Thermal width and quarkonium dissociation in an EFT framework

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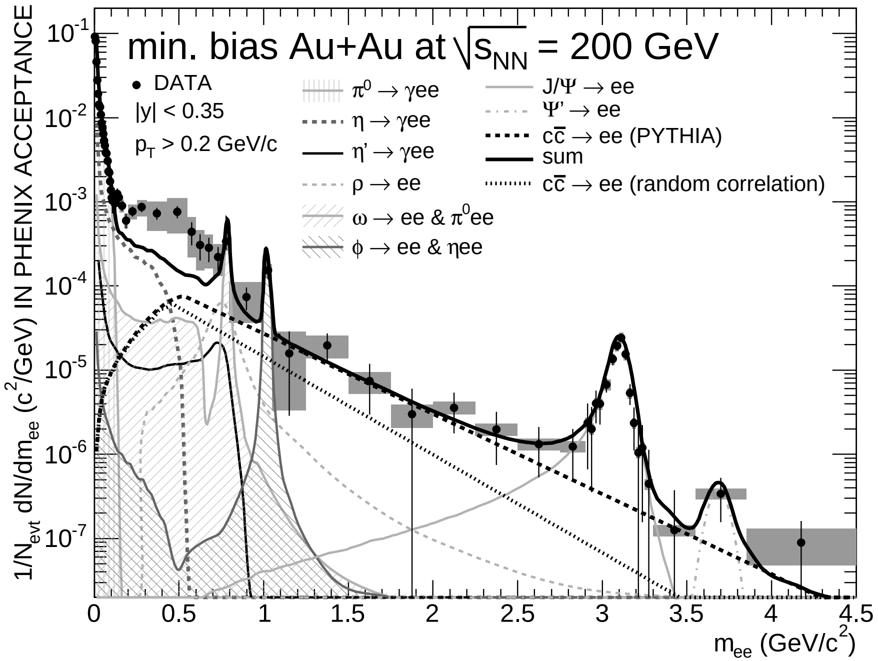
Quarkonium as a quark-gluon plasma probe

In 1986, Matsui and Satz suggest quarkonium dissociation as an ideal probe of the medium formed in heavy-ion collisions.

- Heavy quarks are formed early in heavy-ion collisions: $1/m \sim 0.1$ fm $\ll 1$ fm.
- Heavy quarkonium formation will be sensitive to the medium.
- The dilepton signal makes the quarkonium a clean experimental probe.



Energy Scales

Quarkonium in a medium is characterized by different energy and momentum scales:

- the scales of a non-relativistic bound state
  (v is the relative heavy-quark velocity):
  m (mass),
  mv (momentum transfer, inverse distance),
  mv^2 (kinetic energy, binding energy, potential V), ...

- the thermodynamical scales:
  πT (temperature),
  m_D (Debye mass, i.e. screening of the chromoelectric interactions), ...

The non-relativistic scales are hierarchically ordered: $m \gg mv \gg mv^2$
We may assume this to be also the case for the thermodynamical scales: $\pi T \gg m_D$

Weak coupling

The best controlled setting is when all scales are perturbative:

$$\text{all energy scales } \gg \Lambda_{\rm QCD}$$

i.e. a weakly-coupled quarkonium in a weakly-coupled quark-gluon plasma.
For a weakly-coupled quarkonium: $v \sim \alpha_{\rm s}$.

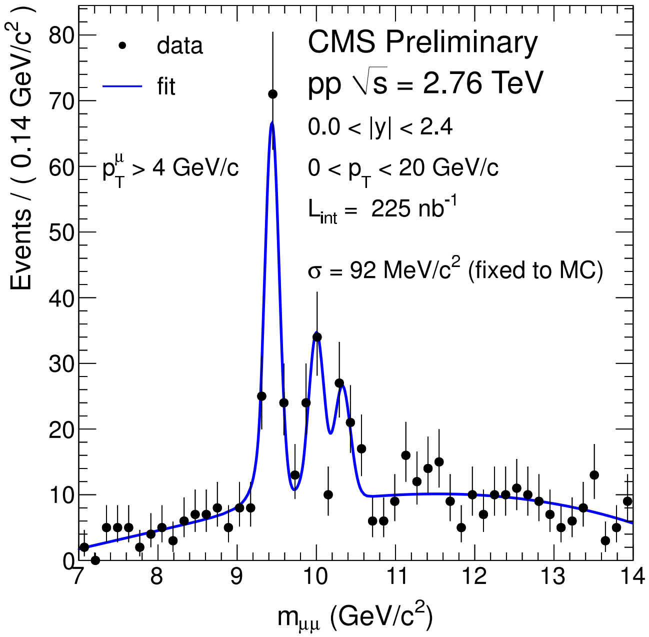
- The quarkonium ground states, produced in the QCD medium of heavy-ion collisions at the LHC, may possibly realize this situation.

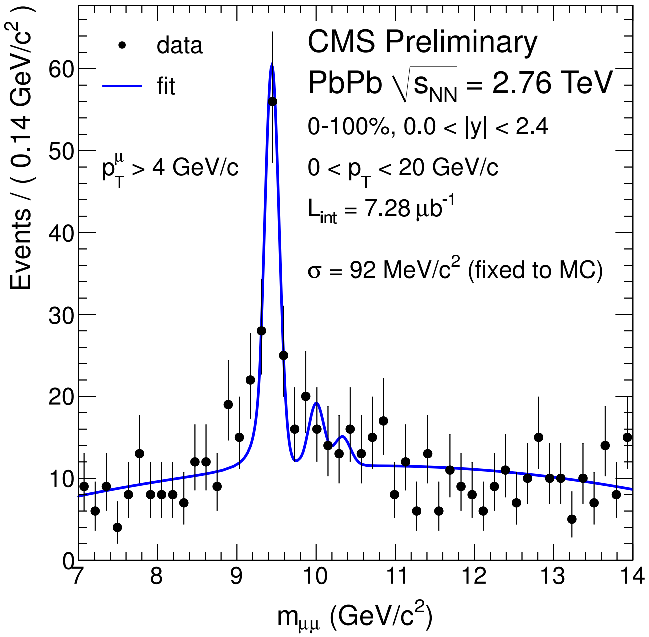
  E.g. for the bottomonium ground state:

  $$m_b \approx 5 \text{ GeV} > m_b\alpha_{\rm s} \approx 1.5 \text{ GeV} > \pi T \approx 1 \text{ GeV} > m_b\alpha_{\rm s}^2 \approx 0.5 \text{ GeV} \sim m_D \gtrsim \Lambda_{\rm QCD}$$

  For excited states, $m_b\alpha_{\rm s}$ becomes closer to m_D and the bottomonium dissociates.

