In-medium QCD forces at high temperature

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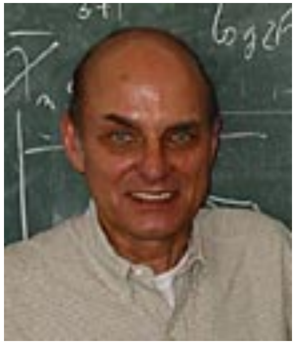
References:
Y.A., A.Rothkopf, PRD85,105011(2012).
Y.A., PRD87,045016(2013).
Short introduction:
Y.A., arXiv:1303.2976 [nucl-th].

Contents

1. “INTRODUCTION”

Key members

- J/Ψ suppression

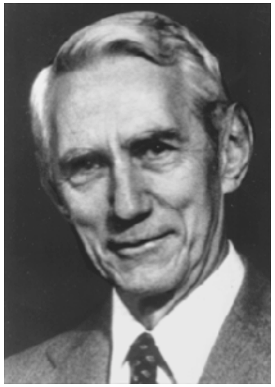
 

Debye

- J/Ψ survival (up to $1.6T_c$?)

Bayes?



Shannon

New paradigm since '07

- Imaginary part of potential

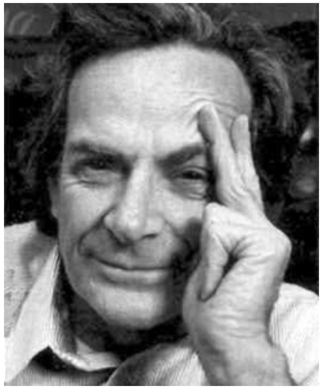
et al

followed by other groups with
- path-integral approach
- lattice QCD+MEM approach
- pNRQCD EFT approach

$$V_{Q\bar{Q}}(R) = -\frac{g^2 C_F}{4\pi}\left(\omega_D + \frac{e^{-\omega_D R}}{R} + iT\varphi(\omega_D R)\right)$$

New members

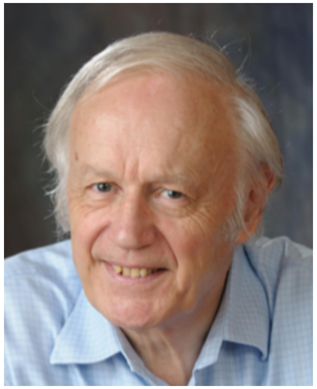
- New does not mean young...



Feynman

Frank Lee
Vernon, Jr.

Caldeira



Leggett

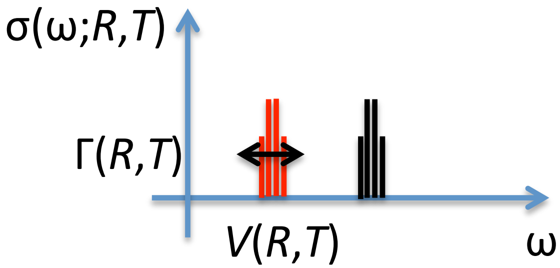
Quartet ~~Magico~~ Fisica (di sistemi quantistici aperti)?

Translated by google

2. STOCHASTIC POTENTIAL

Complex potential

- Definition



$$\langle \Psi(t;R) \rangle_T \propto \langle J(t;R) J^\dagger(0;R) \rangle_T$$

$$\propto \sum_{n,m} \left| \langle m | J^\dagger(0;R) | n \rangle \right|^2 e^{-\beta E_n(R)} \exp\left[i \{ E_n(R) - E_m(R) \} t \right]$$

$$\sim \exp\left[-i \{ V(R,T) - i\Gamma(R,T)/2 \} t \right]$$

Laine et al (07), Beraudo et al (08), Brambilla et al (10), Rothkopf et al (12)

- Stochastic interpretation

$$\Psi(t+\Delta t, R) = \exp\left[-i\Delta t \left\{ V(R) + \Theta(t,R) \right\} \right] \Psi(t,R),$$

Unitary evolution

$$\langle \Theta(t,R) \rangle = 0, \quad \langle \Theta(t,R) \Theta(t',R') \rangle = \Gamma(R,R') \delta_{tt'} / \Delta t,$$

