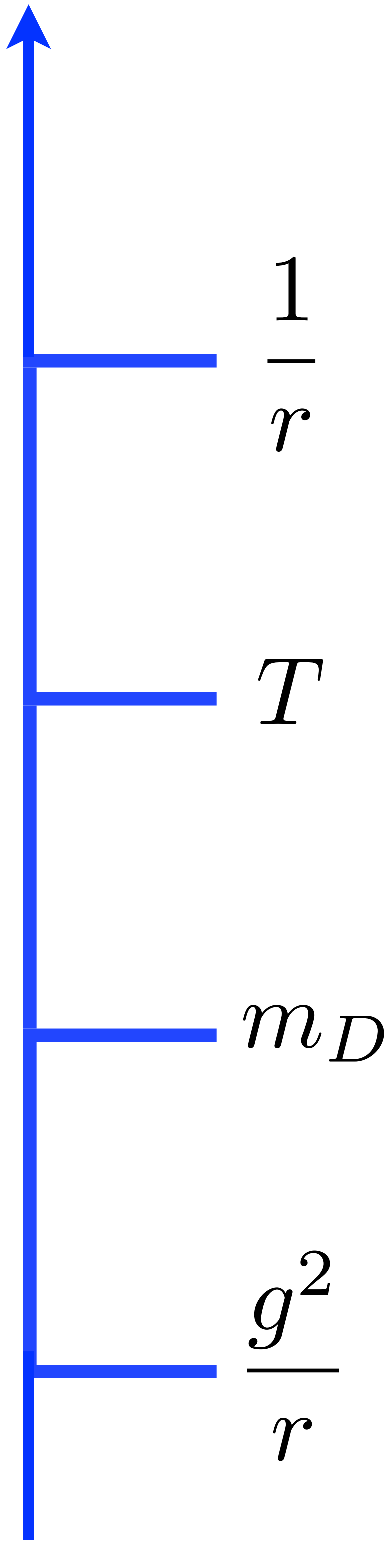
- It corresponds to two Polyakov lines connected by an adjoint spacelike Wilson line
- The restored gauge invariance comes at a price: no longer a simple QQbar free energy and additional divergences

Motivation

- Understand the Polyakov loop correlator in terms of singlet and octet contributions in the EFT framework
- Renormalize the cyclic loop
- Future: program of comparison between perturbation theory and lattice for quarkonium-related quantities

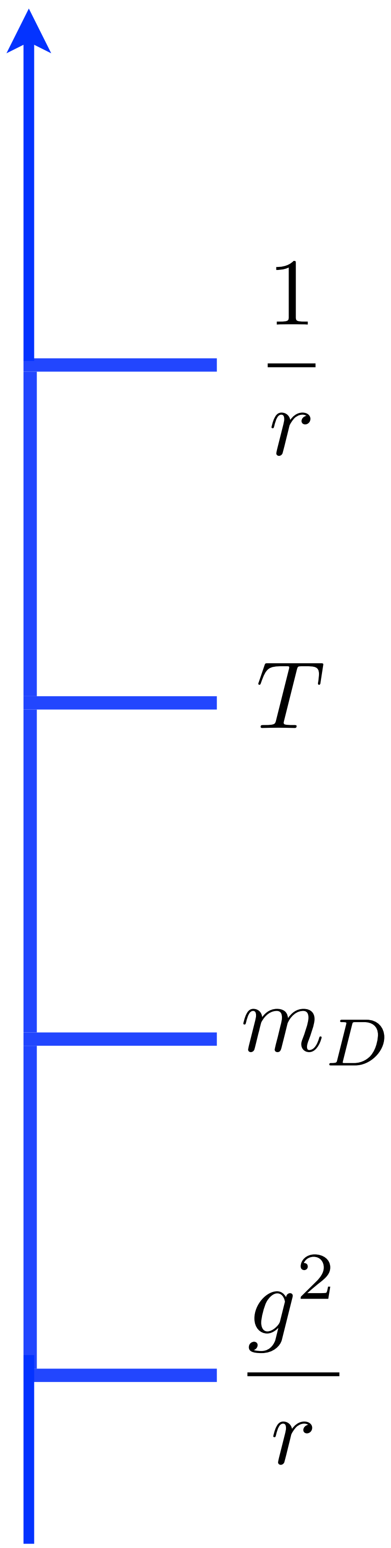
The Polyakov loop correlator

Our perturbative calculation



- The correlator was computed by Nadkarni in 1986 up to order g^6 within EQCD, i. e. $1/r \sim m_D$
  Nadkarni **PRD33** (1986)

