Following heavy quarks in the gluon plasma

with

Saumen Datta, Rajiv Gavai, Pushan Majumdar

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Debasish Banerjee

Albert Einstein Center for Fundamental Physics
Universität Bern

Heavy quarks and quarkonia in thermal QCD, ECT*, Trento
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Outline

Why be Poirot?

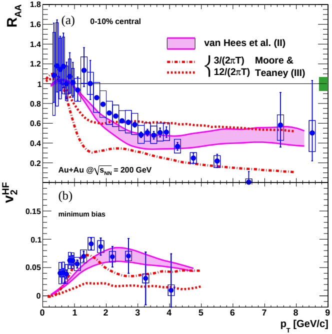
- $m_C, m_B \gg T_{\rm RHIC}, T_{LHC}$ $\rightarrow$ produced during pre-equilibriated state of the collision $\rightarrow$ probe for early time physics
- Perturbative arguments suggest energy loss mechanism to be very different for heavy quarks (HQ) from that of light quarks.
- Gluon bremstrahlung dominates for light quark jets Baier et. al. (1996); supressed for heavy quark jets Dokshitzer, Kharzeev (2001)
- For HQ, collisional loss is at least as important as radiative loss for ~ 5 GeV, and more at lower momenta. Moore,Teaney (2005); Mustafa (2005)
- Weak coupling calculations relate thermalization time for heavy quarks (τ_R^H) and light quarks: τ_R^L: $\tau_R^H = \frac{M}{T}\tau_R^L$
  $M \rightarrow$ HQ mass and $T \rightarrow$ temperature of the medium.
  (For $T \sim 250$ MeV and charm $M \sim 1.5$ GeV, this is about a factor 6!)
- Early elliptic flow $\rightarrow$ azimuthal anistropy parameter v_2 is sensitive to this
- Expect mass ordering of the elliptic flow: $v_2^h \gg v_2^D \gg v_2^B$;
  Experimentally: $v_2^D \lesssim v_2^h$! Suggest early thermalization of charm quarks

How much is D worth?

- $E_K \sim T$, $p \sim \sqrt{MT} \gg T \longrightarrow$ changes little in a single collision; successive collisions uncorrelated $\rightarrow$ Langevin description for motion of HQ in the medium

  Svetitsky (1988), Moore Teaney (2005), Mustafa(2005)
- Calculate v_2 in terms of the diffusion constant (D) of HQ in the medium
- D is a parameter that can be tuned to match the experimental results

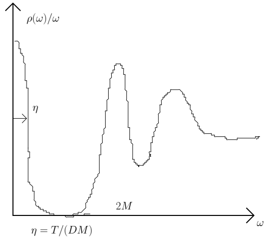
v_2 of charmed mesons, and their p_T dependence well described, but requires small D Moore Teaney (2005)
An order of magnitude lower than leading order PT!

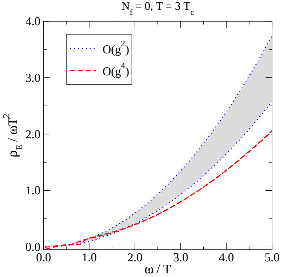


Right: A. Adare et. al. (PHENIX) 2010

Ms NP (non-perturbative) agrees to help

- Expert in Euclidean correlation functions, it used the operator $\bar{Q}\gamma_i Q$
- D needs to be extracted from the the width of the narrow transport peak at low-ω. Difficult!
- In the static limit, propagation of HQ replaced by Wilson lines Casladerrey-Solana, Teaney (2006)
- Can be reformulated as an correlation function of color electric fields Caron-Huot, Laine, Moore (2009)
- NLO-PT shows the corresponding $\rho(\omega)$ is smooth at low-ω $\rightarrow$ good news for lattice! Brunier, Laine, Langelage, Mether (2010)





Langevin formalism in NR-QCD formulation

Moore and Teaney(2005) Caron-Huot,Laine,Moore(2009)

- For heavy quarks $M \gg T$ moving in the plasma, average thermal momentum $p \sim \sqrt{MT} \gg T$
- $\mathcal{O}(M/T)$ collisions by the quasiparticles of the plasma needed to change the motion of the quarks

$$\frac{dp_{\mathrm{HQ}}}{dt} = \xi(t) - \eta_D p_{\mathrm{HQ}}; \quad \langle \xi(t)\xi(t') \rangle = \kappa \delta(t - t')$$

