

Ab Initio Calculation of Finite Temperature Charmonium Potentials

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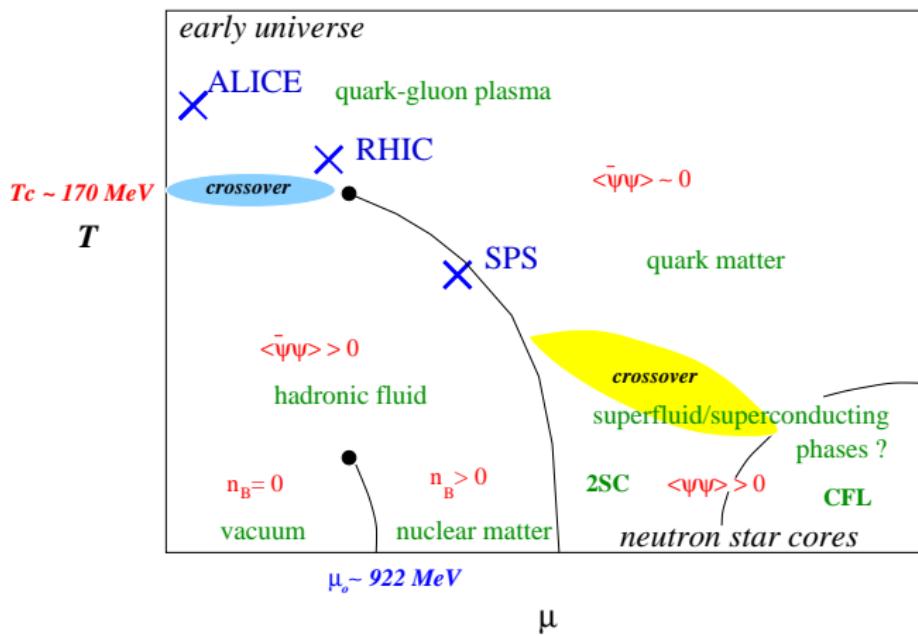
with Chris Allton and Jon-Ivar Skullerud

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ECT*

April 4, 2013

QCD Phase Diagram

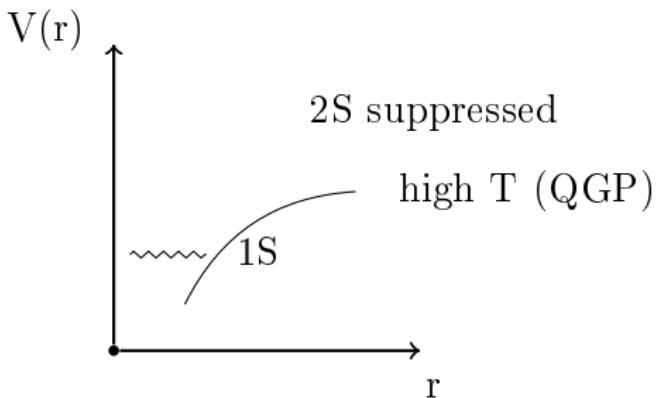
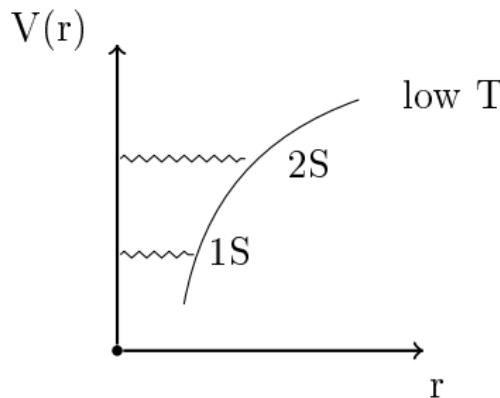


