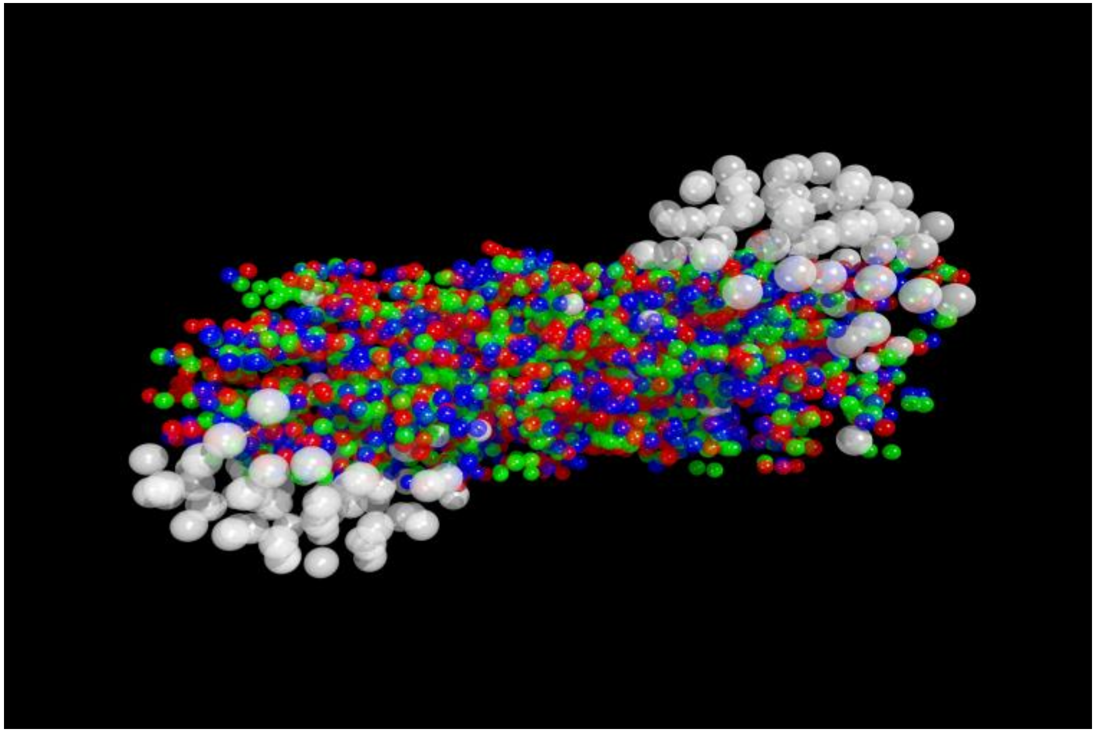
University of Catania
INFN-LNS

Heavy flavor Suppression : Langevin vs Boltzmann



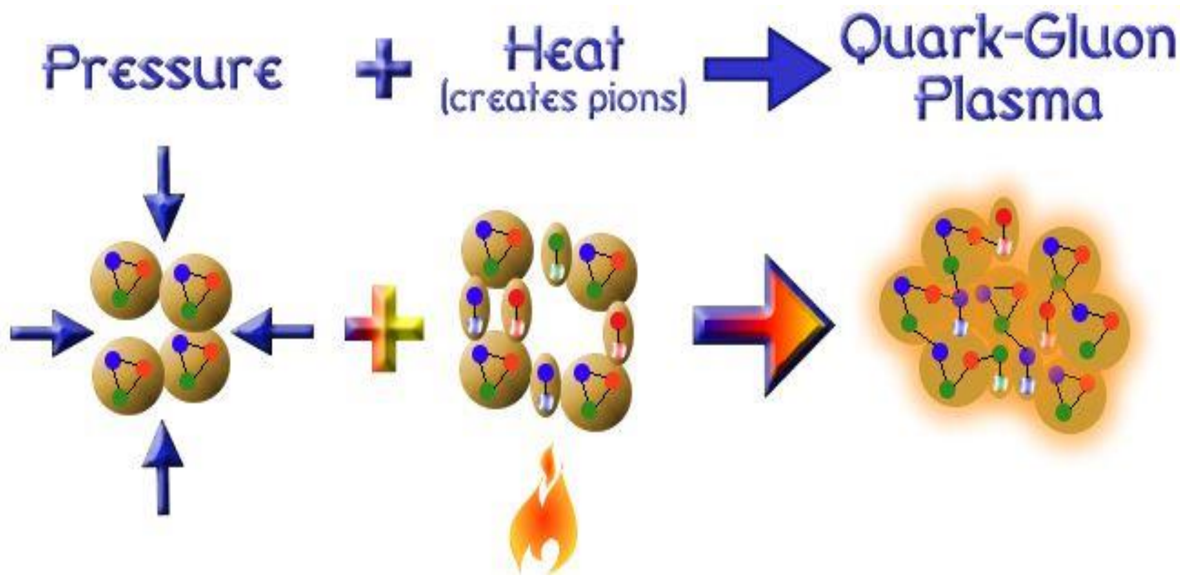
S. K. Das, F. Scardina
V. Greco, S. Plumari

OUTLINE OF OUR TALK..............

- Introduction
- Langevin Equation and the Thermalization Issue
- Boltzmann Equation and the Thermalization Issue
- Nuclear Suppression: Langevin vs Boltzmann
- Summary and outlook