$\xi(t) \longrightarrow$ random force; $\eta_D \longrightarrow$ drag
$\kappa \longrightarrow$ strength of the stochastic interaction: Property of the medium
Relaxation governed by η_D; relaxation time $\tau_R = 1/\eta_D$

- Momentum diffusion coefficient: $\kappa = \frac{1}{3} \int_{-\infty}^{\infty} dt \sum_i \langle \xi_i(t)\xi_i(0) \rangle$
- With HQ in the static limit and in Euclidean time, the following correlation function needs to be calculated:

$$G_E(\tau) = -\frac{1}{3} \sum_{i=1}^{3} \frac{\langle \mathrm{Re\ Tr}[U(\beta,\tau) gE_i(\tau) U(\tau,0) gE_i(0)] \rangle}{\langle \mathrm{Re\ Tr}[U(\beta,0)] \rangle}$$

From $G(\tau)$ to $\rho(\omega)$

Need to solve an integral equation to get κ from $G(\tau)$:

$$G_E(\tau) = \int_0^\infty \frac{d\omega}{\pi} \rho(\omega) \frac{\cosh[(\frac{\beta}{2} - \tau)\omega]}{\sinh[\frac{\beta\omega}{2}]}$$

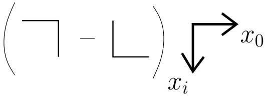
$$\kappa = \lim_{\omega \to 0} \frac{2T}{\omega} \rho(\omega); \quad D = \frac{2T^2}{\kappa}$$

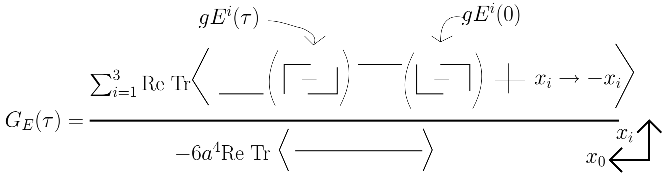
- In general, the inversion problem is ill-defined
- Usually, some assumptions on $\rho(\omega)$ to get any meaningful output
- MEM has been used for this in literature $\rightarrow$ requires a large no of points, as well as per-mille errorbars on data
- Such accuracy much more difficult with gauge field observables than with meson correlation functions
- For our case, will parametrize $\rho(\omega)$ with small number of parameters, and subsequently extract them using fitting

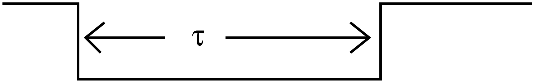
Outline

The night-time mission

- Note $g\,E_i \equiv [D_0, D_i] = D_0 D_i - D_i D_0$. Replace this by (suggested in Caron-Huot, Laine, Moore (2009))



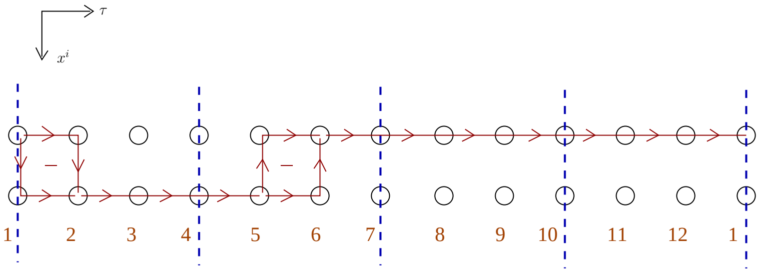




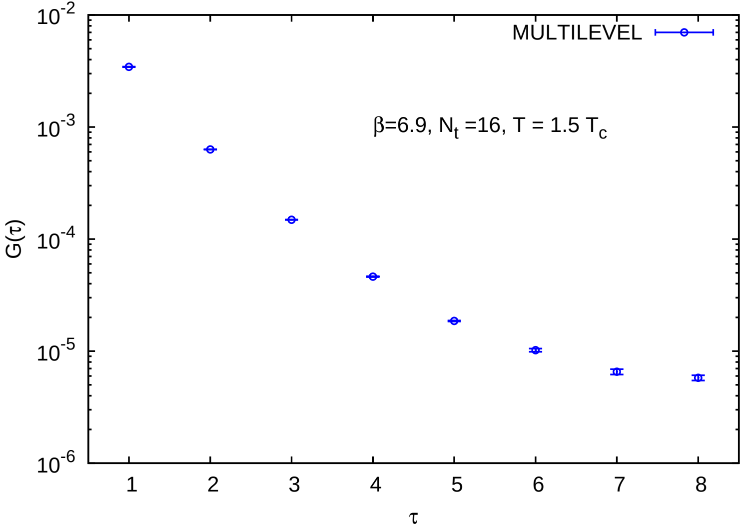
$$G_E(\tau) = 2C(\tau) - C(\tau+1) - C(\tau-1)$$

