Universität Bielefeld

Quarkonium Spectral and Transport properties from Lattice QCD

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Heavy Quarks and Quarkonia in Thermal QCD

ECT* Trento

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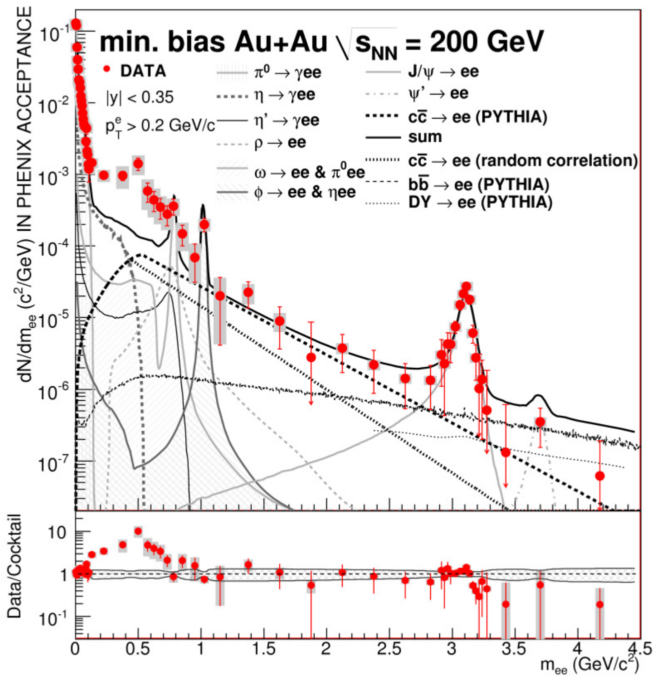
Motivation – PHENIX/STAR results for the low-mass dilepton rates

pp-data well understood by hadronic cocktail

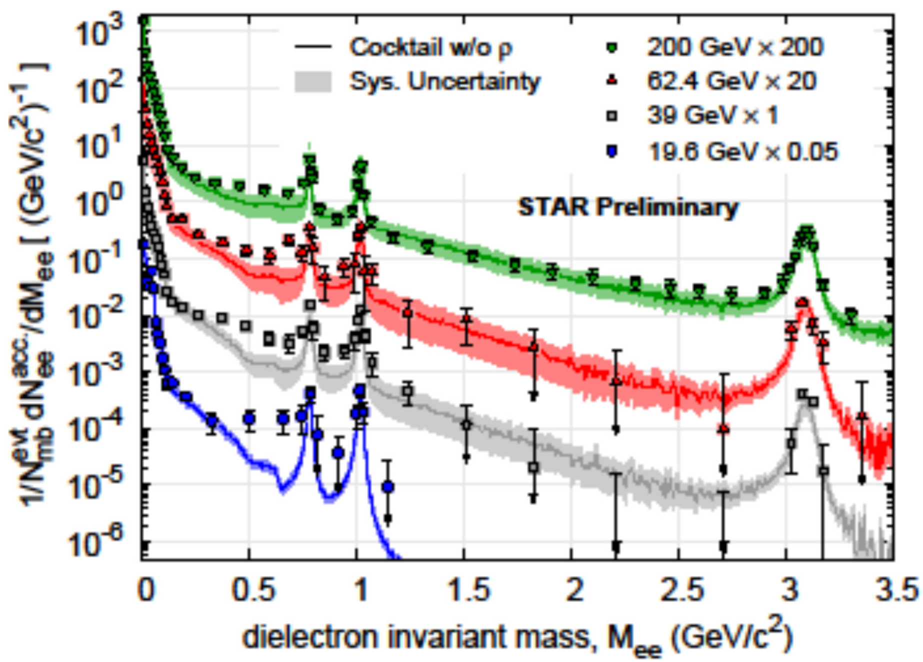
large enhancement in Au+Au between 150-750 MeV

indications for thermal effects!?

Need to understand the contribution from QGP$\rightarrow$ spectral functions from lattice QCD



[PHENIX PRC81, 034911 (2010)]



[STAR preliminary, arXiv:1210.5549]

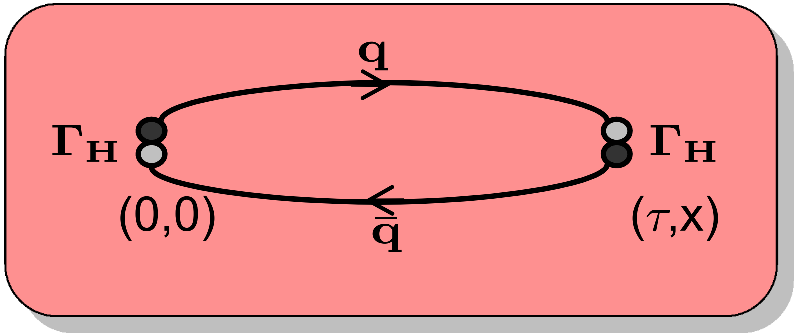
Dileptonrate directly related to vector spectral function:

$$\frac{\mathrm{d}W}{\mathrm{d}\omega \mathrm{d}^3p} = \frac{5\alpha^2}{54\pi^3}\frac{1}{\omega^2(e^{\omega/T}-1)}\ \rho_{\mathbf{V}}(\omega,\vec{\mathbf{p}},\mathbf{T})$$

Vector correlation functions at high temperature

$$G(\tau,\vec{p},T) = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\rho(\omega,\vec{p},T)K(\tau,\omega,T)$$

$$K(\tau,\omega,T) = \frac{\cosh\left(\omega(\tau-\frac{1}{2T})\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$



Lattice observables:

$$G_{\mu\nu}(\tau,\vec{x}) = \langle J_\mu(\tau,\vec{x})J_\nu^\dagger(0,\vec{0})\rangle$$

$$J_\mu(\tau,\vec{x}) = 2\kappa Z_V \bar{\psi}(\tau,\vec{x})\Gamma_\mu\psi(\tau,\vec{x})$$

← local, non-conserved current,
needs to be renormalized

$$G_{\mu\nu}(\tau,\vec{p}) = \sum_{\vec{x}} G_{\mu\nu}(\tau,\vec{x})e^{i\vec{p}\vec{x}}$$

← only $\vec{p}=0$ used here

How to extract spectral properties from correlation functions?

Spectral functions at high temperature

Free theory (massless case):

free non-interacting vector spectral function (infinite temperature):

$$\begin{aligned}\rho_{00}^{free}(\omega) &= 2\pi T^2 \omega \delta(\omega) \\ \rho_{ii}^{free}(\omega) &= 2\pi T^2 \omega \delta(\omega) + \frac{3}{2\pi}\,\omega^2 \tanh(\omega/4T)\end{aligned}$$

δ-functions exactly cancel in $\rho_V(\omega)$=-$\rho_{00}(\omega)$+$\rho_{ii}(\omega)$

With interactions (but without bound states):

while ρ_{00} is protected, the δ-funtion in ρ_{ii} gets smeared:

$\kappa = \frac{\alpha_s}{\pi}$ at leading order

Ansatz:

$$\begin{aligned}\rho_{00}(\omega) &= 2\pi \chi_q \omega \delta(\omega) \\ \rho_{ii}(\omega) &= 2\chi_q c_{BW} \frac{\omega \Gamma/2}{\omega^2 + (\Gamma/2)^2} + \frac{3}{2\pi}(1+\kappa)\,\omega^2\, \tanh(\omega/4T)\end{aligned}$$

Ansatz with 3-4 parameters: $(\chi_q), c_{BW}, \Gamma, \kappa$

["Thermal dilepton rate and electrical conductivity…", H.T.-Ding, OK et al., PRD83 (2011) 034504]

Electrical Conductivity

Electrical Conductivity $\longleftrightarrow$ slope of spectral function at ω=0 (Kubo formula)

$$\frac{\sigma}{T} = \frac{C_{em}}{6} \lim_{\omega \to 0} \frac{\rho_{ii}(\omega, \vec{p}=0, T)}{\omega T}$$

$$C_{em} = e^2 \sum_{f=1}^{n_f} Q_f^2 \quad = \quad \begin{array}{lll} 5/9\ e^2 & \text{for} & n_f = 2 \\ 6/9\ e^2 & \text{for} & n_f = 3 \end{array}$$

Using our Ansatz for $\rho_{ii}(\omega)$:

$$\frac{\sigma}{T} = \frac{2}{3} \frac{\chi_q}{T^2} \frac{T}{\Gamma} c_{BW}\ C_{em}$$

previous studies using staggered fermions (need to distinguish ρ_{even} and ρ_{odd}):

S.Gupta, PLB 597 (2004) 57: N_τ=8-14, $N_\sigma \leq$ 44
G.Aarts et al., PRL 99 (2007) 022002: N_τ=16,24, N_σ=64

Vector correlation function on large & fine lattices

Quenched SU(3) gauge configurations at T/T_c=1.5 (separated by 500 updates)

Lattice size $N_\sigma^3 N_\tau$ with N_σ = 32 – 128
N_τ = 16, 24, 32, 48 Temperature: $T = \frac{1}{aN_\tau}$

Non-perturbatively O(a) clover improved Wilson fermions

Non-perturbative renormalization constants

Quark masses close to the chiral limit, $\kappa \simeq \kappa_c \Leftrightarrow m_{\overline{MS}}/T[\mu=2\text{GeV}] \approx 0.1$

Volume dependence

N_τ	N_σ	β	c_{SW}	κ	Z_V	$1/a$[GeV]	a[fm]	#conf
16	32	6.872	1.4124	0.13495	0.829	6.43	0.031	60
16	48	6.872	1.4124	0.13495	0.829	6.43	0.031	62
16	64	6.872	1.4124	0.13495	0.829	6.43	0.031	77
16	128	6.872	1.4124	0.13495	0.829	6.43	0.031	129
24	128	7.192	1.3673	0.13440	0.842	9.65	0.020	156
32	128	7.457	1.3389	0.13390	0.851	12.86	0.015	255
48	128	7.793	1.3104	0.13340	0.861	19.30	0.010	431

cut-off dependence & continuum extrapolation

close to continuum

Light Quarks - Vector Correlation Function – continuum extrapolation

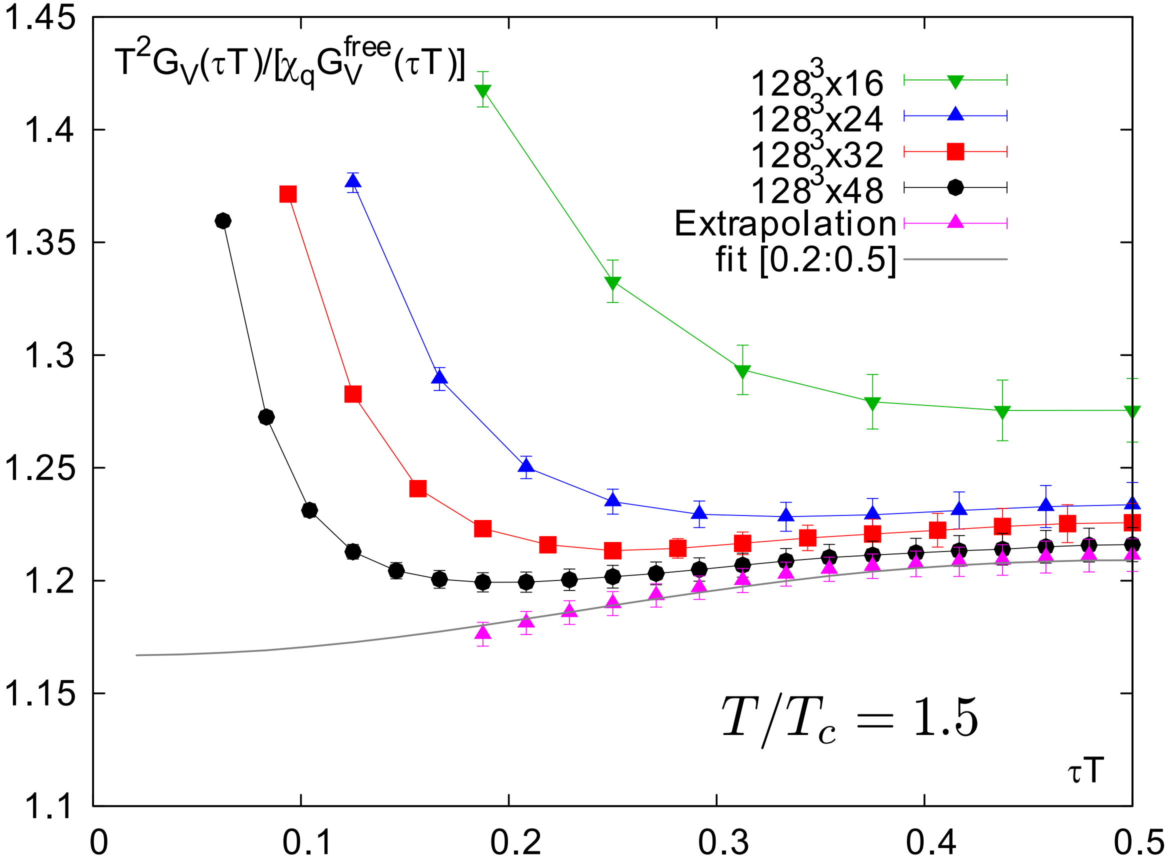
Use our Ansatz for the spectral function