Our perturbative calculation



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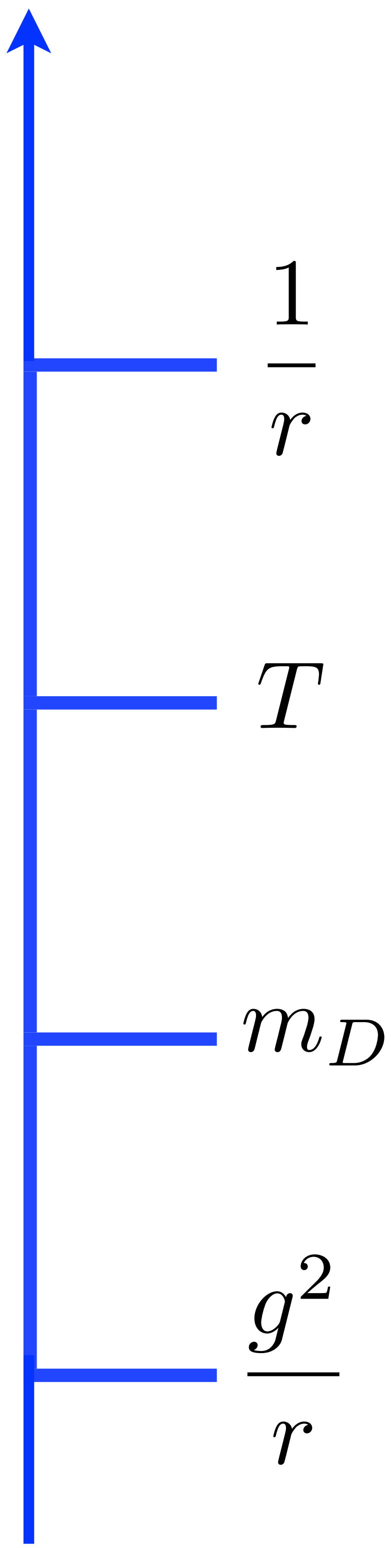
- We performed instead our computation assuming this hierarchy:

$$\frac{1}{r} \gg T \gg m_D \gg \frac{g^2}{r}$$

Our perturbative calculation



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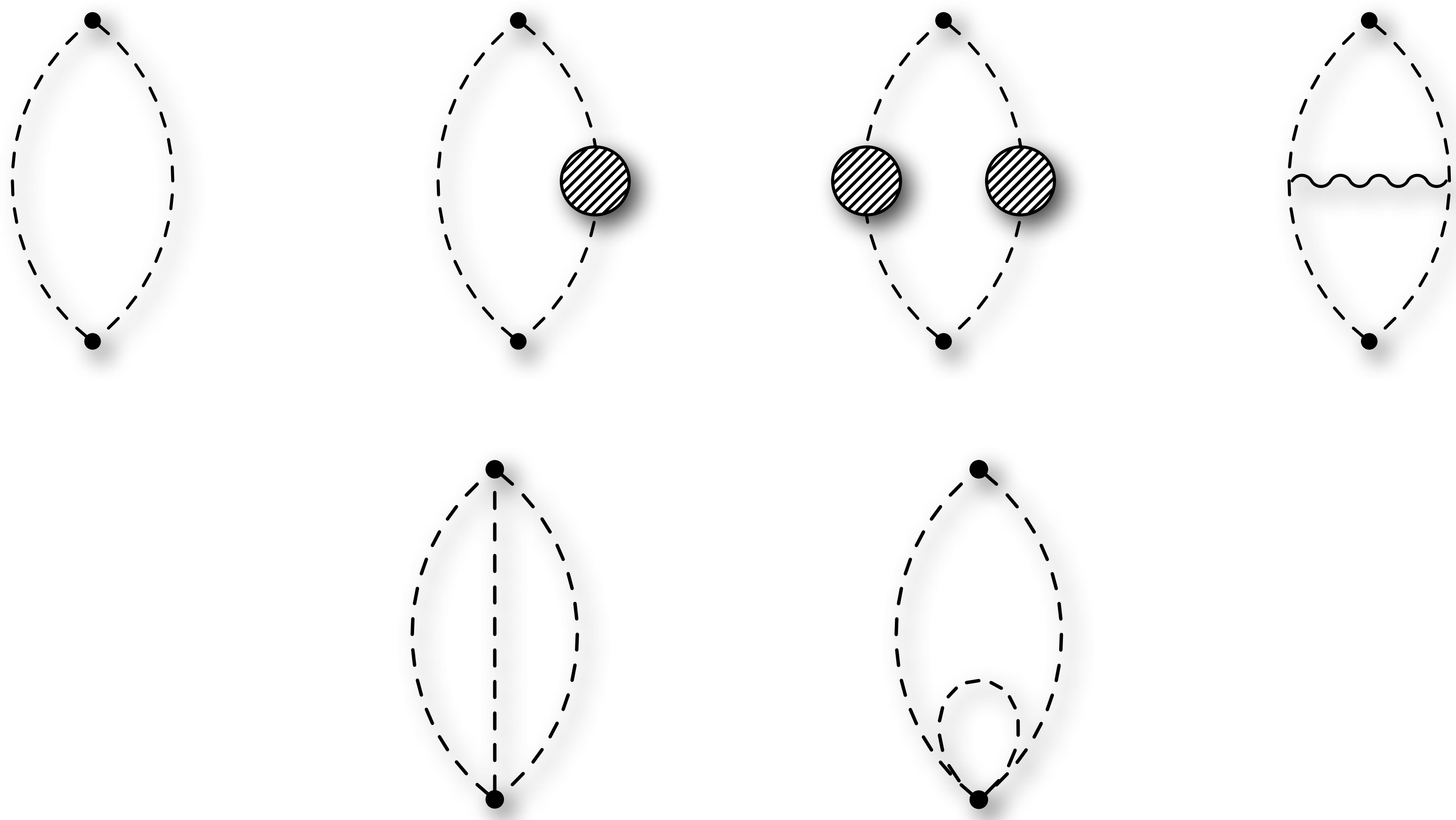
- We performed instead our computation assuming this hierarchy:

$$\frac{1}{r} \gg T \gg m_D \gg \frac{g^2}{r}$$

- rT is an additional expansion parameter, we included terms up to $g^6(rT)^0$

The perturbative result

- The hierarchy is implemented by separating the contribution of each momentum region by appropriate expansions and resummations in the integrals