Ms NP gives Poirot a clever idea

- Known that the signal for the Polyakov loop becomes exponentially supressed for large N_t
- Reliable extraction of κ needs large N_t
- Use of Multilevel algorithm Lüscher Weisz (2001 & 2002) essential
- Downside: requires large memory



And the idea works!



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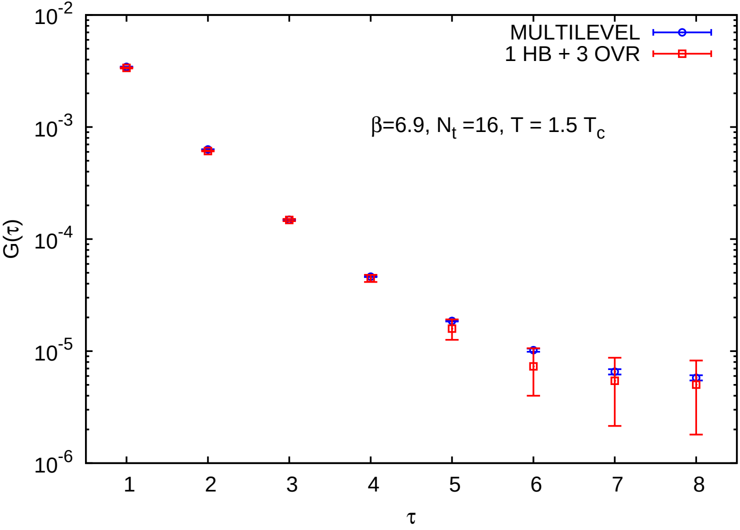


Data from the mission

- Explored $N_t = 12 - 24$
- For finite volume analysis: $N_s/N_t = 2 - 4$
- Temperature range from just above T_c to $3T_c$
- Reliable extraction possible only for $N_t \geq 20$
- Typical stats: several hundred independent configs, each with several thousand multilevel updates
- Correlation function have a few % error-bars at the largest τ for $N_t \sim 20$
- Very fine lattices: typical lattice spacings 0.02 - 0.03 fm

β	6.76	6.80	6.90	7.192	7.255
N_t	20	20	20	24	20
T/T_c	1.04	1.09	1.24	1.5	1.96

Outline

Parametrization strategy

- LO perturbative form of $\rho(\omega) \sim b\omega^3$
- In the $\omega \to 0$ limit need $\rho(\omega) \sim a\omega$ to see diffusion in $\mathcal{N} = 4$ SYM plasma Casalderrey-Solana, Teaney 2006
- Ansatz: $\rho_1(\omega) = a\omega\Theta(\Lambda - \omega) \;+\; b\omega^3$
- Calculations in classical lattice gauge theory suggest

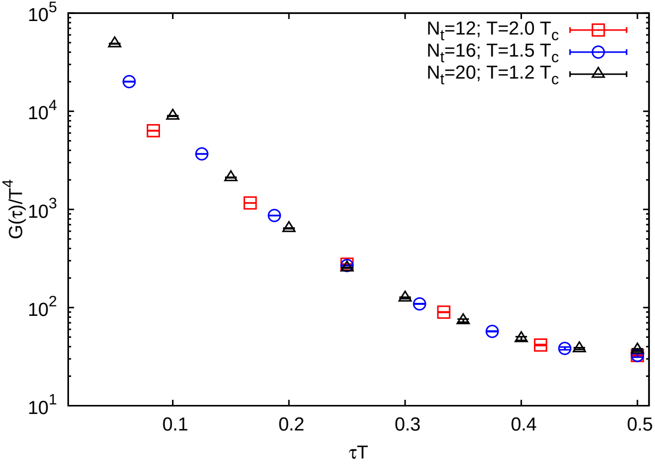
$$\rho(\omega) \;\sim\; c \tanh\frac{\omega\beta}{2} \qquad \text{for } \;\omega a \ll 1.$$

