Heavy Quarkonium Spectral Functions with Complex Potential from Gauge/Gravity Duality

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Reference: T.H., K. Nawa, T. Hatsuda, arXiv:1211.4942[hep-ph]

Introduction

○ Propagation of heavy quark (quarkonia) inside QGP

- Transport coefficients

$$\boxed{\begin{aligned} \mathrm{d}p &= -\eta_D p\,\mathrm{d}t + \xi(t)\mathrm{d}t \\ \langle \xi(t)\xi(t') \rangle &= 2\gamma T^3 \delta(t-t') \end{aligned}}$$

- Decay widths of quarkonia

$$\boxed{\frac{\mathrm{d}N}{\mathrm{d}t} = -\Gamma(N - N_{\mathrm{eq}})}$$

Introduction

○ Quarkonium spectral function

• Thermal perturbation theory + NRQCD

Laine , Philipsen, Tassler, Romatschke (2007)
Beraudo, Blaizot, Ratti (2008)
N. Brambilla, J. Ghiglieri, A. Vairo and P. Petreczky (2008)
Brambilla, Escobedo, Ghiglieri, Soto, Vairo (2010)

Introduction

○ Quarkonium spectral function

• Lattice QCD simulations + Bayesian technique (MEM)

Asakawa, Hatsuda (2004)
Datta, Karsch, Petreczky, Wetzorke (2004)
Umeda, Nomura, Matsufuru (2005)
Aarts, Alltona, Kim, Lombardo, Oktay, Ryan, Sinclair, Skullerud (2011)
Banerjee, Datta, Gavai, Majumdar (2012)
Ding, Francis, Kaczmarek, Karsch, Satz, Soeldner (2012)

• QCD sum rules

Gubler, Morita, Oka (2011)
Suzuki, Gubler, Morita, Oka (2013).

Introduction

○ Quarkonium spectral function

- Gauge/Gravity duality + NRQCD/MEM

○ Complex potential from gauge/gravity duality

- Potential model in leading order $N_{\rm c},\ \lambda$ expansion
- MEM potential approach

In-Medium $Q\bar{Q}$ potential from Wilson loop

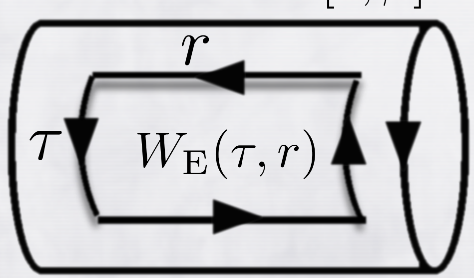
○ Time-dependent and complex potential

$$\boxed{V_{Q\bar{Q}}(t,r) = i\partial_t \ln W_{\mathrm{E}}(it,r)}$$

- How fast reaches asymptotic profile?
- Magnitude of Im part?

○ Euclid Wilson loop from gauge/gravity duality

$\tau \in [0, \beta]$



Gravity dual description of Wilson loop

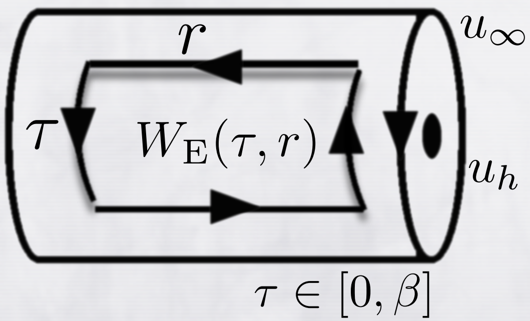
○ Extremum of Nambu-Goto action $S_{\rm NG}$ Maldacena, PRL 80 4859 (1998)

$$\boxed{W_E(\tau, r) = \mathrm{e}^{-S_{\rm NG}(\tau, r, X^\mu)}}$$

- Finite T ⟷ AdS black hole metric in 5-dim

$$\boxed{\mathrm{d}s^2 = l_s^2\left[\frac{u^2}{R^2}\left\{f(u)\mathrm{d}\tau'^2 + \mathrm{d}\mathbf{x}^2\right\} + \frac{R^2}{u^2}\frac{\mathrm{d}u^2}{f(u)}\right]}$$

$$f(u) = 1 - (\pi T R^2)^4/u^4$$
$$\underline{R = (2\lambda)^{1/4},\ \lambda = g_{\rm YM}^2 N_{\rm c}}$$



Gravity dual description of Wilson loop

○ Equation of motion ($u \to u/\pi T R^2$) with rectangular contour

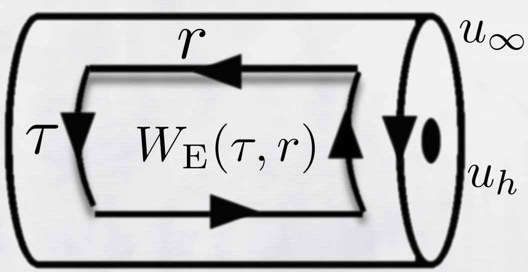
$$
\begin{aligned}
&2u(t,r)^4 u'(t,r)\dot{u}(t,r)\dot{u}'(t,r) + u(t,r)^4\left(1 - u(t,r)^4 - u'(t,r)^2\right)\ddot{u}(t,r) \\
&-\left(1 + u(t,r)^8 + u(t,r)^4\left(-2 + \dot{u}(t,r)^2\right)\right)u''(t,r) + 2u(t,r)^3\left(1 + 2u(t,r)^4\right)\dot{u}(t,r)^2 \\
&-4u(t,r)^3\left(1 + u(t,r)^4\right)u'(t,r)^2 + 2u(t,r)^3 - 4u(t,r)^7 + 2u(t,r)^{11} = 0
\end{aligned}
$$