The perturbative result

- The hierarchy is implemented by separating the contribution of each momentum region by appropriate expansions and resummations in the integrals

$$P_c(r,T) \equiv C_{\rm PL}(r,T) + L_F^2(T)$$

- Up to order $g^6(rT)^0$ we have

$$\begin{aligned} C_{\rm PL}(r,T) \quad &= \quad \frac{N^2-1}{8N^2}\left\{\frac{\alpha_{\rm s}(1/r)^2}{(rT)^2} - 2\frac{\alpha_{\rm s}^2}{rT}\frac{m_D}{T}\right. \\ &\quad + \frac{\alpha_{\rm s}^3}{(rT)^3}\frac{N^2-2}{6N} + \frac{1}{2\pi}\frac{\alpha_{\rm s}^3}{(rT)^2}\left(\frac{31}{9}C_A - \frac{10}{9}n_f + 2\gamma_E\beta_0\right) \\ &\quad + \frac{\alpha_{\rm s}^3}{rT}\left[C_A\left(-2\ln\frac{m_D^2}{T^2} + 2 - \frac{\pi^2}{4}\right) + 2n_f\ln 2\right] \\ &\quad \left. + \alpha_{\rm s}^2\frac{m_D^2}{T^2} - \frac{2}{9}\pi\alpha_{\rm s}^3 C_A\right\} + \mathcal{O}\left(g^6(rT), \frac{g^7}{(rT)^2}\right) \end{aligned}$$

The EFT approach

- We proceed to create an EFT framework that
  - enables us to re-obtain the same results in terms of colour singlet and colour octet correlators
  - gives a more transparent interpretation of the previous result
- Obtained by integrating out *1/r*, the largest scale, yielding Euclidean potential non-relativistic QCD (pNRQCD)

At the scale 1/r

- In pNRQCD the Polyakov loop correlator is given by

$$C_{\mathrm{PL}}(r,T) = \frac{1}{N^2}\Big[Z_s\langle S(\mathbf{r},\mathbf{0},1/T)S^\dagger(\mathbf{r},\mathbf{0},0)\rangle + Z_o\langle O^a(\mathbf{r},\mathbf{0},1/T)O^{a\,\dagger}(\mathbf{r},\mathbf{0},0)\rangle$$
$$+\mathcal{O}\left(\alpha_{\mathrm{s}}^3(rT)^4\right)\Big] - \langle L_F\rangle^2.$$

Higher-dimensional operators with more gauge fields are suppressed.

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Higher-dimensional operators with more gauge fields are suppressed.

- If we match to the previous determination of $C_{\rm PL}(r,T)$ we get

$$Z_s = Z_o = 1$$
$$\langle S(\mathbf{r},\mathbf{0},1/T)S^\dagger(\mathbf{r},\mathbf{0},0)\rangle|_{1/r} = e^{-V_s(r)/T}$$
$$\langle O^a(\mathbf{r},\mathbf{0},1/T)O^{a\,\dagger}(\mathbf{r},\mathbf{0},0)\rangle|_{1/r} = (N^2-1)e^{-V_o(r)/T}$$

which is coherent with the spectral decomposition $\quad P_c = \sum_n e^{-E_n/T}$