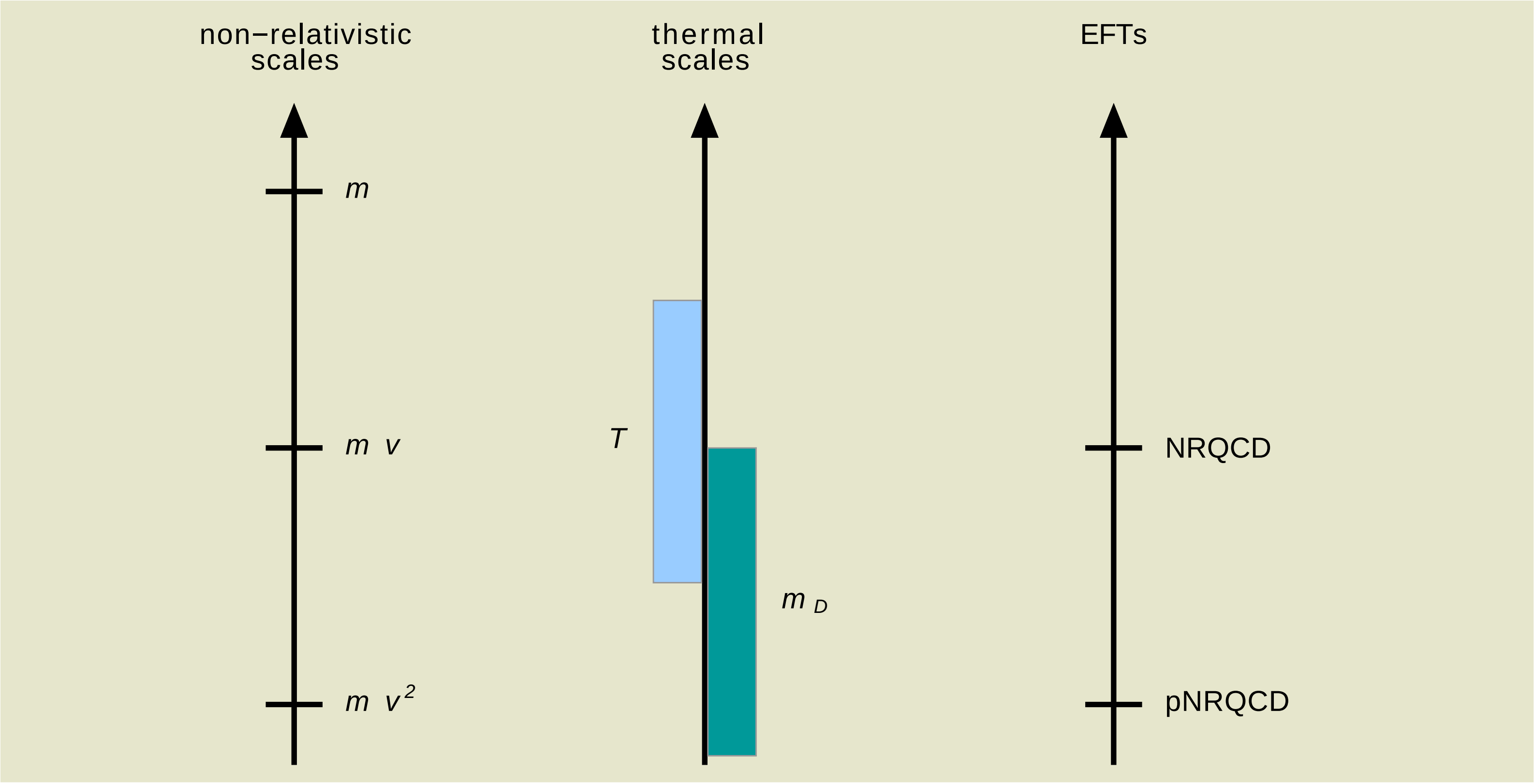
Υ suppression at CMS





Non-relativistic Effective Field Theories at finite T

The existence of a hierarchy of energy scales calls for a description of the system (quarkonium at rest in a thermal bath) in terms of a hierarchy of EFTs.



For larger temperatures the quarkonium does not form.

NRQCD

NRQCD is obtained by integrating out modes associated with the scale m and possibly with thermal scales larger than mv.

- The Lagrangian is organized as an expansion in $1/m$:

$$\mathcal{L} = \psi^\dagger \left(iD_0 + \frac{\mathbf{D}^2}{2m} + \dots \right) \psi + \chi^\dagger \left(iD_0 - \frac{\mathbf{D}^2}{2m} + \dots \right) \chi + \dots$$
$$+ \mathcal{L}_{\rm light}$$

  ψ (χ) is the field that annihilates (creates) the (anti)fermion.

- $\mathcal{L}_{\rm light}$ describes the propagation of gluons and light quarks.
  It depends on the version of NRQCD.

- Thermodynamical scales may be set to zero while matching at the scale m.

pNRQCD

pNRQCD is obtained by integrating out modes associated with the scale mv and possibly with thermal scales larger than mv^2.

- The degrees of freedom of pNRQCD are quark-antiquark states (color singlet S, color octet O), low energy gluons and light quarks propagating in the medium.
- The Lagrangian is organized as an expansion in $1/m$ and r:

$$\mathcal{L} = \int d^3r \, \mathrm{Tr}\left\{ \mathrm{S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s + \dots \right) \mathrm{S} + \mathrm{O}^\dagger \left(iD_0 - \frac{\mathbf{p}^2}{m} - V_o + \dots \right) \mathrm{O} \right\}$$
$$\mathrm{Tr}\left\{ \mathrm{O}^\dagger \mathbf{r} \cdot g\mathbf{E}\, \mathrm{S} + \mathrm{H.c.} \right\} + \frac{1}{2}\mathrm{Tr}\left\{ \mathrm{O}^\dagger \mathbf{r} \cdot g\mathbf{E}\, \mathrm{O} + \mathrm{c.c.} \right\} + \cdots$$
$$+\mathcal{L}_{\mathrm{light}}$$

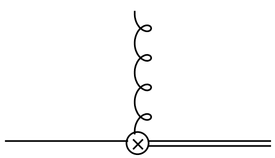
- $\mathcal{L}_{\mathrm{light}}$ describes the in-medium propagation of gluons and light quarks. It depends on the version of pNRQCD.
- At leading order in r, the singlet S satisfies a Schrödinger equation. The explicit form of the potential depends on the version of pNRQCD.

pNRQCD

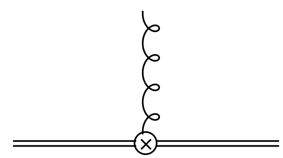
- Feynman rules:

$$= \theta(t)\, e^{-itH_s}$$

$$= \theta(t)\, e^{-itH_o} \left(e^{-i\int dt\, A^{\rm adj}} \right)$$



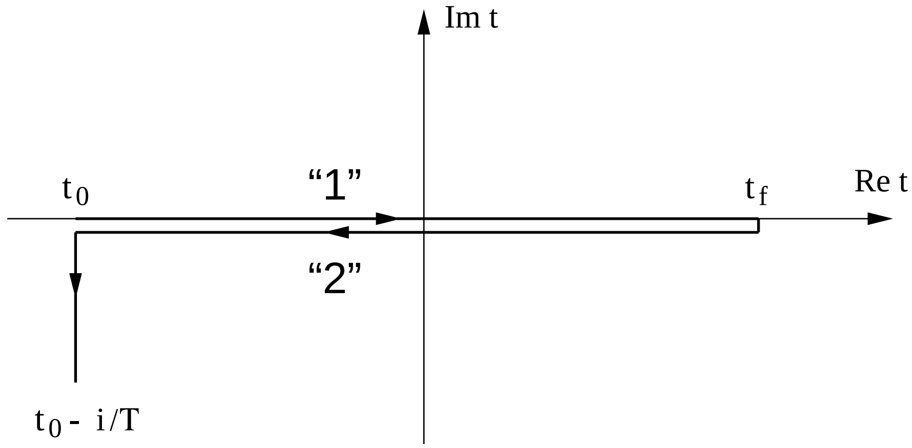
$$= \mathrm{O}^\dagger \mathbf{r} \cdot g\mathbf{E}\, \mathrm{S}$$



$$= \mathrm{O}^\dagger \{\mathbf{r} \cdot g\mathbf{E},\, \mathrm{O}\}$$

Real time

The contour of the partition function is modified to allow for real time:



In real time, the degrees of freedom double ("1" and "2").

- In the heavy-quark sector, the second degrees of freedom, labeled "2", decouple from the physical degrees of freedom, labeled "1".
- The second degrees of freedom contribute to the gluon self energy.
  The discontinuity of the "11" gluon self energy is encoded in
  $\Pi_S^{\mu\nu}(k_0, k) = [1 + 2n_{\rm B}(|k_0|)]\,{\rm sgn}(k_0)\,[\Pi_R^{\mu\nu}(k_0, k) - \Pi_A^{\mu\nu}(k_0, k)]$.

Dissociation mechanisms at LO

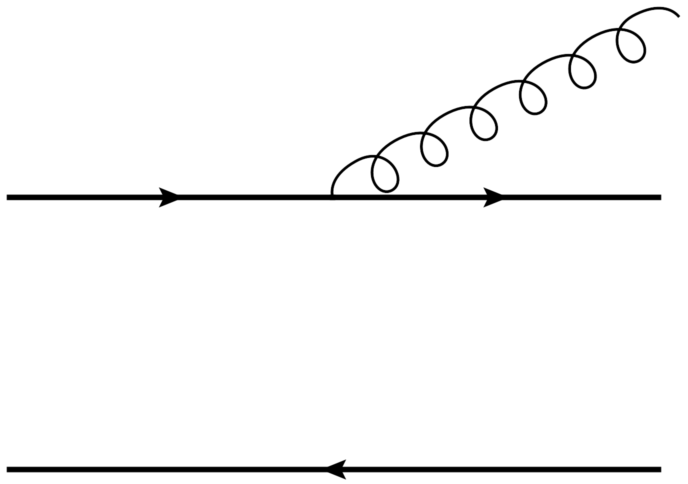
Two distinct dissociation mechanisms may be identified at leading order:

- gluodissociation,
  which is the dominant mechanism for $mv^2 \gg m_D$;
- dissociation by inelastic parton scattering,
  which is the dominant mechanism for $mv^2 \ll m_D$.

Beyond leading order the two mechanisms are intertwined and distinguishing between them becomes unphysical, whereas the physical quantity is the total decay width.

Gluodissociation

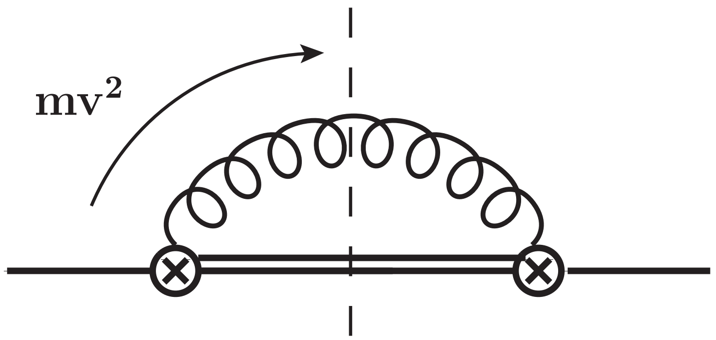
Gluodissociation is the dissociation of quarkonium by absorption of a gluon from the medium.