Hands 2001

Quarkonia Suppression - Color-Debye Screening

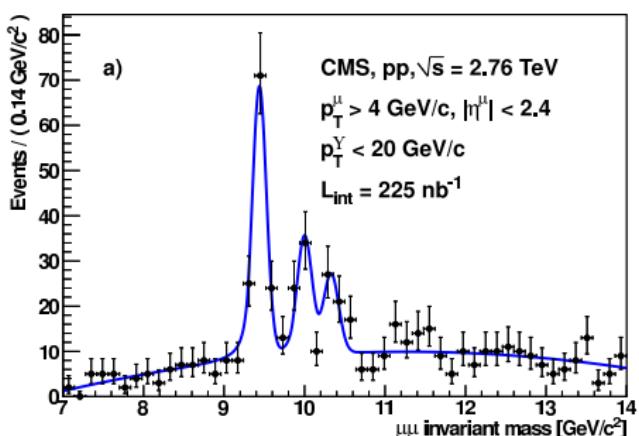
At large color charge density, inter-quark potential is modified.

$$V(r) \rightarrow -\frac{\alpha}{r} e^{-r/r_D(T)}$$
 Matsui and Satz 1986

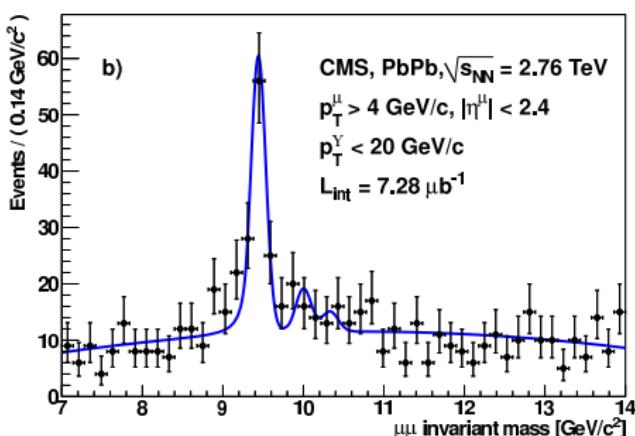


Υ Suppression at the LHC

Proton-Proton



Lead-Lead



Lattice QCD Approach

$$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}[\psi, \bar{\psi}, A] e^{-S_{QCD}[\psi, \bar{\psi}, A]} O[\psi, \bar{\psi}, A],$$

$$\langle O \rangle \approx \frac{1}{N} \sum_{U_n}^{U_N} O[U_n]$$

U_n sampled with probability $\propto e^{-S_{LQCD}}$

Finite Temperature

Compare the partition function of statistical mechanics,

$$Z = \sum_s e^{-\beta E_s},$$

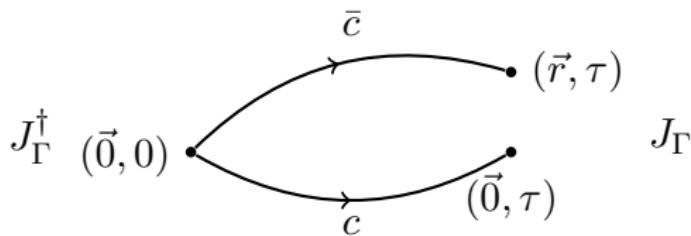
with that of LQCD in a suggestive form,

$$Z = \sum_j e^{-N_\tau a_\tau E_j},$$

to relate T to N_τ ,

$$N_\tau a_\tau \propto \frac{1}{T}.$$

Local-Extended Meson Correlators



Define general charmonium interpolator,

$$J_\Gamma(x; \mathbf{r}) = \bar{c}(x)\Gamma U(x, x + \mathbf{r})c(x + \mathbf{r})$$

then correlation functions can be written,

$$\begin{aligned} C_\Gamma(\mathbf{r}, \tau) &= \sum_{\mathbf{x}} \langle J_\Gamma(\mathbf{x}, \tau; \mathbf{r}) J_\Gamma^\dagger(0; \mathbf{0}) \rangle \\ &= \sum_j \frac{\psi_j^*(\mathbf{0}) \psi_j(\mathbf{r})}{2E_j} \left(e^{-E_j \tau} + e^{-E_j(N_\tau - \tau)} \right) \end{aligned}$$

