Chemical equilibration of heavy quarks

Dietrich Bödeker, Bielefeld U.

work with Mikko Laine

JHEP 1207 (2012) 130, ibid. 1301 (2013) 037

Outline

relaxation to equilibrium

chemical equilibration in perturbation theory

defining heavy quark number density non-perturbatively

heavy quark annihilation in imaginary time NRQCD

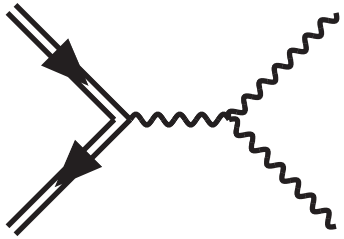
Sommerfeld effect

chemical equilibrium

heavy quarks Q stable within QCD

kinetic equilibration by scattering of Q with light quarks, gluons

chemical equilibration by $Q\bar{Q}$-annihilation



similar to dark matter decoupling in the early Universe

General consideration

system in thermal equilibrium

exception: one slowly relaxing quantity n

deviations from thermal equilibrium

$$\delta n \equiv n - n_{\rm eq}$$

time evolution

$$\delta \dot{n} = -\Gamma \delta n$$

Γ only depends on the medium properties, i.e. temperature T

Γ from Boltzmann equation

weak coupling, leading order in α_{S}: Boltzmann equation

$T \ll M \Rightarrow$

$$\dot{n} = c\left(n_{\mathrm{eq}}^2 - n^2\right) \equiv \dot{n}_{\mathrm{gain}} + \dot{n}_{\mathrm{loss}}$$

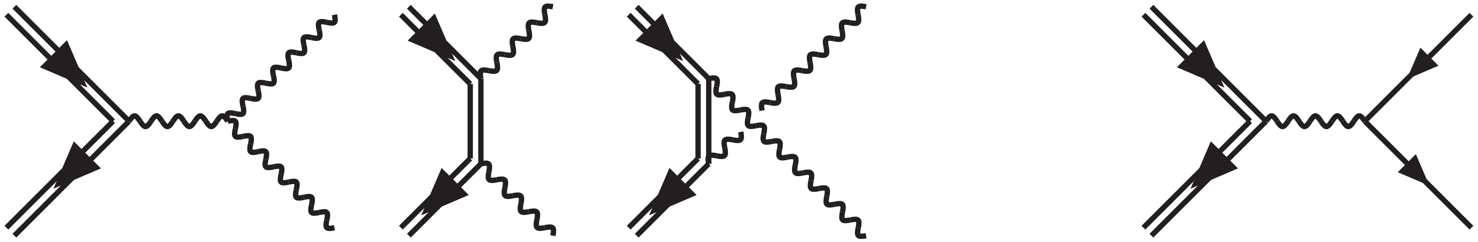
[Lee, Weinberg; Bernstein, Brown, Feinberg]

linearize in $\delta n \quad \Rightarrow$

$$\Gamma = \frac{2\dot{n}_{\mathrm{gain}}}{n_{\mathrm{eq}}}$$

Γ from Boltzmann equation (cont'd)

tree level



heavy quark limit $T \ll M$

$$\Gamma \approx \frac{g^4 C_F}{8\pi M^2}\left(N_f + 2C_F - \frac{N_c}{2}\right)\left(\frac{TM}{2\pi}\right)^{\frac{3}{2}} e^{-M/T}$$

[$N_f = 0$: Rafelski, Müller 1986]

$N_f = 3$, $M = 1.5$GeV $\alpha_s = 0.3$ gives

$\Gamma^{-1}(T = 600$ MeV$) = 10$ fm, $\Gamma^{-1}(T = 400$ MeV$) = 60$ fm

higher order corrections? non-perturbative computation?