- Difficult to solve PDE analytically

➔Take variational approach

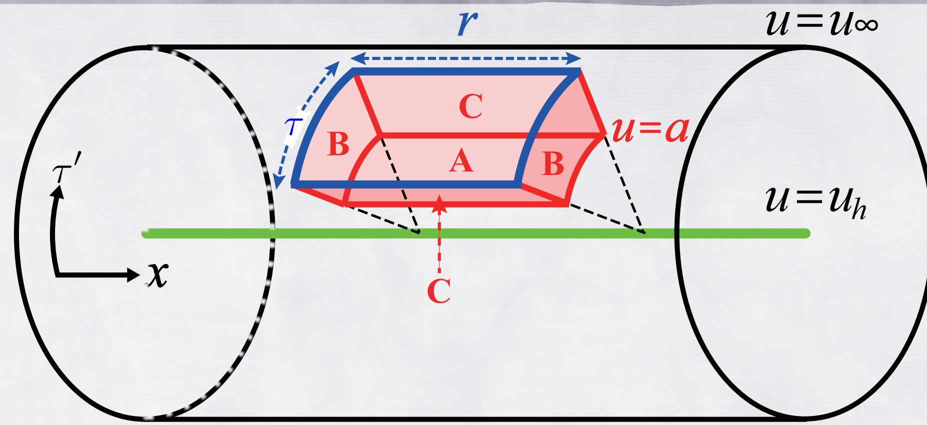
$f(u) = 1 - (\pi T R^2)^4/u^4$

$R = (2\lambda)^{1/4}$, $\lambda = g_{\rm YM}^2 N_{\rm c}$

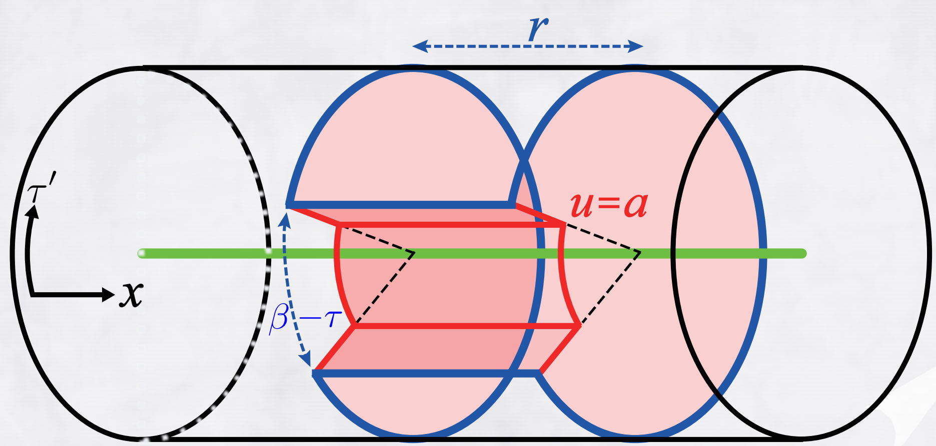


$\tau \in [0, \beta]$

Box-type ansatz



Configuration I

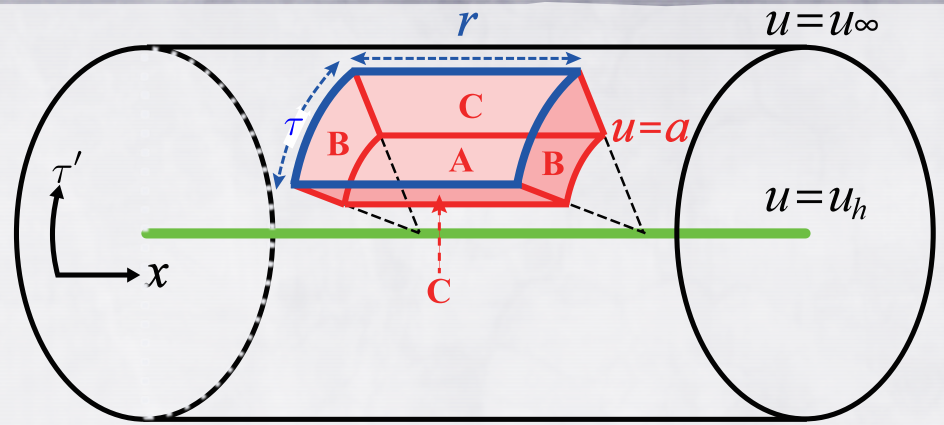


Configuration III

Box-type ansatz

○ Extremum condtion

$$\boxed{\frac{a}{R^2} - \frac{1}{\tau} = \frac{1}{r}\sqrt{1 - \frac{u_h^4}{a^4}}}$$



Configuration I

○ Regulated NG action

$$S_{\mathrm{NG}}^{(\mathrm{I})}(\tau, r, a) = \frac{1}{2\pi}\Big[\tau r \frac{a^2}{R^2}\sqrt{f(a)} - 2\tau(a - u_\infty) + 2r\Big(\int_a^{u_\infty} \mathrm{d}u\, \frac{1}{\sqrt{f(u)}}\Big) - 2(\tau + r)u_\infty\Big]$$

$$= \frac{1}{2\pi}\Big[\tau r \frac{a^2}{R^2}\sqrt{f(a)} - 2\tau a - 2raF(-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}; \frac{1}{a^4})\Big]$$

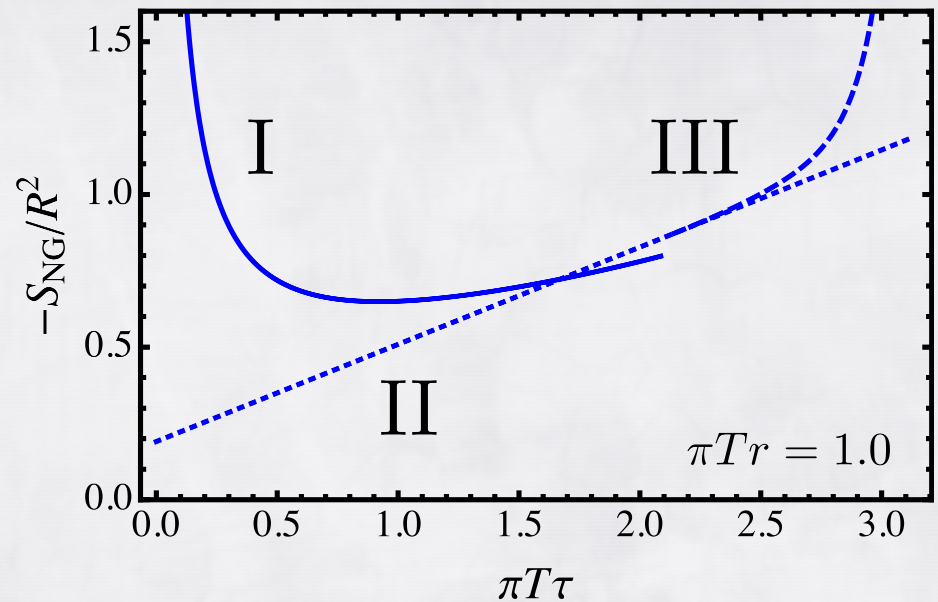
- Subtract out quark self-energy

Box-type ansatz

○ Wilson loop

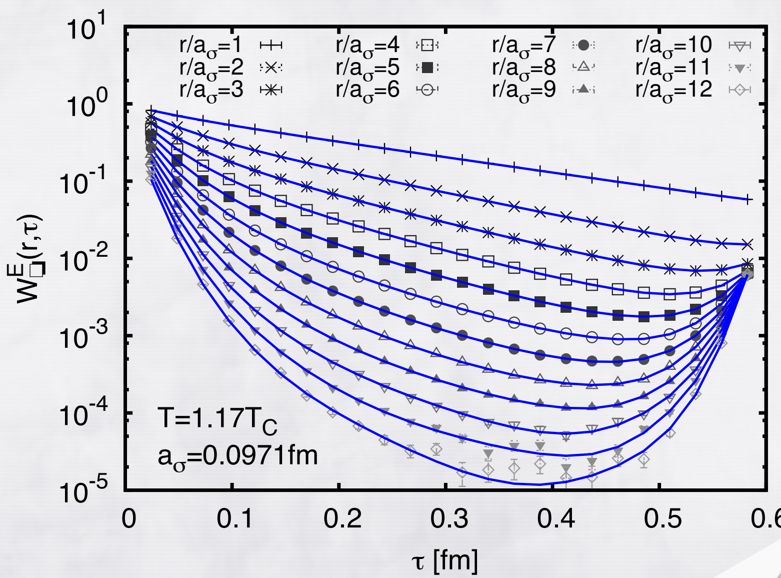
- W/o cusp contributions

- Non-smoothness:
Artifact of ansatz?