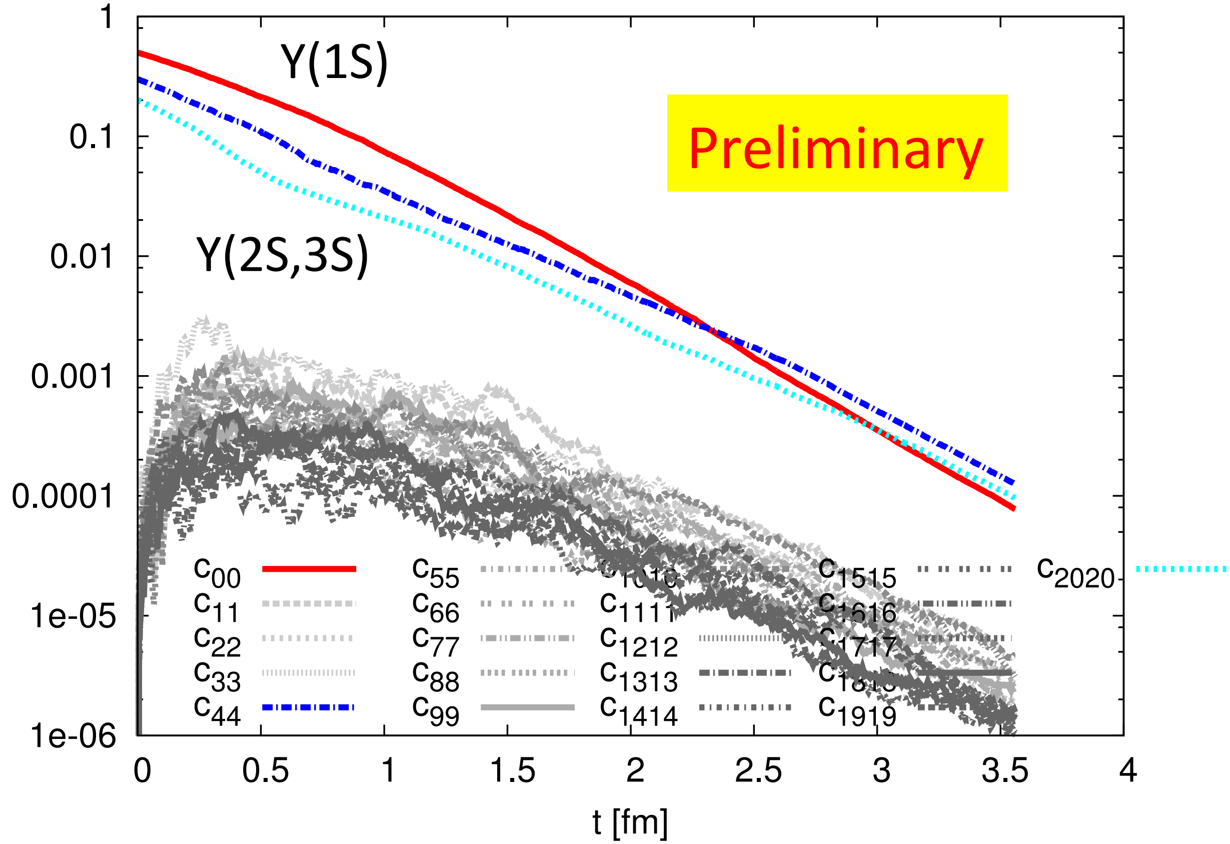
$$\Rightarrow i \frac{\partial}{\partial t} \langle \Psi(t,R) \rangle_T = \left\{ V(R) - \frac{i}{2} \Gamma(R,R) \right\} \langle \Psi(t,R) \rangle_T .$$

Imaginary part from averaging the random phase

Akamatsu & Rothkopf (12)