- The exchanged gluon is lightlike or timelike.
- The process happens when the gluon has an energy of order mv^2.

Gludissociation

From the optical theorem, the gludissociation width follows from cutting the gluon propagator in the following pNRQCD diagram



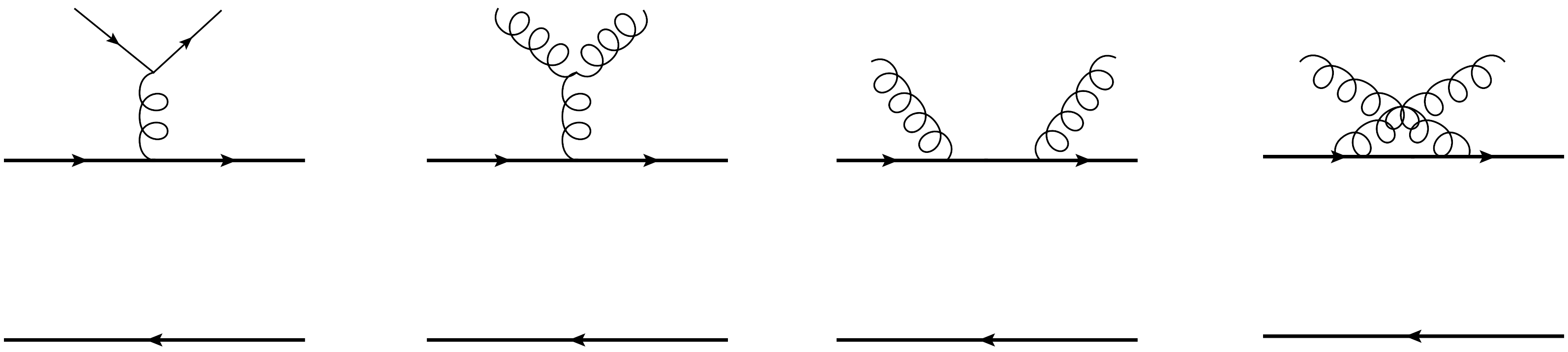
For a quarkonium at rest with respect to the medium, the width has the form

$$\Gamma_{nl} = \int_{q_{\min}} \frac{d^3q}{(2\pi)^3}\, n_{\rm B}(q)\, \sigma^{nl}_{\rm gluo}(q)\,.$$

- $\sigma^{nl}_{\rm gluo}$ is the in-vacuum cross section $(Q\overline{Q})_{nl} \to Q + \overline{Q} + g$.
- Gludissociation is also known as singlet-to-octet break up.

Dissociation by inelastic parton scattering

Dissociation by inelastic parton scattering is the dissociation of quarkonium by scattering with gluons and light-quarks in the medium.



- The exchanged gluon is spacelike.
- In Coulomb gauge, external thermal gluons are transverse.
- In the NRQCD power counting, each external transverse gluon is suppressed by T/m.

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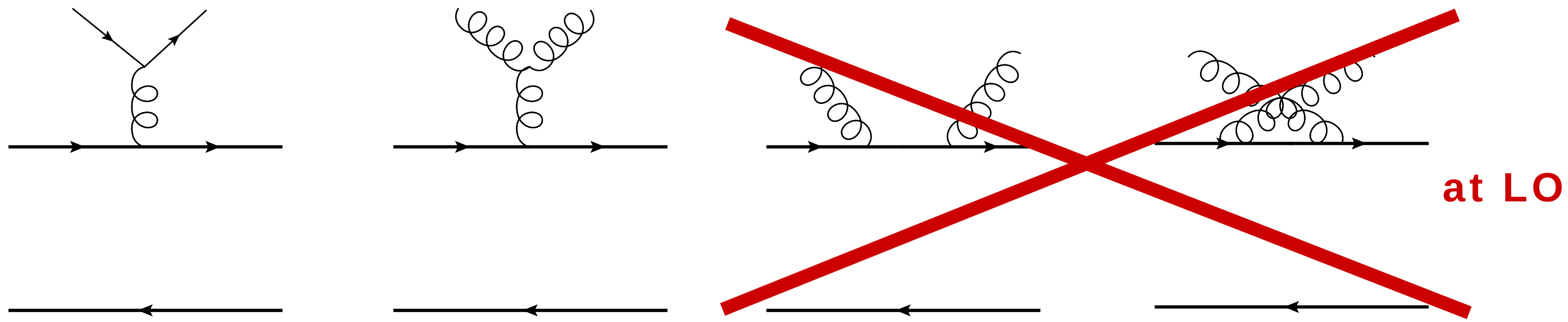
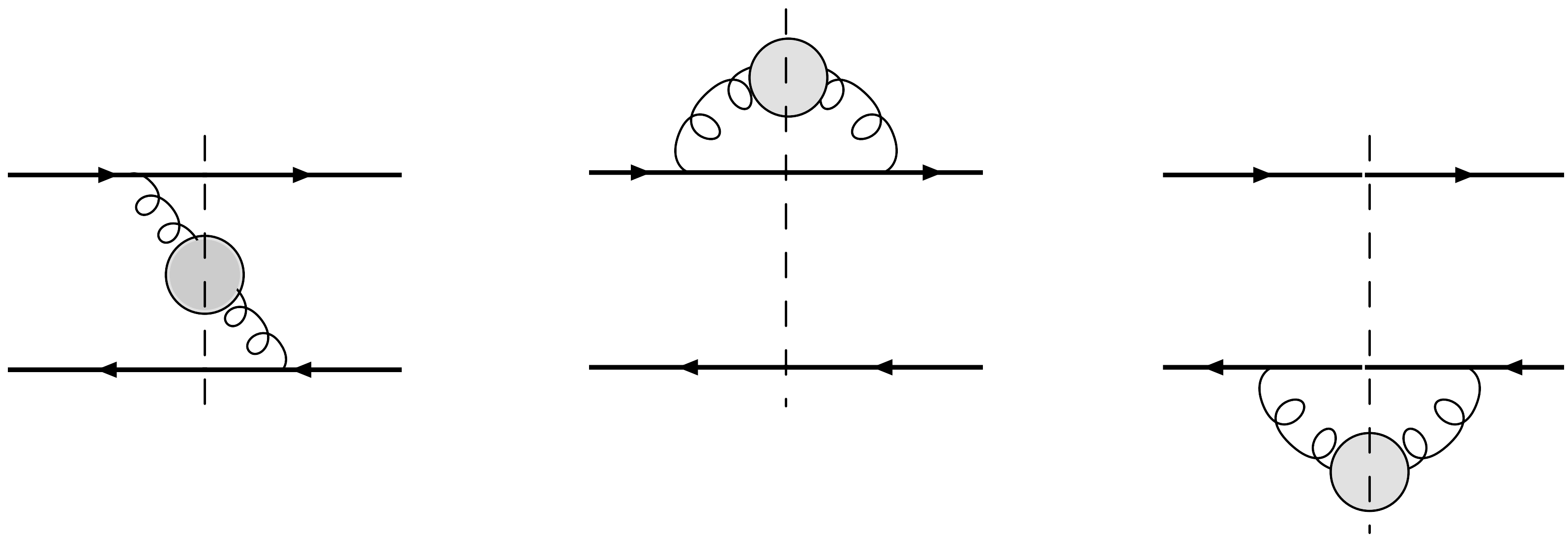


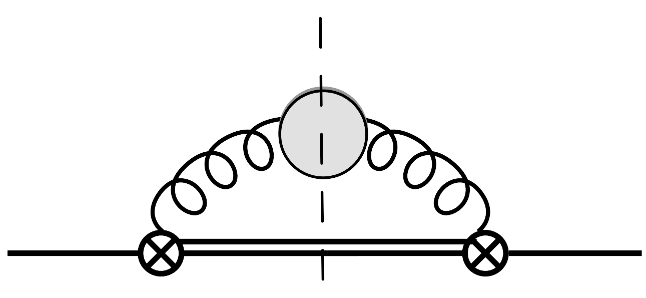
- The exchanged gluon is spacelike.
- In Coulomb gauge, external thermal gluons are transverse.
- In the NRQCD power counting, each external transverse gluon is suppressed by T/m.

Dissociation by inelastic parton scattering

From the optical theorem, the thermal width follows from cutting the gluon self-energy in the following NRQCD diagrams (momentum of the gluon $\gtrsim mv$)



and/or pNRQCD diagram (momentum of the gluon $\ll mv$)



- Dissociation by inelastic parton scattering is also known as Landau damping.