- Short time behavior

→ Analytic continuation of Configuration I



Rothkopf *et. al.*, PRL 108 162001 (2012)

Time-dependent $Q\bar{Q}$ potential at finite T

○ Analytic continuation of Wilson loop

- Extremum eq. with $\tau \to it$

$$\bar{a} = a/R^2$$

$$\bar{a} + \frac{i}{t} = \frac{1}{r}\sqrt{1 - \left(\frac{\pi T}{\bar{a}}\right)^4}$$

- Unique solution $\bar{a}_{\mathrm{I}}$

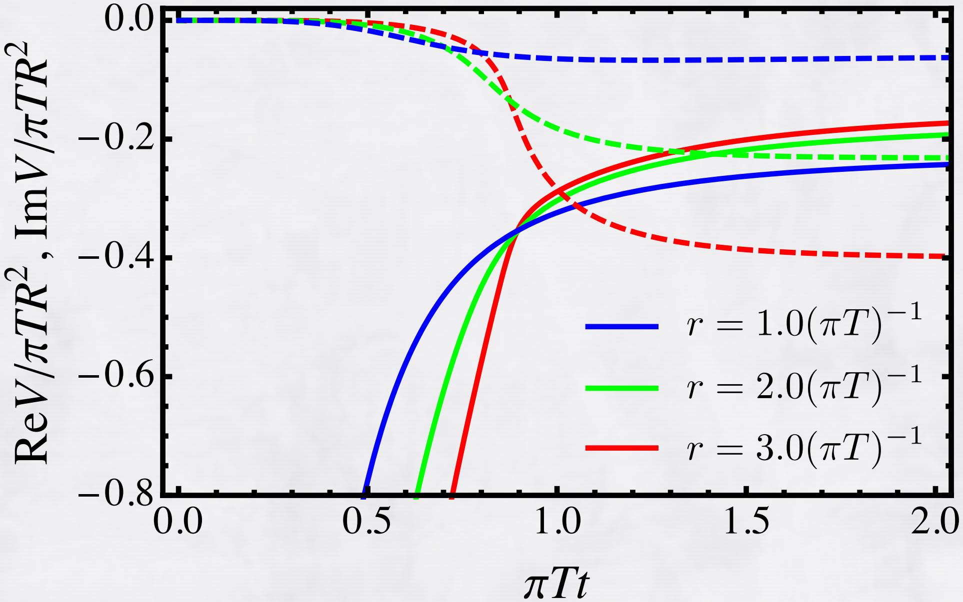
○ Time-dependent complex potential

$$\begin{aligned} V_{Q\bar{Q}}(t,r) &= i\partial_t \ln W_{\mathrm{E}}^{(\mathrm{I})}(it,r) \\ &= \frac{R^2}{2\pi}\Big(r^2\bar{a}_{\mathrm{I}}^3(it,r) - ir^2\bar{a}_{\mathrm{I}}^2(it,r)/t - 2\bar{a}_{\mathrm{I}}(it,r)\Big) \end{aligned}$$

- Proportional to $\sqrt{\lambda}$

Time-dependent $Q\bar{Q}$ potential at finite T

○ t -dependence



- Reaches static profile at

$$\boxed{t_{\mathrm{eq}} \sim 1/\pi T}$$

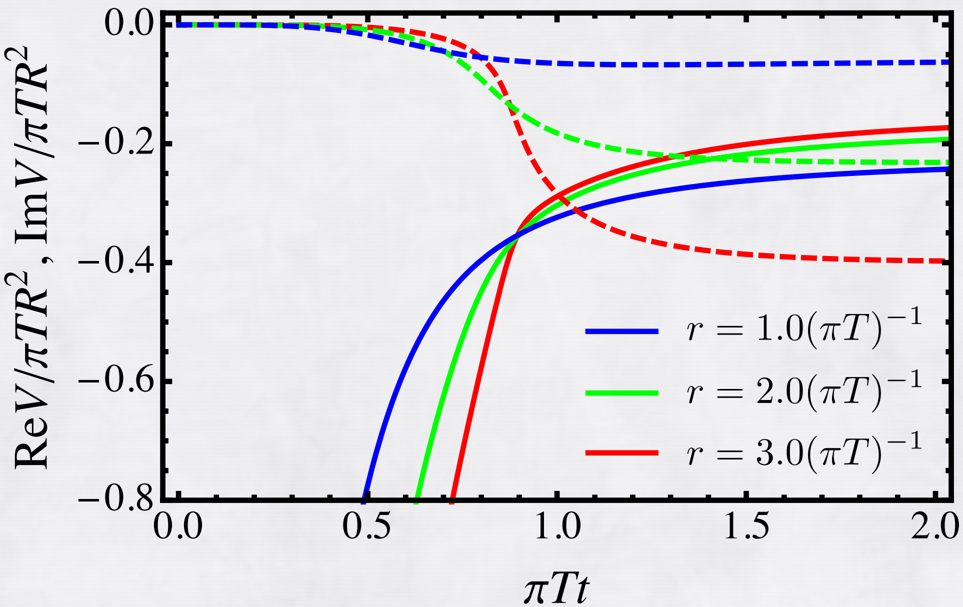
HTL: $t_{\mathrm{eq}} \sim 10/gT$
$\sim 5/T \ (g = 2)$

Laine *et. al.*, 2007

Solve PDE numerically+MEM?

Time-dependent $Q\bar{Q}$ potential at finite T

○ t-dependence



- Reaches static profile at

$$\boxed{t_{\mathrm{eq}} \sim 1/\pi T}$$

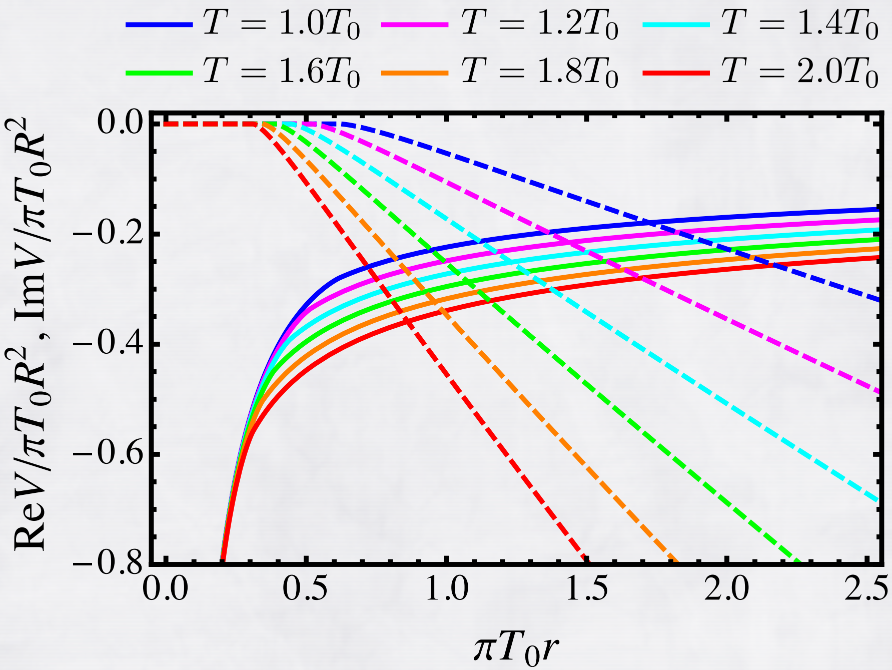
HTL: $t_{\mathrm{eq}} \sim 10/gT$
$\sim 5/T \ (g = 2)$