["Thermal dilepton rate and electrical conductivity…", H.T.-Ding, OK et al., PRD83 (2011) 034504]

$$\rho_{00}(\omega) = 2\pi\chi_q\omega\delta(\omega)$$

$$\rho_{ii}(\omega) = 2\chi_q c_{BW}\frac{\omega\Gamma/2}{\omega^2+(\Gamma/2)^2} + \frac{3}{2\pi}(1+\kappa)\,\omega^2\ \tanh(\omega/4T)$$

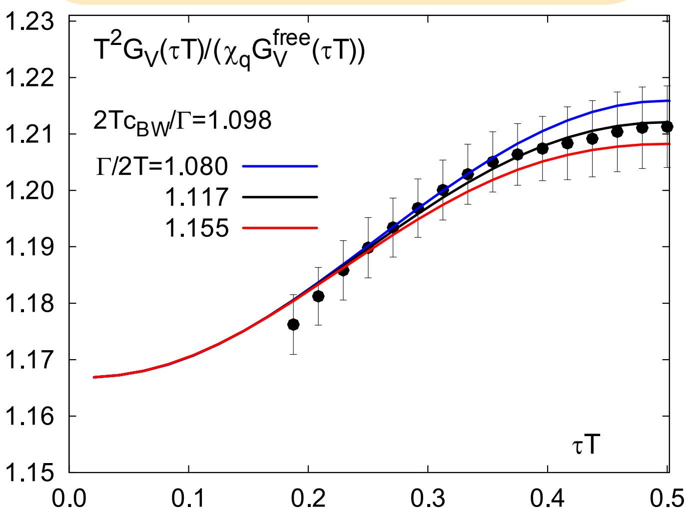
and fit to the continuum extrapolated values $\dfrac{\mathbf{G_V(\tau,T)}}{\mathbf{\bar{G}_{00}G_V^{free}(\tau,T)}}$ & $\mathbf{G_V^{(2)}}$



$$\frac{2C_{BW}\chi_q}{\Gamma} = 1.098(27)$$

$$\frac{\Gamma}{T} = 2.235(75)$$

$$\kappa = 0.0465(27)$$



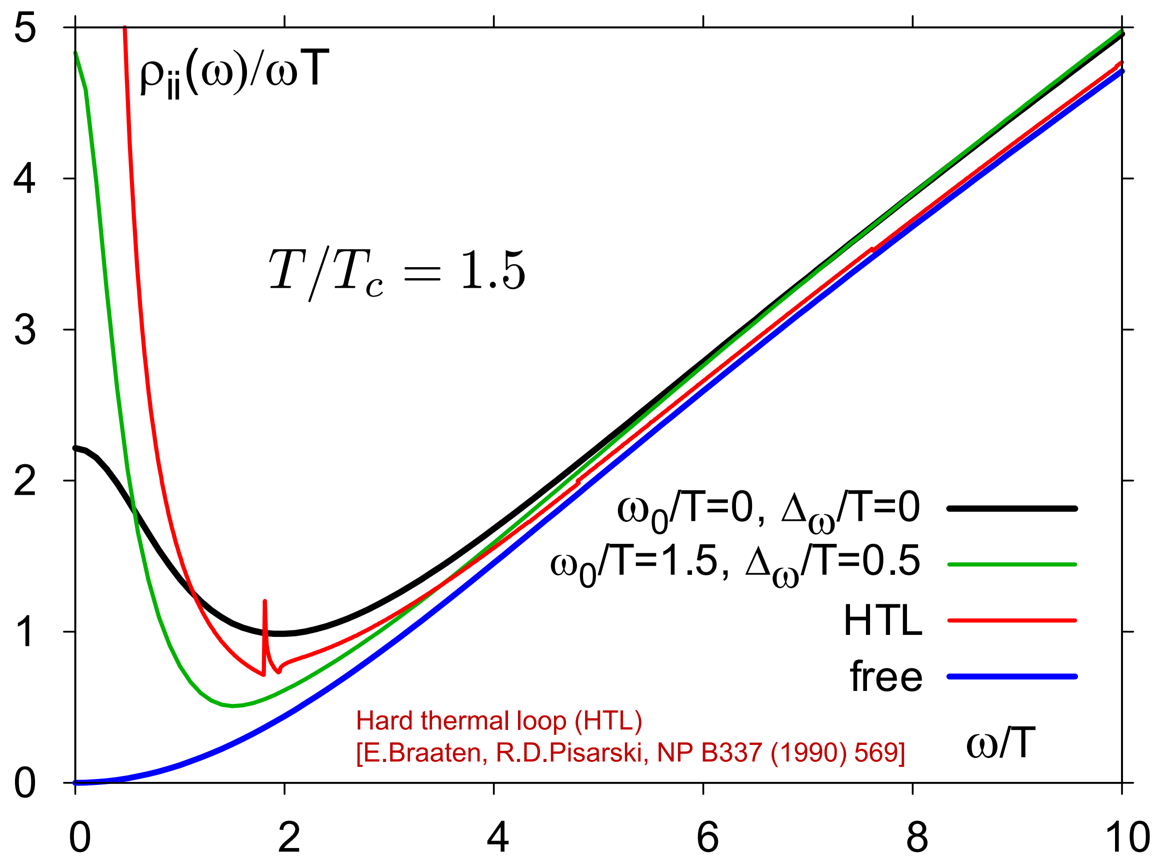
Light Quarks - Spectral function and electrical conductivity

Use our Ansatz for the spectral function

["Thermal dilepton rate and electrical conductivity…", H.T.-Ding, OK et al., PRD83 (2011) 034504]

$$\rho_{00}(\omega) = 2\pi\chi_q\omega\delta(\omega)$$

$$\rho_{ii}(\omega) = 2\chi_q c_{BW}\frac{\omega\Gamma/2}{\omega^2+(\Gamma/2)^2} + \frac{3}{2\pi}(1+\kappa)\,\omega^2\;\tanh(\omega/4T)\times\Theta(\omega_0,\Delta_\omega)$$



Analysis of the systematic errors

using truncation of the large ω contribution

$$\Theta(\omega_0,\Delta_\omega) = \left(1+e^{(\omega_0^2-\omega^2)/\omega\Delta_\omega}\right)^{-1}$$

$$\frac{\sigma}{T} = \frac{C_{em}}{6}\lim_{\omega\to 0}\frac{\rho_{ii}(\omega,\vec{p}=0,T)}{\omega T}$$

electrical conductivity

$$1/3 \le \frac{1}{C_{em}}\frac{\sigma}{T} \le 1$$

Light Quarks - Dilepton rates and electrical conductivity

["Thermal dilepton rate and electrical conductivity…", H.T.-Ding, OK et al., PRD83 (2011) 034504]

Dileptonrate directly related to vector spectral function:

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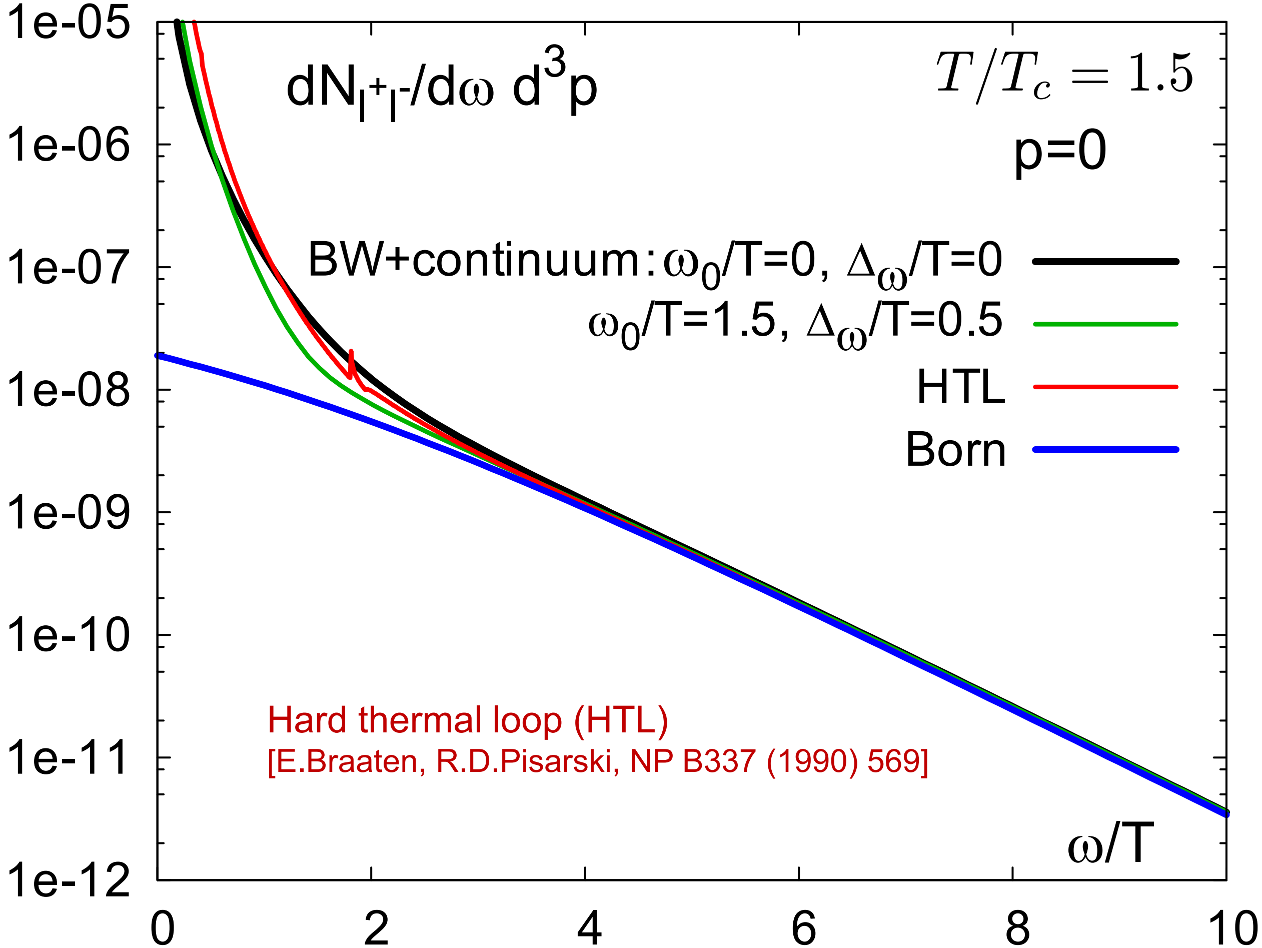


Preliminary results at 1.1 T_c

[M.Müller et al., arXiv:1301.7436]

PRACE-Project:
Thermal Dilepton Rates and
Electrical Conductivity in the QGP
(JUGENE Bluegene/P in Jülich)

$1.09T_c$ Lattices (all $N_\sigma/N_\tau = 3$)

$1/T = a \cdot N_\tau$

N_τ	N_σ	β	κ	$1/a$[GeV]	$\approx a$[fm]	#conf
32	96	7.192	0.13440	9.65	0.020	223
48	144	7.544	0.13383	14.21	0.015	226
64	192	7.793	0.13345	19.30	0.010	165

study of T-dependence of dilepton rates and electrical conductivity

fixed aspect ratio to allow continuum limit at finite momentum:

$$\frac{\vec{p}}{T} = 2\pi\vec{k}\frac{N_\tau}{N_\sigma}$$

Light Quarks - Spectral function and electrical conductivity

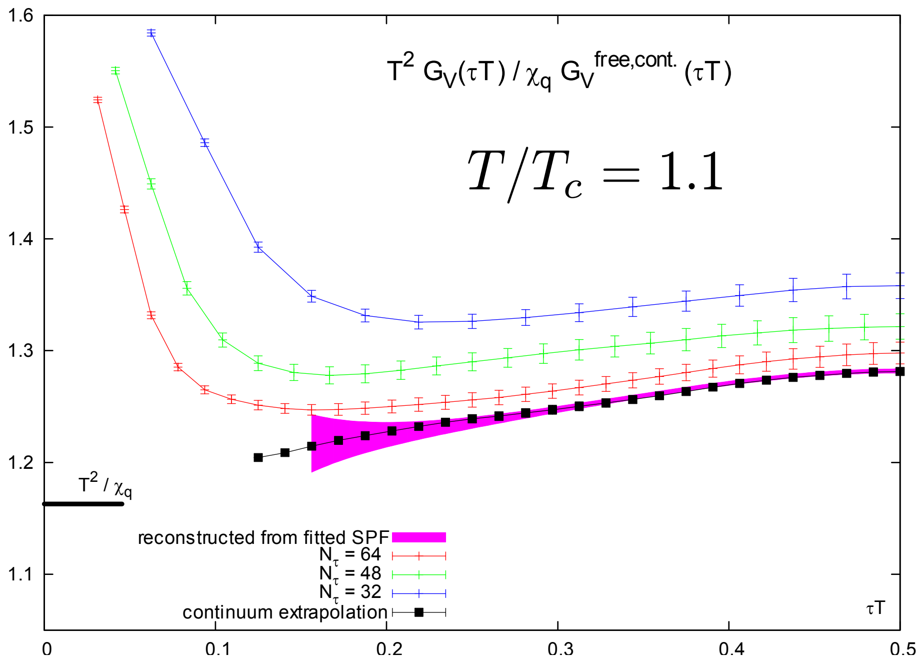
Use our Ansatz for the spectral function