Dissociation by inelastic parton scattering

For a quarkonium at rest with respect to the medium, the thermal width has the form

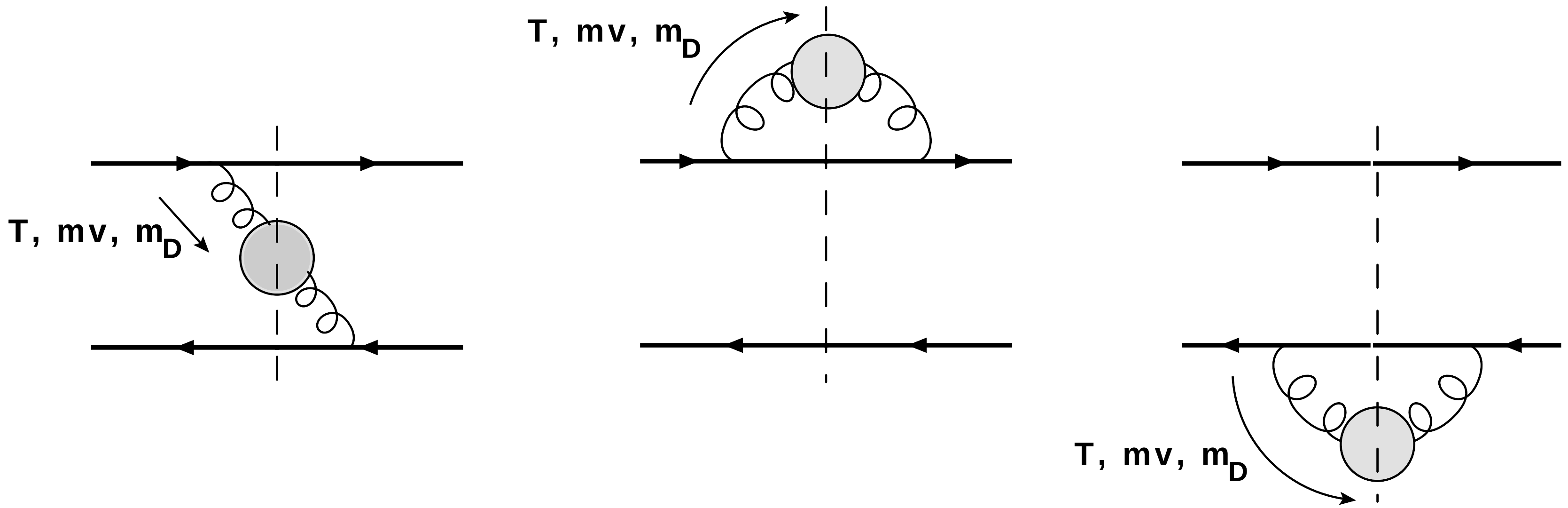
$$\Gamma_{nl} = \sum_p \int_{q_{\rm min}} \frac{d^3q}{(2\pi)^3} \, f_p(q) \, [1 \pm f_p(q)] \, \sigma_p^{nl}(q)$$

where the sum runs over the different incoming light partons and $f_g = n_{\rm B}$ or $f_q = n_{\rm F}$.

- σ_p^{nl} is the in-medium cross section $(Q\overline{Q})_{nl} + p \to Q + \overline{Q} + p$.
- The convolution formula correctly accounts for Pauli blocking in the fermionic case (minus sign).
- The formula differs from the gluodissociation formula.

The temperature region $T \gg mv \sim m_D$

The gluon self-energy contributes to the imaginary part of the pNRQCD potential through:



Similarly for the real part.

The temperature region $T \gg mv \sim m_D$

$$\mathrm{Re}\, V_s(r) = -C_F\, \alpha_{\rm s} \left(\frac{e^{-m_D r}}{r} + m_D \right)$$

$$\mathrm{Im}\, V_s(r) = -\frac{g^2 T C_F m_D^2}{2\pi} \int_0^\infty \frac{dt\, t}{(t^2+m_D^2)^2} \left(1 - \frac{\sin(tr)}{tr} \right)$$

- The real part of the potential is a screened Coulomb potential.
- The imaginary part of the potential stems from cutting the self energy of a space-like gluon. Hence it describes quarkonium dissociation by inelastic parton scattering.
- The imaginary part of the potential is larger than the real one: in this temperature region the quarkonium does not form.

The temperature region $T \gg mv \gg m_D$

In this temperature region:

- $\mathrm{Re}\, V_s(r) \sim \alpha_{\rm s}/r$, quarkonium is a Coulombic bound state (e.g. $\langle \mathbf{r}|1S\rangle = 1/(\sqrt{\pi} a_0^{3/2}) \exp(-r/a_0)$, where $a_0 = 2/(mC_F\alpha_{\rm s})$);

- quarkonium dissociates at a temperature such that $\mathrm{Im}\, V_s(r) \sim \mathrm{Re}\, V_s(r) \sim \alpha_{\rm s}/r$:

$$\pi T_{\rm dissociation} \sim m g^{4/3}$$

  which in the $\Upsilon(1S)$ case is about 450 MeV.
  (Note that $\pi T \approx$ 1 GeV is below the dissociation temperature.)

- The interaction is screened when $\langle 1/r \rangle \sim m_D$, hence

$$\pi T_{\rm screening} \sim m g \gg \pi T_{\rm dissociation}$$

The temperature region $T \gg mv \gg m_D$

$$\Gamma_{nl} = -2 \langle n, l | \mathrm{Im}\, V_s(r) | n, l \rangle$$

where $|n, l\rangle$ is the eigenstate of $\mathbf{p}^2/m + \mathrm{Re}\, V_s(r)$ that identifies the quarkonium.

$$m_D^2 = -\frac{ik}{2\pi T} \Pi_S^{00}(0, k \ll T) = \frac{2g^2}{T} \int \frac{d^3q}{(2\pi)^3} n_f \, n_{\mathrm{F}}(q) \left[1 - n_{\mathrm{F}}(q)\right] + N_c \, n_{\mathrm{B}}(q) \left[1 + n_{\mathrm{B}}(q)\right]$$

implies the convolution formula $\Gamma_{nl} = \sum_p \int_{q_{\min}} \frac{d^3q}{(2\pi)^3} \, f_p(q) \, \left[1 \pm f_p(q)\right] \, \sigma_p^{nl}(q)$.

The temperature region $T \gg mv \gg m_D$: $1S$ dissociation by parton scattering

$$\sigma_p^{1S} = \sigma_{cp} f(m_D a_0)$$

$$\sigma_{cq} \equiv 8\pi C_F n_f \, \alpha_{\rm s}^2 \, a_0^2, \quad \sigma_{cg} \equiv 8\pi C_F N_c \, \alpha_{\rm s}^2 \, a_0^2$$

$$f(m_D a_0) \equiv \frac{2}{(m_D a_0)^2} \left[1 - 4 \frac{(m_D a_0)^4 - 16 + 8(m_D a_0)^2 \ln\left(4/(m_D a_0)^2\right)}{((m_D a_0)^2 - 4)^3} \right]$$

- The cross section for inelastic parton scattering is not related, even at leading order, with a zero temperature process. The underlying reason is the infrared sensitivity of the cross section at the momentum scale mv. This IR sensitivity is cured by the HTL resummation at the scale m_D and signaled by $\ln(m_D a_0)$.
- The cross section is momentum independent.

Quasi-free approximation

The quasi-free approximation amounts at replacing σ_p^{nl} by $2\,\sigma_p^Q$; σ_p^Q is the free in-vacuum cross section $p + Q \to p + Q$ (with thermal masses for IR regularization).

- The quasi-free approximation amounts at neglecting interference terms between the different heavy-quark lines in the amplitude square. Interference terms are the ones sensitive to the bound state. This corresponds in approximating