- Cusp contribution damps more rapidly

Laine *et. al.*, 2007

Static profile at finite T

○ Profile at $t \to \infty$



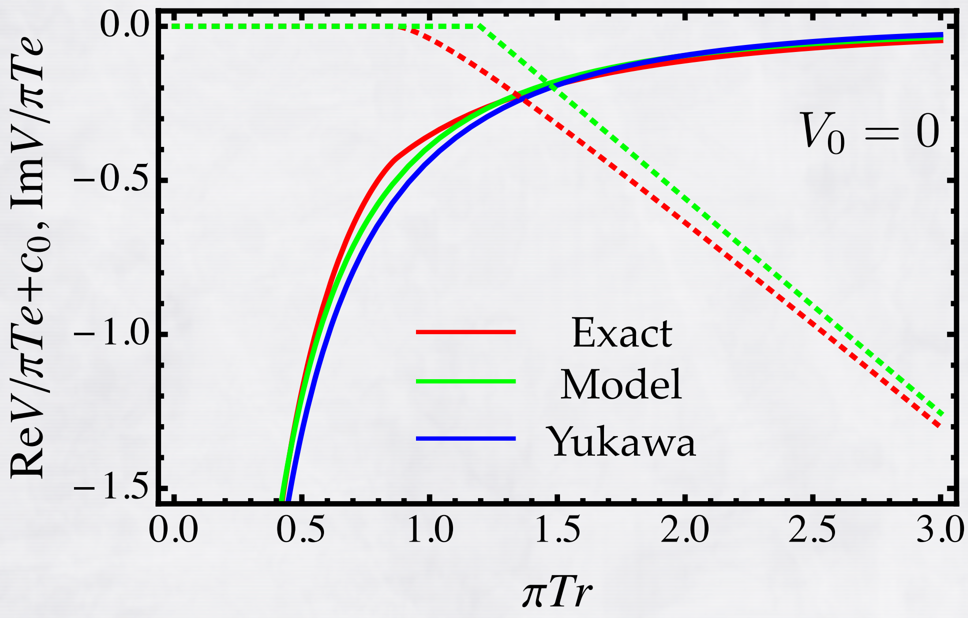
- ReV: Coulomb type irrespective of T
  Deeper as T increases (lattice QCD: Maezawa et.al., 2012)

- ImV: Emerges at $r_{\mathrm{th}} = (4/27)^{1/4}/\pi T$
  Developes linearly (~ rT^2)

(*complex stringy coordinate*: Kovchegov *et. al.*, 2008)

Static profile at finite T

○ Analytic calculation (Kovchegov *et. al.*, 2008)



$$\boxed{\begin{aligned} e &= \sqrt{2\lambda}/(2\pi c_0^2) \\ c_0 &= \Gamma(1/4)^2/(2\pi)^{3/2} \end{aligned}}$$

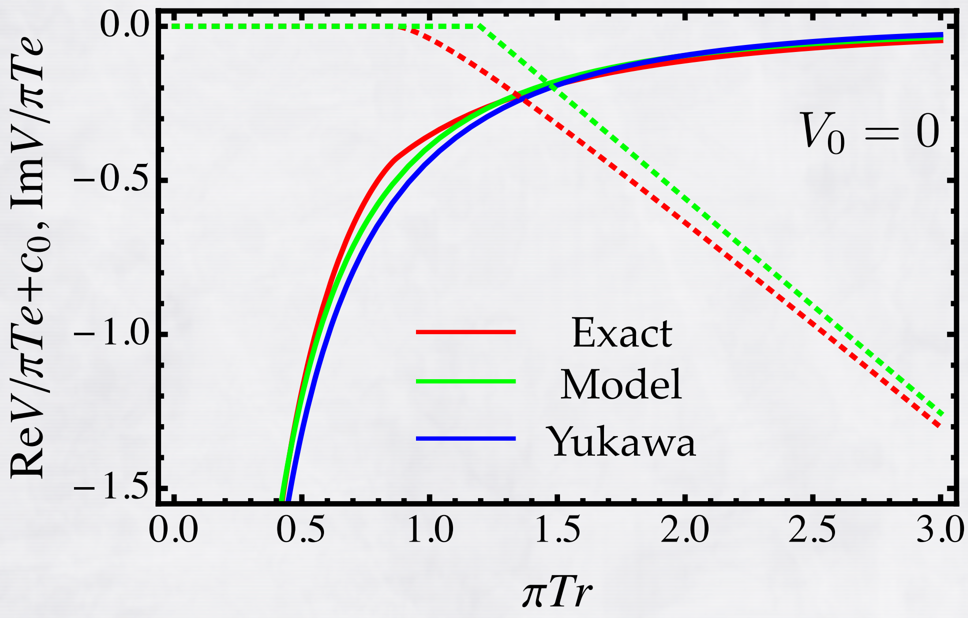
○ Model $\mathrm{Re}V_{Q\bar{Q}}(r,T) = -e[\pi c_0 T + \dfrac{r_0^3}{r(r_0+r)^3}] + V_0$

$$\mathrm{Im}V_{Q\bar{Q}}(r,T) = -e\pi^2 c_0^2 T^2 (r - r_1)\Theta(r - r_1)$$

$$\boxed{r_0 = (\pi c_0^5/2)^{1/3}/T,\ r_1 = 1/(\pi c_0 T)}$$

Static profile at finite T

○ Analytic calculation (Kovchegov *et. al.*, 2008)



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$$\mathrm{Im}V_{Q\bar{Q}}(r,T) = -e\pi^2 c_0^2 T^2 (r-r_1)\Theta(r-r_1)$$

- Color screening with Debye mass $\pi c_0 T$
- “Dissociation” distance: $\mathrm{Im}V_{Q\bar{Q}} > \mathrm{Re}V_{Q\bar{Q}}$ at $r > r_{\mathrm{diss}} = 1.5/\pi T$

SPFs from Schrödinger equation

- Potential at $T=0$

$$V_{Q\bar{Q}}(r)\Big|_{T=0} - V_0 = \begin{cases} -\sqrt{2\lambda}/(2\pi c_0^2 r) & (r < r_{\rm c} = 2^{1/4}/(\pi c_0 T_{\rm c})) \\ \frac{\pi}{2}\sqrt{\lambda}T_{\rm c}^2(r-2r_{\rm c}) & (r > r_{\rm c}) \end{cases}$$

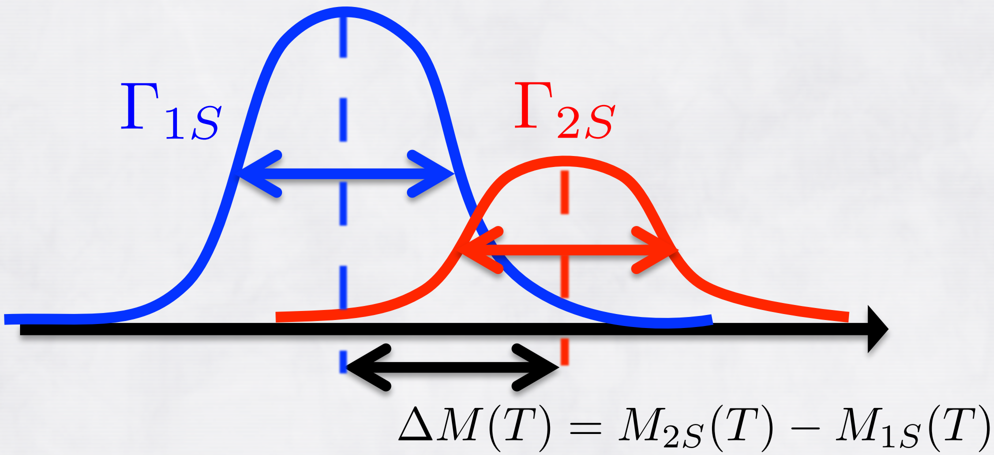
- Parameters

$T_{\rm c}=270$ (MeV)	M (GeV)	λ	$\sqrt{\sigma}$ (MeV)	V_0 (GeV)
Charm	1.92	5.08	508	-0.540
Bottom	4.68	4.40	490	0.630