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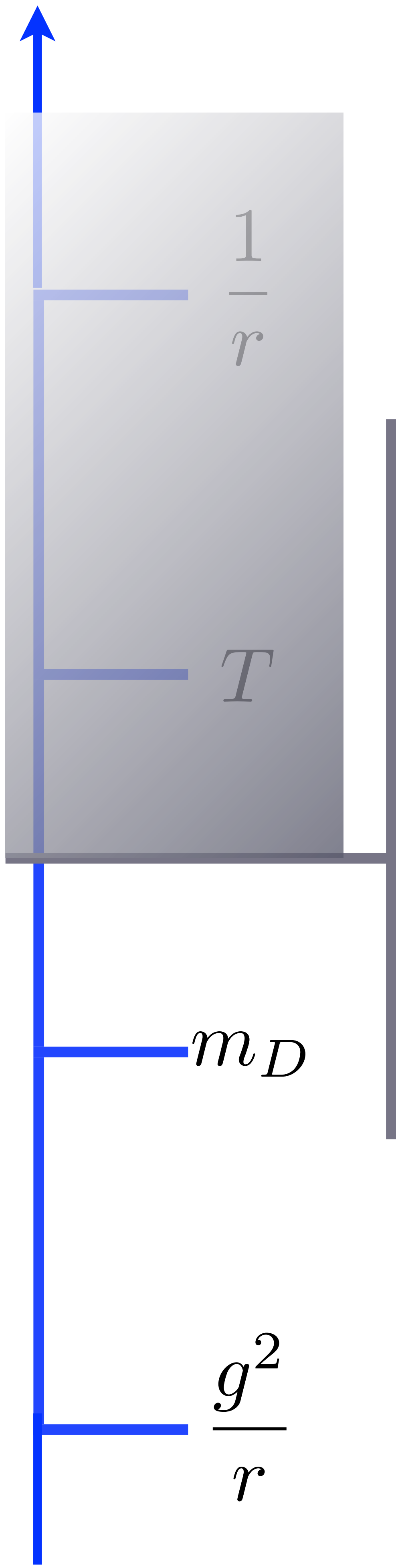
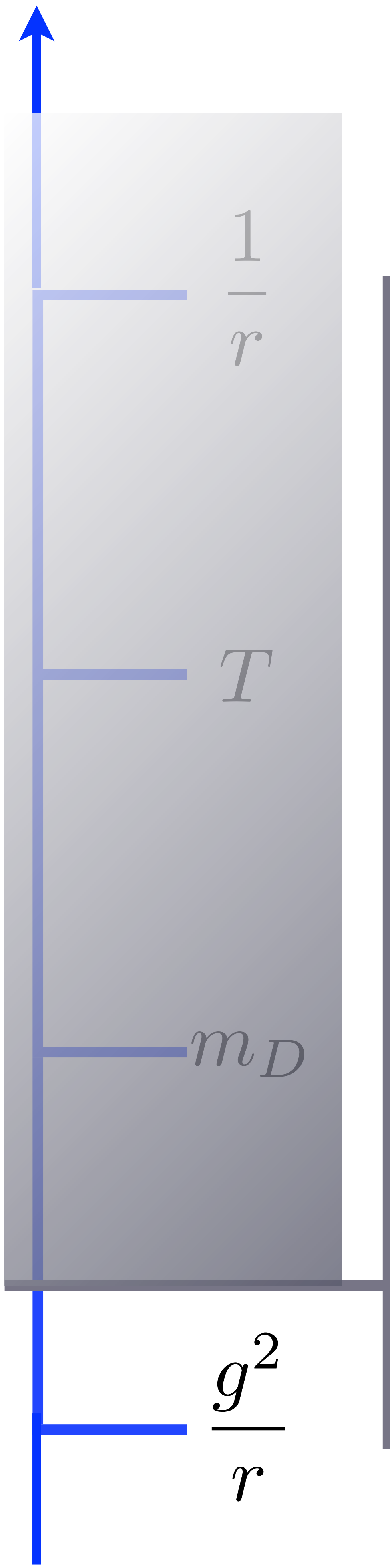
- If we instead assume the spectral decomposition, then the matching provides a non-trivial verification of the two-loop octet potential



Integrating out the temperature

$$\langle S(\mathbf{r},\mathbf{0},1/T)S^\dagger(\mathbf{r},\mathbf{0},0)\rangle|_{1/r,T} = e^{-f_s(r,T)/T}$$

$$\langle O^a(\mathbf{r},\mathbf{0},1/T)O^{a\,\dagger}(\mathbf{r},\mathbf{0},0)\rangle|_{1/r,T} = (N^2-1)e^{-f_o(r,T)/T}$$

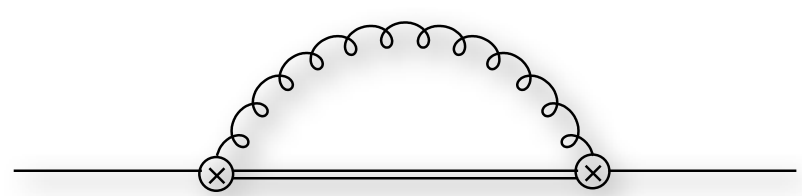


Integrating out the Debye mass

$$\langle S(\mathbf{r},\mathbf{0},1/T)S^\dagger(\mathbf{r},\mathbf{0},0)\rangle|_{1/r,T,m_D} = e^{-f_s(r,T,m_D)/T}$$

$$\langle O^a(\mathbf{r},\mathbf{0},1/T)O^{a\,\dagger}(\mathbf{r},\mathbf{0},0)\rangle|_{1/r,T,m_D} = (N^2-1)e^{-f_o(r,T,m_D)/T}$$

- f_s and f_o may be interpreted as singlet and octet free energies in pNRQCD
- They are obtained by evaluating loop diagrams in pNRQCD



Integrating out T and m_D

- For the singlet we have

$$
\begin{aligned}
f_s(r,T,m_D) &= V_s(r) \\
&+\frac{2}{9}\pi N C_F \alpha_s^2 r T^2 \left[1+\sum c_n^{\rm NS}(rT)^{2n+2}\right] - \frac{\pi}{36}N^2 C_F \alpha_s^3 T \\
&-\left(\frac{3}{2}\zeta(3) C_F \frac{\alpha_s}{\pi}(r m_D)^2 T - \frac{2}{3}\zeta(3) N C_F \alpha_s^2 r^2 T^3\right)\left[1+\sum c_n^{\rm S}(rT)^{2n+2}\right] \\
&+C_F \frac{\alpha_s}{6} r^2 m_D^3 + T\mathcal{O}\left(g^6(rT), \frac{g^8}{rT}\right)
\end{aligned}
$$