Numerical simulation

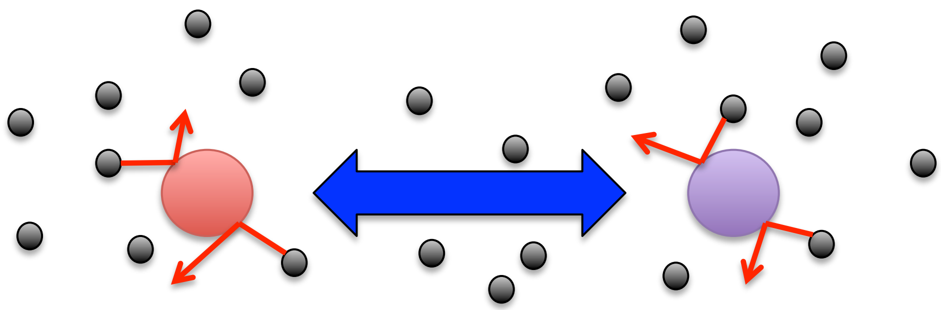
- Stochastic evolution



3. INFLUENCE FUNCTIONAL

Classical picture

- $M=\infty$ or $M<\infty$ matters



Debye screened force

$M=\infty$ +

Fluctuation

} (stochastic/complex) Potential force →Hamiltonian dynamics

$M<\infty$ Drag force --------- Non-potential force →Not Hamiltonian dynamics

Langevin dynamics

Key words: Forces, open quantum system, influence functional

Open quantum system

- Basics

$$\begin{cases} \text{sys = heavy quarks} \\ \text{env = gluon, light quarks} \end{cases}$$

Hilbert space $\mathrm{H}_{\mathrm{tot}} = \mathrm{H}_{\mathrm{sys}} \otimes \mathrm{H}_{\mathrm{env}}$

von Neumann equation $i\frac{d}{dt}\hat{\rho}_{\mathrm{tot}}(t) = \left[\hat{H}_{\mathrm{tot}}, \hat{\rho}_{\mathrm{tot}}(t)\right]$

⬇ Trace out the environment

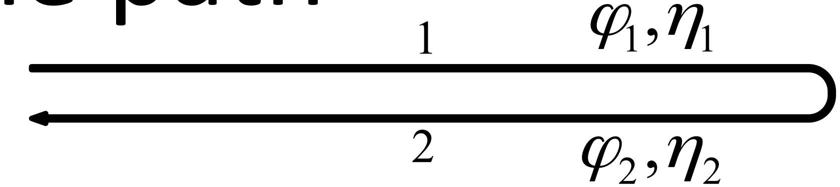
Reduced density matrix $\hat{\rho}_{\mathrm{red}}(t) \equiv \mathrm{Tr}_{\mathrm{env}}\left[\hat{\rho}_{\mathrm{tot}}(t)\right]$

Master equation $i\frac{d}{dt}\hat{\rho}_{\mathrm{red}}(t) = ?$

(Markovian limit)

Closed-time path

$$\rho[\varphi_1^{\text{ini}},\varphi_2^{\text{ini}}]$$



- Partition function

$$Z[\eta_1,\eta_2]\sim\int D[\psi_{1,2}\underline{q_{1,2}A_{1,2}}]\rho[\psi_1^* q_1^* A_1|^{\text{ini}},\psi_2 q_2 A_2|^{\text{ini}}]$$
$$\times\exp\left[iS[\psi_1]-iS[\psi_2]+i\int\psi_1\eta_1-i\int\psi_2\eta_2\right]$$
$$\underline{\times\exp\left[iS[q_1A_1]-iS[q_2A_2]+ig\int j_1A_1-ig\int j_2A_2\right]}$$

$\rho_{\text{tot}}=\rho_{\text{env}}^{\text{eq}}\otimes\rho_{\text{sys}}$ Factorized initial density matrix

$$\to\rho_{\text{tot}}[\psi_1^* q_1^* A_1|^{\text{ini}},\psi_2 q_2 A_2|^{\text{ini}}]=\underline{\rho_{\text{env}}^{\text{eq}}[q_1^* A_1|^{\text{ini}},q_2A_2|^{\text{ini}}]}\cdot\rho_{\text{sys}}[\psi_1^{*\text{ini}},\psi_2^{\text{ini}}]$$

Influence functional   Feynman & Vernon (63)

$$\text{—}=Z_{qA}[j_1,j_2]\equiv\exp\left[iS_{\text{FV}}[j_1,j_2]\right]$$
$$=\exp\left[-g^2/2\int j_1G_A^{\text{F}}j_1+j_2G_A^{\tilde{\text{F}}}j_2-j_1G_A^{>}j_2-j_2G_A^{<}j_1+\int g^3G_A^{(3)}jjj+g^4G_A^{(4)}jjjj+\cdots\right]$$

