Heavy flavor spectra in AA collision within a Langevin approach

work in collaboration with A. De Pace, M. Monteno and F. Prino

Andrea Beraudo

Physics Department - Theory Unit - CERN

Heavy quarks and quarkonia in thermal QCD,
ECT*, Trento, 2^{nd}-5^{th} April 2013

Outline

- Heavy flavor in elementary collisions as *benchmark*
  - of our understanding of pQCD,
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  - tomography of the produced matter ($T(x)$, $\epsilon(x)$, $\hat{q}$, dN_g/dy...)
- A particular approach: the relativistic Langevin equation;

Heavy-flavor production in pQCD

The large mass M of c and b quarks makes a pQCD calculation of $Q\overline{Q}$ production possible:

- It sets a *minimal off-shellness* of the intermediate propagators (diagrams don't diverge);
- It sets a *hard scale* for the evaluation of $\alpha_s(\mu)$ (speeding the convergence of the perturbative series);
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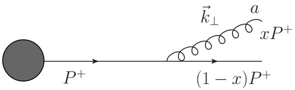
Both the *total cross-section* $\sigma_{\mathrm{Q\overline{Q}}}^{\mathrm{tot}}$ and the *invariant single-particle spectrum* $E(d\sigma_Q/d^3p)$ are well-defined quantities which can be calculated in pQCD

Suppression of collinear radiation



Massless case

$$d\sigma^{\rm rad} = d\sigma^{\rm hard}\frac{\alpha_s}{\pi^2}C_F\frac{dx}{x}\frac{d\mathbf{k}_\perp}{\mathbf{k}_\perp^2}$$

Due to collinear gluon-radiation ($\sim d\theta/\theta$), partonic cross-sections of hard processes are not well defined, but require the introduction of a "cutoff" (*factorization scale* μ_F) to regularize collinear divergences. Only hadronic cross-section

$$d\sigma_h \equiv \sum_f d\sigma_f(\mu_F)\otimes D_f^h(z,\mu_F)$$

are *collinear-safe* observables.

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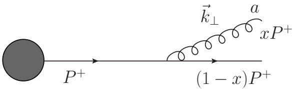


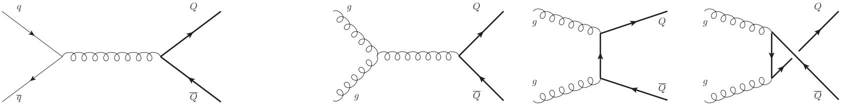
Massive case

$$d\sigma^{\mathrm{rad}} = d\sigma^{\mathrm{hard}} \frac{\alpha_s}{\pi^2} C_F \frac{dx}{x} d\mathbf{k}_\perp \frac{\mathbf{k}_\perp^2}{[\mathbf{k}_\perp^2 + x^2 M^2]^2}$$

Gluon radiation at angles $\theta < M/E$ is suppressed (*dead-cone effect*!) and heavy-quark production is well-defined even at the partonic (for what concerns the final state) level.

Leading Order contribution

- The LO processes are:

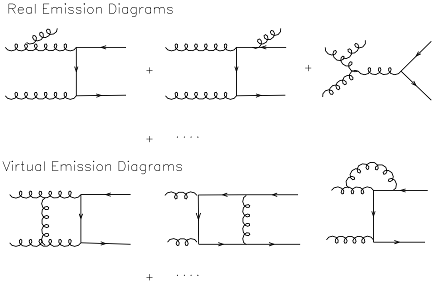


- The propagators introduce in the amplitudes the denominators:

$$(p_1 + p_2)^2 = 2m_T^2(1 + \cosh \Delta y)$$
$$(p_3 - p_1)^2 = -m_T^2 \left(1 + e^{-\Delta y}\right)$$
$$(p_3 - p_2)^2 = -m_T^2 \left(1 + e^{\Delta y}\right)$$

  - *Minimal off-shellness* $\sim m_T^2$;
  - Q and $\overline{Q}$ close in rapidity.

Next to Leading Order process



- Real emission: $|\mathcal{M}_{\mathrm{real}}|^2 \sim \mathcal{O}(\alpha_s^3)$
- Virtual corrections: $2\mathrm{Re}\mathcal{M}_0\mathcal{M}^*_{\mathrm{virt}} \sim \mathcal{O}(\alpha_s^3)$

- NLO calculation gives the $\mathcal{O}(\alpha_s^3)$ result for $\sigma^{\mathrm{tot}}_{Q\overline{Q}}$ and $E(d\sigma_Q)/d^3p$;
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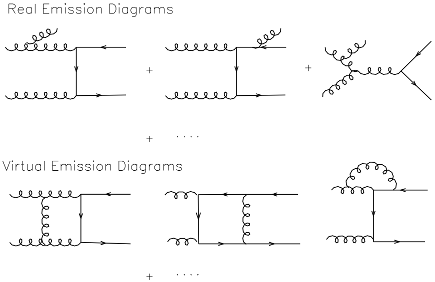