Γ from equilibrium correlation function

thermal fluctuations $\delta n \equiv n - n_{\rm eq}$

$$\delta \dot{n} = -\Gamma \delta n + \xi$$

ξ = white noise

correlation function

$$C(t) \equiv \left\langle \delta n(t) \delta n(0) \right\rangle = \left\langle \delta n^2 \right\rangle \exp\left(-\Gamma |t|\right)$$

$$C(\omega) = \frac{2\Gamma}{\omega^2 + \Gamma^2} \langle \delta n^2 \rangle \simeq \frac{2\Gamma}{\omega^2} \langle \delta n^2 \rangle \qquad (\Gamma \ll \omega \ll \omega_{\rm microscopic})$$

Non-perturbative defintion of particle number

there's no particle number density operator in relativistic field theory

idea: in the non-relativistic limit

- particle number $\times M \simeq$ energy in Q, $\overline{Q}$
- energy in Q, $\overline{Q} \simeq H_Q$

$$H_Q \equiv \int d^3x T_Q^{00}, \qquad T^{\mu\nu} = T_Q^{\mu\nu} + T_{\text{light}}^{\mu\nu}$$

so we suggest to *define*

$$n = \frac{1}{M} H_Q$$

Non-perturbative defintion of particle number (cont'd)

definition serves to extract a time scale

at weak coupling $\rightsquigarrow$ same result as Boltzmann equation

non-perturbative computation using NRQCD?

NRQCD

heavy quarks have nothing to annihilate into

4-fermion operators with $d = 6$ [Bodwin, Braaten, Lepage 1994]

$$\delta \mathscr{L}_M = \frac{f_1(^1S_0)}{M^2}\,\mathscr{O}_1(^1S_0) + \frac{f_1(^3S_1)}{M^2}\,\mathscr{O}_1(^3S_1) + \frac{f_8(^1S_0)}{M^2}\,\mathscr{O}_8(^1S_0) + \frac{f_8(^3S_1)}{M^2}\,\mathscr{O}_8(^1S_1)$$

$$\mathscr{O}_1(^1S_0) \equiv \psi^\dagger \chi\, \chi^\dagger \psi \qquad \mathscr{O}_1(^3S_1) \;\equiv\; \psi^\dagger \boldsymbol{\sigma} \chi \cdot \chi^\dagger \boldsymbol{\sigma} \psi$$

$$\mathscr{O}_8(^1S_0) \equiv \psi^\dagger T^a \chi\, \chi^\dagger T^a \psi \qquad \mathscr{O}_8(^3S_1) \;\equiv\; \psi^\dagger \boldsymbol{\sigma} T^a \chi \cdot \chi^\dagger \boldsymbol{\sigma} T^a \psi$$

annihilation described *in real time* by

$$\mathrm{Im}(f_1),\; \mathrm{Im}(f_8) = O(\alpha_s^2) \neq 0$$

NRQCD and imaginary time

imaginary part from discontinuity across real ω-axis

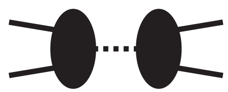
how to include it in imaginary time formulation?

spectral representation

$$\delta S = \int_{\boldsymbol{x},\Omega_i} \left[\psi^\dagger(\Omega_1,\boldsymbol{x})\chi(\Omega_2,\boldsymbol{x})\right] \left[\int dz \frac{\rho(z)}{\Omega_1+\Omega_2-z}\right] \left[\chi^\dagger(\Omega_3,\boldsymbol{x})\psi(\Omega_4,\boldsymbol{x})\right]$$