Integrating out T and m_D

- For the octet

$$
\begin{aligned}
f_o(r,T,m_D) \;=\;& V_o(r) \\
& -\frac{C_A\alpha_{\rm s}}{2}m_D + \frac{1}{48}C_A^2\alpha_{\rm s}^2\frac{m_D^2}{T} \\
& -\frac{C_A\alpha_{\rm s}^2}{2}T\left[C_A\left(-\ln\frac{T^2}{m_D^2}+\frac{1}{2}\right) - n_f\ln 2 + b_1\,g + b_2\,g^2 + a\,\alpha_{\rm s}\right] \\
& -\frac{\pi}{9}\alpha_{\rm s}^2 rT^2\left[1+\sum c_n^{\rm NS}(rT)^{2n+2}\right] - \frac{\pi}{72}N\alpha_{\rm s}^3 T \\
& +\left(\frac{3}{4N}\zeta(3)\frac{\alpha_{\rm s}}{\pi}(rm_D)^2T - \frac{1}{3}\zeta(3)\alpha_{\rm s}^2r^2T^3\right)\left[1+\sum c_n^{\rm S}(rT)^{2n+2}\right] \\
& -\frac{1}{N}\frac{\alpha_{\rm s}}{12}r^2m_D^3 + T\mathcal{O}\left(g^6(rT),\frac{g^8}{rT}\right)
\end{aligned}
$$

Final results

- In the Polyakov loop correlator $C_{PL}(r,T)$, large cancellations occur between f_s, f_o and the (fundamental) Polyakov loop

$$\langle L_R \rangle = 1 + \frac{C_R \alpha_s}{2}\frac{m_D}{T} + \frac{C_R \alpha_s^2}{2}\left[C_A\left(\ln\frac{m_D^2}{T^2}+\frac{1}{2}\right) - n_f \ln 2 + a\,\alpha_s + b_1\, g + b_2\, g^2\right] + \left(3C_R^2 - \frac{C_R C_A}{2}\right)\frac{\alpha_s^2}{24}\left(\frac{m_D}{T}\right)^2 + \mathcal{O}\left(g^7\right).$$

- They lead to the previous result for $C_{PL}(r,T)$ to order $g^6(rT)^0$.

Comparison with the literature

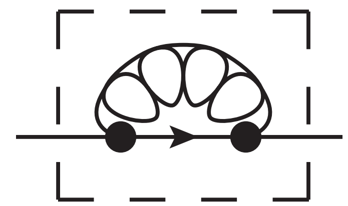
- Recently the singlet static potential at finite temperature has been determined in a pNRQCD EFT framework in real-time.

- The real-time potential has real and imaginary parts. The singlet free energy f_s we have introduced does not agree completely with the real part of the real-time potential Re$V_s(r)$ in the same hierarchy. The difference can be traced back to the different boundary conditions in the two cases, i.e. cyclic imaginary time vs. real large time.
  Brambilla JG Petreczky Vairo **PRD78** (2008)
  Brambilla Escobedo JG Soto Vairo **JHEP1009** (2010)

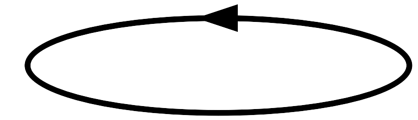
The cyclic Wilson loop

Renormalization of Wilson loops

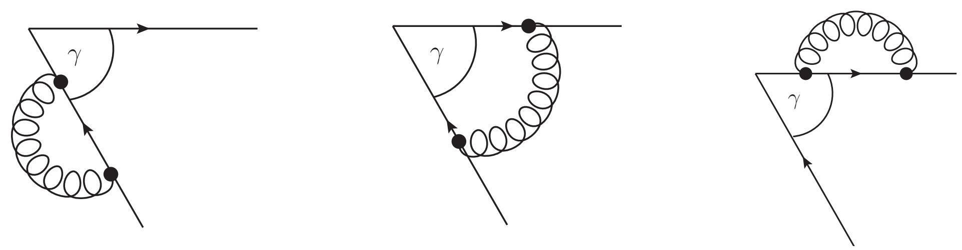
- All Wilson lines have a linear UV divergence proportional to their length:



⇒A Wilson loop with a smooth, nonintersecting contour is finite in DR after charge renormalization



- Cusps in the contour introduce UV *cusp divergences,* renormalized multiplicatively through the *cusp anomalous dimension,* which only depends on the angle. Known in QCD to NLO



$$\frac{\alpha_s C_F}{2\pi\epsilon}(1+(\pi-\gamma)\cot\gamma)$$

Polyakov **NPB84** (1980) Dotsenko Vergeles **NPB169** (1980) Brandt Neri Sato **PRD24** (1981) Korchemsky Radyushkin **NPB283** (1987)

Taxonomy of Wilson loops

Loop	Divergence	Renormalization
Smooth, non-intersecting	linear	multiplicative
rectangular, non-cyclic	linear+cusp (log)	multiplicative

The divergence in the cyclic loop

- Burnier Laine Vepsäläinen computed the loop for *rT~1* in **JHEP1001**. After charge renormalization the result was still UV divergent at order g^4