- Also used the following fit form to cross-check:

$$\rho_2(\omega) \;=\; c \tanh\frac{\omega\beta}{2}\,\Theta(\Lambda - \omega) \;+\; b\omega^3.$$

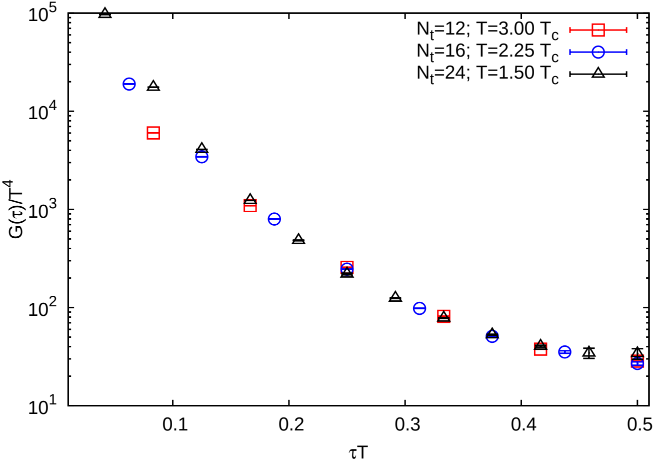
- Not feasable to do a 3-param fit. κ and Λ strongly correlated
- Keep Λ fixed, and do a full covariance matrix fit for $\tau a \in [N_t/4, N_t/2]$

Correlation functions



- Small-τ affected by lattice artefacts
- Large-τ region shows scaling: hint of continuum physics?

Correlation functions



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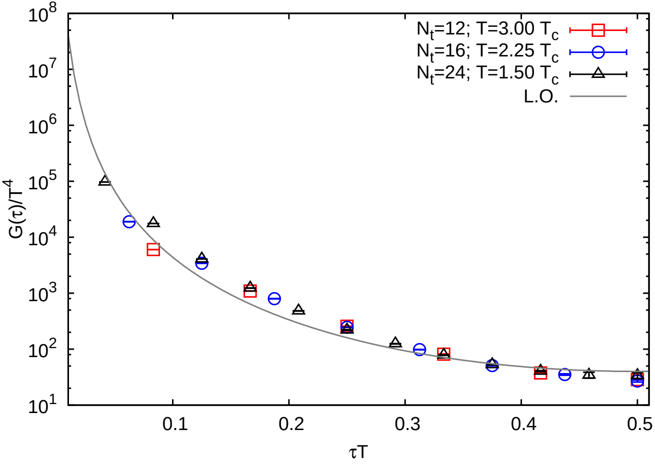
The LO contribution



Large N_t needed for reliable extraction!

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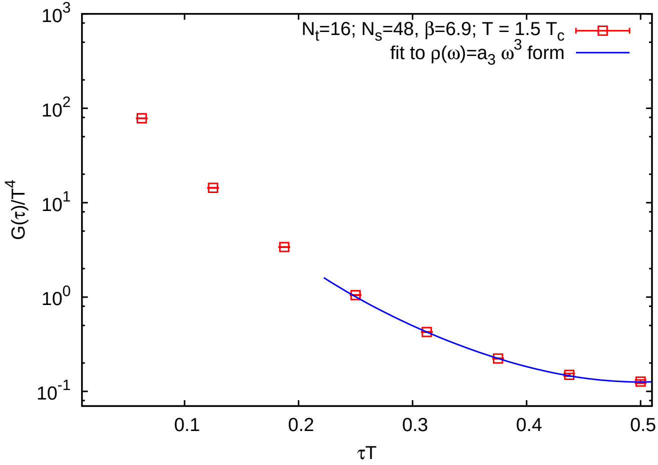

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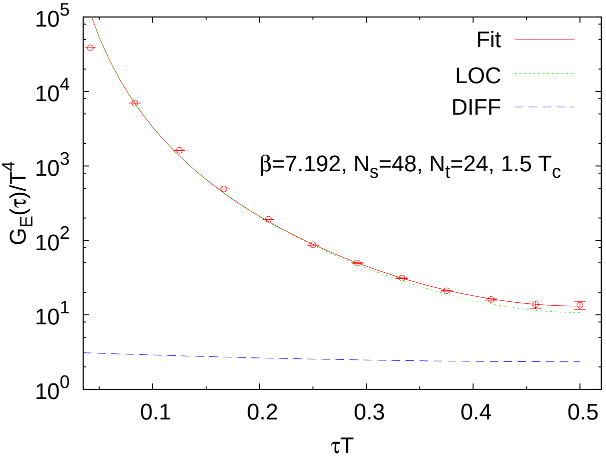
The diffusive part