$$f(m_D a_0) \approx \frac{2}{(m_D a_0)^2}$$

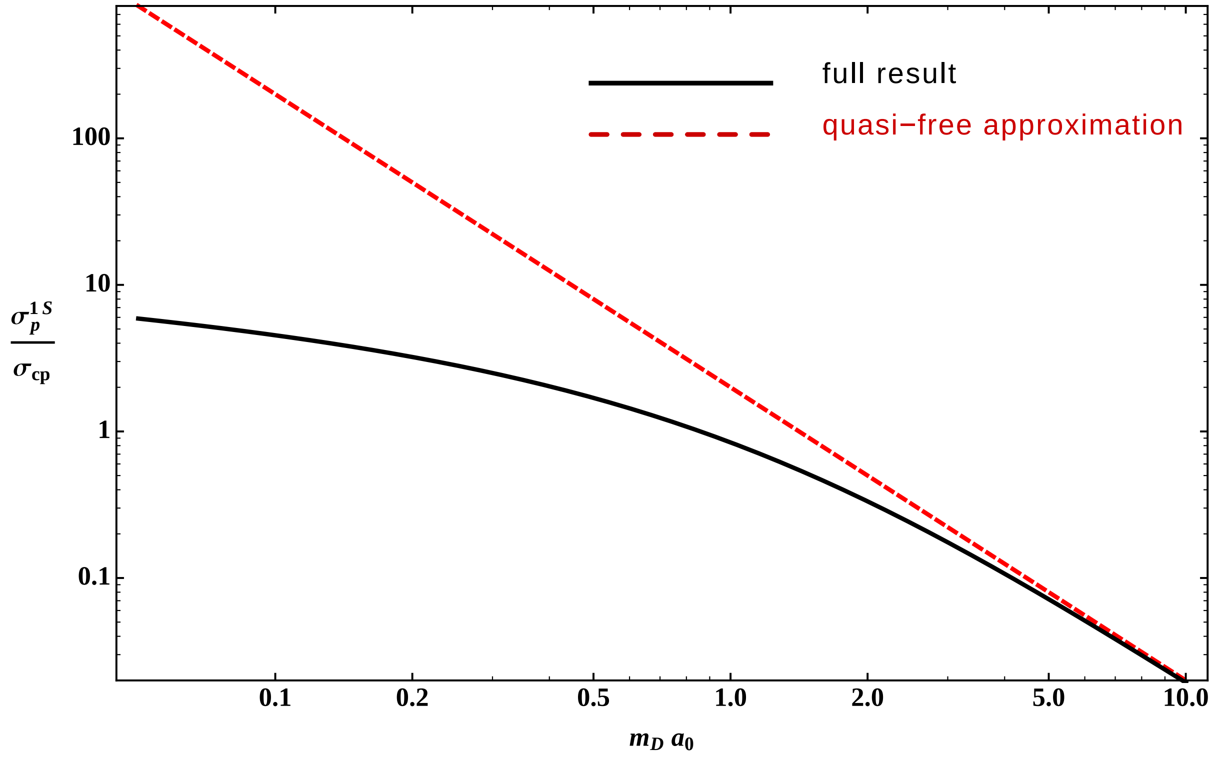
- The approximation holds only for $m_D \gg 1/a_0$, which requires $T \gg mg \gg T_{\text{dissociation}}$.

- For temperatures where the approximation holds, quarkonium is dissociated. Otherwise, the approximation is largely violated by bound-state effects. E.g. for $m_D \ll 1/a_0$

$$f(m_D a_0) \approx \frac{2}{(m_D a_0)^2} \left[\cancel{1} - \cancel{1} + (m_D a_0)^2 \left(-\frac{3}{4} + \ln(2/(m_D a_0)) \right) + \dots \right]$$

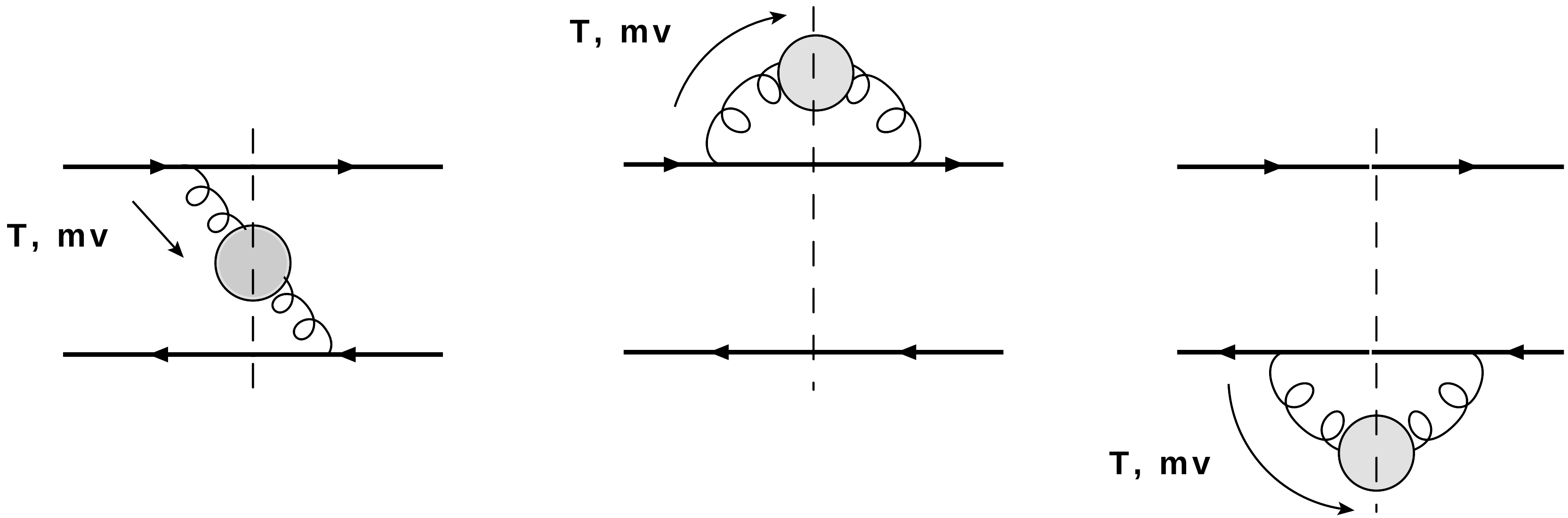
Quasi-free approximation vs full result

In a weak-coupling framework, the quasi-free approximation is not justified for the whole range of temperatures where a quarkonium can exist.

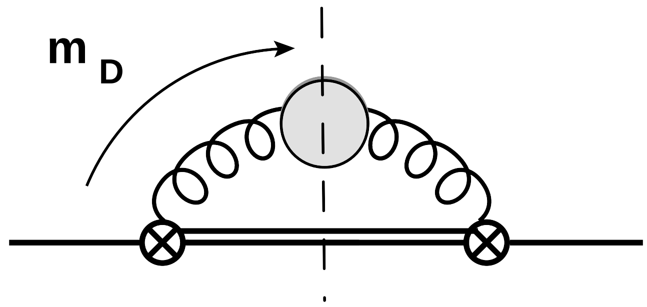


The temperature region $T \sim mv \gg m_D$

The gluon self-energy contributes to the imaginary part of the pNRQCD potential through:



and the pNRQCD diagram



The temperature region $T \sim mv \gg m_D$

$$\mathrm{Re}\, V_s(r) = -C_F\,\frac{\alpha_s}{r}$$

$$\mathrm{Im}\, V_s(r) = -\frac{g^4 C_F n_f}{\pi}\left\{\frac{r^2T^3}{144}\left(\frac{8}{3} - 2\gamma_E - \ln(r^2 m_D^2)\right)\right.$$

$$+\int\frac{d^3q}{(2\pi)^3} n_\mathrm{F}(q)[1-n_\mathrm{F}(q)]\left[\int_{2q}^{\infty}\frac{dt}{t^3}\left(\frac{\sin(tr)}{tr}-1\right)+\int_0^{2q}\frac{dt}{4tq^2}\left(\frac{\sin(tr)}{tr}-1\right)\right]\left.\vphantom{\frac{1}{1}}\right\}$$

$$-\frac{g^4 C_F N_c}{\pi}\left\{\frac{r^2T^3}{72}\left(\frac{8}{3} - 2\gamma_E - \ln(r^2 m_D^2)\right)\right.$$

$$+\int\frac{d^3q}{(2\pi)^3} n_\mathrm{B}(q)[1+n_\mathrm{B}(q)]\left[\int_{2q}^{\infty}\frac{dt}{t^3}\left(\frac{\sin(tr)}{tr}-1\right)+\int_0^{2q}\frac{dt}{2tq^2}\left(\frac{\sin(tr)}{tr}-1\right)\right.$$

$$\left.\left.-\frac{1}{4q^2}\left(\frac{\sin^2(qr)}{(qr)^2}-1\right)\right]\right\}$$

quark

gluon

- The real part of the potential is the Coulomb potential
- The imaginary part of the potential gets contributions from the scales $mv \sim T$ and m_D. The term $\ln(rm_D)$ signals the cancellation of divergences between the two scales.

The temperature region $T \sim mv \gg m_D$: $1S$ dissociation by parton scattering

$$\sigma_p^{1S} = \sigma_{cp} h_p(m_D a_0, q a_0)$$

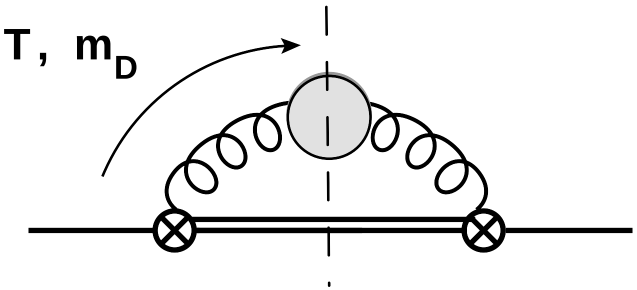
$$h_q(m_D a_0, q a_0) \equiv -\ln\left(\frac{(m_D a_0)^2}{4}\right) - \frac{3}{2} + \ln\left(\frac{(q a_0)^2}{1+(q a_0)^2}\right) - \frac{1}{2(q a_0)^2}\ln(1+(q a_0)^2)$$

$$h_g(m_D a_0, q a_0) \equiv -\ln\left(\frac{(m_D a_0)^2}{4}\right) - \frac{3}{2} + \ln\left(\frac{(q a_0)^2}{1+(q a_0)^2}\right) + \frac{1}{2(1+(q a_0)^2)} - \frac{1}{(q a_0)^2}\ln(1+(q a_0)^2)$$

- The cross section is momentum dependent.