Dissociation temperature

○ Breit-Wigner function

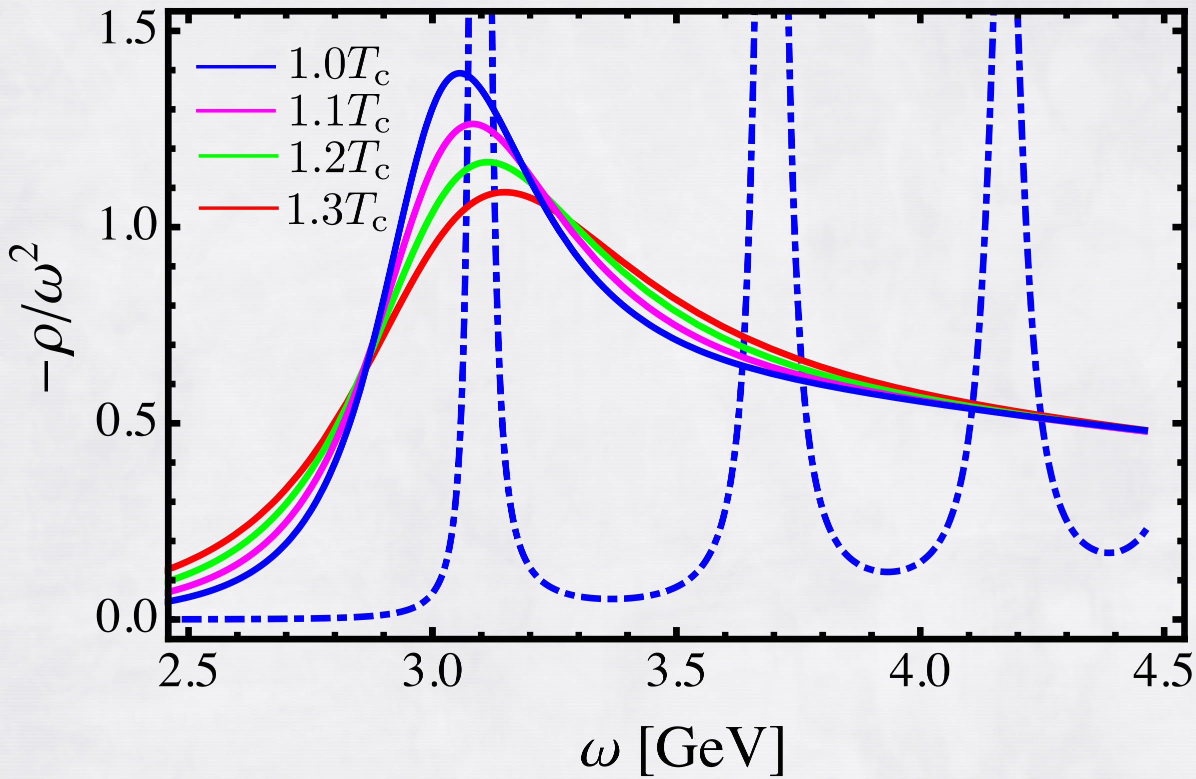
$$\frac{-\rho_{1S}(\omega)}{\omega^2} = A_{1S}(T)\frac{2}{\pi\Gamma_{1S}(T)}\frac{(\Gamma_{1S}(T)/2)^2}{(\omega - M_{1S}(T))^2 + (\Gamma_{1S}(T)/2)^2}$$



○ Simplified form $(\Gamma_{1S}(T) \sim \Gamma_{2S}(T))$

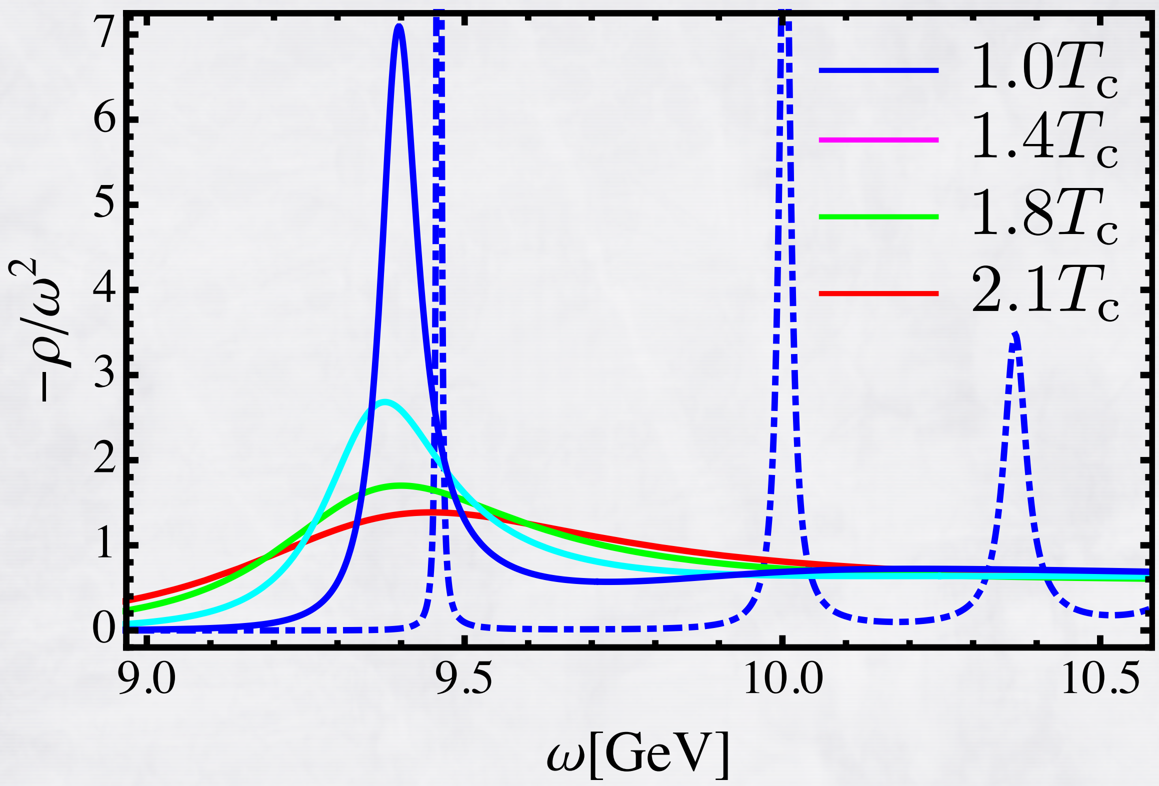
$$\boxed{\Gamma_{1S}(T_{\text{diss}}) = \Delta M(0)}$$

SPF in vector channel for charmonium



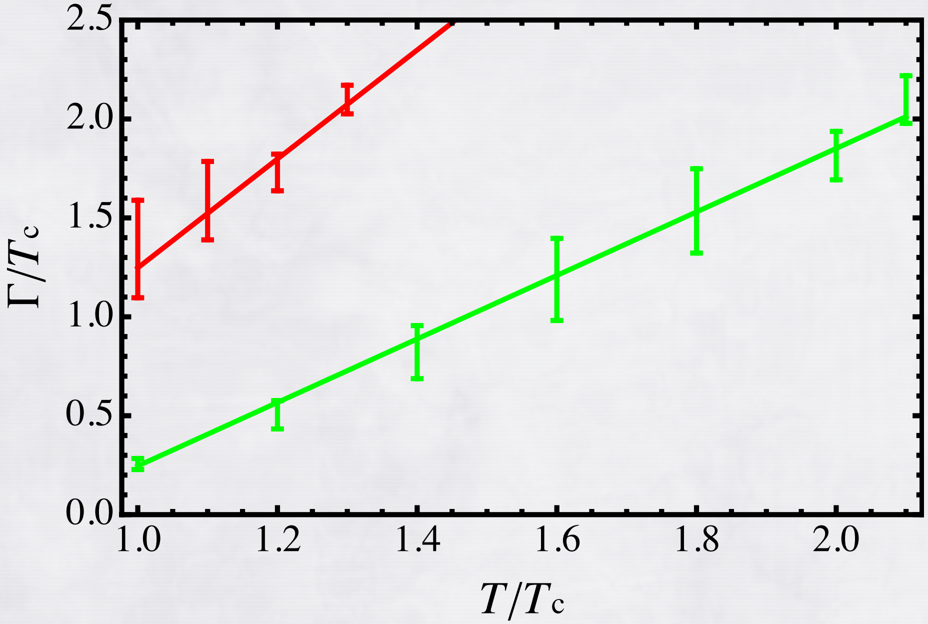
- $1S$ peak dissolves at $T_\mathrm{diss} = 1.3T_\mathrm{c}$
- Excited states already disappear at T_c