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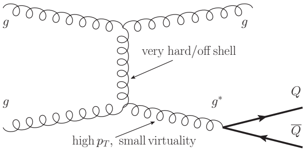
- Output of hard event can be interfaced with a Parton Shower (PYTHIA or HERWIG)

NLO calculation: gluon-splitting contribution



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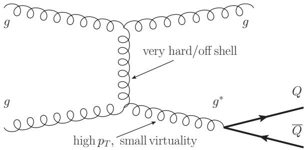

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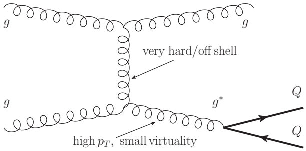

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$Q\overline{Q}$ multiplicity in a gluon jet of transverse energy p_T: $\sim \alpha_s \ln(p_T/M)$

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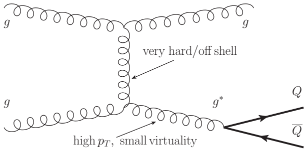

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The NLO calculation *contains* an $\alpha_s \ln(p_T/M)$ term, *potentially large*!

Resummation of (Next to) Leading Logs: FONLL

- Using the above result as the initial condition of the DGLAP evolution for the D_g^Q FF:

$$D_g^Q(z,\mu_0) = \frac{\alpha_s}{2\pi}\frac{1}{2}[z^2+(1-z)^2]\ln\frac{\mu_0^2}{M^2}$$

amounts to resumming all $[\alpha_s \ln(p_T/M)]^n$ terms ($\alpha_s[\alpha_s \ln(p_T/M)]^n$ with NLO splitting functions)

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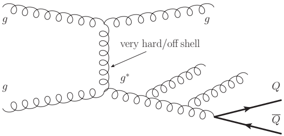

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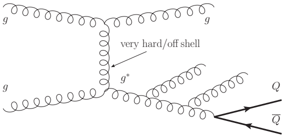
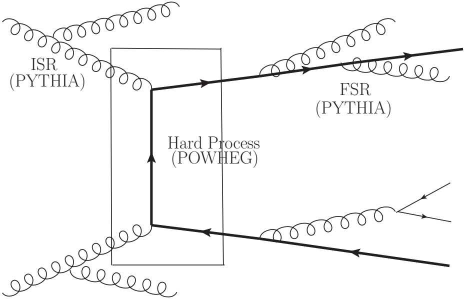


$Q\overline{Q}$ from the shower of light partons produced in the hard event!

- A code like FONLL provides a calculation of $d\sigma_Q$ at this accuracy!

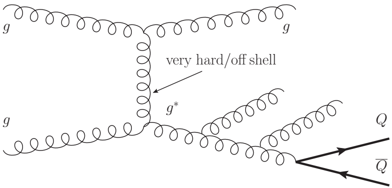
NLO calculation + Parton Shower



- A different strategy is to interface the output of a NLO event-generator for the hard process with a parton-shower describing Initial and Final State Radiation.
- This provides a *fully exclusive information on the final state*

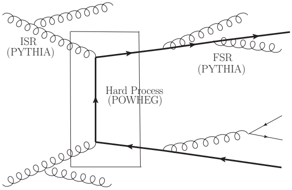
FONLL vs POWHEG+PS

FONLL



- It is a *calculation*
- It provides NNL accuracy, resumming large $\ln(p_T/M)$
- It includes processes missed by POWHEG (hard events with light partons)

POWHEG+PS



- It is an *event generator*
- Results compatible with FONLL
- It is a more flexible tool, allowing to address more differential observables (e.g. $Q\overline{Q}$ correlations)

Heavy quark production in pQCD: some references

- For a general introduction: M. Mangano, hep-ph/9711337 (lectures);
- For POWHEG: S. Frixione, P. Nason and G. Ridolfi, JHEP 0709 (2007) 126;
- For FONLL: M. Cacciari, M. Greco and P. Nason, JHEP 9805 (1998) 007.
- For a systematic comparison (POWHEG vs MC@NLO vs FONLL): M. Cacciari *et al.*, JHEP 1210 (2012) 137.

Heavy flavour: experimental observables

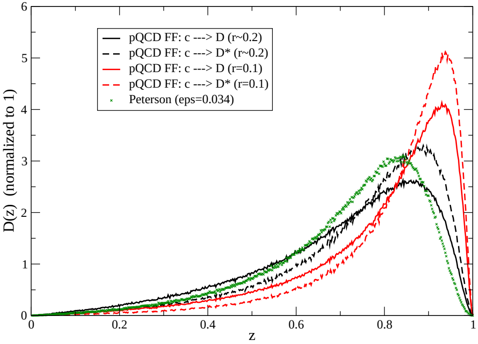
- D and B mesons;
- Non-prompt J/ψ's ($B \to J/\psi\, X$)
- Heavy-flavour electrons, from the decays
  - of charm (e_c)

$$D \to X \nu e$$

  - of beauty (e_b)

$$\begin{aligned} B &\to D\nu e \\ B &\to D\nu e \to X\nu e\nu e \\ B &\to DY \to X\nu e Y \end{aligned}$$

Results: D and B mesons @ 7 TeV



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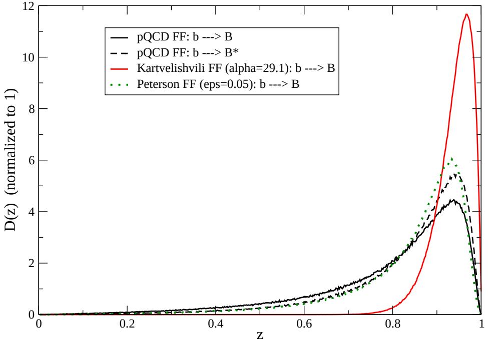

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- B-meson FF from Kartvelishvili *et al.*, PLB 78 (1978) 615.