Time-Dependent Potential

HALQCD Collaboration 2012

Consider only forward moving contribution of $C_\Gamma(\mathbf{r}, \tau)$,

$$\begin{aligned} C_\Gamma(\mathbf{r}, \tau) &= \sum_j \frac{\psi_j^*(\mathbf{0})\psi_j(\mathbf{r})}{2E_j} e^{-E_j\tau} \\ &= \sum_j \Psi_j(\mathbf{r}) e^{-E_j\tau} \end{aligned}$$

Differentiate w.r.t. τ ,

$$\frac{\partial}{\partial \tau} C_\Gamma(\mathbf{r}, \tau) = - \sum_j E_j \Psi_j(\mathbf{r}) e^{-E_j\tau}$$

Now consider Schrödinger equation for $\Psi_j(\mathbf{r})$,

$$\left(-\frac{\nabla^2}{2\mu} + V_\Gamma(\mathbf{r}) \right) \Psi_j(\mathbf{r}) = E_j \Psi_j(\mathbf{r})$$

$$\begin{aligned}\frac{\partial}{\partial \tau} C_{\Gamma}(\mathbf{r}, \tau) &= \sum_j \left(\frac{\nabla^2}{2\mu} - V_{\Gamma}(\mathbf{r}) \right) \Psi_j(\mathbf{r}) e^{-E_j \tau} \\ &= \left(\frac{\nabla^2}{2\mu} - V_{\Gamma}(\mathbf{r}) \right) \sum_j \Psi_j(\mathbf{r}) e^{-E_j \tau} \\ &= \left(\frac{\nabla^2}{2\mu} - V_{\Gamma}(\mathbf{r}) \right) C(\mathbf{r}, \tau)\end{aligned}$$
$$\implies V_{\Gamma}(\mathbf{r}) = \left(\frac{\nabla^2 C_{\Gamma}(\mathbf{r}, \tau)}{2\mu} - \frac{\partial C_{\Gamma}(\mathbf{r}, \tau)}{\partial \tau} \right) \frac{1}{C_{\Gamma}(\mathbf{r}, \tau)}$$

S-Waves

The S-wave potential can be expressed as:

$$V_{\Gamma}(\mathbf{r}) = V_C(\mathbf{r}) + s_1 \cdot s_2 V_S(\mathbf{r}).$$

$s_1 \cdot s_2 = -3/4, 1/4$ for the pseudoscalar and vector respectively.

Hence,

$$V_C(\mathbf{r}) = \frac{1}{4}V_{PS}(\mathbf{r}) + \frac{3}{4}V_V(\mathbf{r})$$

and

$$V_S(\mathbf{r}) = V_V(\mathbf{r}) - V_{PS}(\mathbf{r})$$

Simulation Details

N_s	N_τ	T (MeV)	T/T_c	N_{cfg}
12	80	90	0.42	250
12	32	230	1.05	1000
12	28	263	1.20	1000
12	24	306	1.40	500
12	20	368	1.68	1000

Ensembles

Two-Plaquette Symanzik gauge action, $N_f = 2$ dynamical sea quarks,
Hamber-Wu fermion action with stout-link smearing,
Anisotropic lattice spacing: $a_s/a_\tau = 6$.