SPF in vector channel for bottomonium



- $1S$ peak dissolves at $T_{\rm diss} = 2.1T_{\rm c}$
- Excited states already disappear at $T_{\rm c}$

Thermal width



Upper value: $|\omega - M_{1S}(0)| \leq \Delta M(0)/2$
Lower value: Left side of peak

	Fit		Input
	λ	Γ_0/T_c	λ
$c\bar{c}$	6.2(7)	1.5(6)	5.08
$b\bar{b}$	4.3(1)	1.36(6)	4.40

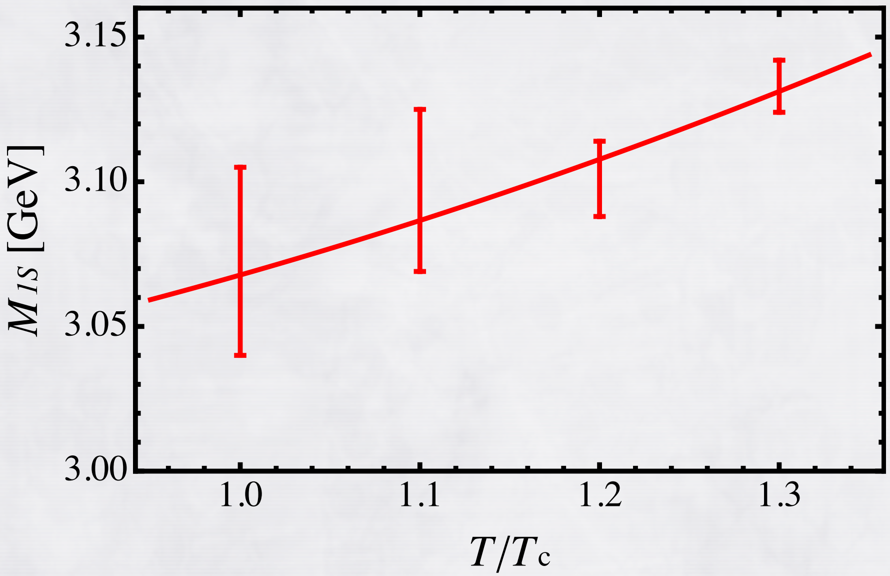
Brambilla et. al., 2010

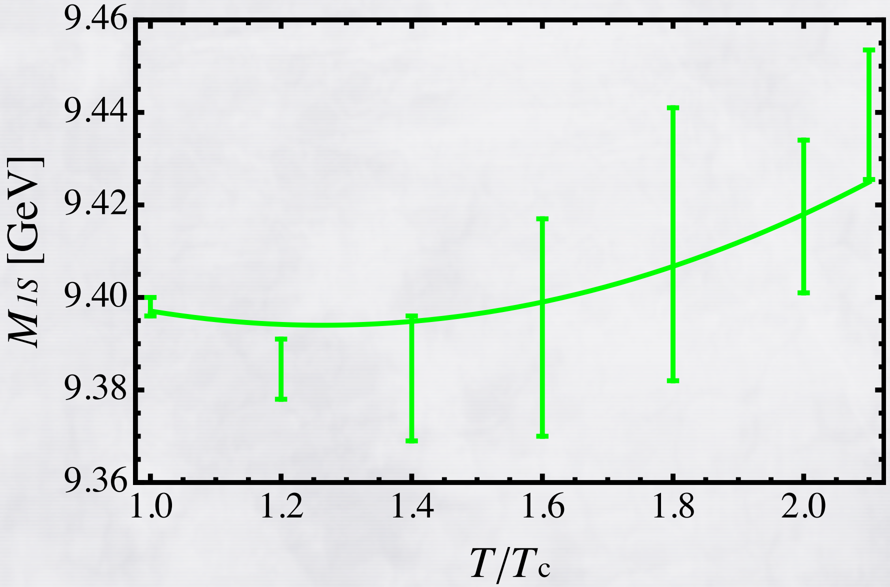
$$(\Gamma_{1S}(T) + \Gamma_0)/T = \frac{16}{3}(1 + \frac{1}{4C_F N_c})^2 (C_F \alpha_s)^3 \sim \lambda^3$$

- Coulomb coefficient $e = \sqrt{2\lambda}/(2\pi c_0^2)$

$$\boxed{(\Gamma_{1S}(T) + \Gamma_0)/T = \frac{16}{3} e^3 \sim \lambda^{3/2}}$$

Mass shift





Upper value: $|\omega - M_{1S}(0)| \leq \Delta M(0)/2$

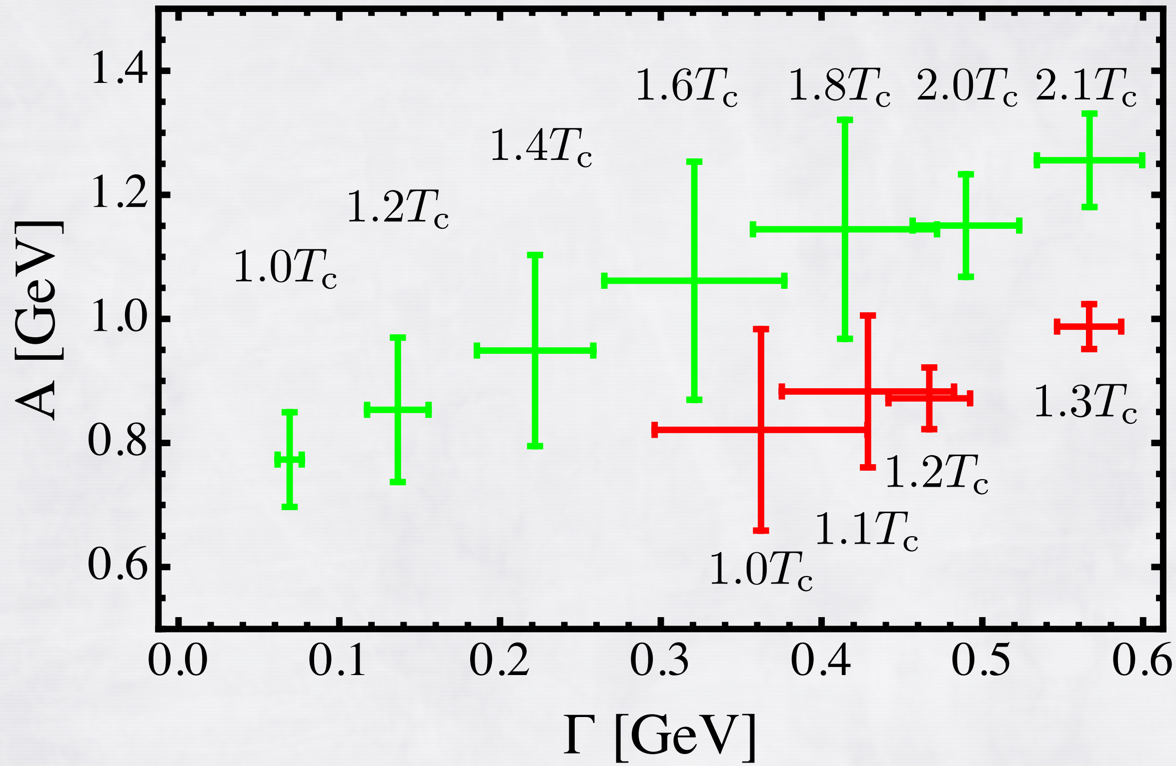
Lower value: Left side of peak

$$\delta M_{1S}(T) = \frac{2\pi}{3}(1 + \frac{N_{\rm c}}{2C_{\rm F}})(C_{\rm F}\alpha_s)\frac{T^2}{M_q}$$

$$\boxed{M_{1S}(T) = M_0 + \frac{2\pi e T_{\rm c}^2}{3M_q}\left(\frac{T}{T_{\rm c}} - T_0\right)^2}$$