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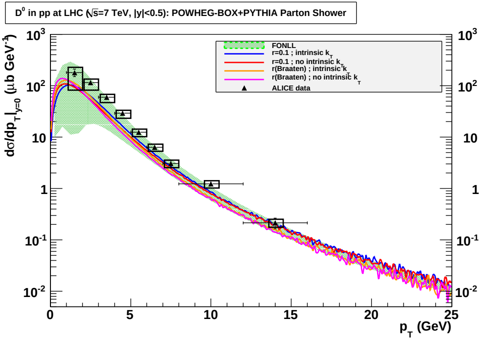

- Our choice (M. Monteno talk at HP2012, arXiv:1208.0705): POWHEG for the *hard event* interfaced with PYTHIA for the *shower* stage;
- With the same default parameters ($m_c = 1.5$ GeV, $m_b = 1.5$ GeV, $\mu_R = \mu_F = m_T$) and FF results in agreement with FONLL.

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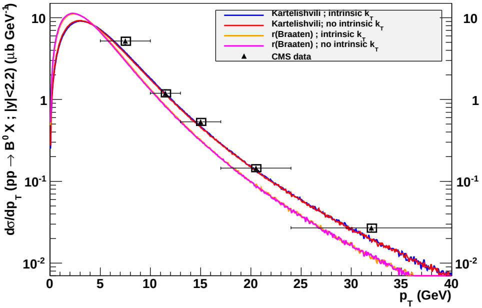

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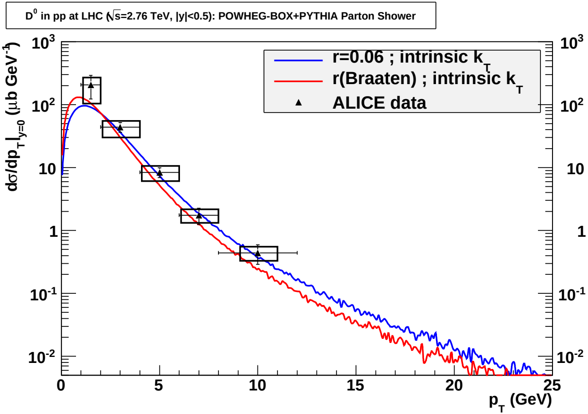

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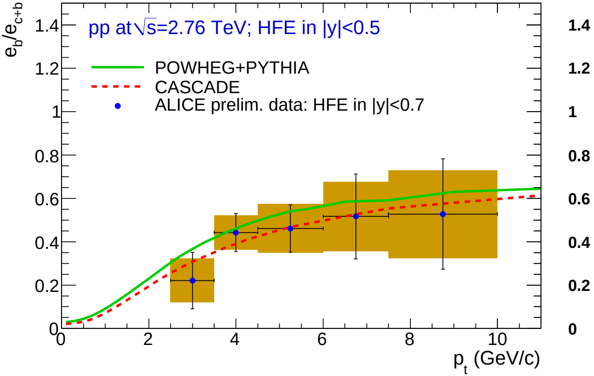

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- and for the heavy-flavour electrons (e_c and e_b)

Heavy quarks as probes of the QGP

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- Initial production: pQCD + possible nuclear effects (nPDFs, k_T-broadening);

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Ideally only the parton-medium interaction should be model-dependent

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In practice each model deals with the other points in a different (often rather schematic) way: difficulty in performing a systematic comparison!

Heavy Flavour in the QGP: the conceptual setup

- Description of soft observables based on hydrodynamics, assuming to deal with a system close to local thermal equilibrim (no matter why);
- Description of jet-quenching based on energy-degradation of external probes (high-p_T partons);

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- Description of heavy-flavour observables requires to employ/develop a setup (transport theory) allowing to deal with more general situations and in particular to describe *how particles would (asymptotically) approach equilibrium*.