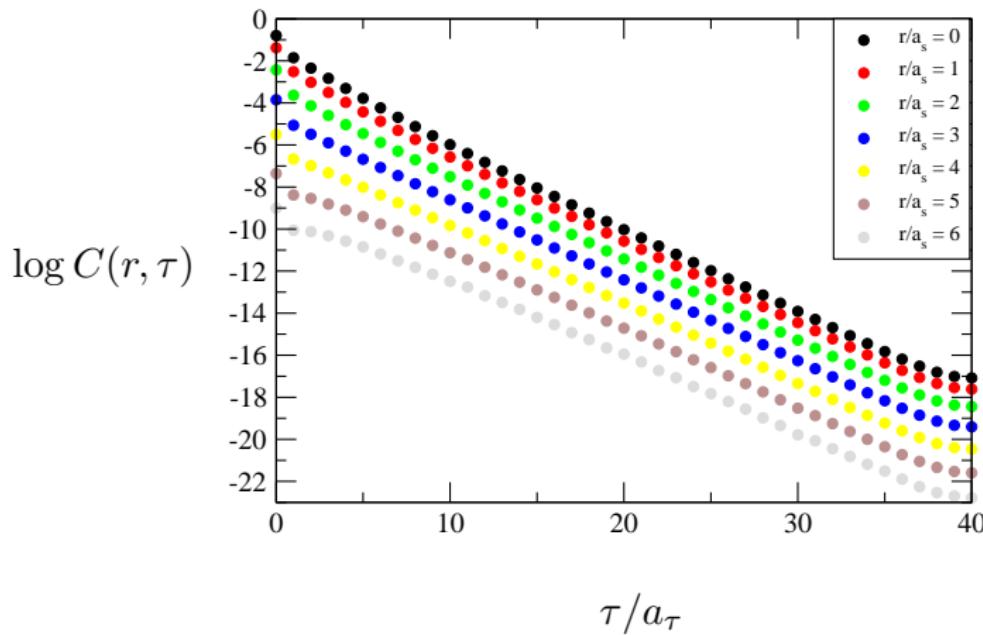
Measurement

Wilson Clover action,
Pseudoscalar effective mass tuned to experimental η_c mass,
Gaussian smear sources.