imaginary time

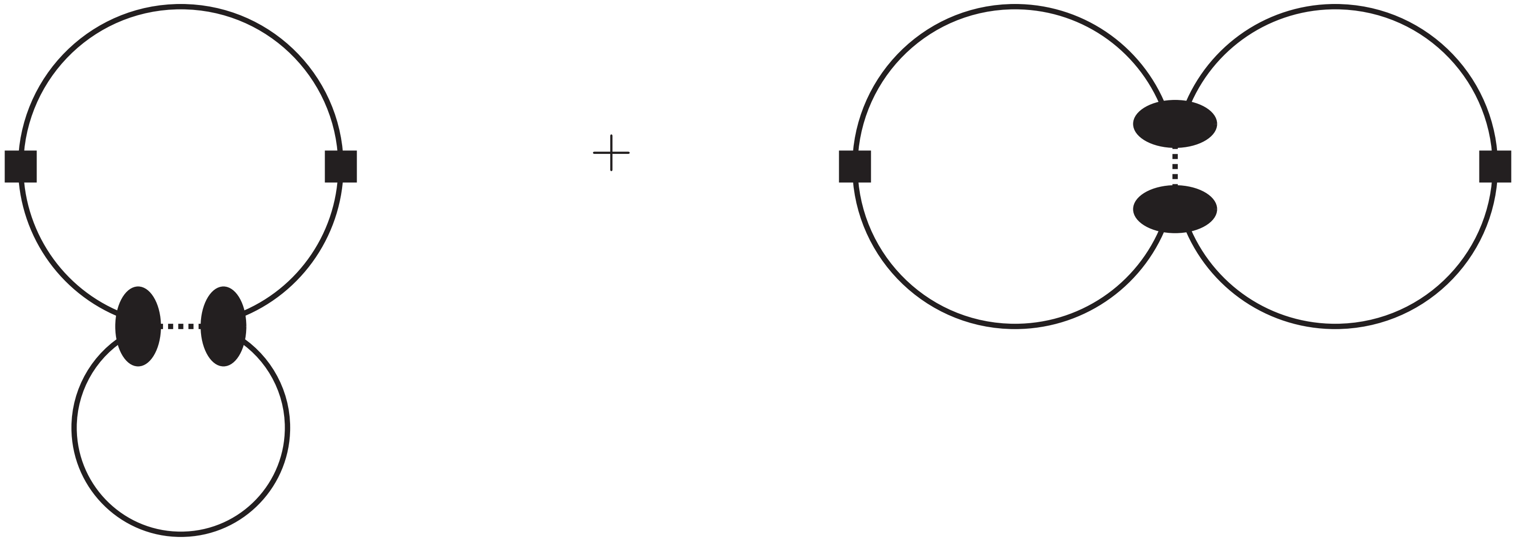
$$\delta S = \int_{\tau_i,\boldsymbol{x}} \left[\psi^\dagger\chi\right](\tau_1,\boldsymbol{x})\, K(\tau_1-\tau_2)\, \left[\chi^\dagger\psi\right](\tau_2,\boldsymbol{x}) \quad =$$



Perturbative evaluation

compute Γ in NRQCD

lowest order



LO contributions from singlet and octet operators

Sommerfeld effect

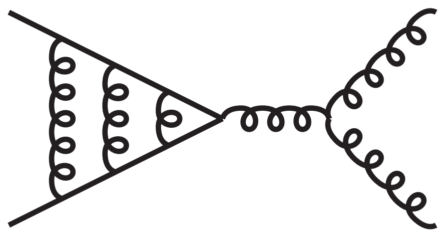
[Sommerfeld 1931; Landau, Lifshitz; Fadin, Khoze, Sjöstrand 1990]

velocity $v \sim \sqrt{T/M} \ll 1$ for heavy quarks

$$\sigma = \sigma_0 \left[1 + O\left(\frac{\alpha_\mathrm{s}}{v}\right) + O\left(\alpha_\mathrm{s} \ln v\right) + O\left(\alpha_\mathrm{s}\right)\right]$$

higher order corrections not suppressed when $v \lesssim \alpha_\mathrm{s} \quad \Leftrightarrow \quad T \lesssim g^4 M$