Transport theory: the Boltzmann equation

Time evolution of HQ phase-space distribution $f_Q(t, \mathbf{x}, \mathbf{p})$:

$$\frac{d}{dt} f_Q(t, \mathbf{x}, \mathbf{p}) = C[f_Q]$$

- Total derivative along particle trajectory

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{x}} + \mathbf{F}\frac{\partial}{\partial \mathbf{p}}$$

  Neglecting $\mathbf{x}$-dependence and mean fields: $\partial_t f_Q(t, \mathbf{p}) = C[f_Q]$

- Collision integral:

$$C[f_Q] = \int d\mathbf{k} [\underbrace{w(\mathbf{p}+\mathbf{k}, \mathbf{k}) f_Q(\mathbf{p}+\mathbf{k})}_{\text{gain term}} - \underbrace{w(\mathbf{p}, \mathbf{k}) f_Q(\mathbf{p})}_{\text{loss term}}]$$

  $w(\mathbf{p}, \mathbf{k})$: HQ transition rate $\mathbf{p} \to \mathbf{p} - \mathbf{k}$

From Boltzmann to Fokker-Planck

Expanding the collision integral for *small momentum exchange*[1] (Landau)

$$C[f_Q] \approx \int d\mathbf{k} \left[k^i \frac{\partial}{\partial p^i} + \frac{1}{2} k^i k^j \frac{\partial^2}{\partial p^i \partial p^j} \right] [w(\mathbf{p}, \mathbf{k}) f_Q(t, \mathbf{p})]$$

[1] B. Svetitsky, PRD 37, 2484 (1988)

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$$\frac{\partial}{\partial t} f_Q(t,\mathbf{p}) = \frac{\partial}{\partial p^i} \left\{ A^i(\mathbf{p}) f_Q(t,\mathbf{p}) + \frac{\partial}{\partial p^j} [B^{ij}(\mathbf{p}) f_Q(t,\mathbf{p})] \right\}$$

where

$$A^i(\mathbf{p}) = \int d\mathbf{k}\, k^i w(\mathbf{p},\mathbf{k}) \longrightarrow \underbrace{A^i(\mathbf{p}) = A(p)\, p^i}_{\text{friction}}$$

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Problem reduced to the *evaluation of three transport coefficients*

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The relativistic Langevin equation

The Fokker-Planck equation can be recast into a form suitable to follow the dynamics of each individual quark: the Langevin equation

$$\frac{\Delta p^i}{\Delta t} = -\underbrace{\eta_D(p)p^i}_{\text{determ.}} + \underbrace{\xi^i(t)}_{\text{stochastic}},$$

with the properties of the noise encoded in

$$\langle \xi^i(\mathbf{p}_t)\xi^j(\mathbf{p}_{t'})\rangle = b^{ij}(\mathbf{p}_t)\frac{\delta_{tt'}}{\Delta t} \qquad b^{ij}(\mathbf{p}) \equiv \kappa_\parallel(p)\hat{p}^i\hat{p}^j + \kappa_\perp(p)(\delta^{ij} - \hat{p}^i\hat{p}^j)$$

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Transport coefficients to calculate:

- *Momentum diffusion* $\kappa_\perp \equiv \frac{1}{2}\frac{\langle \Delta p_\perp^2 \rangle}{\Delta t}$ and $\kappa_\parallel \equiv \frac{\langle \Delta p_\parallel^2 \rangle}{\Delta t}$;
- *Friction* term (dependent on the discretization scheme!)

$$\eta_D{}^{\mathrm{Ito}}(p) = \frac{\kappa_\parallel(p)}{2TE_p} - \frac{1}{E_p^2}\left[(1-v^2)\frac{\partial \kappa_\parallel(p)}{\partial v^2} + \frac{d-1}{2}\frac{\kappa_\parallel(p) - \kappa_\perp(p)}{v^2}\right]$$

fixed in order to insure approach to equilibrium (Einstein relation):

The background medium

The fields $u^\mu(x)$ and $T(x)$ so far were taken from the output of two longitudinally boost-invariant ("Hubble-law" longitudinal expansion $v_z = z/t$)

$$x^\mu = (\tau \cosh\eta, \mathbf{r}_\perp, \tau \sinh\eta) \quad \text{with} \quad \tau \equiv \sqrt{t^2 - z^2}$$

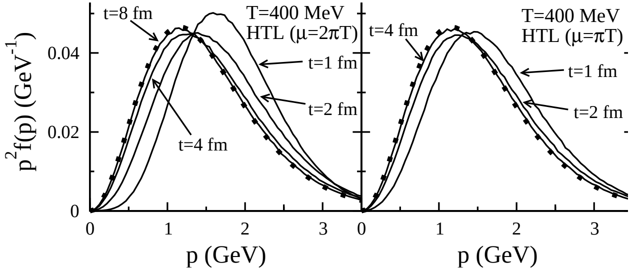
$$u^\mu = \gamma_\perp(\cosh\eta, \mathbf{u}_\perp, \sinh\eta) \quad \text{with} \quad \gamma_\perp \equiv \frac{1}{\sqrt{1-\mathbf{u}_\perp^2}}$$

hydro codes[2].

- $u^\mu(x)$ used to perform the update each time in the fluid rest-frame;
- $T(x)$ allows to fix at each step the value of the transport coefficients.