The temperature region $mv \gg T \gg m_D \gg mv^2$: $1S$ dissociation by parton scattering

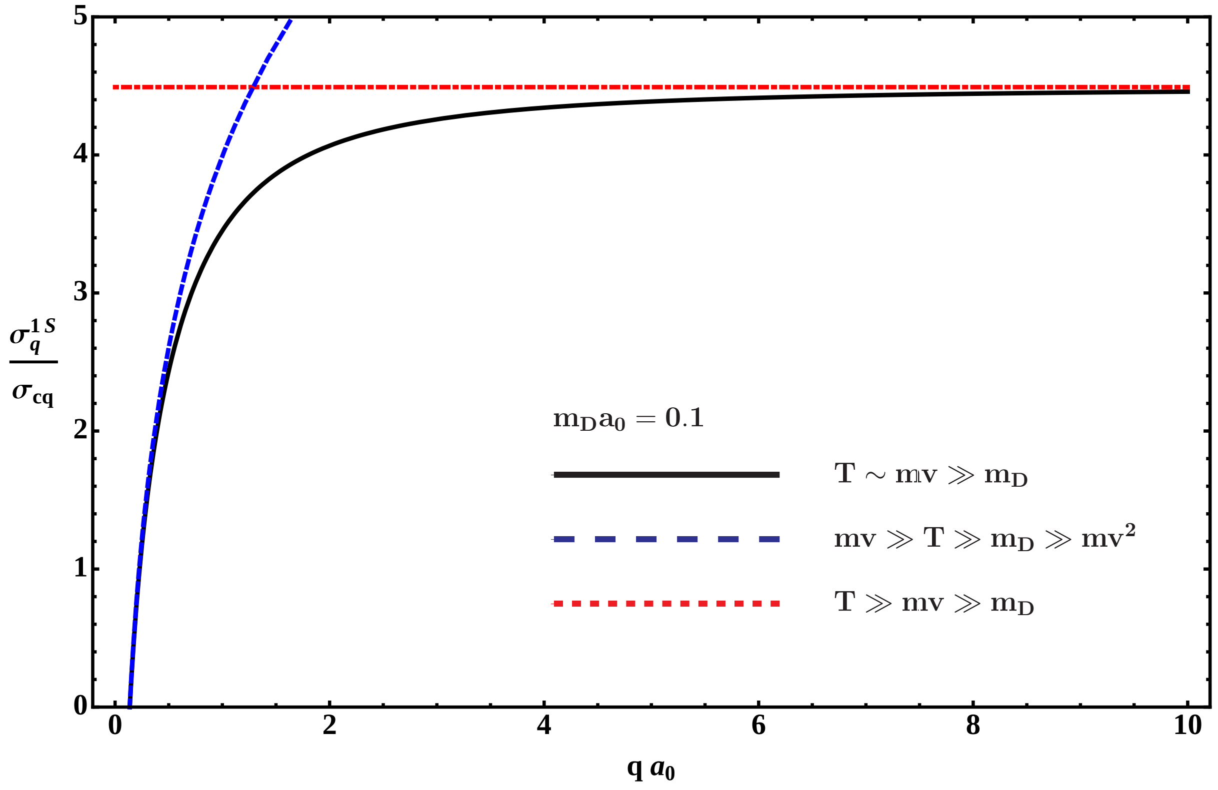
The gluon self-energy contributes to the imaginary part of the pNRQCD potential through:



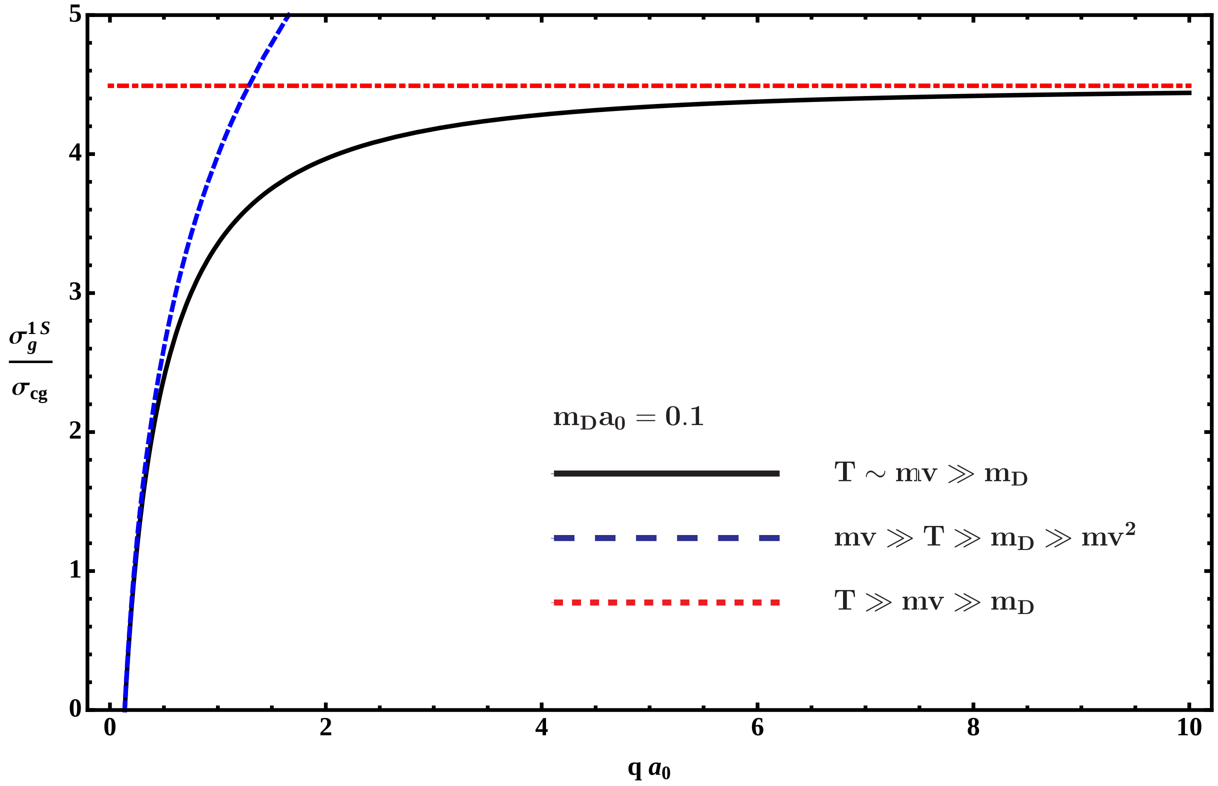
leading to dissociation cross sections for $1S$ Coulombic states:

$$\sigma_p^{1S}(q) = \sigma_{cp} \left[\ln \left(\frac{4q^2}{m_D^2} \right) - 2 \right]$$

Dissociation by quark inelastic scattering

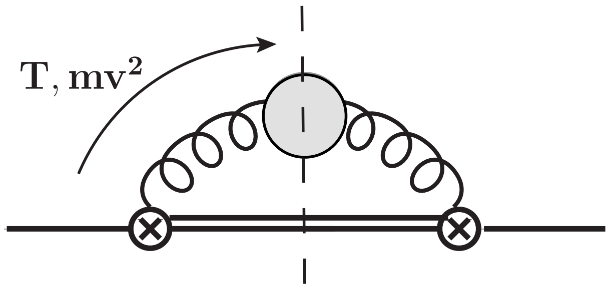


Dissociation by gluon inelastic scattering

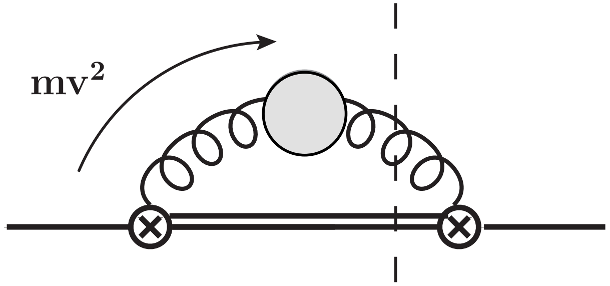


The temperature region $mv \gg T \gg mv^2 \gg m_D$

The gluon self-energy gives rise to a quarkonium width through:



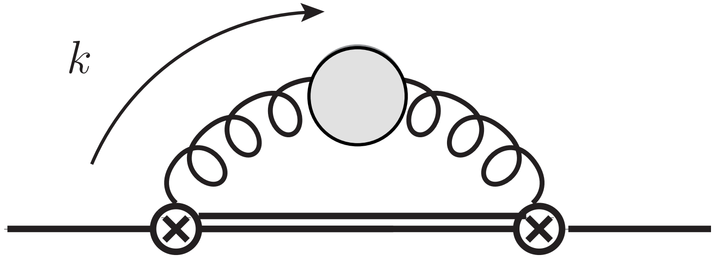
which contributes to dissociation by inelastic parton scattering, and through



which contributes to gluodissociation.