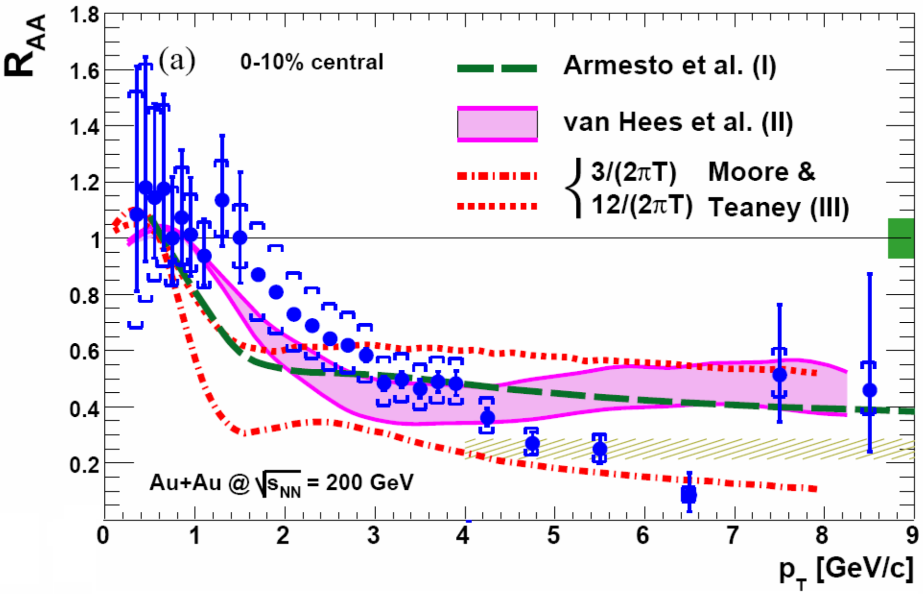
Introduction

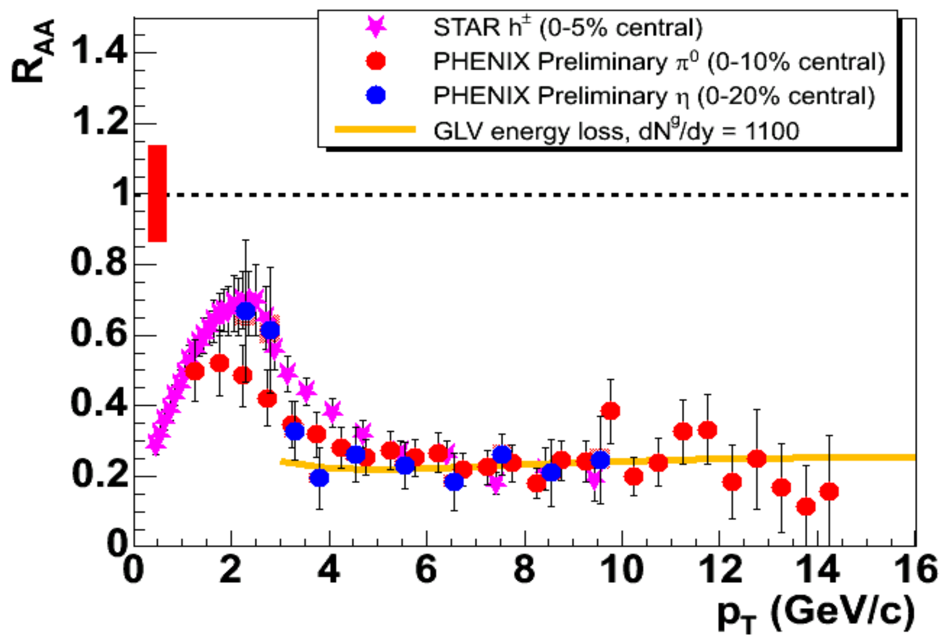
At very high temperature and density hadrons melt to a new phase of matter called Quark Gluon Plasma (QGP).