Influence functional

- LO pQCD, NR limit, slow dynamics

Stochastic potential (finite in M→∞)

$$S_{1+2} = S_{\rm kin}^{\rm NR}[Q_{1(c)}] - S_{\rm kin}^{\rm NR}[Q_{2(c)}] + S_{\rm FV}^{\rm LONR}[j_1, j_2] + \cdots$$

$$S_{\rm FV}^{\rm LONR}[j_1, j_2] = -\frac{1}{2}\int_{t,\vec{x},\vec{y}} (\rho_{1a}, \rho_{2a})_{(t,\vec{x})} \begin{bmatrix} V(\vec{x}-\vec{y}) & -iD(\vec{x}-\vec{y}) \\ -iD(\vec{x}-\vec{y}) & -V^*(\vec{x}-\vec{y}) \end{bmatrix} \begin{pmatrix} \rho_{1a} \\ \rho_{2a} \end{pmatrix}_{(t,\vec{y})}$$

$$-\int_{t,\vec{x},\vec{y}} \left\{ \frac{\vec{\nabla} D(\vec{x}-\vec{y})}{4T} \cdot \left(\vec{j}_{1a}(t,\vec{x})\rho_{2a}(t,\vec{y}) + \rho_{1a}(t,\vec{x})\vec{j}_{2a}(t,\vec{y}) \right) \right\}$$

Drag force (vanishes in M→∞)

$$-g^2 \left\{ \overline{G}^R_{00,ab}(\vec{x}-\vec{y}) + i\overline{G}^>_{00,ab}(\vec{x}-\vec{y}) \right\} \equiv V(\vec{x}-\vec{y})\delta_{ab}$$

$$-g^2 \overline{G}^>_{00,ab}(\vec{x}-\vec{y}) \equiv D(\vec{x}-\vec{y})\delta_{ab} = \mathrm{Im} V(\vec{x}-\vec{y})\delta_{ab}$$

4. REAL-TIME DYNAMICS

Functional differential equation

- Path integral → Schroedinger equation

$$\rho_{\mathrm{red}}[t, Q^*_{1(c)}, \tilde{Q}^*_{2(c)}] \sim \int_{-\infty}^{t, Q^*_{1(c)}, \tilde{Q}^*_{2(c)}} D[Q^{(*)}_{1(c)}, \tilde{Q}^{(*)}_{2(c)}] \rho_{\mathrm{sys}}[Q^{*\mathrm{ini}}_{1(c)}, \tilde{Q}^{*\mathrm{ini}}_{2(c)}]$$

$$\times \exp\Big[iS_{\mathrm{NR}}[Q^{(*)}_{1(c)}] - iS_{\mathrm{NR}}[\tilde{Q}^{(*)}_{2(c)}] + iS^{\mathrm{LOFV}}[j_1, j_2] + \cdots \Big]$$

$$\left\{\hat{Q}_1(\vec{x}), \hat{Q}_1^\dagger(\vec{y})\right\} = \left\{\hat{Q}_{1c}(\vec{x}), \hat{Q}_{1c}^\dagger(\vec{y})\right\} = \delta(\vec{x} - \vec{y}) \Leftrightarrow Q_{1(c)} = \frac{\delta}{\delta Q^*_{1(c)}}$$

$$\left\{\hat{\tilde{Q}}_2(\vec{x}), \hat{\tilde{Q}}_2^\dagger(\vec{y})\right\} = \left\{\hat{\tilde{Q}}_{2c}(\vec{x}), \hat{\tilde{Q}}_{2c}^\dagger(\vec{y})\right\} = -\delta(\vec{x} - \vec{y}) \Leftrightarrow \tilde{Q}_{2(c)} = -\frac{\delta}{\delta \tilde{Q}^*_{2(c)}}$$

$$i \frac{\partial}{\partial t} \rho_{\mathrm{red}}[t, Q^*_{1(c)}, \tilde{Q}^*_{2(c)}] = H^{\mathrm{func}}_{1+2}[Q^*_{1(c)}, \tilde{Q}^*_{2(c)}] \rho_{\mathrm{red}}[t, Q^*_{1(c)}, \tilde{Q}^*_{2(c)}]$$