[2]P.F. Kolb, J. Sollfrank and U. Heinz, Phys. Rev. C **62** (2000) 054909
P. Romatschke and U.Romatschke, Phys. Rev. Lett. **99** (2007) 172301

A first check: thermalization in a static medium



For $t \gg 1/\eta_D$ one approaches a relativistic Maxwell-Jüttner distribution[3]

$$f_{\mathrm{MJ}}(p) \equiv \frac{e^{-E_p/T}}{4\pi M^2 T\, K_2(M/T)}, \qquad \text{with} \int d^3p\, f_{\mathrm{MJ}}(p) = 1$$

(Test with a sample of c quarks with $p_0 = 2$ GeV/c)

[3] A.B., A. De Pace, W.M. Alberico and A. Molinari, NPA 831, 59 (2009)

The Langevin equation provides a link between *what is possible to calculate in QCD* (transport coefficients) and *what one actually measures* (final p_T spectra)

[4] Our approach: W.M. Alberico *et al.*, Eur.Phys.J. C71 (2011) 1666

The Langevin equation provides a link between *what is possible to calculate in QCD* (transport coefficients) and *what one actually measures* (final p_T spectra)

Evaluation of transport coefficients:

- Weak-coupling hot-QCD calculations[4]
- Non perturbative approaches
  - Lattice-QCD
  - AdS/CFT correspondence
  - Resonant scattering

[4] Our approach: W.M. Alberico *et al.*, Eur.Phys.J. C71 (2011) 1666

Transport coefficients: perturbative evaluation

It's the stage where the various models differ!

We account for the effect of $2 \to 2$ collisions in the medium

[5] Similar strategy for the evaluation of dE/dx in S. Peigne and A. Peshier, Phys.Rev.D77:114017 (2008).

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We account for the effect of $2 \to 2$ collisions in the medium

Intermediate cutoff $|t|^* \sim m_D^2$[5] *separating the contributions of*

- hard collisions ($|t| > |t|^*$): kinetic pQCD calculation
- soft collisions ($|t| < |t|^*$): Hard Thermal Loop approximation (*resummation of medium effects*)

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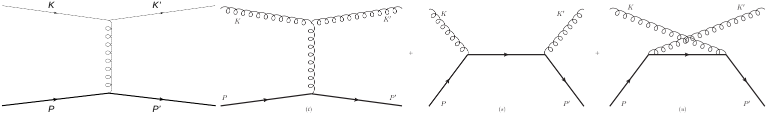
Transport coefficients $\kappa_{T/L}(p)$: hard contribution

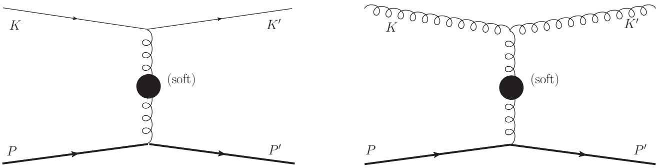


$$\kappa_T^{g/q(\mathrm{hard})} = \frac{1}{2}\frac{1}{2E}\int_k \frac{n_{B/F}(k)}{2k}\int_{k'}\frac{1 \pm n_{B/F}(k')}{2k'}\int_{p'}\frac{1}{2E'}\theta(|t|-|t|^*)\times$$
$$\times (2\pi)^4\delta^{(4)}(P+K-P'-K')\left|\overline{\mathcal{M}}_{g/q}(s,t)\right|^2 q_T^2$$

$$\kappa_L^{g/q(\mathrm{hard})} = \frac{1}{2E}\int_k \frac{n_{B/F}(k)}{2k}\int_{k'}\frac{1 \pm n_{B/F}(k')}{2k'}\int_{p'}\frac{1}{2E'}\theta(|t|-|t|^*)\times$$
$$\times (2\pi)^4\delta^{(4)}(P+K-P'-K')\left|\overline{\mathcal{M}}_{g/q}(s,t)\right|^2 q_L^2$$

where: $(|t| \equiv q^2 - \omega^2)$

Transport coefficients $\kappa_{T/L}(p)$: soft contribution



When the exchanged 4-momentum is **soft** the t-channel gluon feels the presence of the medium and requires **resummation**.

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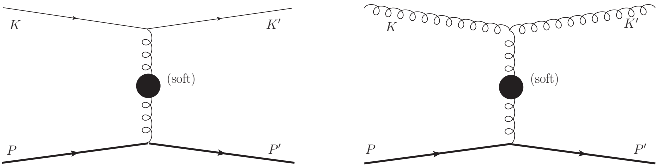


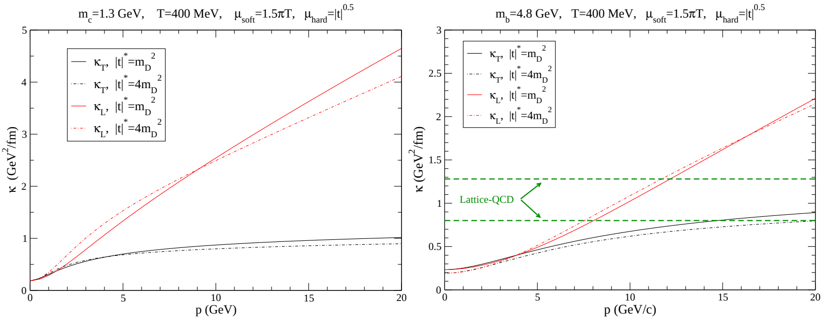
When the exchanged 4-momentum is **soft** the t-channel gluon feels the presence of the medium and requires **resummation**.
The *blob* represents the *dressed gluon propagator*, which has longitudinal and transverse components:

$$\Delta_L(z,q) = \frac{-1}{q^2 + \Pi_L(z,q)}, \quad \Delta_T(z,q) = \frac{-1}{z^2 - q^2 - \Pi_T(z,q)},$$

where *medium effects* are embedded in the HTL gluon self-energy.