$$\ln\left(\frac{\psi_{\rm W}(r)}{|\psi_{\rm P}|^2}\right) \approx \mathcal{G}_{\rm DR}\left(\frac{1}{\epsilon},\frac{\bar\mu}{T},rT\right)\frac{C_F\exp(-m_{\rm E}r)}{4\pi Tr} - \frac{g^4C_FN_{\rm c}}{(4\pi)^2}\frac{\exp(-2m_{\rm E}r)}{8T^2r^2}$$
$$+\frac{g^4C_FN_{\rm c}}{(4\pi)^2}\left\{\frac{2{\rm Li}_2(e^{-2\pi Tr})+{\rm Li}_2(e^{-4\pi Tr})}{(2\pi Tr)^2}\right.$$
$$\left.+\frac{1}{\pi Tr}\int_1^\infty {\rm d}x\left[\frac{1}{x^2}\ln\left(1-e^{-2\pi Trx}\right)+\left(\frac{1}{x^2}-\frac{1}{2x^4}\right)\ln\left(1-e^{-4\pi Trx}\right)\right]\right\}$$
$$+\frac{g^4C_FN_{\rm f}}{(4\pi)^2}\left[\frac{1}{2\pi Tr}\int_1^\infty{\rm d}x\left(\frac{1}{x^2}-\frac{1}{x^4}\right)\ln\frac{1+e^{-2\pi Trx}}{1-e^{-2\pi Trx}}\right]+\mathcal{O}(g^5)\ .$$

$$\mathcal{G}_{\rm DR}\left(\frac{1}{\epsilon},\frac{\bar\mu}{T},rT\right)\overset{m_{\rm E}r\ll1}{\approx} g^2\left\{1+\frac{g^2}{(4\pi)^2}\left[4N_{\rm c}\left(\frac{1}{\epsilon}+\ln\frac{\bar\mu^2}{T^2}+\mathcal{O}(1)\right)\right]\right\}$$

The divergence in the cyclic loop

- We perform a calculation for *rT≪1*, focusing only on the UV aspects and on the contribution from the scale *1/r*.

$$\ln W_c = \frac{C_F\alpha_s}{rT}\left\{1+\frac{\alpha_s}{4\pi}\left[\left(\frac{31}{9}C_A-\frac{20}{9}T_F n_f\right)+\beta_0\left(\ln\mu^2 r^2+2\gamma_E\right)\right]\right\}$$
$$+\frac{4\pi C_F\alpha_s}{T}\int_k \frac{e^{i\mathbf{r}\cdot\mathbf{k}}-1}{(\mathbf{k}^2)^2}\left(-\Pi^{(T)}_{00\,\mathrm{CG}}(0,\mathbf{k})\right)+C_F C_A\alpha_s^2$$
$$+\frac{4C_F C_A\alpha_s^2}{T}\int_k \frac{e^{i\mathbf{r}\cdot\mathbf{k}}}{\mathbf{k}^2}\left[\frac{1}{\epsilon}+1+\gamma_E+\ln\pi+\ln\mu^2 r^2\right]$$
$$+\frac{2C_F C_A\alpha_s^2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n\zeta(2n)}{n(4n^2-1)}(rT)^{2n-1}$$

The divergent terms agree. The divergence is UV and cannot be renormalized multiplicatively

Taxonomy of Wilson loops

Loop	Divergence	Renormalization
Smooth, non-intersecting	linear	multiplicative
rectangular, non-cyclic	linear+cusp (log)	multiplicative
cyclic (W_c)	linear+??? (log)	???

Origin of the divergence

- In Coulomb gauge the singlet free energy is finite

$$\ln\langle \mathrm{Tr} L(\mathbf{r}) L^\dagger(\mathbf{0})\rangle = \frac{C_F\alpha_s}{rT}\left\{1+\frac{\alpha_s}{4\pi}\left[\left(\frac{31}{9}C_A-\frac{20}{9}T_F n_f\right)+\beta_0\left(\ln\mu^2r^2+2\gamma_E\right)\right]\right\}$$

$$+\frac{4\pi C_F\alpha_s}{T}\int_k \frac{e^{i\mathbf{r}\cdot\mathbf{k}}-1}{(\mathbf{k}^2)^2}\left(-\Pi^{(T)}_{00\,\mathrm{CG}}(0,\mathbf{k})\right)$$

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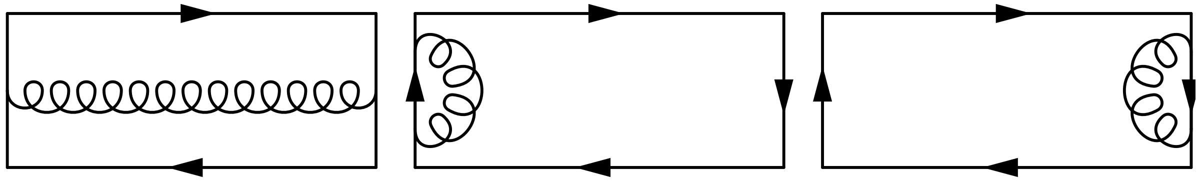
$$+\frac{4\pi C_F\alpha_s}{T}\int_k \frac{e^{i\mathbf{r}\cdot\mathbf{k}}-1}{(\mathbf{k}^2)^2}\left(-\Pi^{(T)}_{00\,\mathrm{CG}}(0,\mathbf{k})\right)$$

- Add the strings: a lot of diagrams cancel because of cyclicity (all those where the two strings are connected on at least one side by the singlet component of a Polyakov line)