Integrating out mv^2

The relevant diagram is



where the loop momentum region is $k_0 \sim mv^2$ and $k \sim mv^2$.

- Gluons are HTL gluons.
- Since $k \sim mv^2 \gg m_D$, the HTL propagators can be expanded in $m_D/mv^2 \ll 1$.

Integrating out mv^2: momentum regions

In the loop with transverse gluons, this type of integral appears

$$\int \frac{d^{D-1}k}{(2\pi)^{D-1}} \int_0^\infty \frac{dk_0}{2\pi} \frac{1}{k_0^2 - k^2 - m_D^2 + i\eta} \left(\frac{1}{E - H_o - k_0 + i\eta} + \frac{1}{E - H_o + k_0 + i\eta} \right)$$

which exhibits two momentum regions for $E \sim mv^2$

- off-shell region: $k_0 - k \sim E$, $\quad k_0 \sim E$, $\quad k \sim E$;
- collinear region: $k_0 - k \sim m_D^2/E$, $\quad k_0 \sim E$, $\quad k \sim E$.

In our energy scale hierarchy, the collinear scale is $mv^2 \gg m_D^2/E \gg mv^3$, i.e. it is smaller than m_D by a factor of $m_D/mv^2 \ll 1$ and still larger than the non-perturbative magnetic mass, which is of order g^2T, by a factor $T/mv^2 \gg 1$.

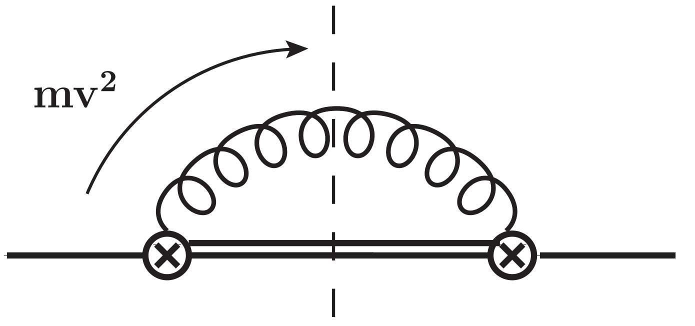
The temperature region $mv \gg T \gg mv^2 \gg m_D$: $1S$ dissociation by parton scattering

The dissociation cross section by parton scattering for $1S$ Coulombic states is

$$\sigma_p^{1S}(q) = \sigma_{cp} \left[\ln\left(\frac{4q^2}{m_D^2}\right) + \ln 2 - 2 \right]$$

The corresponding parton-scattering decay width is of order $\alpha_{\rm s} T \times (m_D/mv)^2$.

The temperature region $mv \gg T \gg mv^2 \gg m_D$: $1S$ gluodissociation at LO



The LO gluodissociation cross section for $1S$ Coulombic states is

$$\sigma^{1S}_{\rm gluo\,LO}(q) = \frac{\alpha_{\rm s} C_F}{3} 2^{10}\pi^2\rho(\rho+2)^2\frac{E_1^4}{mq^5}\left(t(q)^2+\rho^2\right)\frac{\exp\left(\frac{4\rho}{t(q)}\arctan\left(t(q)\right)\right)}{e^{\frac{2\pi\rho}{t(q)}}-1}$$

where $\rho \equiv 1/(N_c^2-1)$, $t(q) \equiv \sqrt{q/|E_1|-1}$ and $E_1 = -mC_F^2\alpha_{\rm s}^2/4$.

- The corresponding gluodissociation decay width is of order $\alpha_{\rm s}T \times (mv^2/mv)^2$.
- The gluodissociation width is larger by a factor $(mv^2/m_D)^2$ than the dissociation width by inelastic parton scattering. Gluodissociation is the dominant process.

Bhanot–Peskin approximation

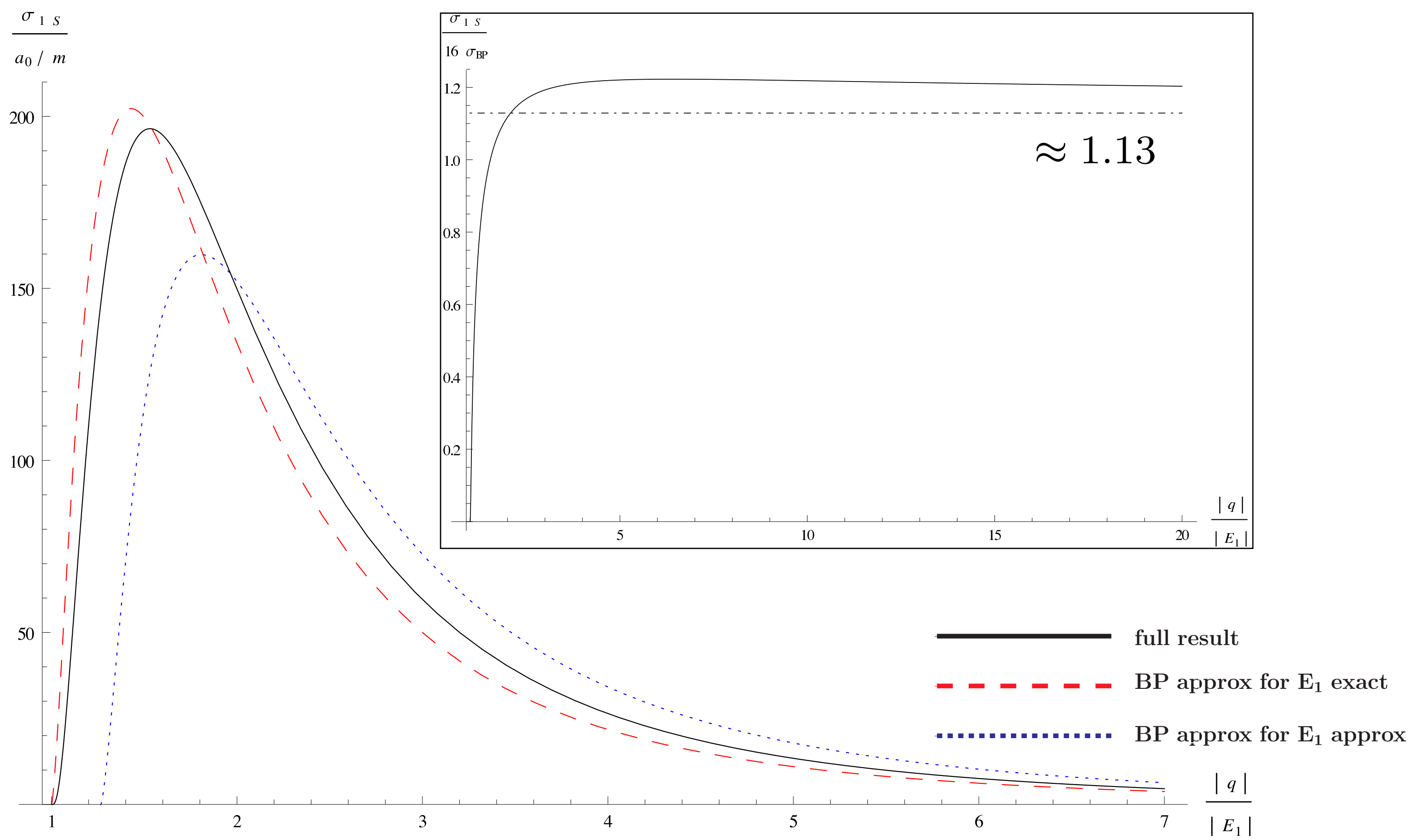
In the large N_c limit:

$$\sigma^{1S}_{\mathrm{gluo\,LO}}(q) \quad \xrightarrow[N_c\to\infty]{} \quad 16\,\frac{2^9\pi\alpha_{\mathrm{s}}}{9}\,\frac{|E_1|^{5/2}}{m}\,\frac{(q+E_1)^{3/2}}{q^5} = 16\,\sigma^{1S}_{\mathrm{BP}}(q)$$