Density matrix for a few HQs

- Remember coherent states

$$\rho_{\rm red}\left[t, Q^*_{1(c)}, \widetilde{Q}^*_{2(c)}\right] = \left\langle Q^*_{1(c)} \right| \hat{\rho}_{\rm red}(t) \left| \widetilde{Q}^*_{2(c)} \right\rangle$$

$$\left\langle Q^*_{1(c)} \right| = \left\langle \Omega \right| \exp\left[-\int_{\vec{x}} \left\{ \hat{Q} Q^*_1 + \hat{Q}_c Q^*_{1(c)} \right\} \right]$$

$$\left| \widetilde{Q}^*_{2(c)} \right\rangle = \exp\left[-\int_{\vec{x}} \left\{ \widetilde{Q}^*_2 \hat{\widetilde{Q}}^\dagger + \widetilde{Q}^*_{2c} \hat{\widetilde{Q}}^\dagger_c \right\} \right] \left| \Omega \right\rangle$$

$$\frac{\delta}{\delta Q^*_1(\vec{x})} \left\langle Q^*_{1(c)} \right| \Bigg|_{Q^*_{1(c)}=0} = \left\langle \Omega \right| \hat{Q}(\vec{x})$$

$$\frac{\delta}{\delta \widetilde{Q}^*_2(\vec{x})} \left| \widetilde{Q}^*_{2(c)} \right\rangle \Bigg|_{\widetilde{Q}^*_{2(c)}=0} = -\hat{Q}^\dagger(\vec{x}) \left| \Omega \right\rangle$$

  - Single HQ

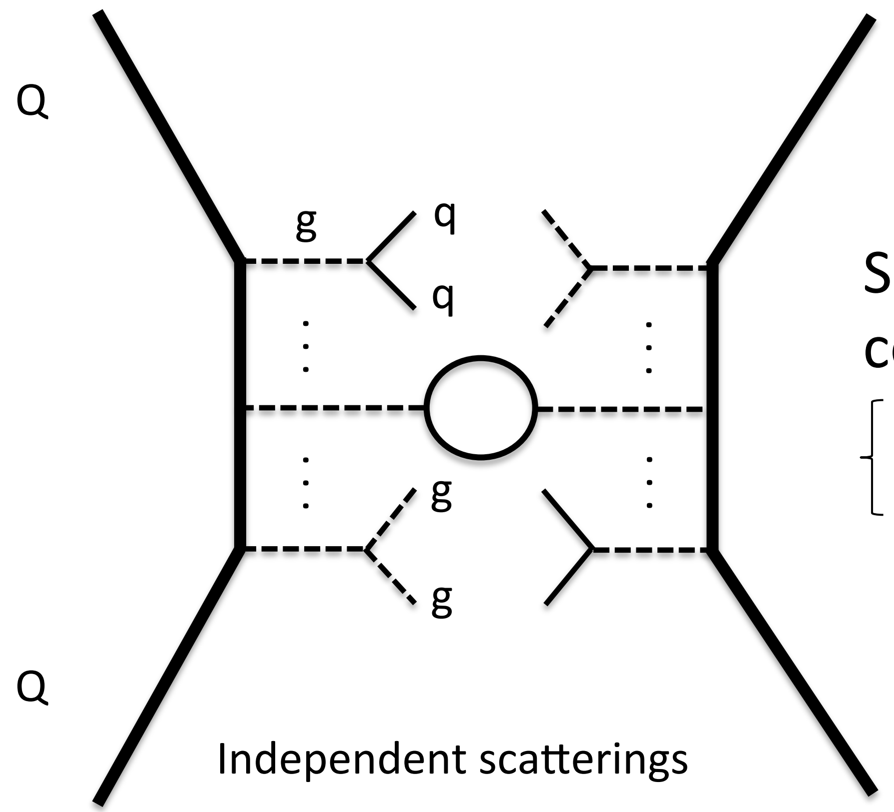
$$\rho_Q(t, \vec{x}, \vec{y}) \propto \left\langle \Omega \right| \hat{Q}(\vec{x}) \hat{\rho}_{\rm red}(t) \hat{Q}^\dagger(\vec{y}) \left| \Omega \right\rangle$$
$$= -\frac{\delta}{\delta Q^*_1(\vec{x})} \frac{\delta}{\delta \widetilde{Q}^*_2(\vec{y})} \rho_{\rm red}\left[t, Q^*_{1(c)}, \widetilde{Q}^*_{2(c)}\right]_{Q^*_{1(c)}=\widetilde{Q}^*_{2(c)}=0}$$