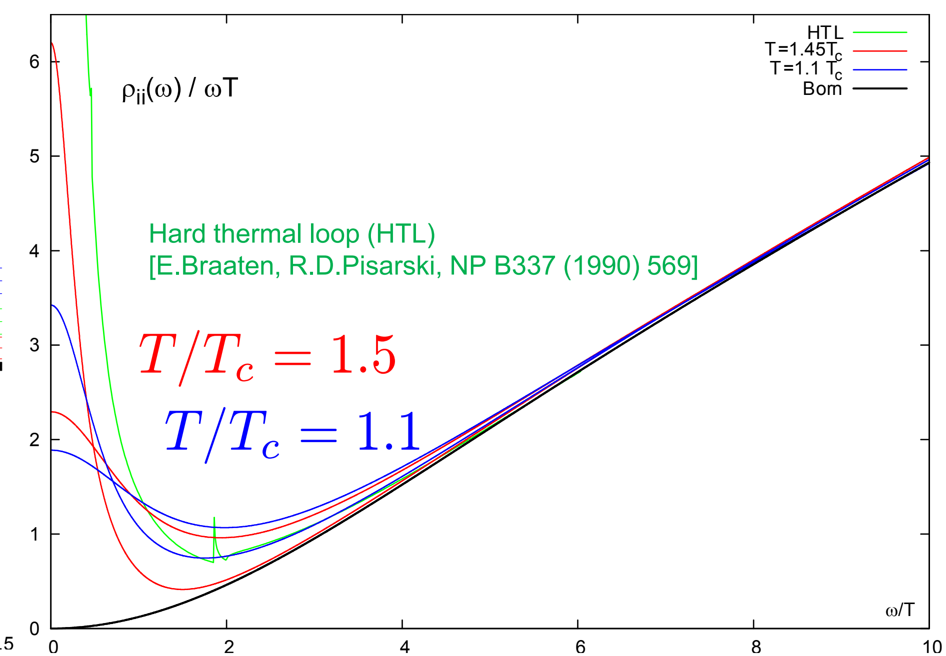
["Thermal dilepton rate and electrical conductivity…", H.T.-Ding, OK et al., PRD83 (2011) 034504 , arXiv:1301.7436]

$$\rho_{00}(\omega) = 2\pi\chi_q\omega\delta(\omega)$$

$$\rho_{ii}(\omega) = 2\chi_q c_{BW}\frac{\omega\Gamma/2}{\omega^2+(\Gamma/2)^2} + \frac{3}{2\pi}(1+\kappa)\,\omega^2\,\tanh(\omega/4T)\times\Theta(\omega_0,\Delta_\omega)$$

$$\frac{\sigma}{T} = \frac{C_{em}}{6}\lim_{\omega\to 0}\frac{\rho_{ii}(\omega,\vec{p}=0,T)}{\omega T}$$





Light Quarks - Dilepton rates and electrical conductivity

["Thermal dilepton rate and electrical conductivity…", H.T.-Ding, OK et al., PRD83 (2011) 034504 , arXiv:1301.7436]

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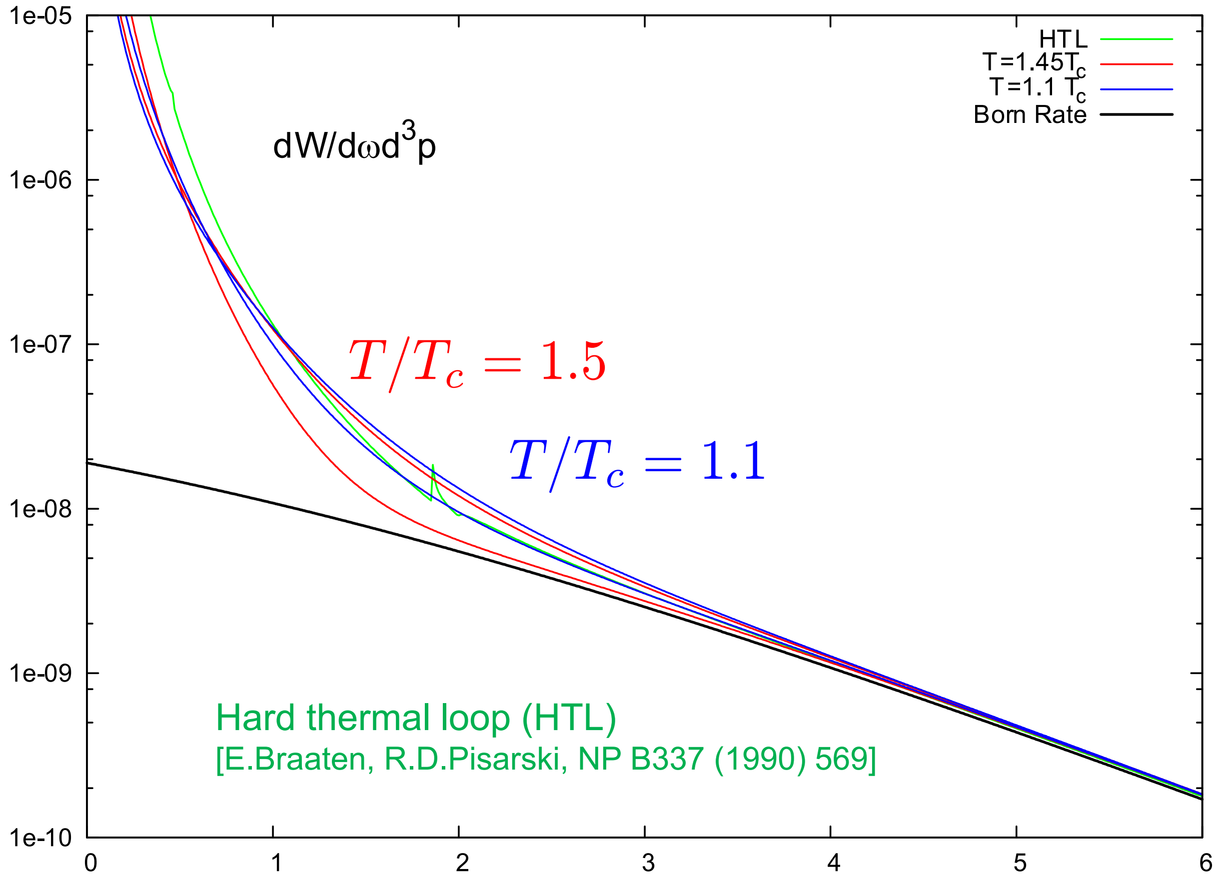
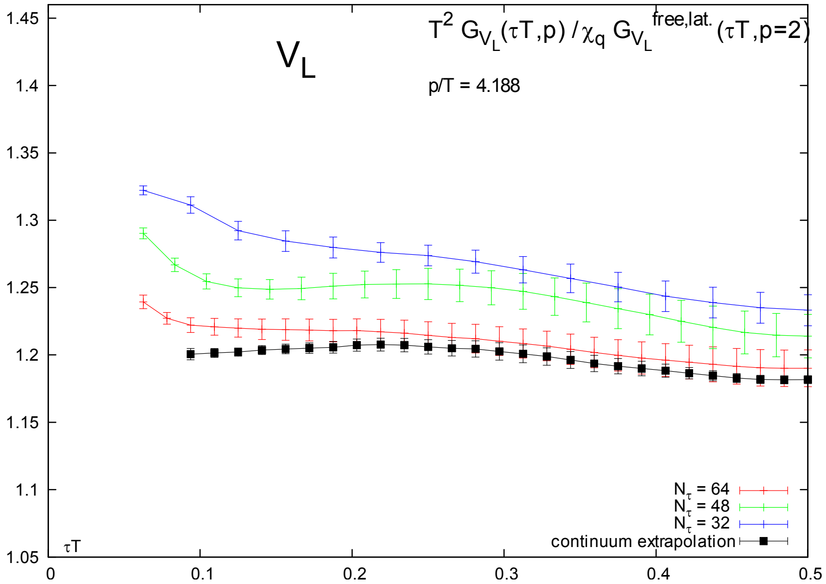
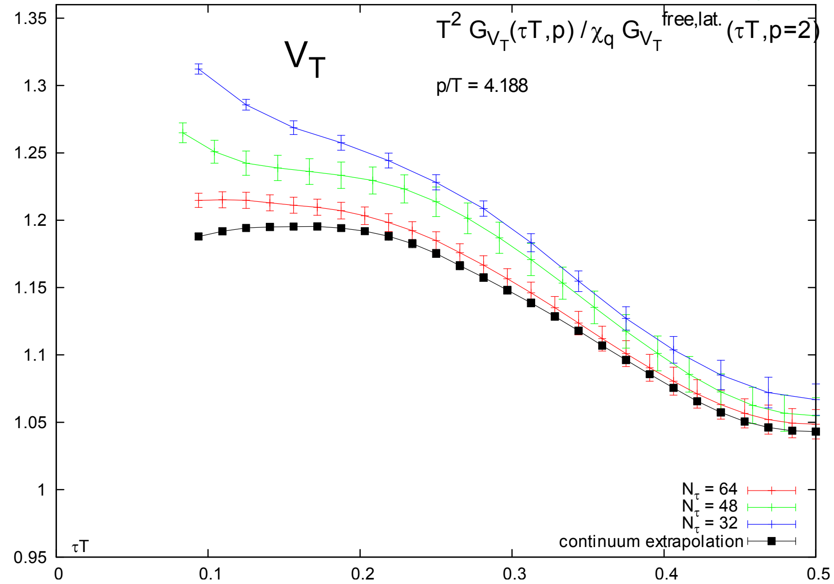


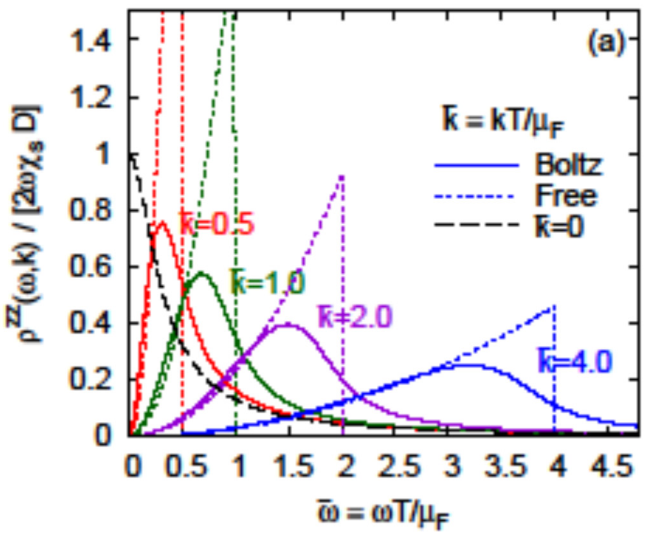
Non-zero momentum

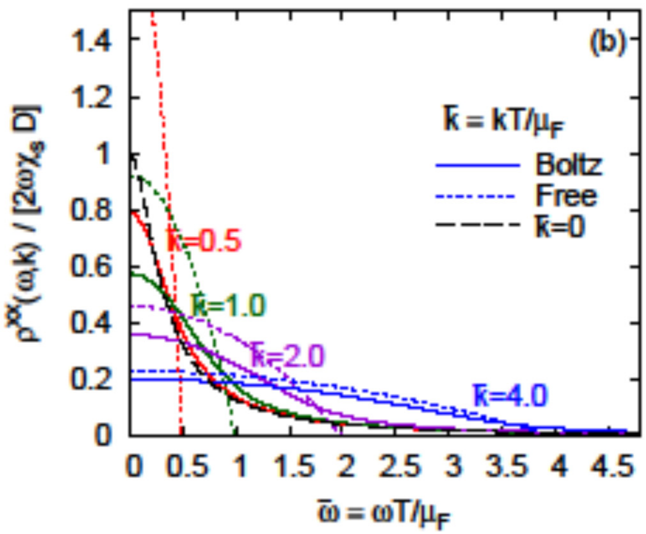
[M.Müller et al., preliminary 2012]





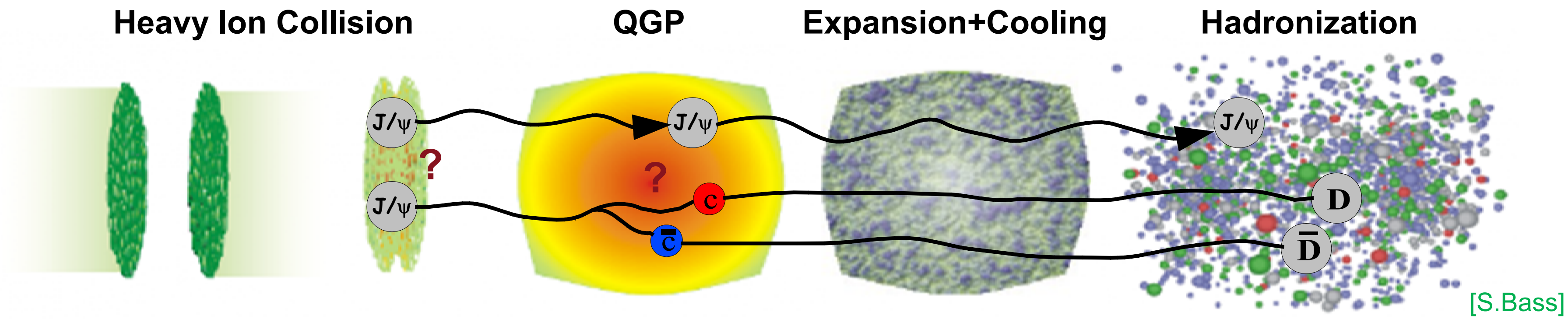
indications for non-trivial behaviour of spectral functions at small frequencies:





[Hong+Teaney, PRC82 (2010)044908]