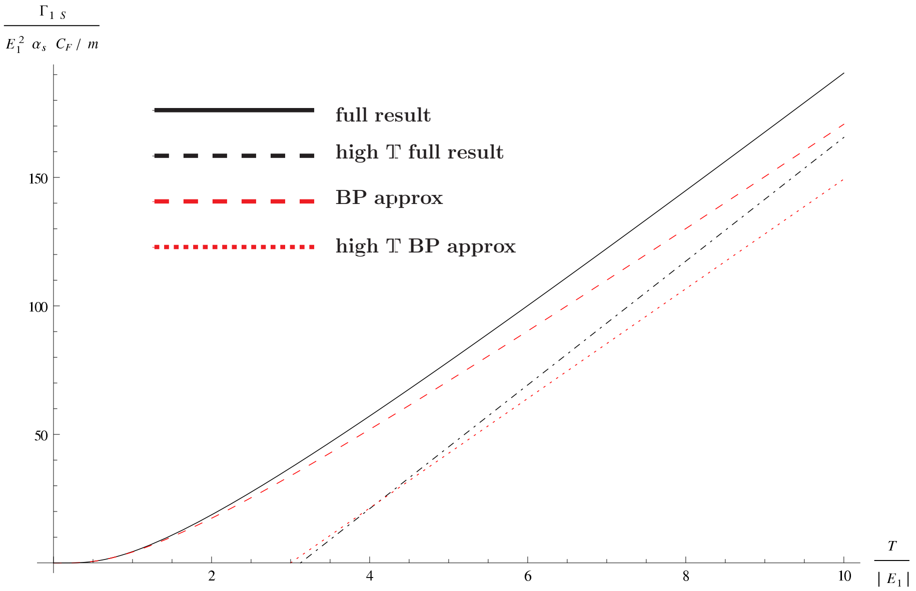
$$\Gamma_{1S\,\mathrm{LO}} \quad \xrightarrow[N_c\to\infty]{} \quad \int_{q\geq|E_1|}\frac{d^3q}{(2\pi)^3}\,n_{\mathrm{B}}(q)\,16\,\sigma^{1S}_{\mathrm{BP}}(q) = \Gamma_{1S,\mathrm{BP}}$$

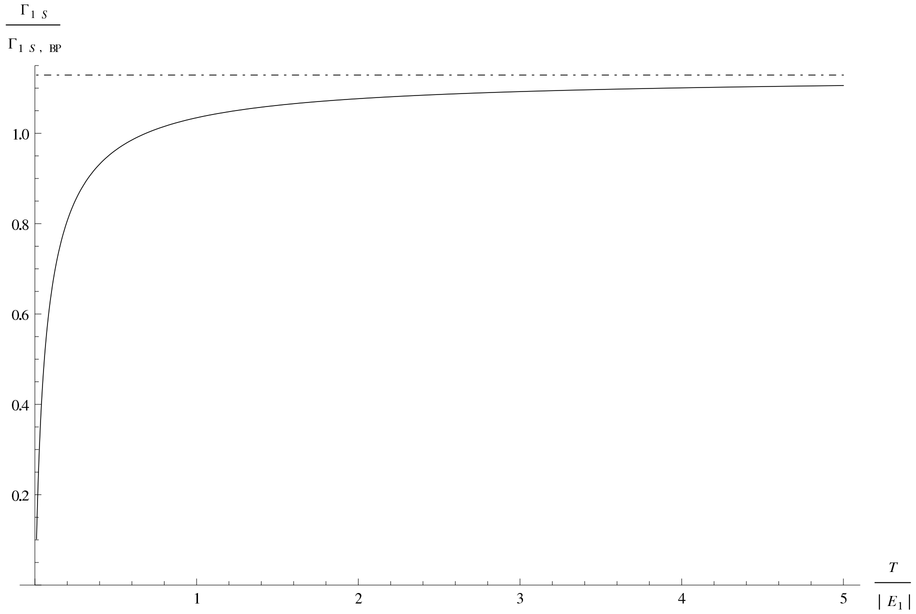
The Bhanot–Peskin approximation corresponds to neglecting final state interactions, i.e. the rescattering of a $Q\overline{Q}$ pair in a color octet configuration (recall $V_o = 1/(2N_c)\times\alpha_{\mathrm{s}}/r$).

Bhanot–Peskin cross section vs full cross section

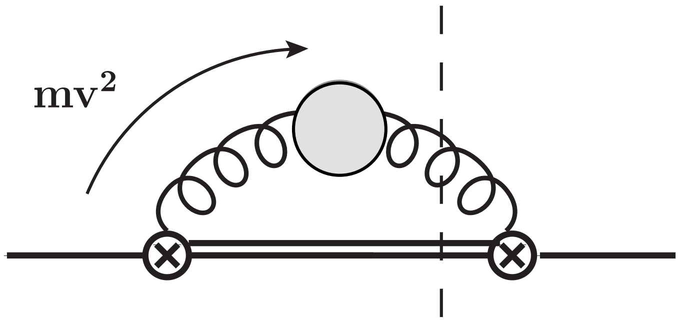


Bhanot–Peskin width vs full width





The temperature region $mv \gg T \gg mv^2 \gg m_D$: gluodissociation at NLO

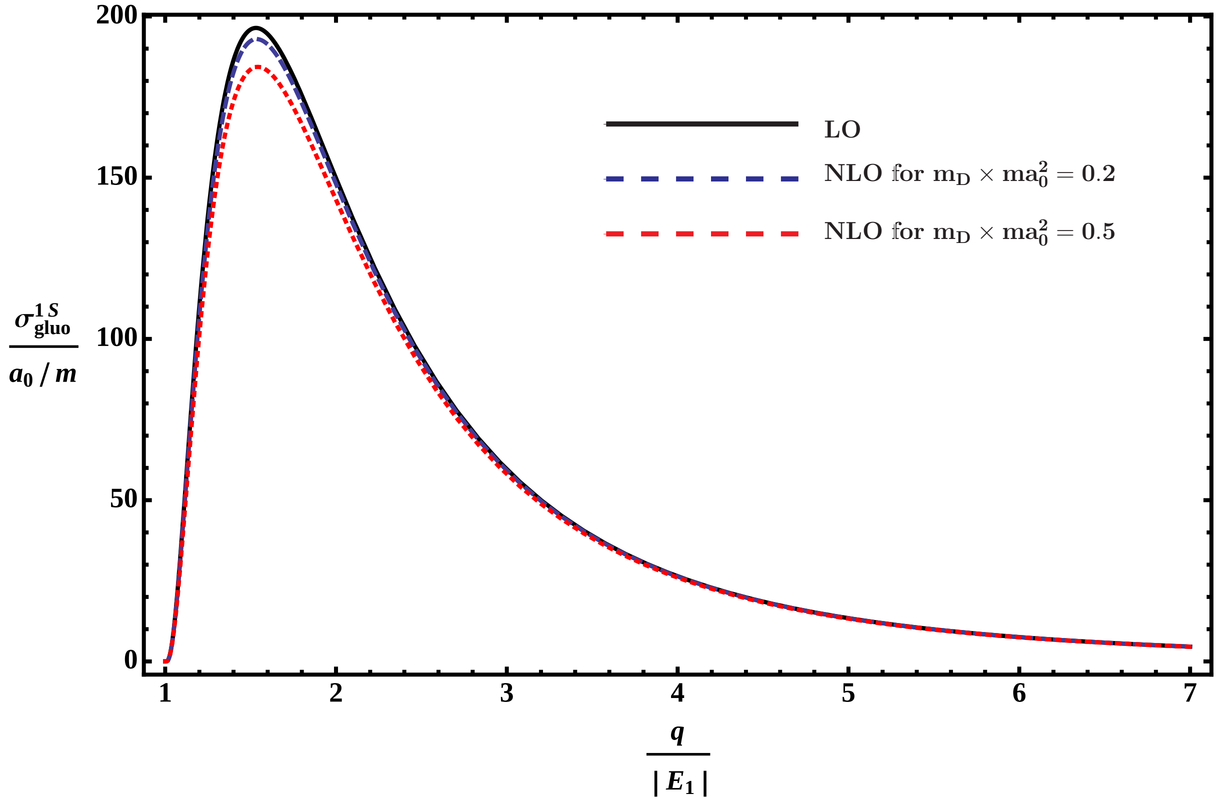


The NLO gluodissociation cross section for $1S$ Coulombic states is

$$\sigma^{nl}_{\rm gluo}(q) = Z(q/m_D)\,\sigma^{nl}_{\rm gluo\,LO}(q)$$

$$Z(q/m_D) = 1 - \frac{m_D^2}{4q^2}\left[\ln(8q^2/m_D^2) - 2\right]$$

$1S$ gluodissociation at LO vs NLO



Conclusions

In a framework that makes close contact with modern effective field theories for non relativistic bound states at zero temperature, one can study the dissociation of a quarkonium in a thermal bath of gluons and light quarks.

In a weakly-coupled framework, the situation is the following.

- For $T < E_{\rm bin}$ the potential coincides with the $T = 0$ potential.
- For $T > E_{\rm bin}$ the potential gets thermal contributions.
- For $E_{\rm bin} > m_D$ quarkonium decays dominantly via gluodissociation (aka singlet-to-octet break up).
- For $m_D > E_{\rm bin}$ quarkonium decays dominantly via inelastic parton scattering (aka Landau damping).
- For $T \sim T_{\rm dissociation} < T_{\rm screening}$, quarkonium cannot be formed.

In a strongly-coupled framework, the hierarchy of non-relativistic scales is preserved, whereas the thermodynamical hierarchy may break down. This requires a non-perturbative definition and evaluation of the potential (real and imaginary).