  - Similar for two HQs, …

$$\rho_{QQ_c}(t, \vec{x}_1, \vec{x}_2, \vec{y}_1, \vec{y}_2), \cdots$$

Physical process

- Scatterings in t-channel



Independent scatterings

Scatterings with hard particles contributes to both

- Diffusion
- Screening

Master equation

- Single HQ

$$i\partial_t \rho_Q(t,\vec{x},\vec{y}) = \left\{ \left(a - a^*\right) M + \left(- \frac{\nabla_x^2 - \nabla_y^2}{2M} \right) \right\} \rho_Q(t,\vec{x},\vec{y})$$

(color traced)

$$+ C_F \left\{ - iD(\vec{x}-\vec{y}) + \frac{\vec{\nabla}_x D(\vec{x}-\vec{y})}{4T} \cdot \frac{\vec{\nabla}_x - \vec{\nabla}_y}{iM} \right\} \rho_Q(t,\vec{x},\vec{y})$$

$$a \equiv 1 + \frac{C_{\mathrm{F}}}{2M} \lim_{r\to 0} V^{(T>0)}(r), \quad V^{(T>0)}(r) \equiv V(r) - V^{(T=0)}(r)$$

$$\left\{ \begin{array}{l} \dfrac{d}{dt}\langle \vec{x} \rangle = \dfrac{\langle \vec{p} \rangle}{M}, \quad \dfrac{d}{dt}\langle \vec{p} \rangle = - \dfrac{\gamma}{2MT} \langle \vec{p} \rangle, \\ \dfrac{d}{dt}\langle E \rangle = - \dfrac{\gamma}{MT} \left(\langle E \rangle - \dfrac{3T}{2} \right). \\ \gamma = \dfrac{C_F}{3} \nabla^2 D(x)\Big|_{x=0} = \dfrac{g^2 C_F}{9} \displaystyle\int \dfrac{d^3k}{(2\pi)^3} k^2 \tilde{G}^{>}_{00,aa}(\omega = 0, k) \end{array} \right.$$

Langevin dynamics

Moore & Teaney (05)

Complex potential

- QQbar

$$\frac{\partial_t \left\langle \Psi_{ij}(t;\vec{x},\vec{y}) \right\rangle_T}{\left\langle \Psi_{ij}(t;\vec{x},\vec{y}) \right\rangle_T} = \frac{\partial_t \left\langle J_{ij}(t;\vec{x},\vec{y}) J^{\dagger}_{kl}(0;\vec{x}_0,\vec{y}_0) \right\rangle_T}{\left\langle J_{ij}(t;\vec{x},\vec{y}) J^{\dagger}_{kl}(0;\vec{x}_0,\vec{y}_0) \right\rangle_T} = V_{ij}(\vec{x}-\vec{y})$$

⇓ Projection onto singlet state

Debye screening w/ imaginary part
(complex potential)

$$V_{\text{singlet}}(R) = 2(a-1)M - C_F V(R) \;= -\frac{g^2 C_F}{4\pi}\left(\omega_D + \frac{e^{-\omega_D R}}{R} + iT\varphi\left(\omega_D R\right)\right)$$

Laine et al (07), Beraudo et al (08), Brambilla et al (10)

Stochastic process

- $M=\infty$

$$
\begin{aligned}
&\exp\left[iS_{\rm FV}^{\rm LONR}[j_1, j_2]\right] \\
&= \exp\left[-i/2\int_{t,\vec{x},\vec{y}} \mathrm{Re}\,V(\vec{x}-\vec{y})\left\{\rho_{1a}(t,\vec{x})\rho_{1a}(t,\vec{y})-\rho_{2a}(t,\vec{x})\rho_{2a}(t,\vec{y})\right\}\right] \\
&\quad\times\left\langle \exp\left[-i\int_{t,\vec{x},\vec{y}} \xi_a(t,\vec{x})\left\{\rho_{1a}(t,\vec{x})-\rho_{2a}(t,\vec{x})\right\}\right]\right\rangle_{\xi}
\end{aligned}
$$

$$
\left\langle \xi_a(t,\vec{x})\xi_b(s,\vec{y})\right\rangle = -\delta_{ab}\delta(t-s)D(\vec{x}-\vec{y})
$$

Debye screening + fluctuation
(stochastic potential)

Akamatsu & Rothkopf (12)

5. SUMMARY

So far and beyond

- LO perturbation
  - Heavy quarks in QGP as open quantum system
  - Unified description of forces
  - Scatterings in t-channel

- NLO perturbation
  - Gluo-dissociation
- Phenomenology at RHIC/LHC