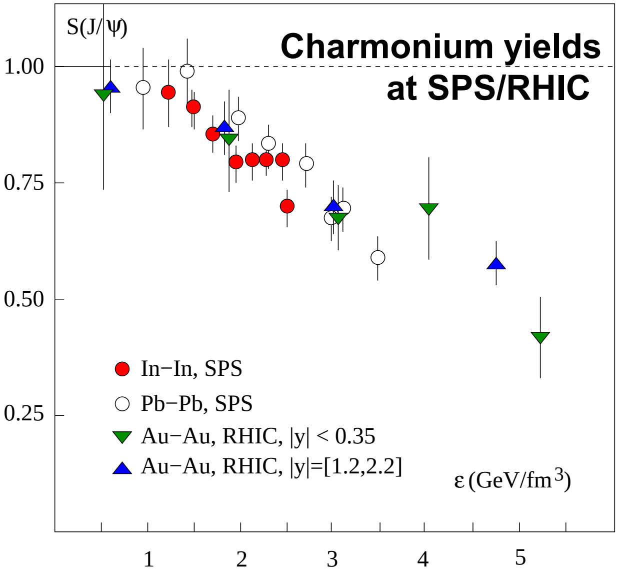
Motivation - Quarkonium in Heavy Ion Collisions



Charmonium+Bottmonium is produced (mainly) in the early stage of the collision

Depending on the Dissociation Temperature

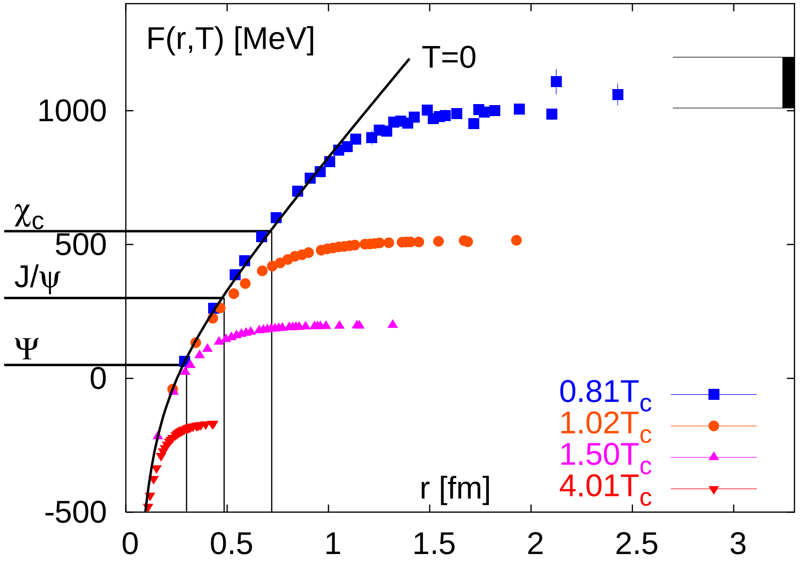
- **remain as bound states in the whole evolution**
- **release their constituents in the plasma**



First estimates on Dissociation Temperatures

from detailed knowledge of Heavy Quark Free Energies and Potential Models

[Kaczmarek, Zantow]



Motivation - Transport Coefficients

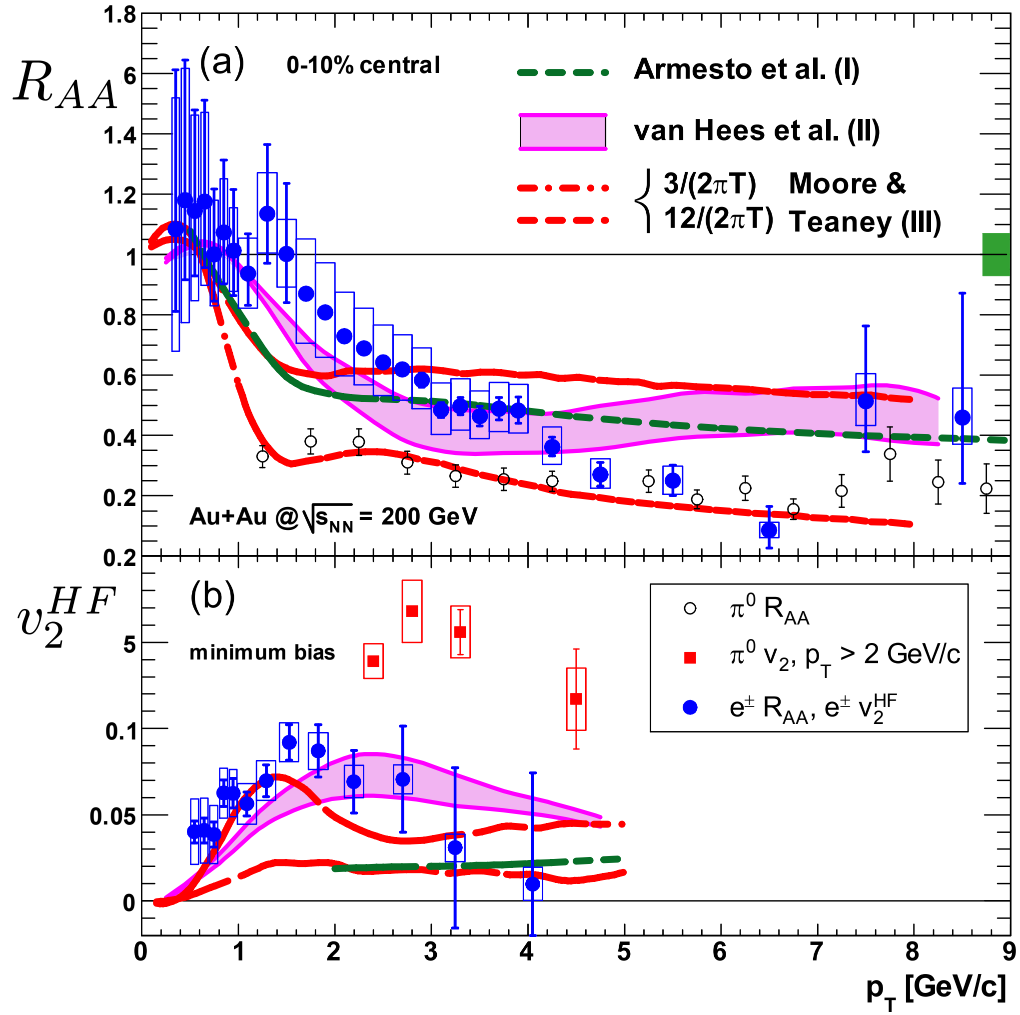
Transport Coefficients are important ingredients into **hydro models for the evolution of the system.**

Usually determined by matching to experiment (see right plot)

here: Heavy Quark Diffusion Constant D

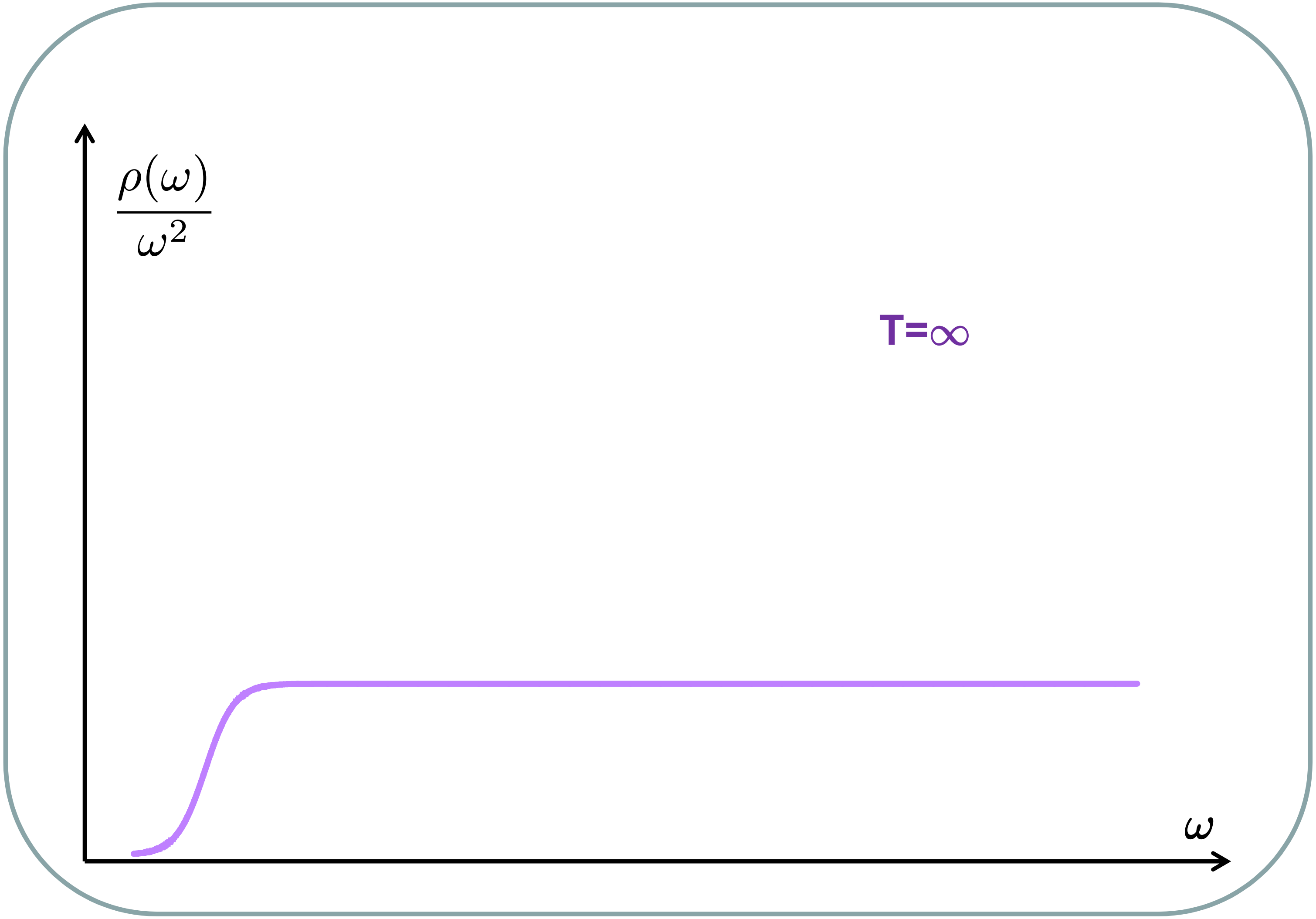
later: Heavy Quark Momentum Diffusion

Need to be determined from QCD using first principle lattice calculations!



[PHENIX Collaboration, Adare et al.,arXiv:1005.1627 & PRL98(2007)172301]

Quarkonium spectral function – What do we expect!?



+ zero-mode contribution at ω=0: $\rho(\omega) = 2\pi\chi_{00}\ \omega\delta(\omega)$

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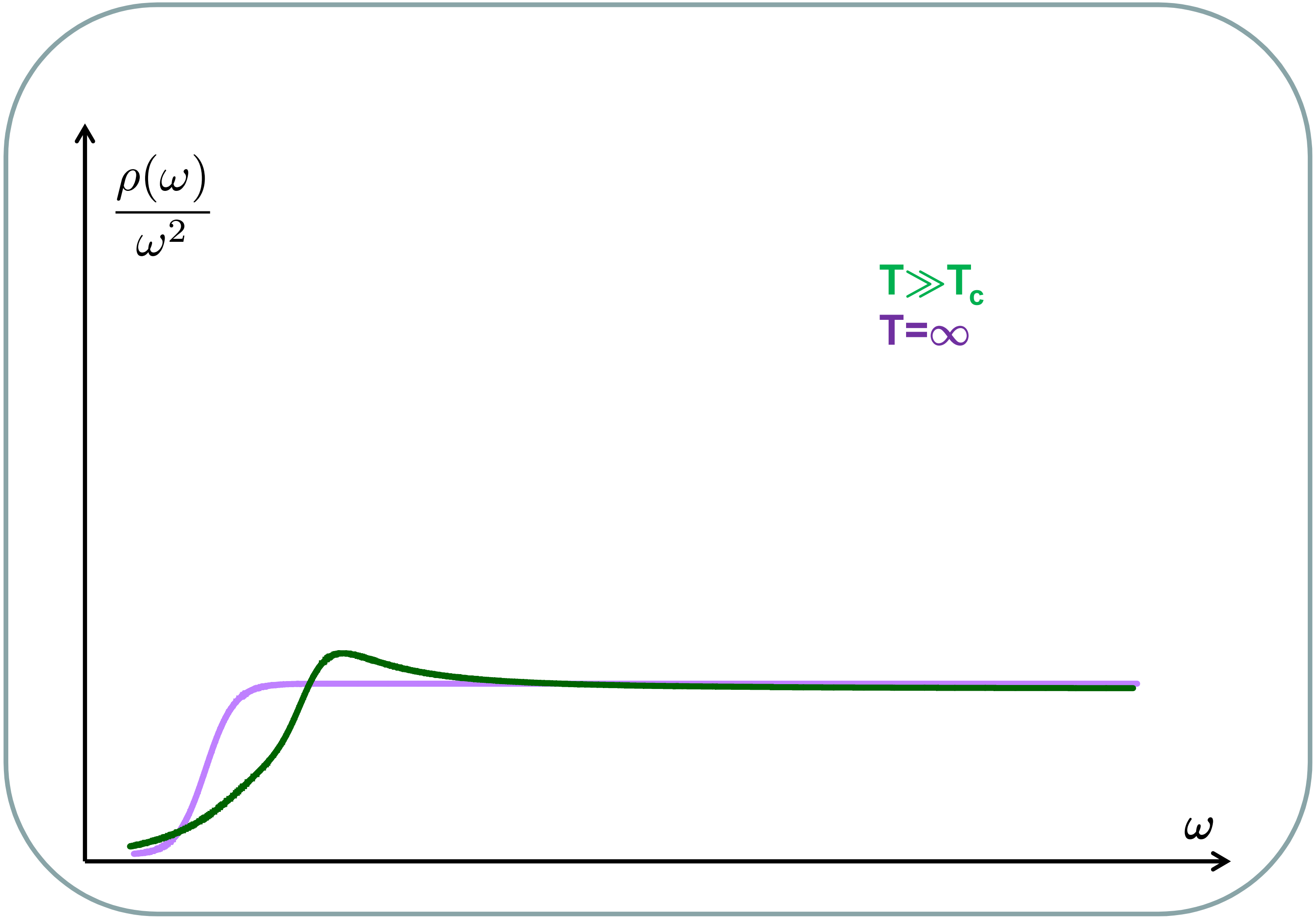