resummation necessary

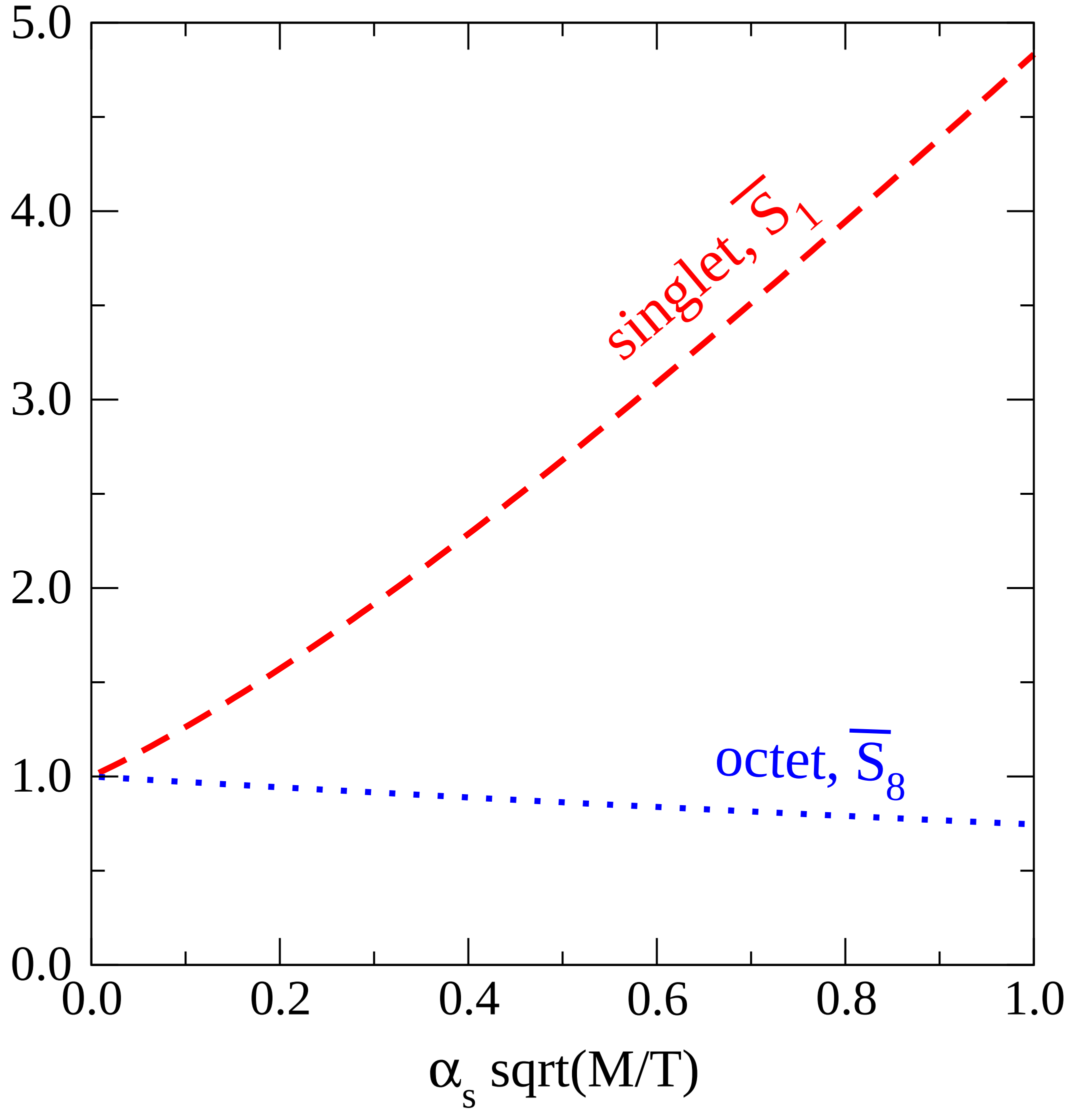


$$|\mathscr{M}_\mathrm{resummed}|^2 = S\,|\mathscr{M}_\mathrm{tree}|^2 \,,$$

Sommerfeld factors

$$S_1 = \frac{X_1}{1 - e^{-X_1}}\,, \quad X_1 \equiv C_f \frac{g^2}{4v} \qquad\qquad S_8 = \frac{X_8}{e^{X_8} - 1}\,, \quad X_8 \equiv \left(\frac{N_c}{2} - C_f\right)\frac{g^2}{4v}$$

Thermally averaged Sommerfeld factors



Sommerfeld resummed rate

$$\Gamma = \frac{g^4 C_f}{8\pi M^2}\left(\frac{MT}{2\pi}\right)^{3/2} e^{-M/T}\left[\left(2C_f - \frac{N_c}{2}\right)\left(\frac{1}{1+r}\bar{S}_1 + \frac{r}{1+r}\bar{S}_8\right) + N_f\,\bar{S}_8\right]$$

$r = (N_c^2 - 4)/2$

for $\alpha_{\rm s} = 0.3$, $M = 1.5$ GeV, $T = 300$ MeV

$\bar{S}_1 = 3.4$, $\bar{S}_8 = 0.8$

but $[\cdots] =$ 4.28 instead of 4.17 without resummation

Summary

heavy quark chemical equilibration is slow

we give non-perturbative definition of heavy quark number

should be useful for non-perturbative calculation

prescription for NRQCD

Sommerfeld effect parametrically important

compensation of enhancement and suppression