Results quoted with $\Lambda = 3T$

Diffusive contribution small: about $\sim 20\%$ for $\tau T = 0.5$

Not possible to see in $N_t = 12, 16$

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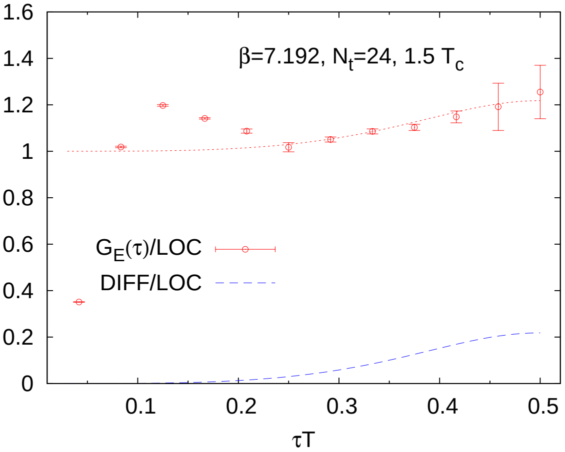

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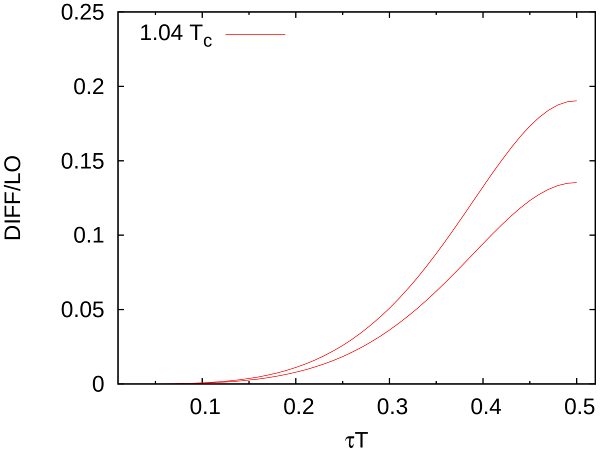
Diffusive part for various T



No significant temperature dependence! Diffusive part reaches to about 5% by $\tau T = 0.3$

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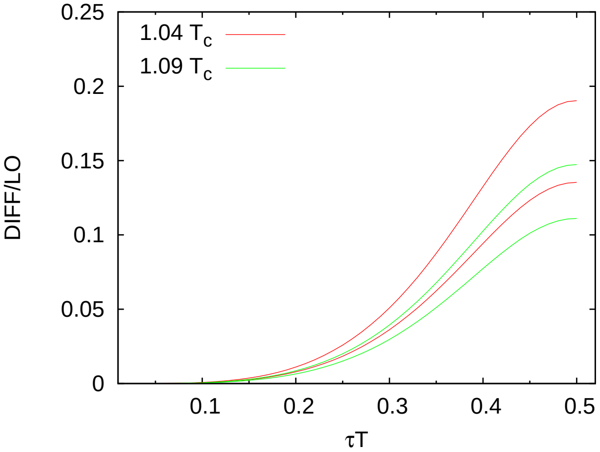

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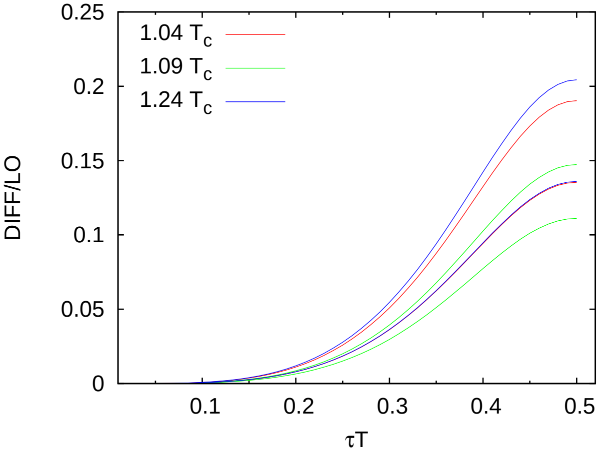