Transport coefficients: numerical results

Combining together the hard and soft contributions...



...the dependence on the intermediate cutoff $|t|^*$ is very mild!

Lattice-QCD transport coefficients

Ongoing efforts to extract transport coefficients from lattice-QCD simulations *assuming a non-relativistic Langevin dynamics of the HQs*

- κ from electric-field correlators[6];
- η_D from current-current correlators, exploting the diffusive dynamics of conserved charges[7]

[6] Solana and Teaney, PRD 74, 085012 (2006)
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General considerations:

- In principle lattice-QCD would provide an "exact" non-perturbative result;
- Difficulties in extracting real-time quantities (transport coefficients) from euclidean $(t = -i\tau)$ simulations;
- Current results limited to the static $(M = \infty)$ or at most non-relativistic limit.

[6] Solana and Teaney, PRD 74, 085012 (2006)
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Lattice-QCD transport coefficients: setup

One consider the non-relativistic limit of the Langevin equation:

$$\frac{dp^i}{dt} = -\eta_D p^i + \xi^i(t), \quad \text{with} \quad \langle \xi^i(t)\xi^j(t')\rangle = \delta^{ij}\delta(t-t')\kappa$$

Hence, in the $p \to 0$ limit:

$$\kappa = \frac{1}{3}\int_{-\infty}^{+\infty} dt \langle \xi^i(t)\xi^i(0)\rangle_{\rm HQ} \approx \frac{1}{3}\int_{-\infty}^{+\infty} dt\, \underbrace{\langle F^i(t)F^i(0)\rangle_{\rm HQ}}_{\equiv D^>(t)},$$

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In the static limit the force is due to the color-electric field:

$$\mathbf{F}(t) = g\int d\mathbf{x} Q^\dagger(t,\mathbf{x})t^a Q(t,\mathbf{x})\mathbf{E}^a(t,\mathbf{x})$$

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In a thermal ensemble $\sigma(\omega) \equiv D^>(\omega) - D^<(\omega) = (1-e^{-\beta\omega})D^>(\omega)$ and

$$\kappa \equiv \lim_{\omega\to 0}\frac{D^>(\omega)}{3} = \lim_{\omega\to 0}\frac{1}{3}\frac{\sigma(\omega)}{1-e^{-\beta\omega}} \underset{\omega\to 0}{\sim} \frac{1}{3}\frac{T}{\omega}\sigma(\omega)$$

Lattice-QCD transport coefficients: results

The spectral function $\sigma(\omega)$ has to be reconstructed starting from the *euclidean electric-field correlator*

$$D_E(\tau) = -\frac{\langle \mathrm{Re}\,\mathrm{Tr}[U(\beta,\tau)gE^i(\tau,\mathbf{0})U(\tau,0)gE^i(0,\mathbf{0})]\rangle}{\langle \mathrm{Re}\,\mathrm{Tr}[U(\beta,0)]\rangle}$$

according to

$$D_E(\tau) = \int_0^{+\infty} \frac{d\omega}{2\pi} \frac{\cosh(\tau - \beta/2)}{\sinh(\beta\omega/2)} \sigma(\omega)$$

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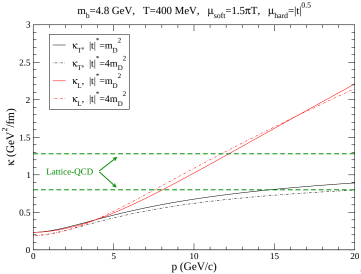
One gets[a]:

$$\kappa \approx 2.5T^3 - 4T^3$$

$\sim$3-5 times larger then the $p=0$ perturbative result

[a] A. Francis *et al.*, PoS LATTICE2011 202; D. Banerjee *et al.*, Phys.Rev. D85 (2012) 014510



POWLANG: results

In the following we will show results obtained within our POWHEG+Langevin setup

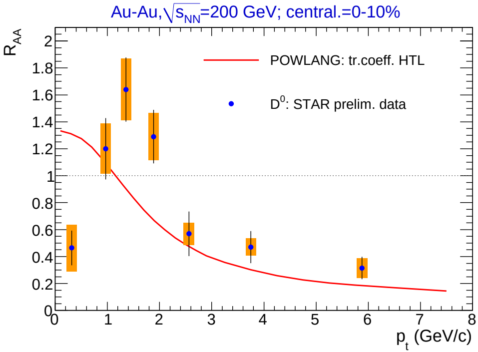
- Formalism developed in Nucl.Phys. A831 (2009) 59 and Eur.Phys.J. C71 (2011) 1666;
- Some for LHC @ 2.76 TeV presented in J.Phys. G38 (2011) 124144 and arXiv:1208.0705;
- All the following plots are part of work in progress