Chroma: Edwards and Joó

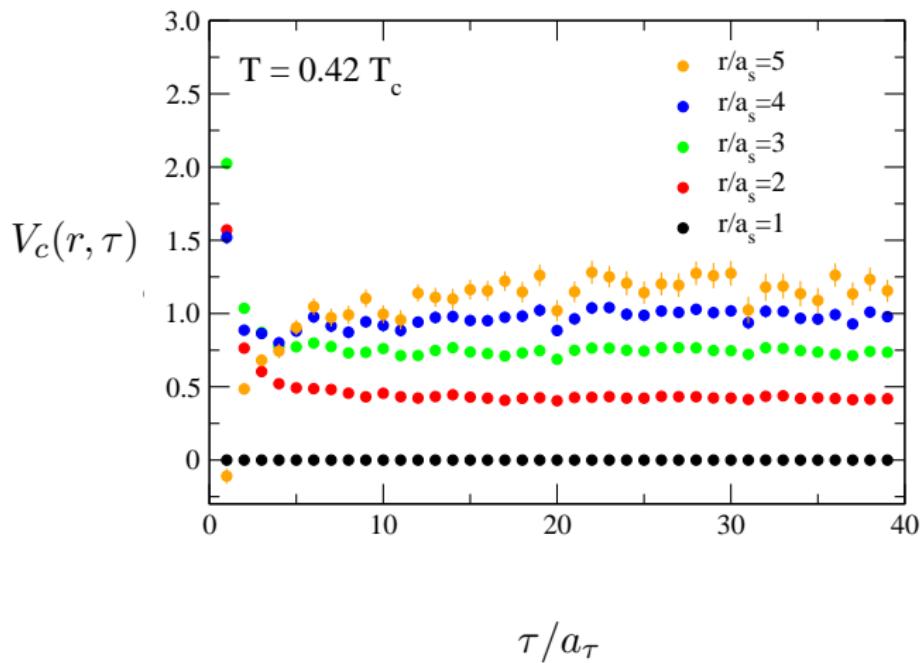
Results

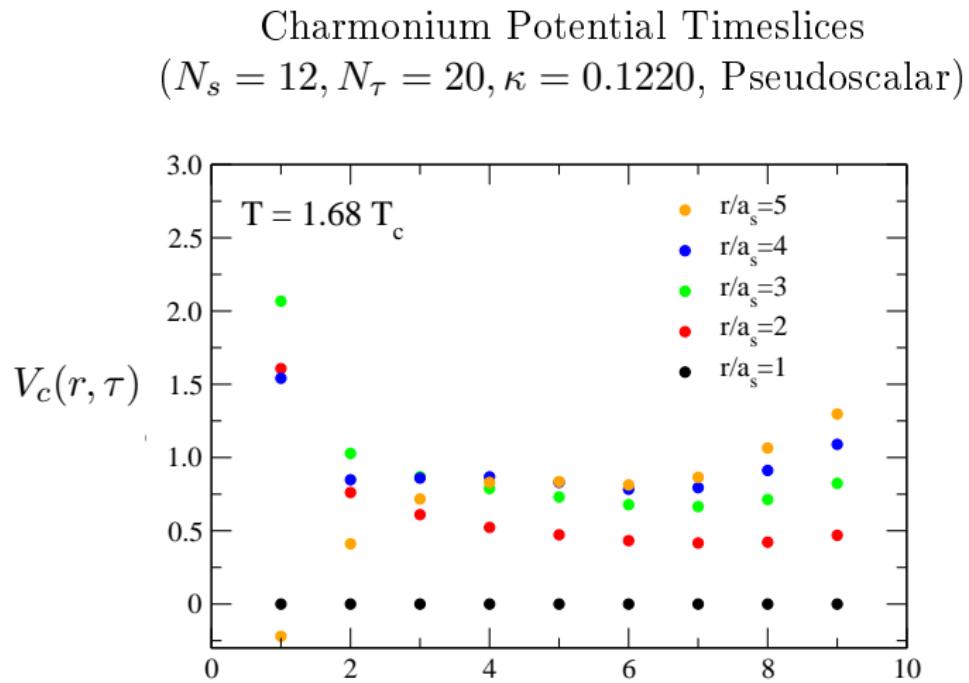
Charmonium Correlation Functions
($N_s = 12, N_\tau = 80, \kappa = 0.1220$, Pseudoscalar)



Charmonium Potential Timeslices

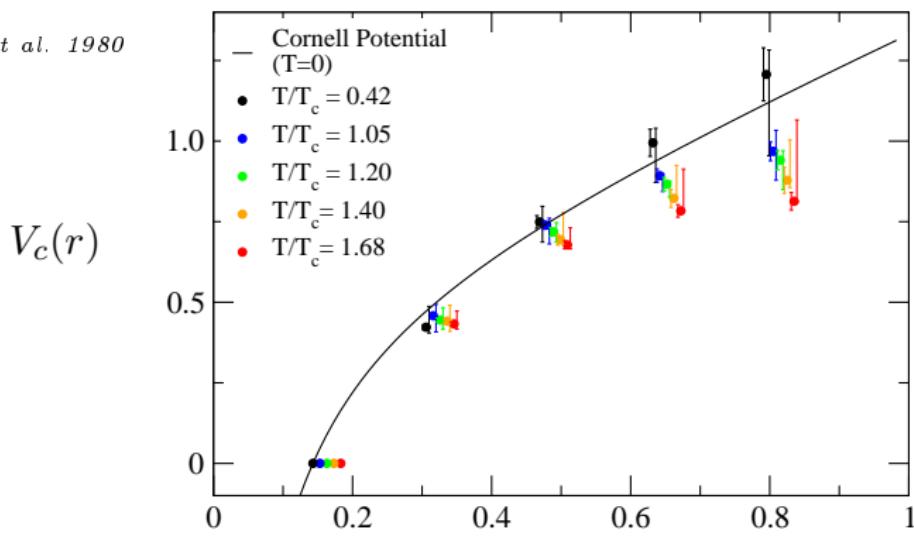
($N_s = 12$, $N_\tau = 80$, $\kappa = 0.1220$, Pseudoscalar)