Scheme-independent cancellation

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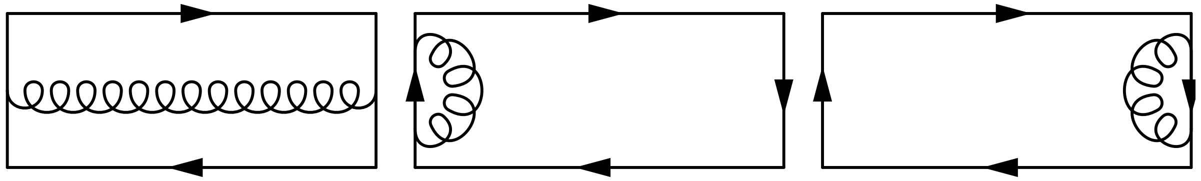
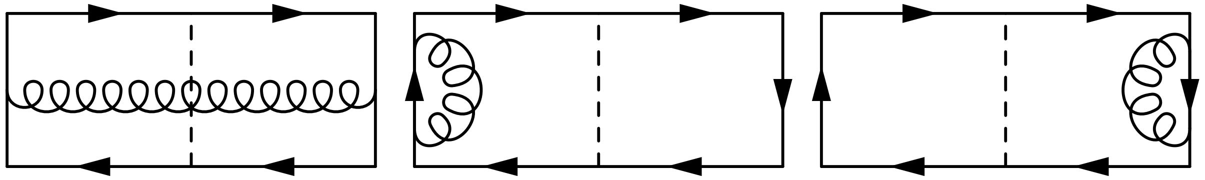


Scheme-independent cancellation

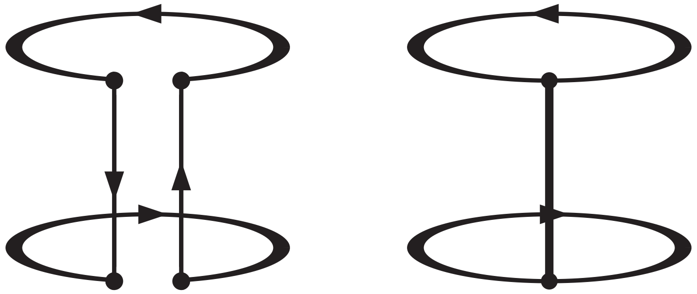
- The divergence is then given by these diagrams



$C_F^2 - C_F C_A/2$ C_F^2 C_F^2

Renormalization

- The divergence is related to the cusp divergence, but not quite the same. Indeed, thinking cylindrically, the cyclic Wilson loop is topologically different from a regular one



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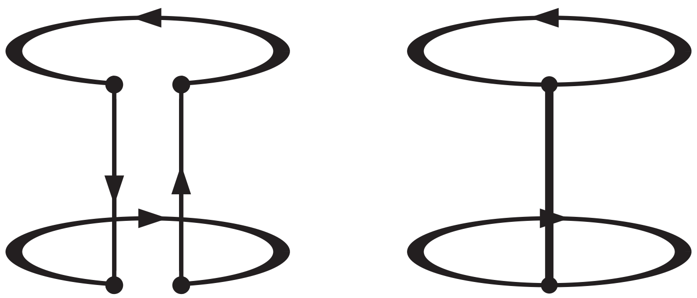

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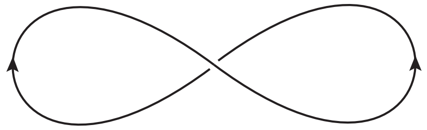
- Wilson loops with intersections are renormalized in matrix form, by considering all possible choices of paths at the intersection



Brandt Neri Sato **PRD24** (1981) Korchemskaya Korchemsky **NPB437** (1995)

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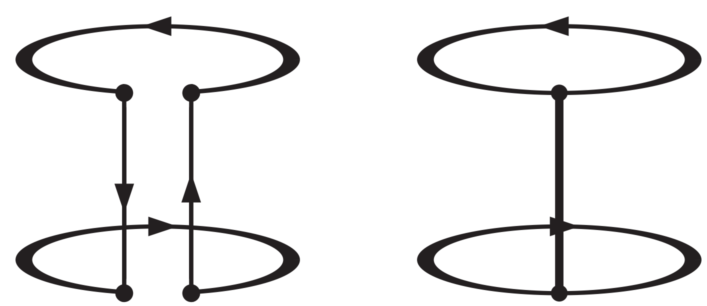

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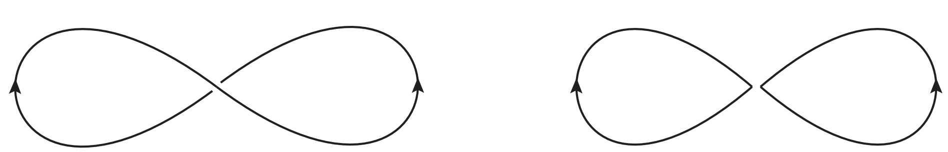
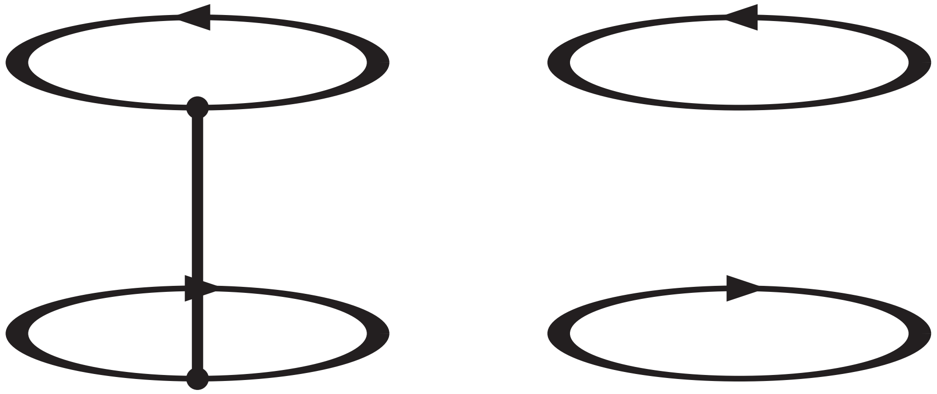


$$W_R^i = Z^{ij}(\theta) W^j$$

Brandt Neri Sato **PRD24** (1981) Korchemskaya Korchemsky **NPB437** (1995)