Initialization and cross-sections

Nuclei	$\sqrt{s_{\rm NN}}$	τ_0 (fm/c)	s_0 (fm^{-3})	T_0 (MeV)
Au-Au	200 GeV	1.0	84	333
Pb-Pb	2.76 TeV	0.6	278	475
Pb-Pb	2.76 TeV	0.1	1668	828

Collision	$\sqrt{s_{\rm NN}}$	$\sigma_{c\bar{c}}$ (mb)	$\sigma_{b\bar{b}}$(mb)
p-p	200 GeV	0.405	1.77×10^{-3}
Au-Au	200 GeV	0.356	2.03×10^{-3}
p-p	2.76 TeV	2.425	0.091
Pb-Pb	2.76 TeV	1.828	0.085

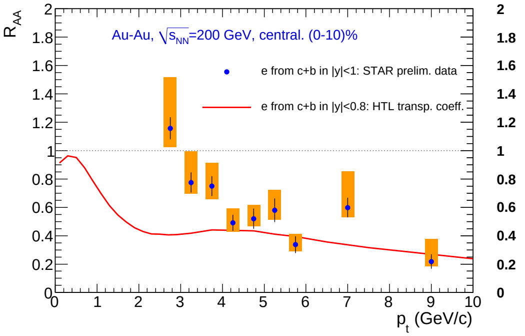
D mesons R_{AA} at RHIC



- Quenching of p_T-spectra nicely reproduced for $p_T \gtrsim 2$ GeV;
- Sharp peak around $p_T \approx 1.5$ GeV: coming from coalescence?

NB peak visible thanks to very fine binning at low-p_T

Heavy-flavour electrons R_{AA} at RHIC



- Rough agreement with the data for $p_T \gtrsim 4$ GeV;
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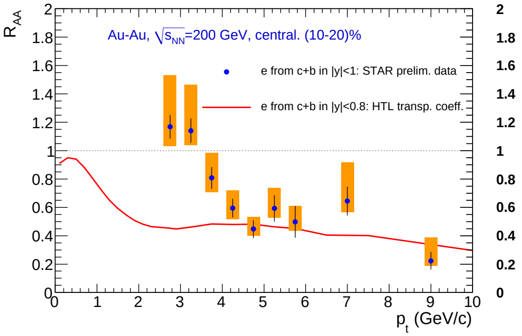

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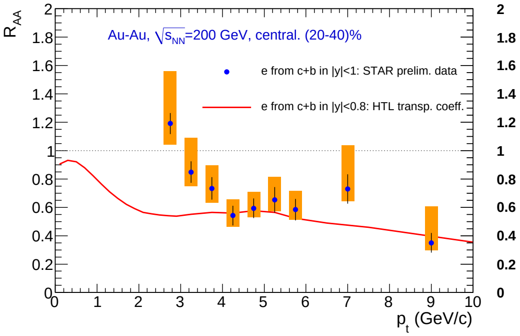

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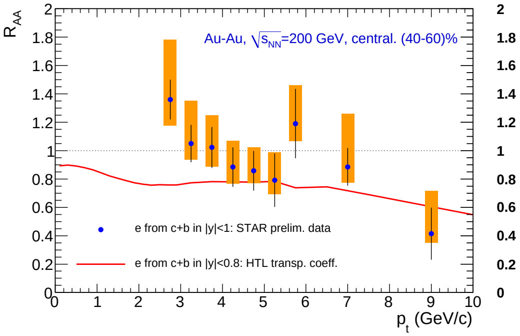

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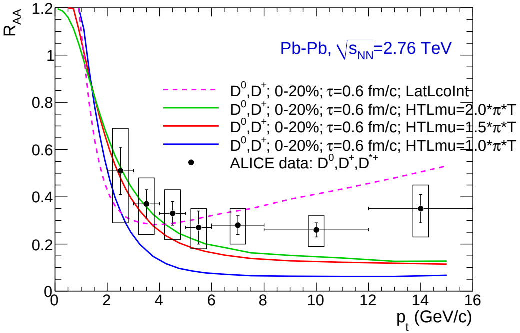

Possibility to discriminate HTL (with $\mu = \pi T - 2\pi T$) and l-QCD results at high-p_T, where however:

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- No info on momentum dependence of $\kappa_{T/L}$ is available from l-QCD

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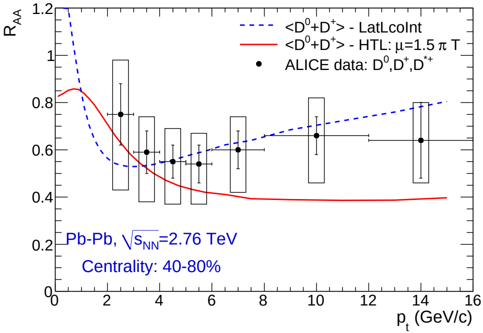