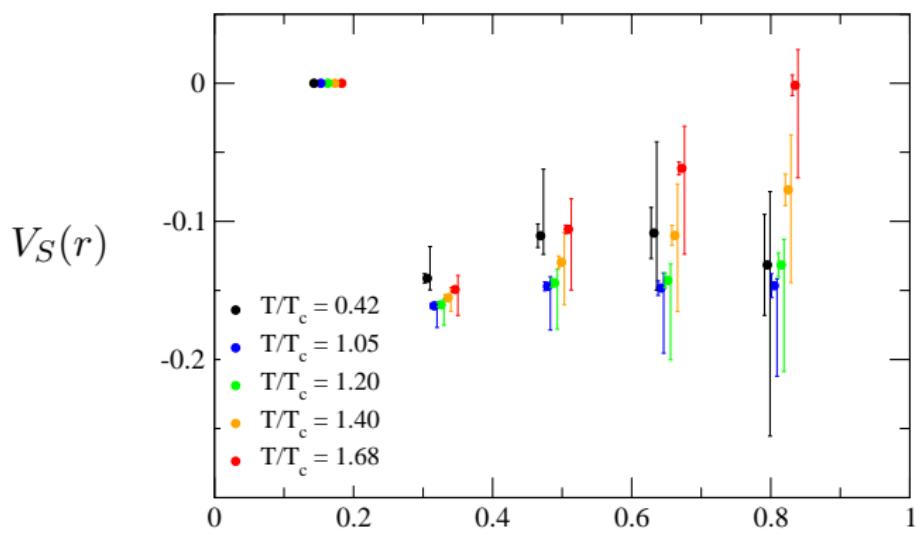


$$\tau/a_\tau$$

Spin-Independent Charmonium Potential

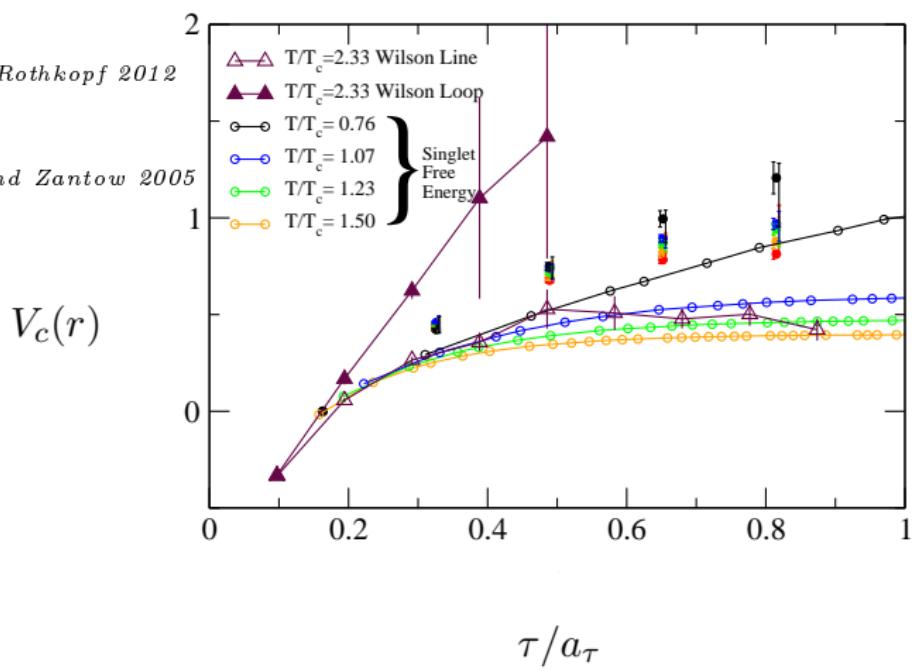
Eichten et al. 1980

Spin-Dependent Charmonium Potential



$$\tau/a_\tau$$

Spin-Independent Charmonium Potential

*Burnier and Rothkopf 2012**Kaczmarek and Zantow 2005*

Conclusion

Time-dependent method produces sensible inter-quark potential.

Temperature dependence of potential is consistent with deconfinement.

Rough agreement with previous studies, but boldly conclude discrepancies are to be expected since this is the first dynamical, finite temperature study of the charmonium potential.

Coming soon... charmonium potential from 2+1 flavour,
 $N_s = 32^3$ configurations, at all possible values of r , not just on-axes values.