+ zero-mode contribution at ω=0: $\rho(\omega) = 2\pi\chi_{00}\ \omega\delta(\omega)$

+ transport peak at small ω: $\rho(\omega \ll T) \simeq 2\chi_{00}\frac{T}{M}\frac{\omega\eta}{\omega^2+\eta^2}\,, \qquad \eta = \frac{T}{MD}$

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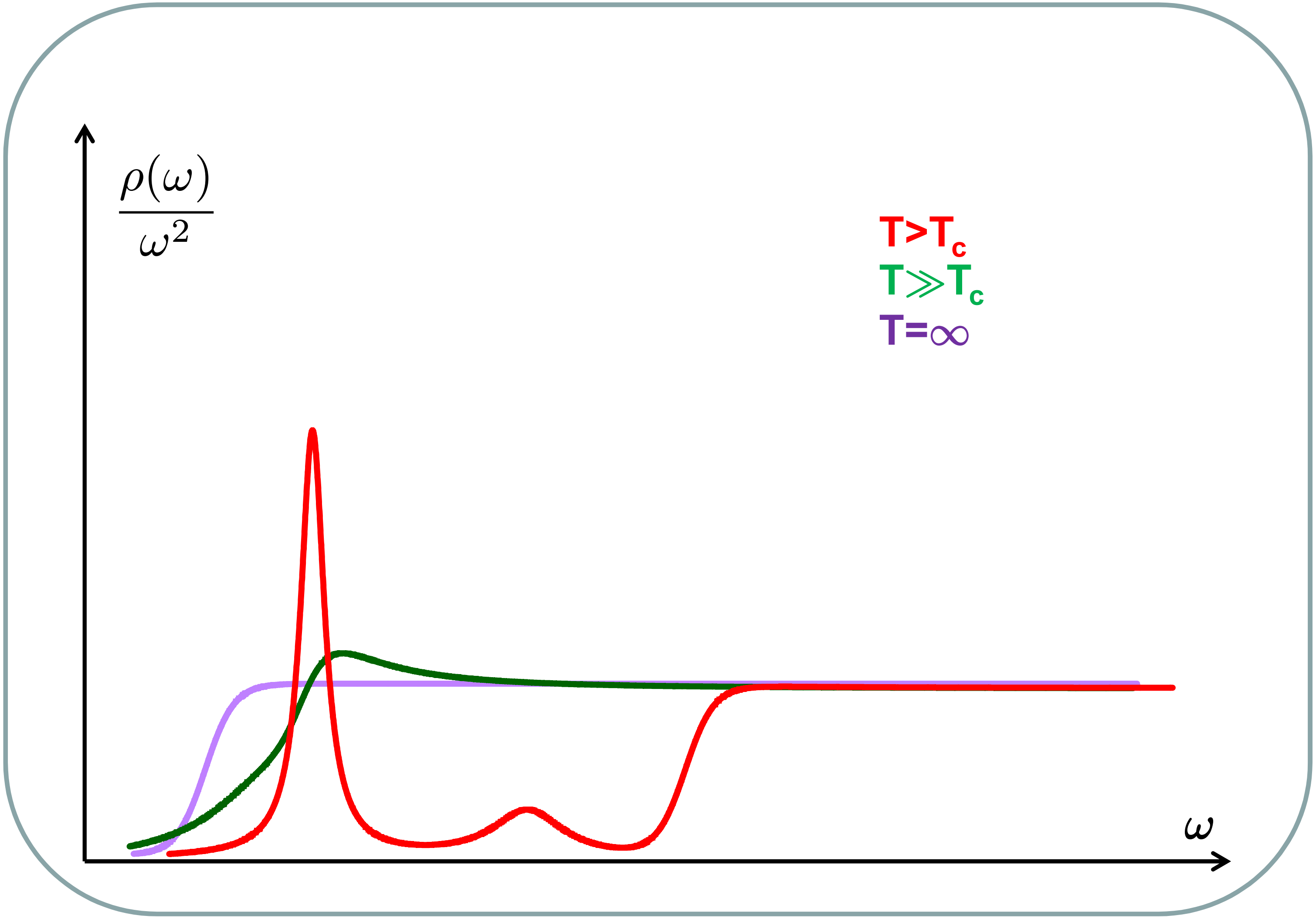

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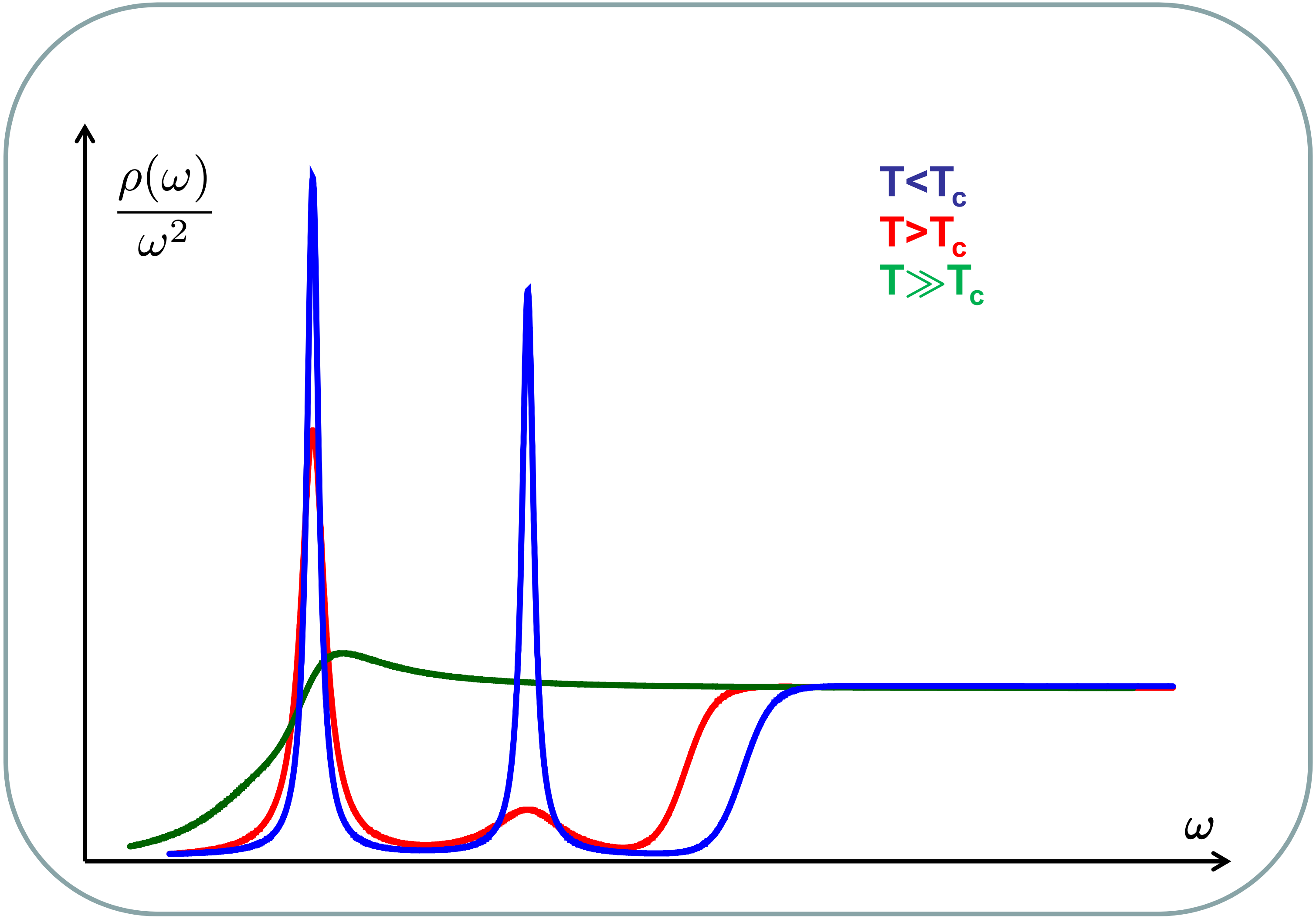

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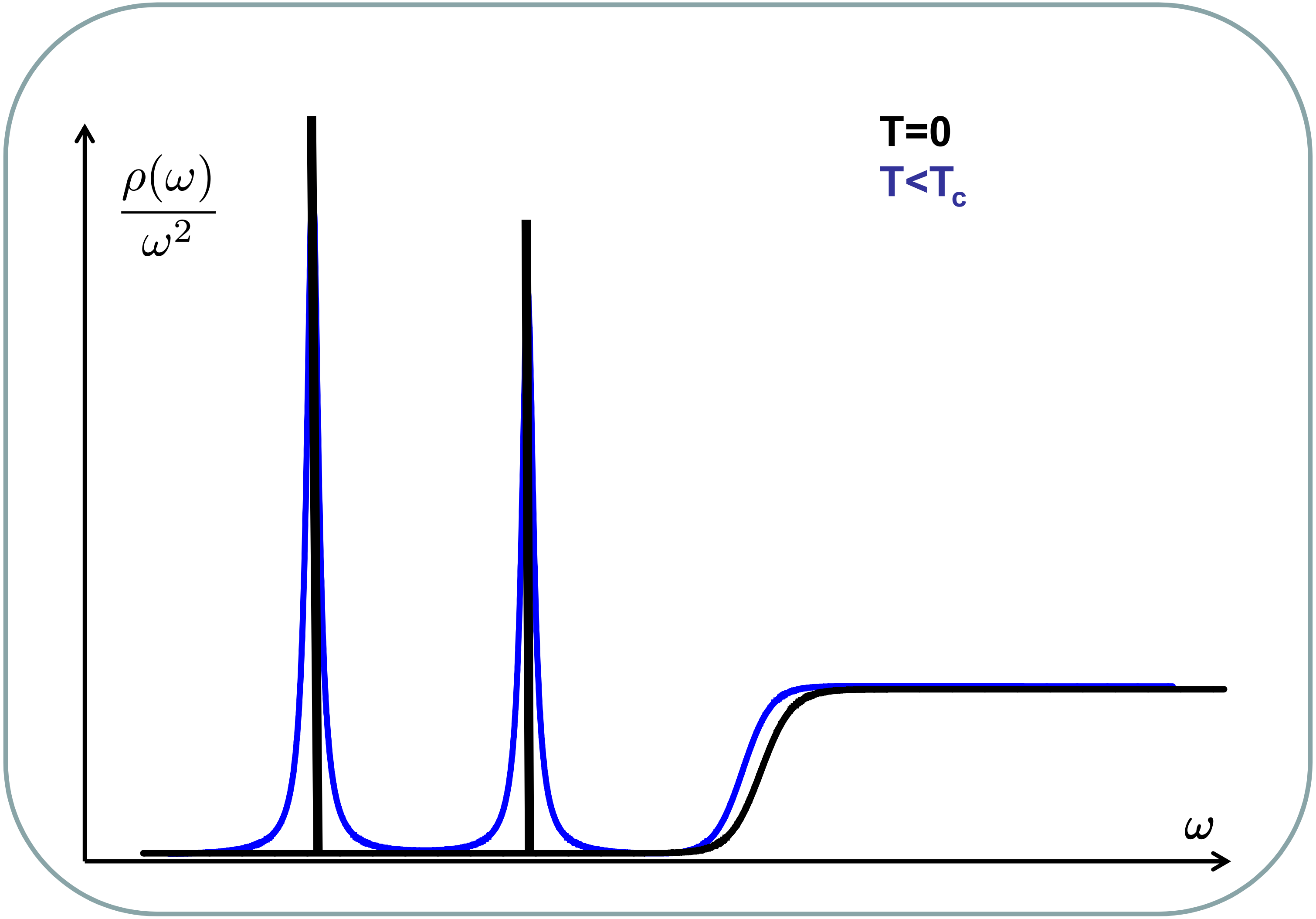