Renormalization

- The procedure is the same in the case of n intersections. In our case in principle n=∞, but in practice there are only two independent paths:



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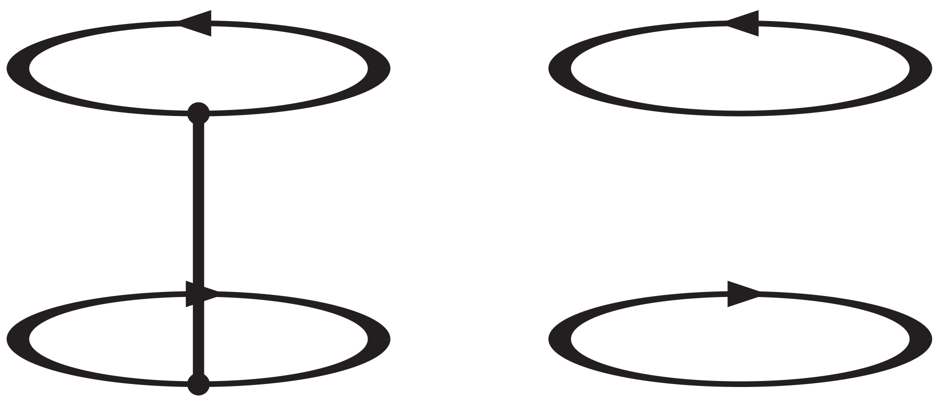

- They are the cyclic loop (W_c) and the correlator of two Polyakov loops (P_c). The latter being finite, the renormalization matrix reads

$$\begin{pmatrix} W_c^R \\ P_c \end{pmatrix} = \begin{pmatrix} Z & (1-Z) \\ 0 & 1 \end{pmatrix} \begin{pmatrix} W_c \\ P_c \end{pmatrix}$$

Intermediate summary

- We have obtained that the cyclic Wilson loop is not renormalized multiplicatively. Due to the periodic boundary conditions, it mixes with the Polyakov loop correlator under renormalization.

$$W_c^R = Z\, W_c + (1 - Z)\, P_c$$

- Alternatively, diagonalize the matrix$\Rightarrow$ W_c-P_c is multiplicatively renormalizeable

- This renormalization prescription is valid at weak and strong coupling

Perturbative renormalization

- The renormalization equation gives

$$\begin{aligned} W_c^R &= Z W_c + (1-Z) P_c \\ &= 1 + \frac{C_F \alpha_s}{rT} + \frac{C_F^2 \alpha_s^2}{2r^2T^2} \\ &\quad + \frac{4\pi C_F \alpha_s}{T} \int_k \frac{e^{i\mathbf{r}\cdot\mathbf{k}}}{\mathbf{k}^2} \left(\frac{C_A \alpha_s}{\pi \epsilon} + Z_1 \alpha_s + \dots \right) + \dots \end{aligned}$$

$$P_c = 1 + \mathcal{O}(g^3)$$

$$Z \equiv 1 + Z_1 \alpha_s + \dots$$

- This implies $Z_1 = -\frac{C_A}{\pi}\left(\frac{1}{\epsilon} - \gamma_E + \ln 4\pi\right)$

- The renormalization procedure has been tested successfully to order g^6, where P_c matters

- Accounting for the different geometries and signatures, it agrees with the result of Korchemskaya Korchemsky **NPB437** (1995). Up to this order

$$Z_{1,2}^{\text{int}} = \frac{C_A}{2C_F} Z_{1,2}^{\text{cusp}}$$

Non-perturbative renormalization

- Dealing directly with W_c is probably complicated. W_c-P_c instead is multiplicatively renormalizeable
- It has linear divergences proportional to r and $1/T$ and intersection log divergences
- Ratios like this should be cutoff-independent

$$\frac{(W_c - P_c)(r)\,(W_c - P_c)(2r_0 - r)}{(W_c - P_c)(r_0)\,(W_c - P_c)(r_0)}$$

another way of comparing PT and lattice

- First measurements of W_c in Bazavov Petreczky 1303.5500

Taxonomy of Wilson loops

Loop	Divergence	Renormalization
Smooth, non-intersecting	linear	multiplicative
rectangular, non-cyclic	linear+cusp (log)	multiplicative
cyclic (W_c)	linear +intersection (log)	mixing with P_c
W_c–P_c	linear+int. (log)	multiplicative

Conclusions

- The Polyakov loop correlator P_c
  - is a well defined, gauge invariant free energy
  - at short distances it can be expressed in an EFT framework
- The cyclic Wilson loop W_c
  - mixes under renormalization with P_c. The difference is multiplicatively renormalizeable
  - is then another well-defined and gauge-invariant operator. Comparisons with lattice are possible, as well as EFT framework