$$\tau_{HQ} > \tau_{LQ}, \quad \tau_{HQ} \sim (M/T)\, \tau_{LQ}$$

Heavy flavor at RHIC

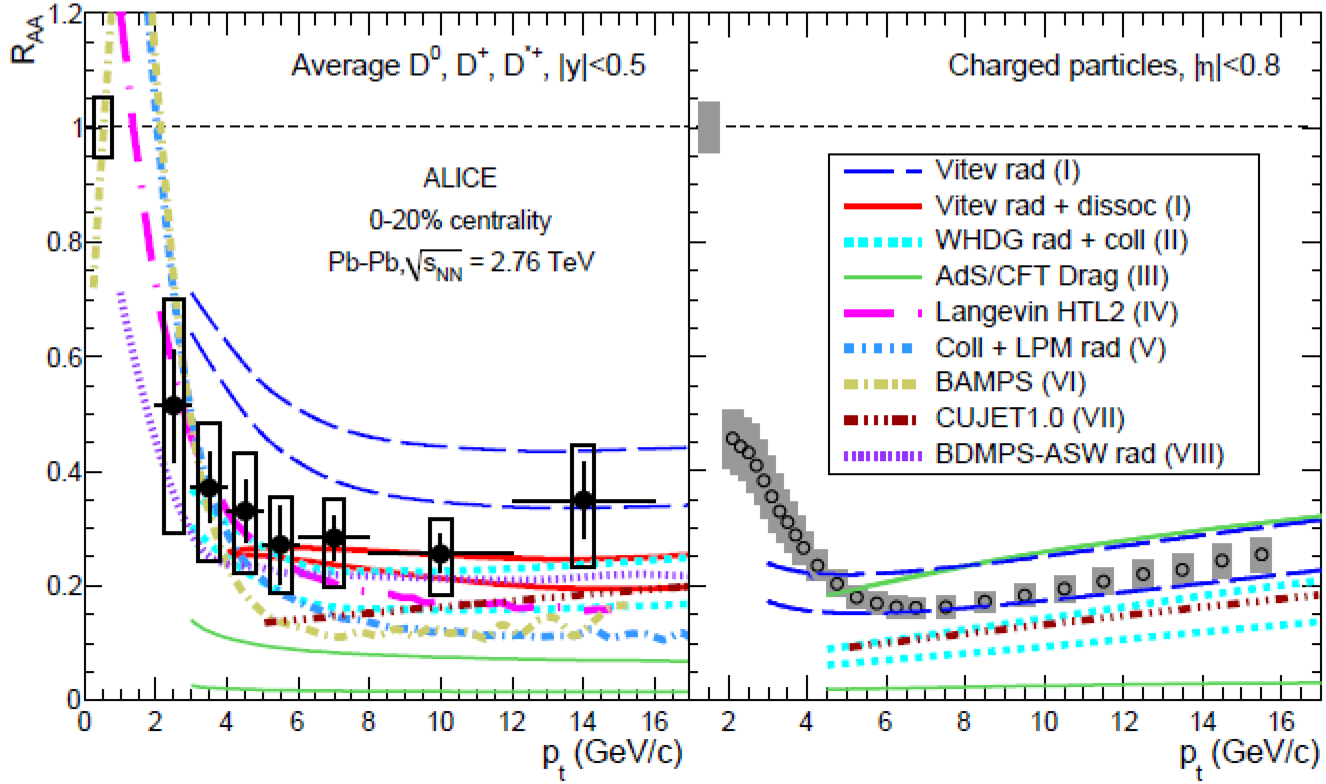




PHENIX: *PRL98(2007)172301*

At RHIC energy heavy flavor suppression is similar to light flavor

Heavy Flavors at LHC



arXiv:1203.2160
ALICE Collaboration

Again at RHIC energy heavy flavor suppression is similar to light flavor

Is the HQ momentum transfer really small !

Boltzmann Kinetic equation

$$\left(\frac{\partial}{\partial t}+\frac{P}{E}\frac{\partial}{\partial x}+\mathrm{F}.\frac{\partial}{\partial p}\right)f\left(x,p,t\right)=\left(\frac{\partial f}{\partial t}\right)_{col}$$

$$R(p,t)=\left(\frac{\partial f}{\partial t}\right)_{col}=\int d^3k\left[\omega(p+k,k)f(p+k)-\omega(p,k)f(p)\right]$$

$$\omega(p,k)=g\int\frac{d^3q}{(2\pi)^3}f'(q)v_{q,p}\sigma_{p,q\to p-k,q+k}$$

→ is rate of collisions which change the momentum of the charmed quark from p to p-k