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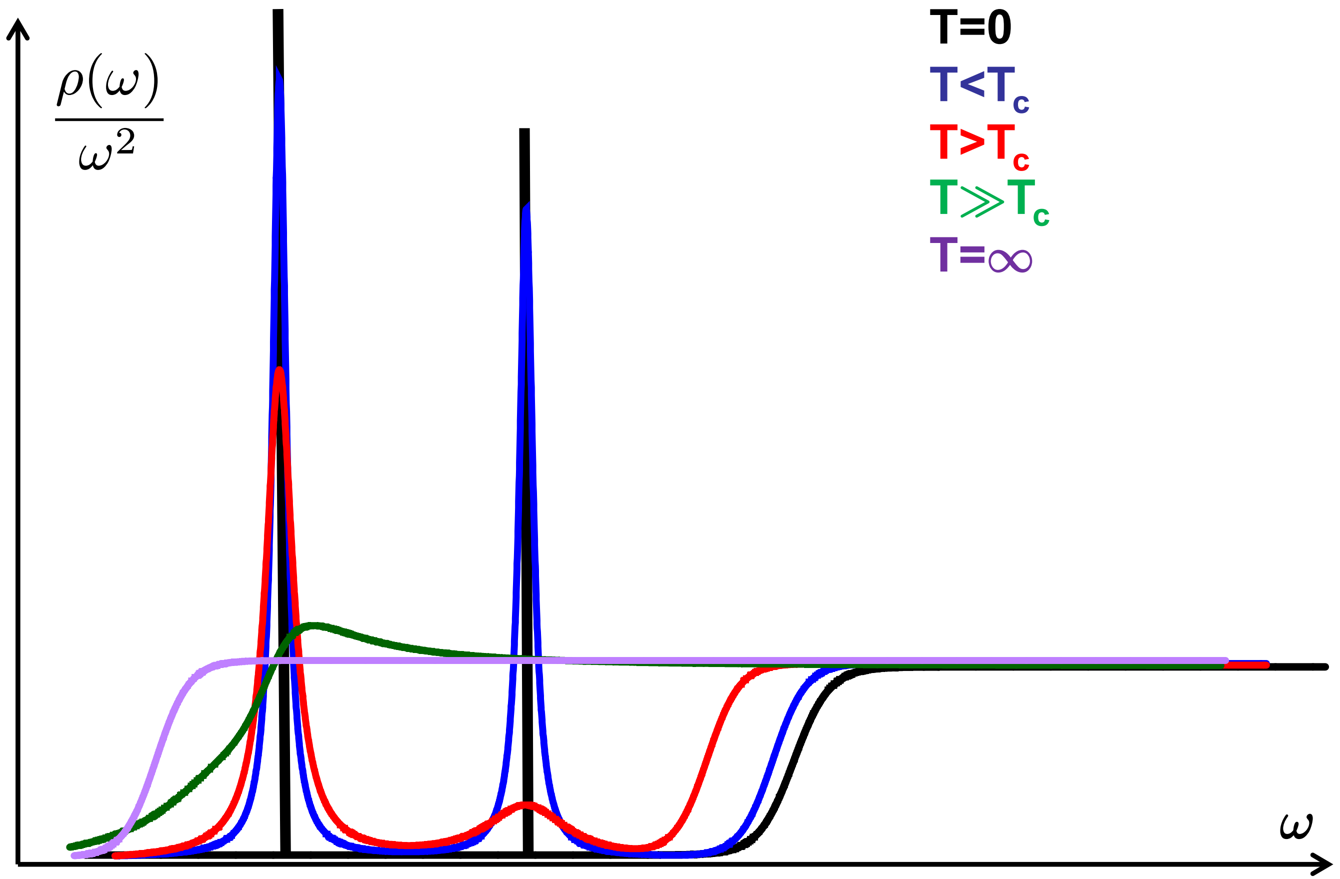

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Spatial Correlation Function and Screening Masses

Correlation functions along the **spatial direction**

$$G(z,T) = \int dxdy \int_0^{1/T} d\tau \langle J(x,y,z,\tau) J(0,0,0,0) \rangle$$

are related to the meson spectral function at **non-zero spatial momentum**

$$G(z,T) = \int_{-\infty}^{\infty} dp_z e^{ip_z z} \int_0^{\infty} d\omega \frac{\sigma(\omega, p_z, T)}{\omega}$$

exponential decay defines **screening mass M_{scr} :** $\quad \xrightarrow[z \gg 1/T]{} \quad \mathbf{e^{-M_{scr} z}}$

bound state contribution

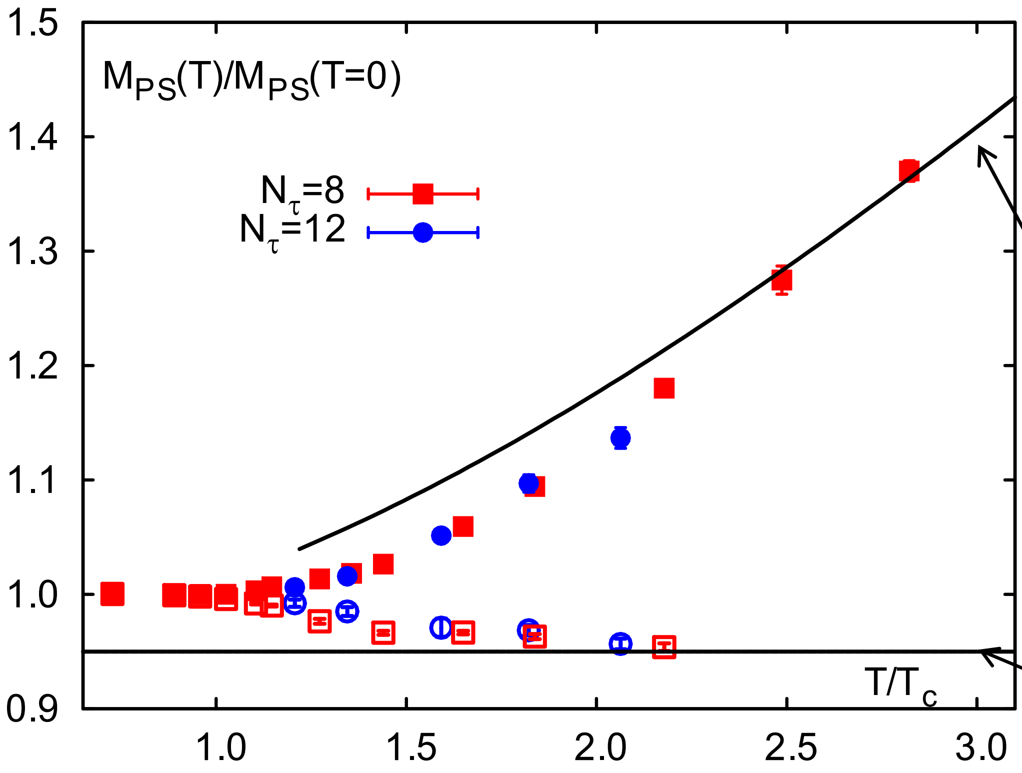
$$\sigma(\omega, p_z, T) \sim \delta(\omega^2 - p_z^2 - M^2)$$

high-T limit (non-interacting free limit)

$$\sigma(\omega, p_z, T) \sim \sigma_{free}(\omega, p_z, T)$$

$M_{scr} = M$ $\longrightarrow$ indications for medium modifications/dissociation $\longleftarrow$ $M_{scr} = 2\sqrt{(\pi T)^2 + {m_c}^2}$

Spatial Correlation Function and Screening Masses



2+1-flavor (p4-improved staggered)

$32^3 \times N_t$ with N_t=8,12 and 32

physical m_s and m_l=ms/10($m_\pi \simeq$220MeV)

anti-periodic boundary conditions (closed symbols)

$$M_{scr} = 2\sqrt{(\pi T)^2 + {m_c}^2}$$

periodic boundary conditions (open symbols)

$$M_{scr} = 2m_c$$

$$M_{scr} = M$$

screening masses for bound states insensitive to boundary conditions due to bosonic nature of the basic degrees of freedom

"… the change in the behavior of the charmonium screening masses around T=1.5T$_c$ is likely due to the melting of the meson states."

["Signatures of charmonium modification in spatial correlation functions", F.Karsch, E.Laermann, S.Mukherjee, P.Petreczky PRD85(2012)114501]

Charmonium spectral function in quenched QCD

[H.T.Ding, OK et al., PRD86(2012)014509]

Quenched SU(3) gauge configurations (separated by 500 updates) at 4 temperatures

Lattice size $N_\sigma^3\, N_\tau$ with $N_\sigma = 128$

$N_\tau = 16, 24, 32, 48, 96$

Non-perturbatively O(a) clover improved Wilson fermions

Non-perturbative renormalization constants

Quark masses close to charm quark mass

	Mass in GeV			
β	J/ψ	η_c	χ_{c1}	χ_{c0}
6.872	3.1127(6)	3.048(2)	3.624(38)	3.540(25)
7.457	3.147(1)(25)	3.082(2)(21)	3.574(8)	3.486(4)
7.793	3.472(2)(114)	3.341(2)(104)	4.02(2)(23)	4.52(2)(37)

cut-off dependence

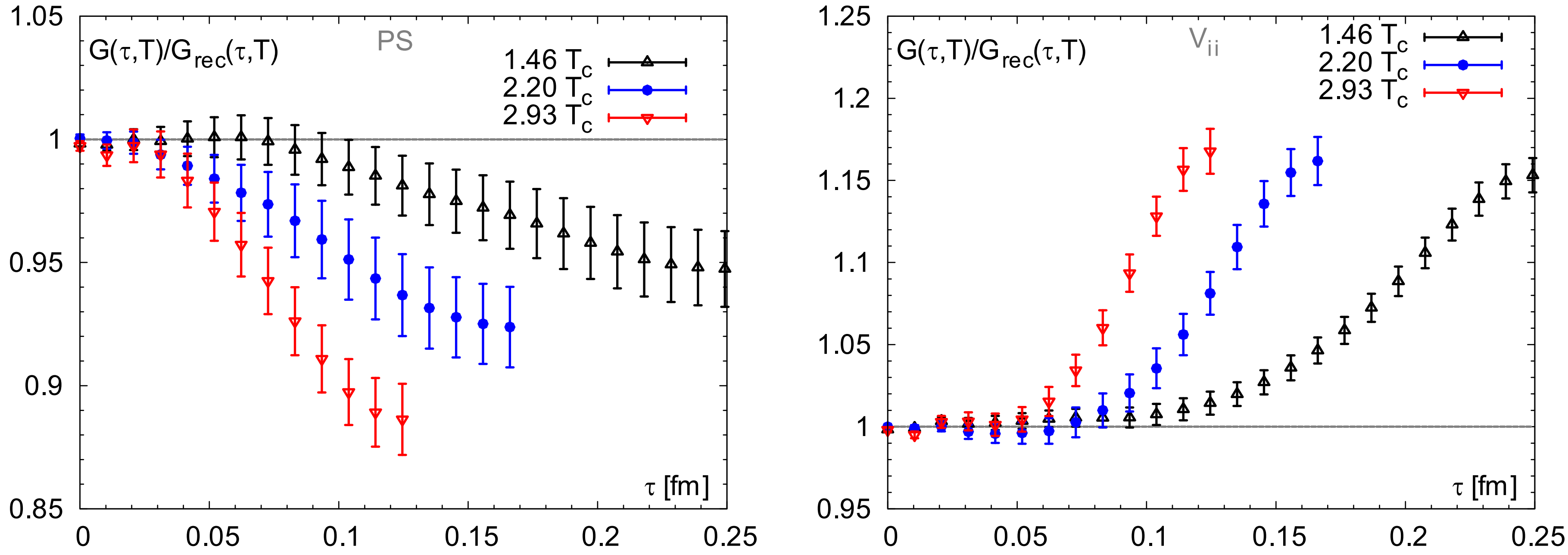
volume dependence

β	a[fm]	a^{-1}[GeV]	L_σ[fm]	c_{SW}	κ	$N_\sigma^3 \times N_\tau$	T/T_c	N_{conf}
6.872	0.031	6.432	3.93	1.412488	0.13035	$128^3 \times 32$	0.74	126
						$128^3 \times 16$	1.49	198
7.457	0.015	12.864	1.96	1.338927	0.13179	$128^3 \times 64$	0.74	179
						$128^3 \times 32$	1.49	250
7.793	0.010	18.974	1.33	1.310381	0.13200	$128^3 \times 96$	0.73	234
						$128^3 \times 48$	1.46	461
						$128^3 \times 32$	2.20	105
						$128^3 \times 24$	2.93	81

close to continuum
($m_c a \ll 1$)

Temperature dependence

Charmonium Correlators vs Reconstructed Correlators



$$G(\tau,T) = \int \sigma(\omega,T)K(\omega,\tau,T)$$

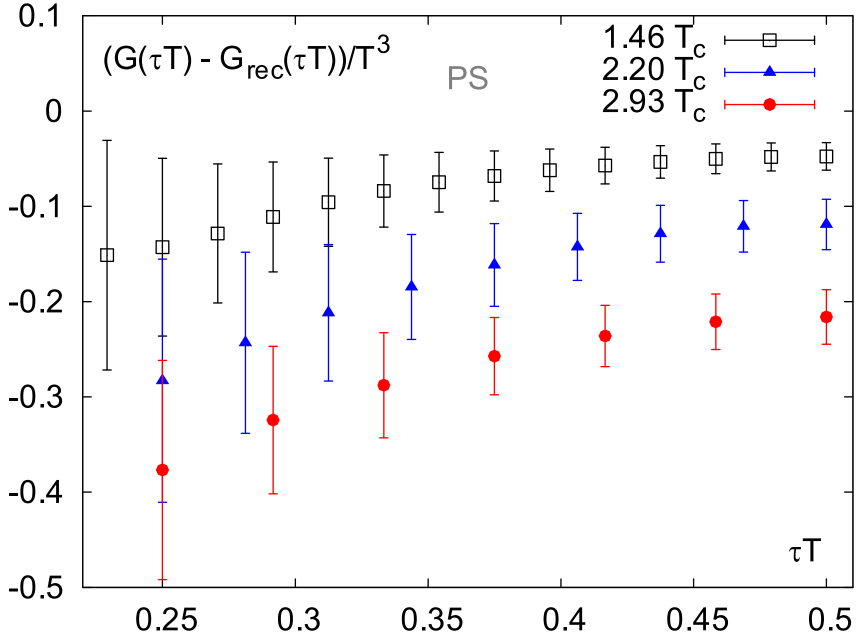
$$G_{rec}(\tau,T) = \int \sigma_0(\omega,0.75T_c)K(\omega,\tau,T)$$