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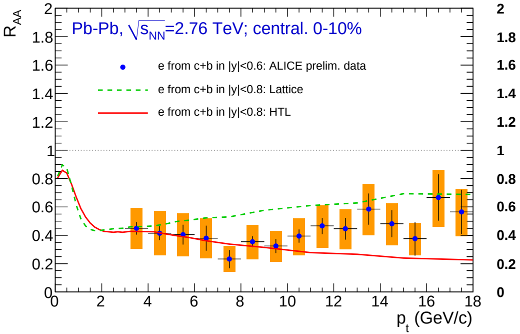


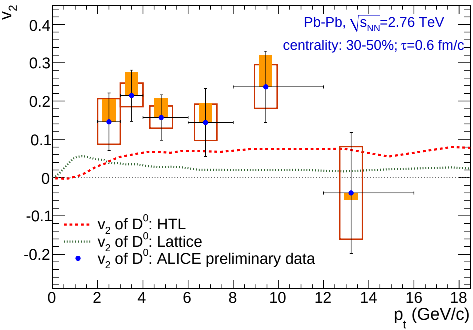
- Good agreement between HTL-Langevin and ALICE data up to ~ 10 GeV;
- For larger p_T data stays beween HTL and l-QCD predictions.

General considerations

Experimental heavy-flavour data at high-p_T always stay between the Langevin results with HTL and l-QCD tranport coefficients, suggesting for $\kappa_L(p)$ a mild rise with the quark momentum, different from

- the strong rise foreseen by the HTL+pQCD result;
- the constant behaviour assumed for the l-QCD case.

Elliptic-flow: *D*-meson v_2 at LHC



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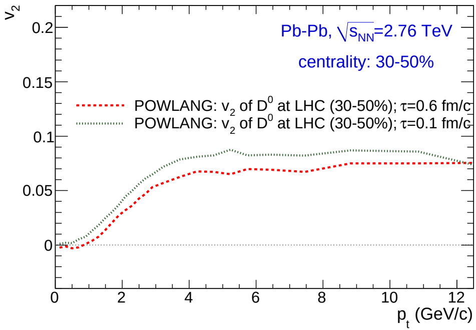

- Langevin outcomes undershoot the data, both with HTL and l-QCD transport coefficients;
- Even assuming a very short thermalization time is not sufficient to reproduce the observed flow at low-moderate p_T.

Beauty in AA collisions

Beauty: a golden probe of the medium

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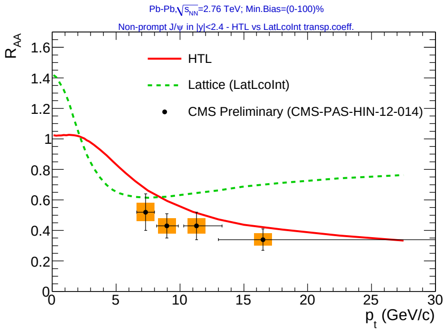
Beauty provides clean information on what happens in the partonic phase!

R_{AA} of displaced J/ψ's at LHC



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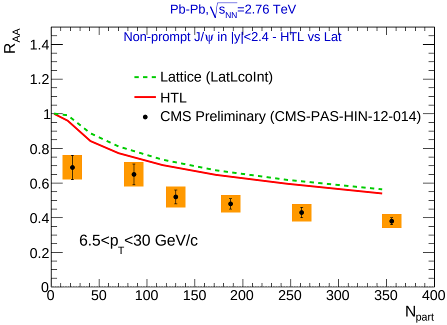

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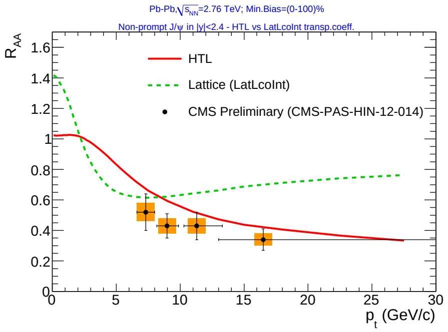

- l-QCD transport coefficients provide a *larger suppression at moderate p_T* wrt perturbative predictions;
- Ignoring momentum-dependence of l-QCD transport coefficients leads to milder suppression at high-p_T wrt HTL results;

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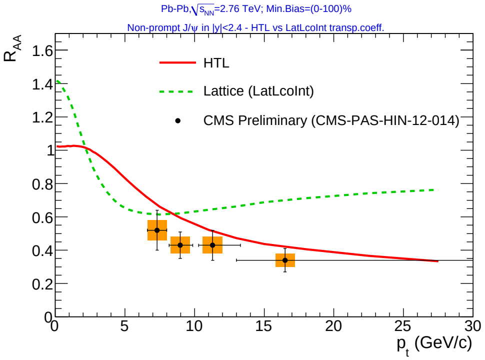

Measurements of B-mesons at low-p_T potentially able to discriminate the two scenarios in a regime in which the uncertainty on the momentum dependence of the transport coefficients shouldn't play a big role

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  - extending the analysis to forward HF observables with a realistic 3+1 hydro background (under development);
  - implementation of coalescence