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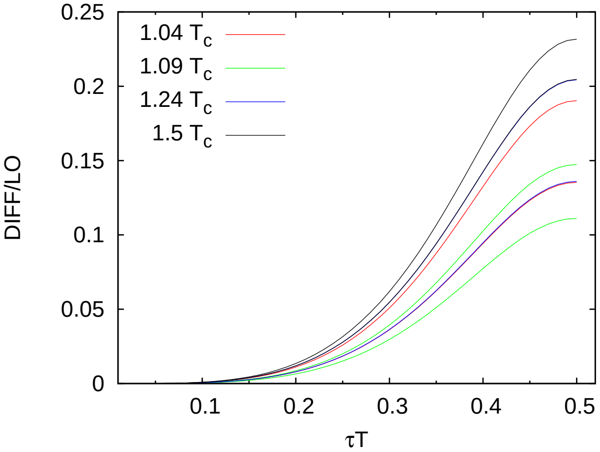

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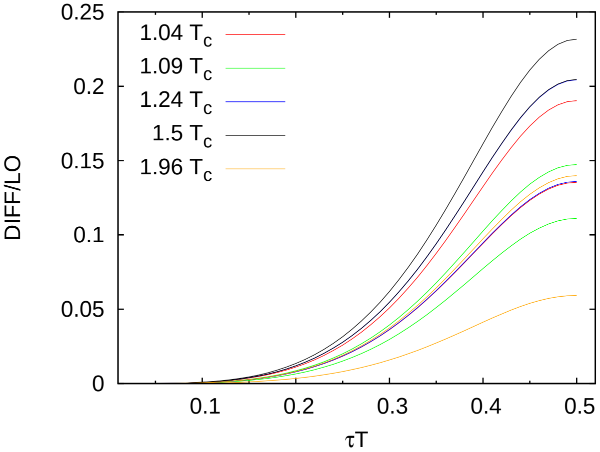

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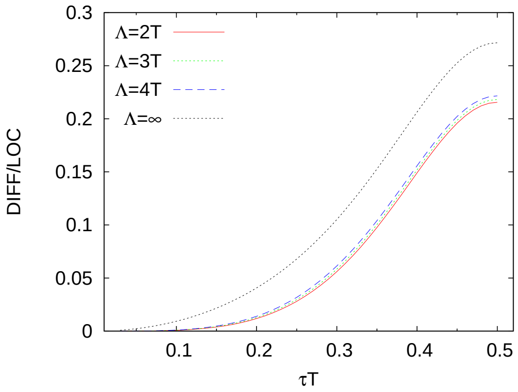
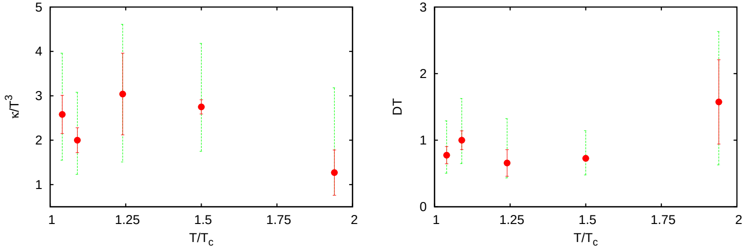
Λ dependence

- Quality of the fit rather insenstive to Λ
- Different Λ $\rightarrow$ different κ without affecting χ^2



- Typically a 30-50% variation in κ ▸ More

Worth of D



- Using Einstein relations, $D = T/(M\eta_D) = 2T^2/\kappa$
- Lower than Meyer,2010 (same formulation, different operators & analysis)
- Agree with preliminary estimates of Francis et. al.,2011 (Same formulation, operators, different analysis); Ding et. al.,2011 (charm correlators, MEM) More

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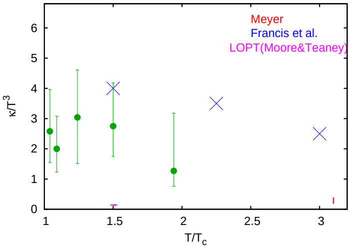

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Systematics: finite volume effects

- Known that appreciable finite volume effects can arise if the spatial size causes deconfinement
- Our lattices ($LT \gtrsim 2$) always satisfy this condition
- Further, low-ω part can have a non-trivial volume dependence
- Results show lack of any significant volume dependence

$$\chi^2/d.o.f. = \frac{1}{N_t/4} \sum_{\tau=\frac{N_t}{2}+1}^{N_t/2} \frac{|G_1(\tau) - G_2(\tau)|}{\sqrt{\sigma_1(\tau)^2 + \sigma_2(\tau)^2}}.$$