	Fit		Input
	T_0	M_0(GeV)	λ
$c\bar{c}$	0.2(2)	3.00(4)	5.08
$b\bar{b}$	1.3(1)	9.394(6)	4.40

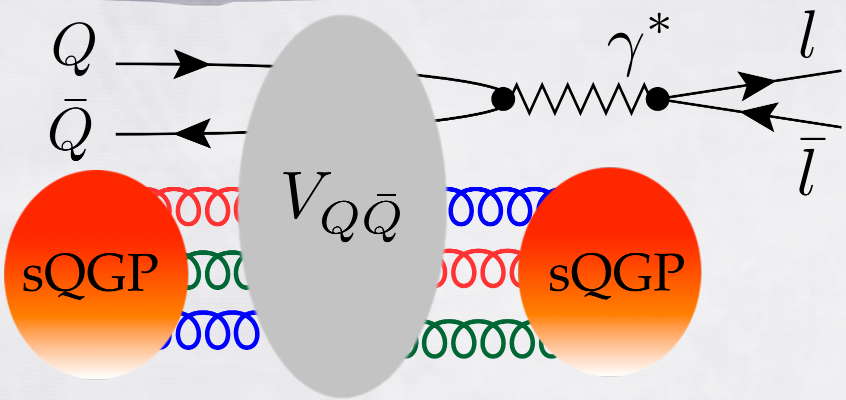
Thermal width vs. Residue



- Upper value: $|\omega - M_{1S}(0)| \leq \Delta M(0)/2$
- Lower value: Left side of peak

- Linear T-dependence
- Enhancement of decay width

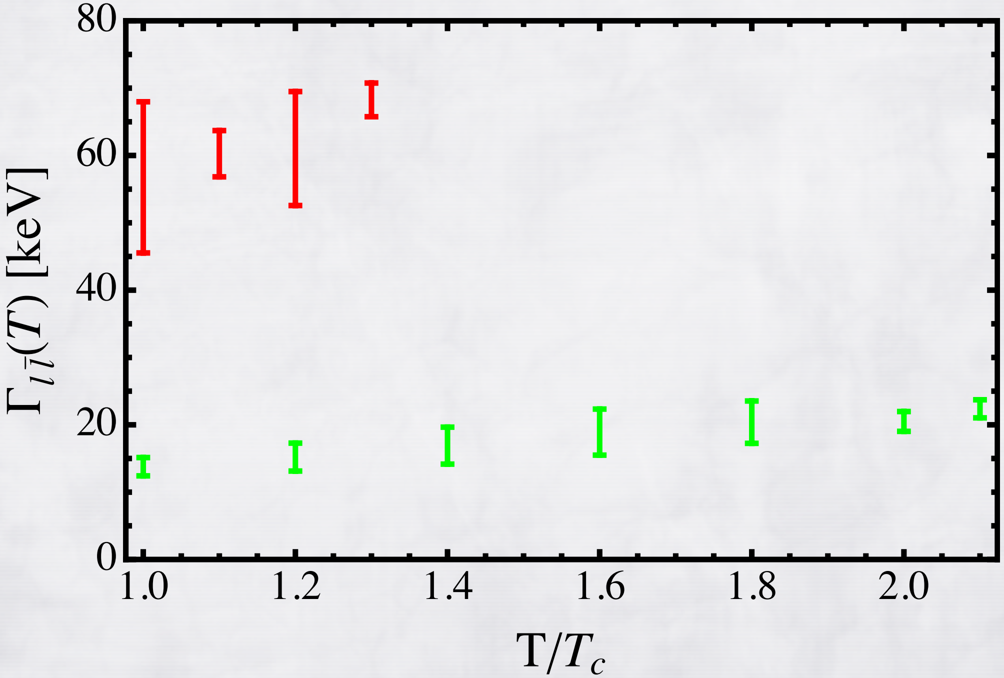
Leptonic decay widths



$$\Gamma_{l\bar{l}}(T) = \frac{8\pi}{9} Q_c^2 \alpha_{\mathrm{QED}}^2 A(T)$$

$$(M_l^2/M_{1S}^2(T) \ll 1)$$

- Weak T- dependence?



Summary

○ Complex $Q\bar{Q}$ potential from gauge/gravity duality

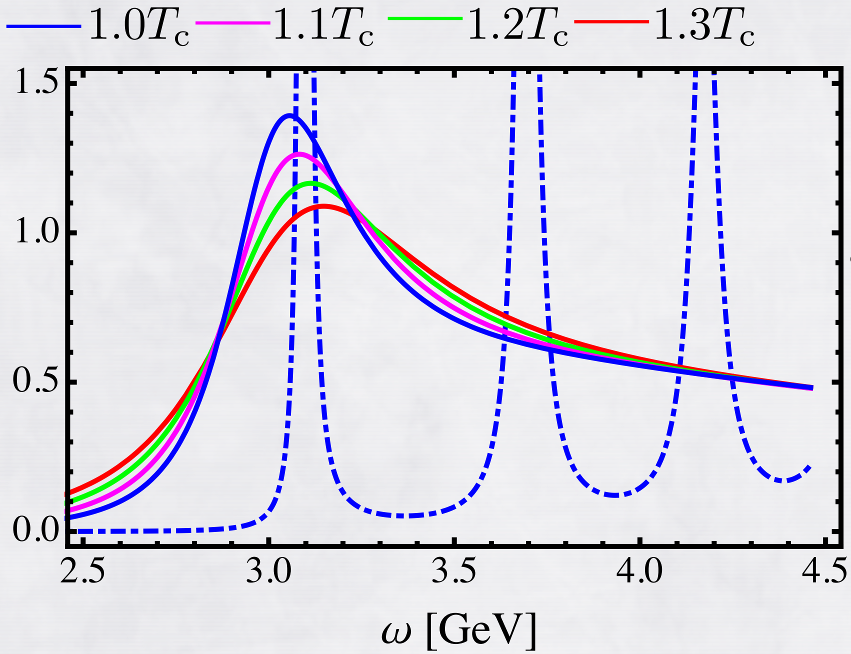
- Model of imaginary part in the strong coupling
- SPFs in vector channel and dissociation temperature