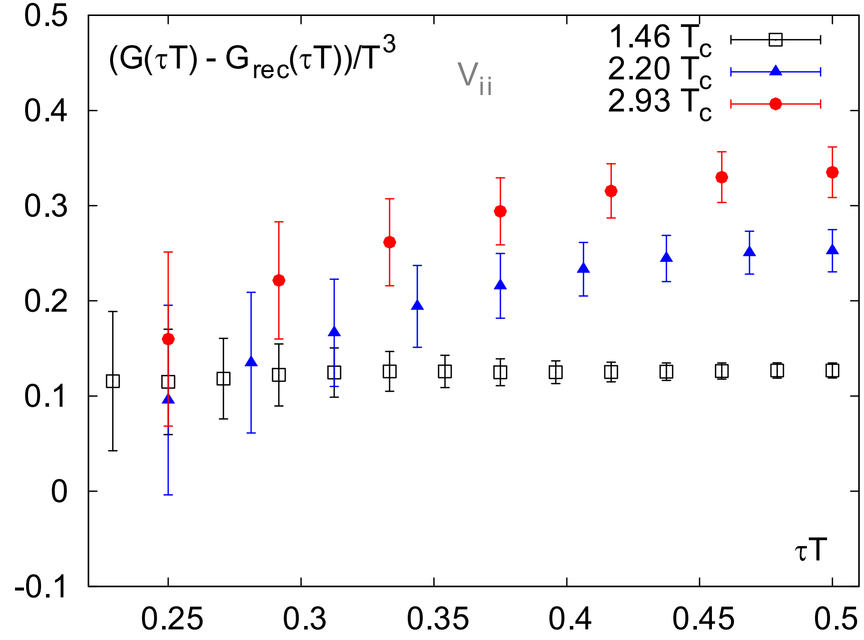
$$G_{\mu\mu}(\tau,T) = G_{ii}(\tau,T) + G_{00}(\tau,T)$$

- main T-effect due to zero-mode contribution
- well described by small ω-part of $\sigma_T(\omega,T)$
- explains the rise in the vector channel
- no zero-mode contribution in PS-channel

(similar to discussions by Umeda, Petreczky)

Charmonium Correlators vs Reconstructed Correlators





- negative difference for all T
- indications for thermal modifications in the bound state frequency region
- remember: no transport contribution in this channel

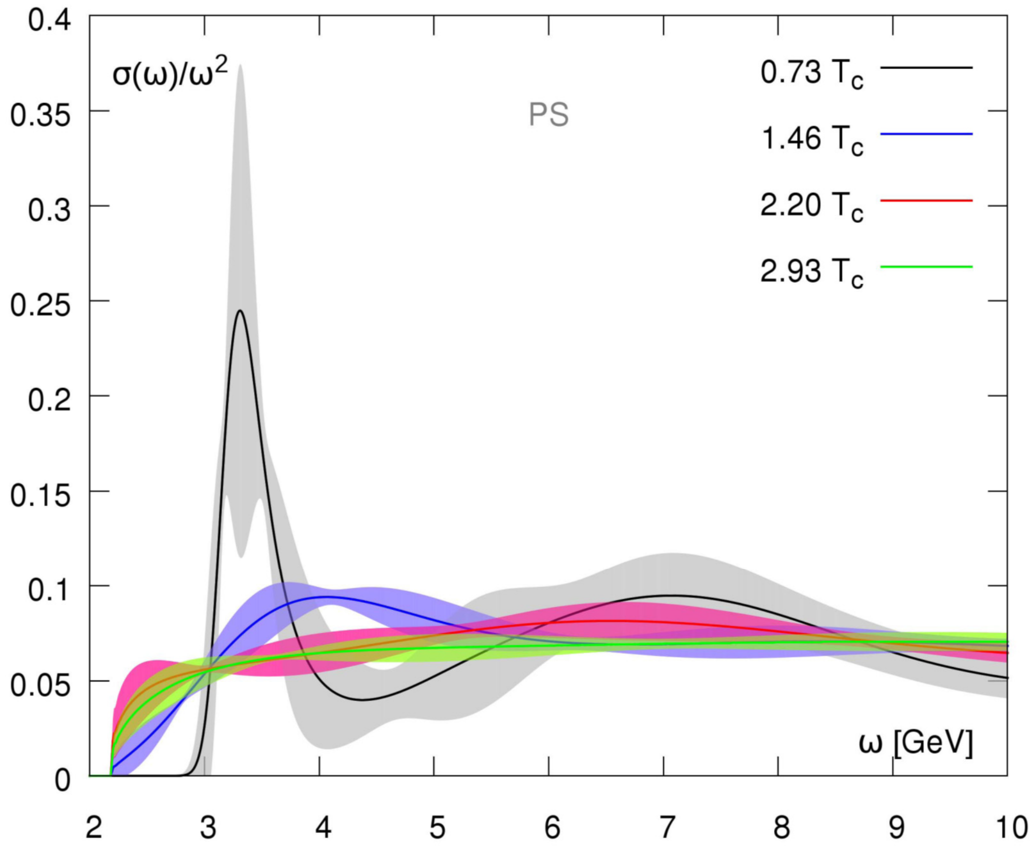
- positive diff. due to small-ω contr.
- positive slope indicates modifications in the bound state frequency region
- remember: small-ω contribution determines transport coefficient

First estimate from fit to vector channel: $2\pi T\, D \approx 0.6 - 3.4$

Charmonium Spectral function

[H.T.Ding, OK et al., PRD86(2012)014509]

from sophisticated Maximum Entropy Method analysis:



statistical error band from Jackknife analysis

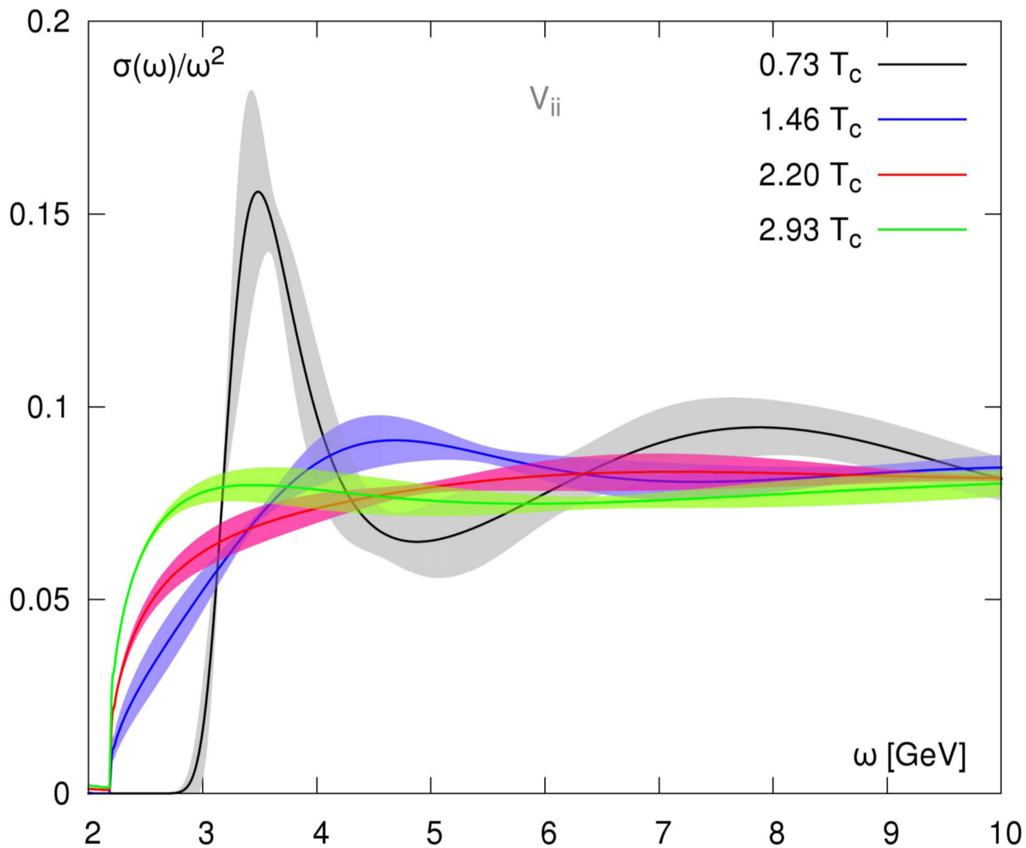
no clear signal for bound states at and above 1.46 T_c

study of the interesting region closer to T_c on the way!

Charmonium Spectral function

[H.T.Ding, OK et al., PRD86(2012)014509]

from sophisticated Maximum Entropy Method analysis:



statistical error band from Jackknife analysis

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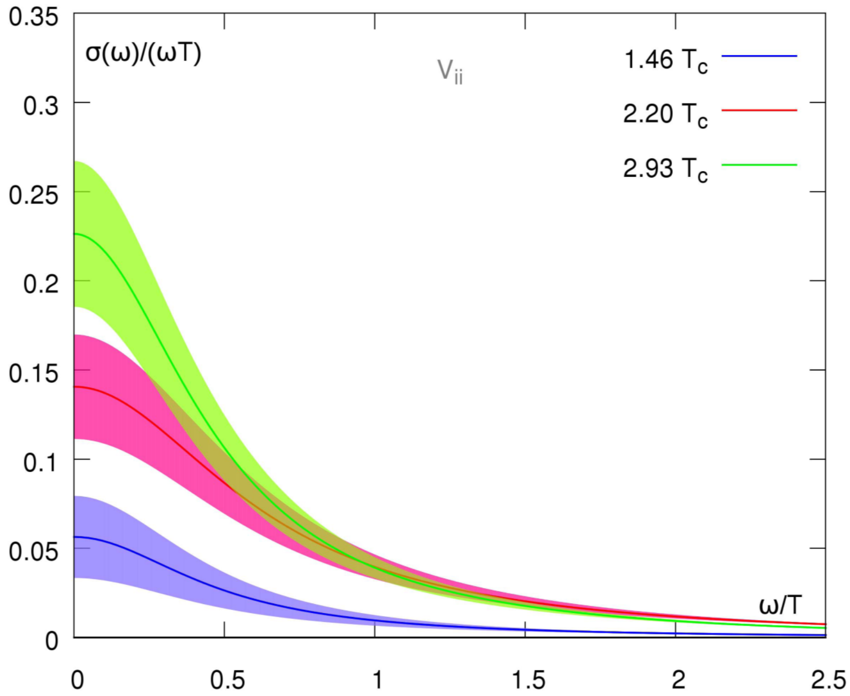
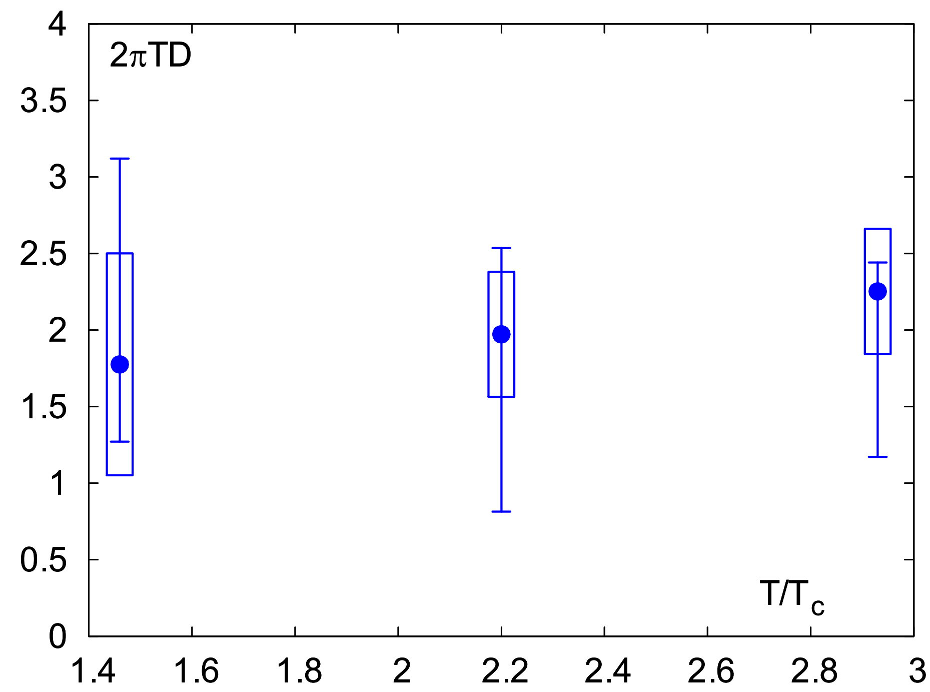
Charmonium Spectral function – Transport Peak

[H.T.Ding, OK et al., PRD86(2012)014509]





$$D = \frac{\pi}{3\chi_{00}} \lim_{\omega\to 0} \frac{\rho_{ii}(\omega, \vec{p}=0, T)}{\omega T}$$