β	N_t	$(LT\|_1, LT\|_2)$	χ^2/d.o.f.
6.4	12	(2, 4)	0.34
6.65	12	(2, 4)	0.75
	16	(2.25, 3)	1.12
6.9	12	(3, 4)	0.24
	16	(2.25, 3)	0.51
	20	(1.8, 2.4)	1.58
7.192	24	(2, 2.33)	0.29

Systematics: Renormalization

- Need to get physical correlator of electric fields:
  $G_E(\tau) = Z(a) G_E^{Lat}(\tau)$
- Non-pert renormalization not available for these operators
- Expected to be dominated by self-energy correction
- Can be taken care of using the tadpole factor:
  $Z_E^{-1} = \left(\frac{1}{N}\langle \mathrm{Tr} U_p \rangle\right)^{\frac{1}{4}}$
- Simplification: $\langle L \rangle$ cancels most of the straight line part;
  $Z(a) = Z_E^2$

Using the tadpole factor for renormalization gives values very close to those obtained by non-pert renorm. for other discretizations at smaller β

Koma, Koma, Wittig (2006), Koma Koma (2007)

Outline

Note from Holmes: PT and AdS/CFT

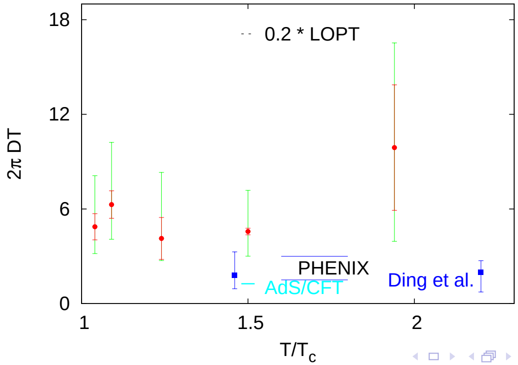
- At very high T, $DT \sim 1/\alpha_S^2$
- LO PT gives a large value for DT Moore,Teaney; (2005) Brunier et. al. (2010)
  At 1.5 T_c, $\alpha_S^{\overline{MS}}(3T) \sim 0.23$; $m_D/T \sim 2.345$ giving $DT \sim 14$
  Not large change for $N_f \neq 0$; order of magnitude greater than the non-pert estimate!
- NLO corrections to κ start at $O(g)$. Calculated for $N_f = 3$ Caron-Huot, Moore (2007): with $\alpha_S \sim 0.2$, $DT \sim 8.4/2\pi$
- While a similar change will bring it close to the non-perturbative estimate of $N_f = 0$, issues with convergence need to be clarified
- On the other hand, computation in AdS/CFT available
  Casalderrey-Solana, Teaney (2006)

$$DT \simeq \frac{0.9}{2\pi}\left(\frac{1.5}{\lambda_{tH}}\right)^{\frac{1}{2}} ;\ \lambda_{tH} = \alpha_S N_c$$

- Note: parametric dependence on α_S different
- Putting $\alpha_S \approx 0.23$ and $N_c = 3$, $DT \approx 0.2$
  Lower than, but in the same ballpark as the non-pt estimate

Was it of any help?

- Non-perturbative results different from PT.
- However, no thermal quarks in the calculation
- Expect scaling with full QCD as function of T/T_c?
- Values in the right ballpark to explain v_2 results from PHENIX in the Langevin formulation



And the case goes on

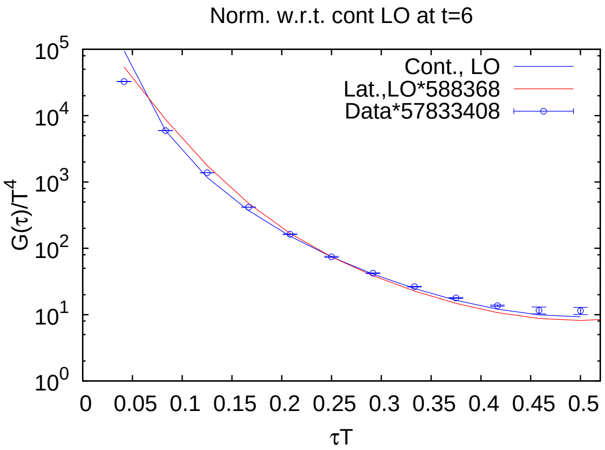
- Calculated momentum diffusion coefficient of heavy quarks in the gluon plasma
- Multilevel algorithm essential for obtaining accurate data
- Essentially used fit ansatz to extract the diffusion constant
- Reasonably close estimates to explain experimental values using the Langevin formulation
- Significanly different from PT. Agreement with estimates of other groups Ding et. al. (2011), Francis et. al. (2011)
- Model independent estimates using subtracted correlation function? Brunier Laine (2012)
- More theoretical control over the renormalization constant desirable
- Finer lattices
- Improved discretizations of electric field?