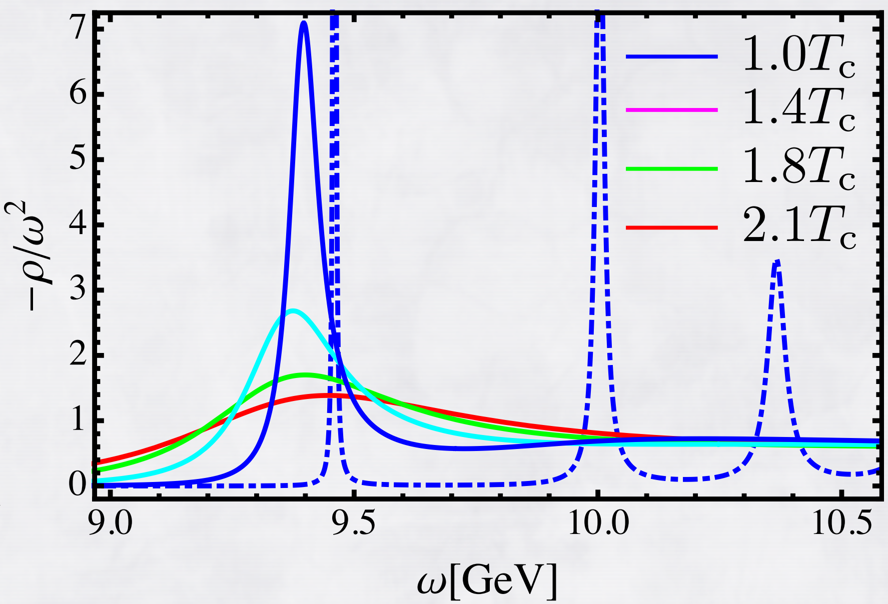
○ Future

- More quantitative analysis of time-dependent potential
- Demonstrate MEM potential approach

Thank you for your kind attention

Spectral functions in vector channel





$\boxed{T_{\rm diss} = 1.3T_{\rm c}}$ $\boxed{T_{\rm diss} = 2.1T_{\rm c}}$

○ Dissociation temperature $T_{\rm diss}$

$$\boxed{\Gamma_{1S}(T_{\rm diss}) = M_{2S}(0) - M_{1S}(0)}$$

○ Excited states disappear above $T_{\rm c}$

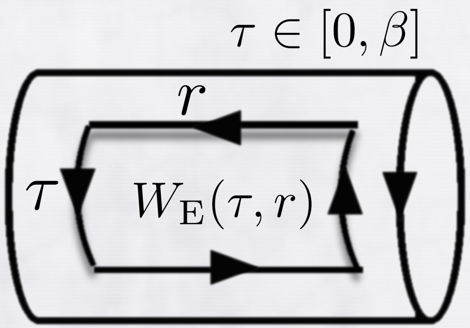
In-Medium $Q\bar{Q}$ potential from Wilson loop

○ Spectral decomposition

$$\boxed{W_{\rm E}(\tau,r) = \lim_{M\to\infty}\int_{-2M}^{\infty} {\rm d}\omega\ {\rm e}^{-\omega\tau}\rho(\omega,r)}$$

$$W_E(\tau,r) \xrightarrow{\tau\to it} W_M(t,r)$$

$$\rho(\omega,r)$$



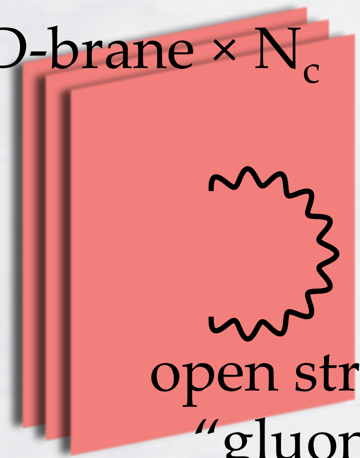
Laine et. al., JHEP 0703 054 (2007); JHEP 0705 028 (2007), Rothkopf et. al, PRL 108 162001 (2012)

Gauge/gravity duality

Maldacena, Adv. Theor. Math. Phys 2 231 (1998)

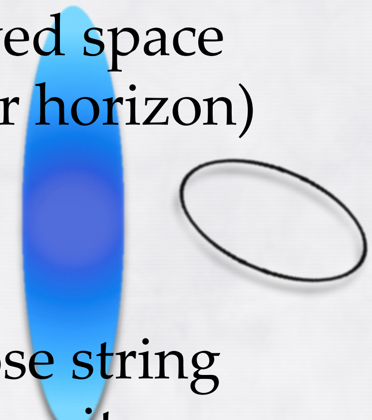
Gauge side | **Gravity side**

D-brane × N_c



open string
"gluon"

curved space
(near horizon)



close string
graviton

Large N_c, λ gauge theory ⟷ Classical SUGRA

SPFs from Schrödinger equation

○ Schrödinger equation

$$\boxed{\left[\frac{\omega'}{M\alpha^2}+\frac{\partial^2}{\partial\bar{r}^2}-\frac{l(l+1)}{\bar{r}^2}-\frac{V_{Q\bar{Q}}(\bar{r})}{M\alpha^2}\right]g_l(\omega',\bar{r},\delta)=0}$$

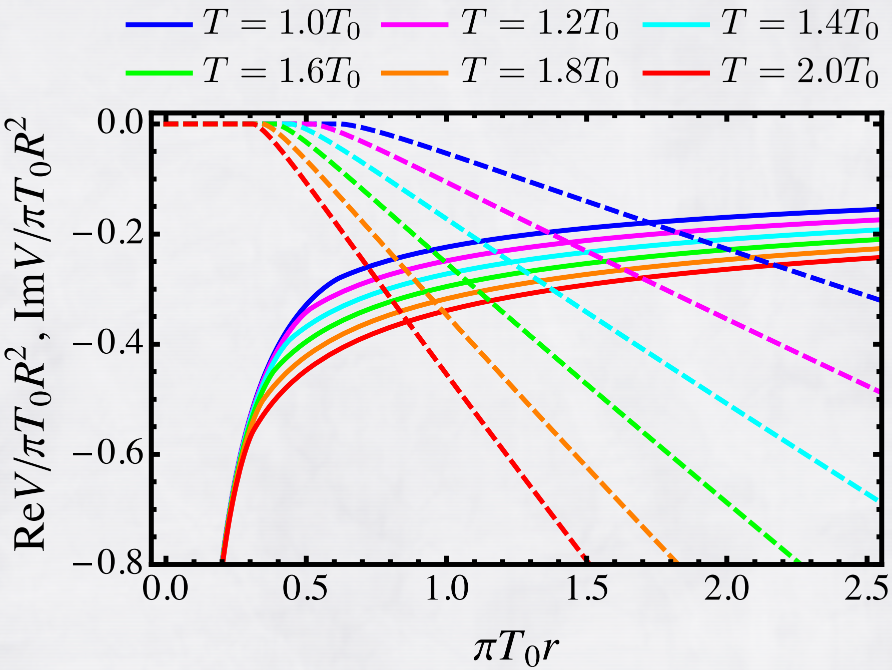
with $$g_0(\omega',\delta,\delta)=\delta-\delta^2/2$$

○ SPFs in vector (scalar) channel

$$\boxed{\frac{\rho_{\mathrm{V}}(\omega')}{M^2}=-\frac{6N_{\mathrm{c}}\alpha}{4\pi}\lim_{\delta\to 0}\int_{\delta}^{\infty}\mathrm{d}\bar{r}\ \mathrm{Im}\left\{\frac{1}{g_0(\omega',\bar{r},\delta)^2}\right\}}$$

Static profile at finite T

○ Profile at $t \to \infty$



- ReV: Coulomb type irrespective of T
  Deeper as T increases

- ImV: Emerges at $r_{\rm th} = (4/27)^{1/4}/\pi T$
  Developes linearly (~ rT^2)

(*complex stringy coordinate* : Kovchegov et. al., PRD 78 115007 (2008))