$$\omega(p+k,k)f(p+k)\approx\omega(p,k)f(p)+k.\frac{\partial}{\partial p}(\omega f)+\frac{1}{2}k_ik_j\frac{\partial^2}{\partial p_i\partial p_j}(\omega f)$$

$$\frac{\partial \mathbf{f}}{\partial \mathbf{t}}=\frac{\partial}{\partial \mathbf{p_i}}\left[\mathbf{A_i}(\mathbf{p})\mathbf{f}+\frac{\partial}{\partial \mathbf{p_j}}\left[\mathbf{B_{ij}}(\mathbf{p})\mathbf{f}\right]\right]$$

B. Svetitsky PRD 37(1987)2484

where we have defined the kernels

,

$$\mathbf{A_i}=\int \mathbf{d^3k}\,\omega(\mathbf{p},\mathbf{k})\mathbf{k_i}\quad\rightarrow\textbf{Drag Coefficient}$$

$$\mathbf{B_{ij}}=\int \mathbf{d^3k}\,\omega(\mathbf{p},\mathbf{k})\mathbf{k_i}\mathbf{k_j}\quad\rightarrow\textbf{Diffusion Coefficient}$$

$$\omega(p+k,k)\,f(p+k) \approx \omega(p,k)\,f(p) + k.\frac{\partial}{\partial p}(\omega f) + \frac{1}{2}k_i k_j \frac{\partial^2}{\partial p_i \partial p_j}(\omega f)$$

Boltzmann Equation

Fokker Planck

It is interesting to study both the equation in a identical environment to ensure the validity of this assumption.