Λ dependence: Which value to quote?

- Why cutoff? Large-ω does not have diffusion
- The Λ dependence in the fit represents a “flat” direction
- Cut-off’s are approximation; no change expected for a smooth variation
- The “flat” direction has a more general nature
- Follow the conservative estimate of letting Λ vary $[2T, \infty]$
- Use systematic error-band
- To quote central value: Determine when the diffusive contribution starts competing with the LO contribution
- This happens around $\Lambda \sim 3T$ for our values
- Alternatively, jump in $\rho(\omega)$ is less when $\Lambda \sim 3T$

Back

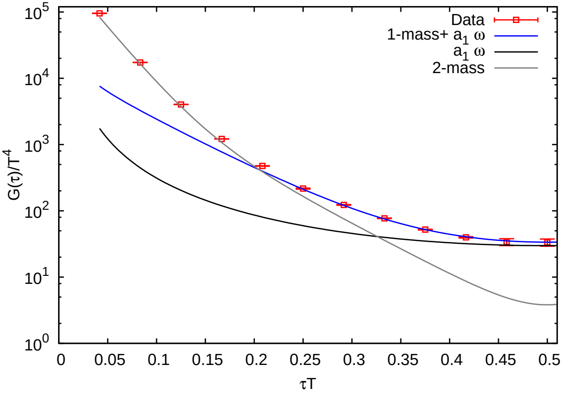
Extra: Lattice PT



cont LO captures short distance effects better than lattice LO PT!

Extra: Other functional forms

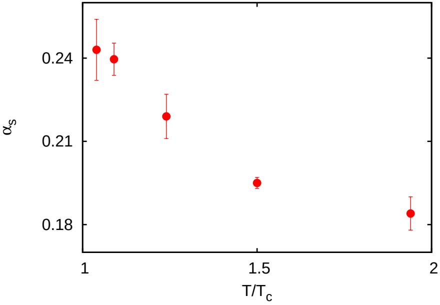
$$G(\tau) = \frac{a_1 \pi}{N_t^2} \frac{1}{\sin^2(\pi\tau/N_t)} + A_1 \cosh(M_1(\tau - 1/2T)) + A_2 \cosh(M_2(\tau - 1/2T))$$



Different behaviour of M_1 and M_2; M_2 does not change with N_t, but $M_1/T \sim 16.5$ between 1.5 - 3 T_c

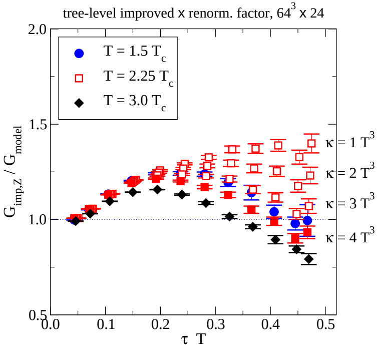
Strong coupling constant α_S

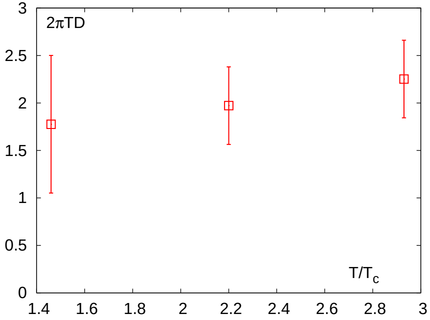
- In LO, $\rho^{LO}(\omega) = \frac{8\alpha_S}{9}\omega^3$
- Use the fit coefficient of ω^3 term to define α_S using the scheme
- Can be related to $\alpha_S^{\bar{MS}}$ using the NLO calculation of Brunier et. al. (2010)



Agrees with a similar calculation of α_S from vector current correlators (Ding et. al., 2010) and other estimates of α_S from static observables

Kaczmarek,Zantow; 2005





Left: Francis et. al. (2011); Right: Ding et. al. (2011) ▸ Back