Perturbative estimate (α_s~0.2, g~1.6):

LO: $2\pi TD \simeq 71.2$

NLO: $2\pi TD \simeq 8.4$

[Moore&Teaney, PRD71(2005)064904,
Caron-Huot&Moore, PRL100(2008)052301]

Strong coupling limit:

$2\pi TD = 1$

[Kovtun, Son & Starinets,JHEP 0310(2004)064]

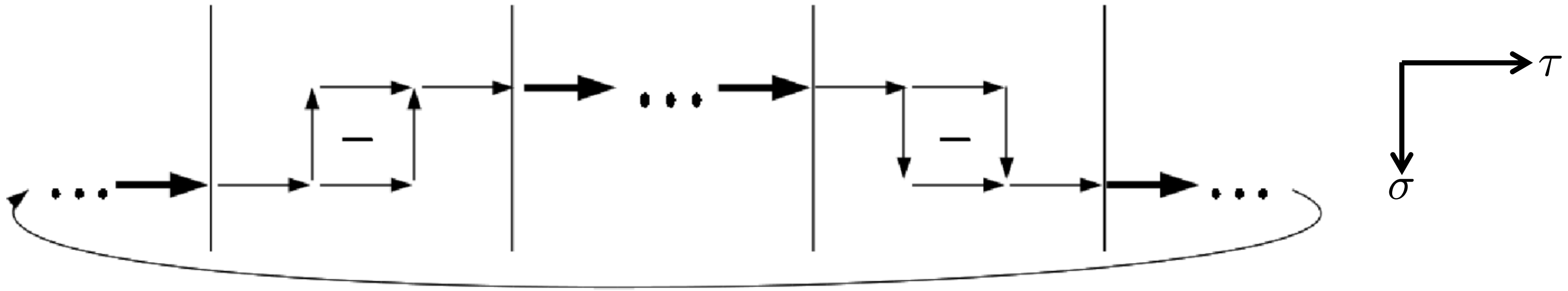
Heavy Quark Momentum Diffusion Constant

[A.Francis,OK,M.Laine,J.Langelage, arXiv:1109.3941]

Heavy Quark Effective Theory (HQET) in the large quark mass limit

leads to a (pure gluonic) “color-electric correlator”

[J.Casalderrey-Solana, D.Teaney, PRD74(2006)085012, S.Caron-Huot,M.Laine,G.D. Moore,JHEP04(2009)053]



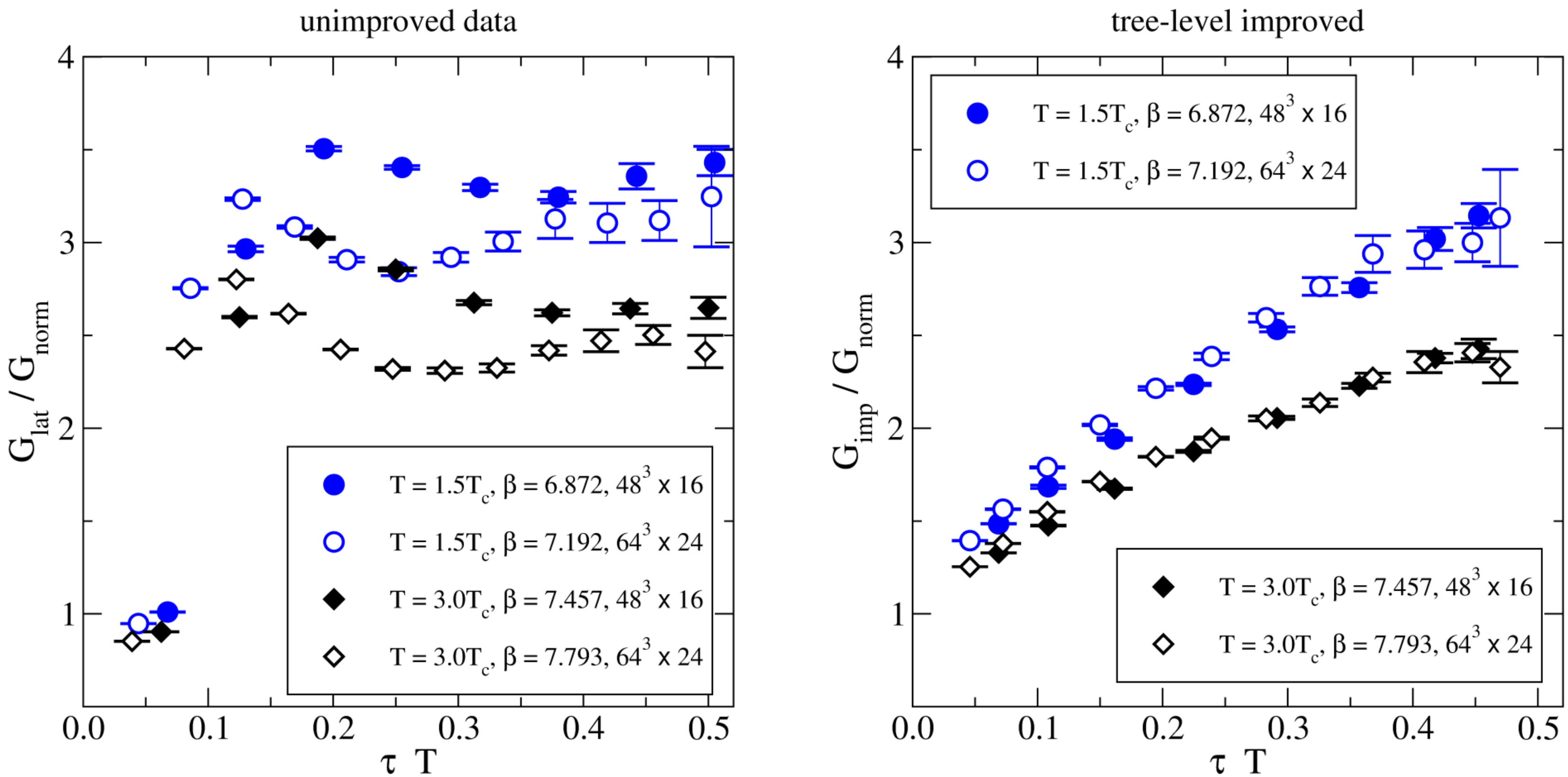
$$G_{\rm E}(\tau) \equiv -\frac{1}{3}\sum_{i=1}^{3}\frac{\left\langle \mathrm{Re\,Tr}\left[U(\tfrac{1}{T};\tau)\, gE_i(\tau,\mathbf{0})\, U(\tau;0)\, gE_i(0,\mathbf{0})\right]\right\rangle}{\left\langle \mathrm{Re\,Tr}[U(\tfrac{1}{T};0)]\right\rangle}$$

Heavy quark (momentum) diffusion:

$$\kappa = \lim_{\omega\to 0}\frac{2T\rho_{\rm E}(\omega)}{\omega} \quad , \qquad D = \frac{2T^2}{\kappa}$$

Heavy Quark Momentum Diffusion Constant

[A.Francis,OK,M.Laine,J.Langelage, arXiv:1109.3941]



due to the gluonic nature of the operator, signal is extremely noisy

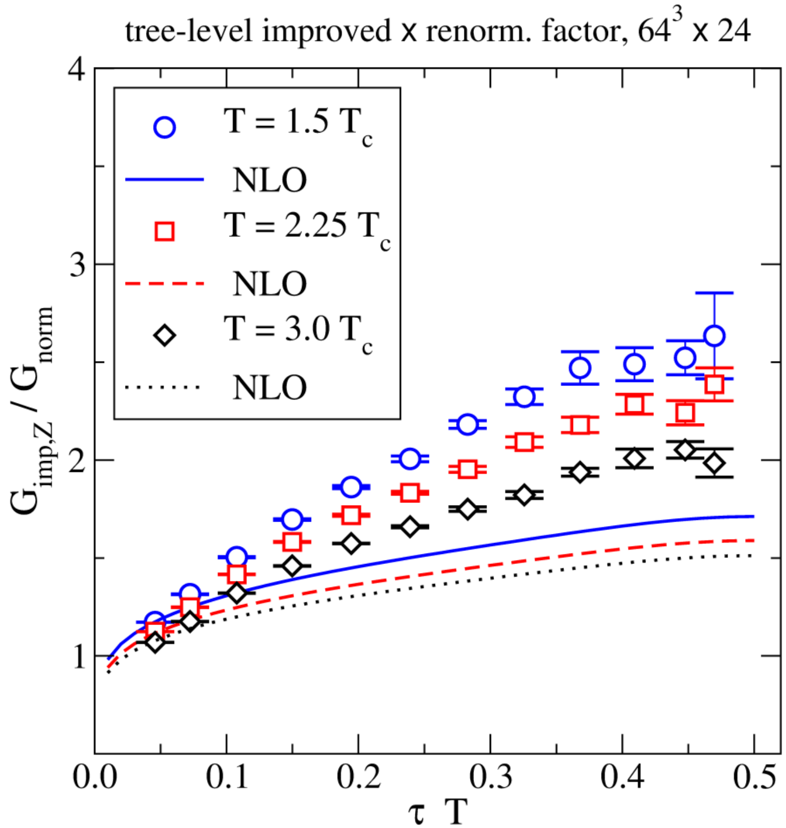
→ multilevel combined with link-integration techniques used to improve the signal

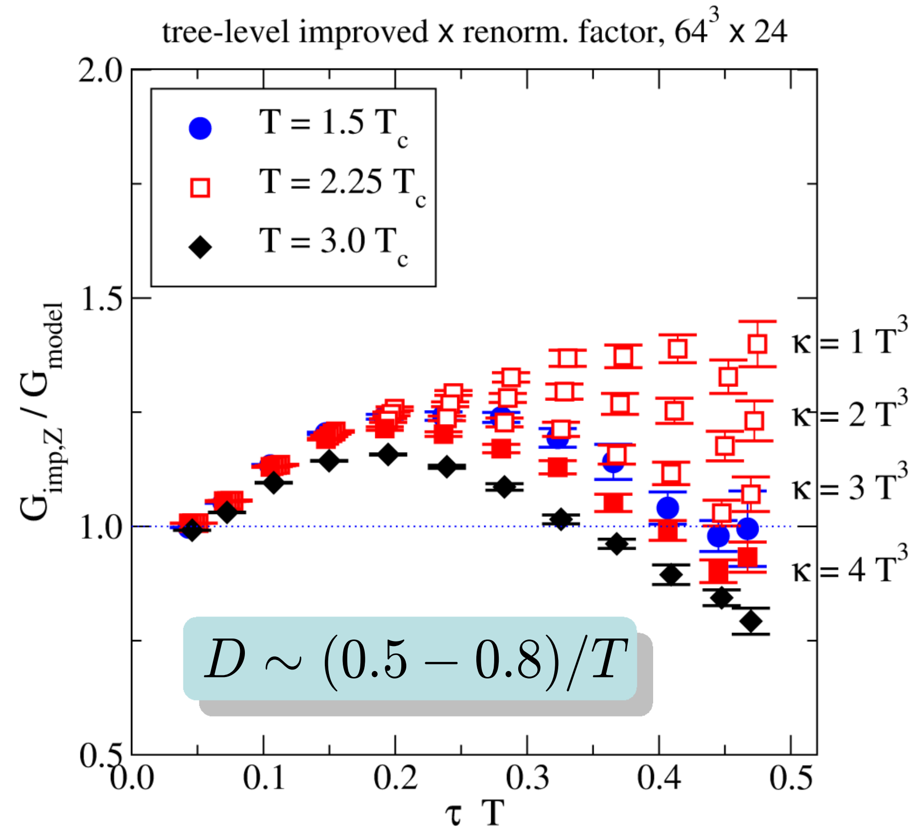
→ tree-level improvement (right figure) to reduce discretization effects

[similar studies by H.B.Meyer, New J.Phys.13 (2011) 035008 and D.Banerjee,S.Datta,R.Gavai,P.Majumdar,PRD85(2012)014510]

Heavy Quark Momentum Diffusion Constant

[A.Francis,OK,M.Laine,J.Langelage, arXiv:1109.3941]





$D \sim (0.5 - 0.8)/T$

Model spectral function: transport contribution + NLO [Y.Burnier et al. JHEP 1008 (2010) 094)]

$$\rho_{\text{model}}(\omega) \equiv \max\left\{\rho_{\text{NLO}}(\omega), \frac{\omega\kappa}{2T}\right\}$$

$$G_{\text{model}}(\tau) \equiv \int_0^\infty \frac{\mathrm{d}\omega}{\pi}\rho_{\text{model}}(\omega)\frac{\cosh\left(\frac{1}{2} - \tau T\right)\frac{\omega}{T}}{\sinh\frac{\omega}{2T}}$$

Still large uncertainties but very promising

→ thermodynamic+continuum limit needed

→ more constraints on the spectral function

→ other operators and observables from EFT?

Conclusions

Charmonium:

[F.Karsch, E.Laermann, S.Mukherjee, P.Petreczky, PRD85(2012)114501]:

"… the change in the behavior of the charmonium screening masses around $T=1.5T_c$ is likely due to the melting of the meson states."

[H.T.Ding, OK et al., PRD86(2012)014509]:

Detailed knowledge of the **vector correlation function** at various T in quenched QCD

→ **continuum extrapolation** of correlation function still needed!

Results so far depend on MEM analysis → Ansätze more difficult due to m_q dependence

→ **Heavy quark diffusion constant:** $2\pi DT \approx 2$

→ **No signs for bound states at and above 1.46 T_c**