Langevin Equation

$$dx_j = \frac{p_j}{E} dt$$

$$dp_j = -\Gamma p_j dt + \sqrt{dt} C_{jk}(t, p + \xi dp)\rho_k$$

where Γ **is the deterministic friction (drag) force**

C_{ij} **is stochastic force in terms of independent Gaussian-normal distributed random variable**

$$\rho = (\rho_x, \rho_y, \rho_z) \quad , \quad P(\rho) = \left(\frac{1}{2\pi}\right)^3 \exp(-\frac{\rho^2}{2})$$

With $<\rho_i(t)\rho_k(t')> = \delta(t-t')\delta_{jk}$

$\xi = 0$ **the pre-point Ito**

$= \frac{1}{2}$ **the mid-point Stratonovic-Fisk**

$= 1$ **the post-point Ito (or H¨anggi-Klimontovich)**

interpretation of the momentum argument of the covariance matrix.

H. v. Hees and R. Rapp
arXiv:0903.1096

Langevin process defined like this is equivalent to the Fokker-Planck equation:

$$\frac{\partial f}{\partial t} + \frac{p_j}{E}\frac{\partial f}{\partial x_j} = \frac{\partial}{\partial p_j}\left[\left(p_j\Gamma - \xi C_{lk}\frac{\partial C_{jk}}{\partial p_l}\right) f\right] + \frac{1}{2}\frac{\partial^2}{\partial p_j \partial p_k}(C_{jl}C_{kl}f)$$

the covariance matrix is related to the diffusion matrix by

$$C_{jk} = \sqrt{2B_0}P_{jk}^{\perp} + \sqrt{2B_1}P_{jk}^{\parallel}$$

and $$A_i = p_j\Gamma - \xi C_{lk}\frac{\partial C_{ij}}{\partial p_l}$$

With $B_0 = B_1 = D$ $$C_{jk} = \sqrt{2D(E)}\delta_{jk}$$

Relativistic dissipation-fluctuation relation

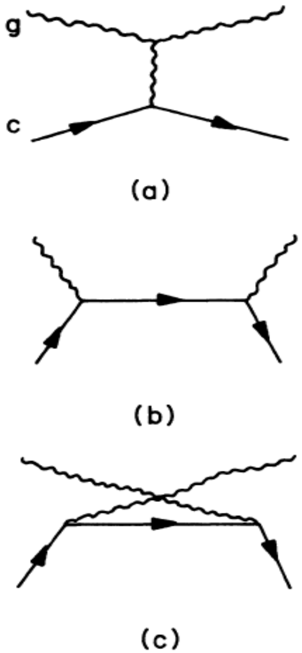
$$A(E)ET - D(E) + T(1-\xi)D'(E) = 0$$

For Collision Process the A_i and B_{ij} can be calculated as following :

$$A_i = \frac{1}{2E_p}\int\frac{d^3q}{(2\pi)^3 2E_q}\int\frac{d^3q'}{(2\pi)^3 2E_{q'}}\int\frac{d^3p'}{(2\pi)^3 2E_{p'}}\frac{1}{\gamma_c}\sum|M|^2(2\pi)^4\delta^4(p+q-p'-q')f(q)[(p-p')_i] = \langle\langle(p-p')_i\rangle\rangle$$

$$B_{ij} = \frac{1}{2}\langle\langle(p-p')_i(p'-p)_j\rangle\rangle$$

Elastic processes



$$gc \rightarrow gc$$

✓ We have introduce a mass into the internal gluon propagator in the t and u-channel-exchange diagrams, to shield the infrared divergence.

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Thermalization in Langevin approach in a static medium

1) Diffusion D=Constant
   Drag A= D/ET from FDT

2) Diffusion D=D(p)
   Drag A(p) =D(p)/ET

3) Diffusion D(p)
   Drag A(p): from FDT + derivative term

$$A(E)ET - D(E) + T(1-\xi)D'(E) = 0$$

4) Diffusion D(p) and Drag A(p) both from pQCD

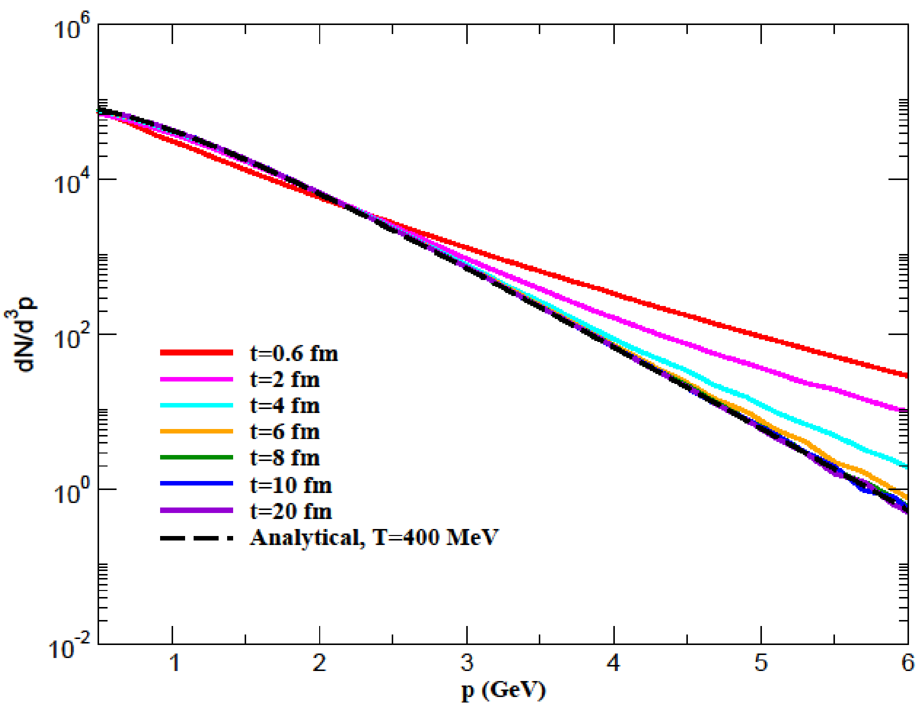
We are solving Langevin Equation in a box.

Case:1

1) D=Constant
   A= D/ET from FDT

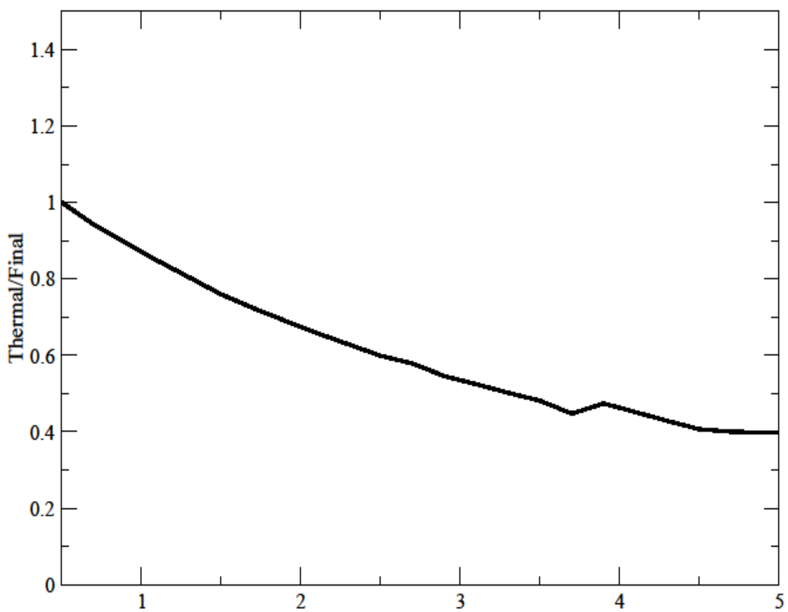
Due to the collision charm approaches to thermal equilibrium with the bulk

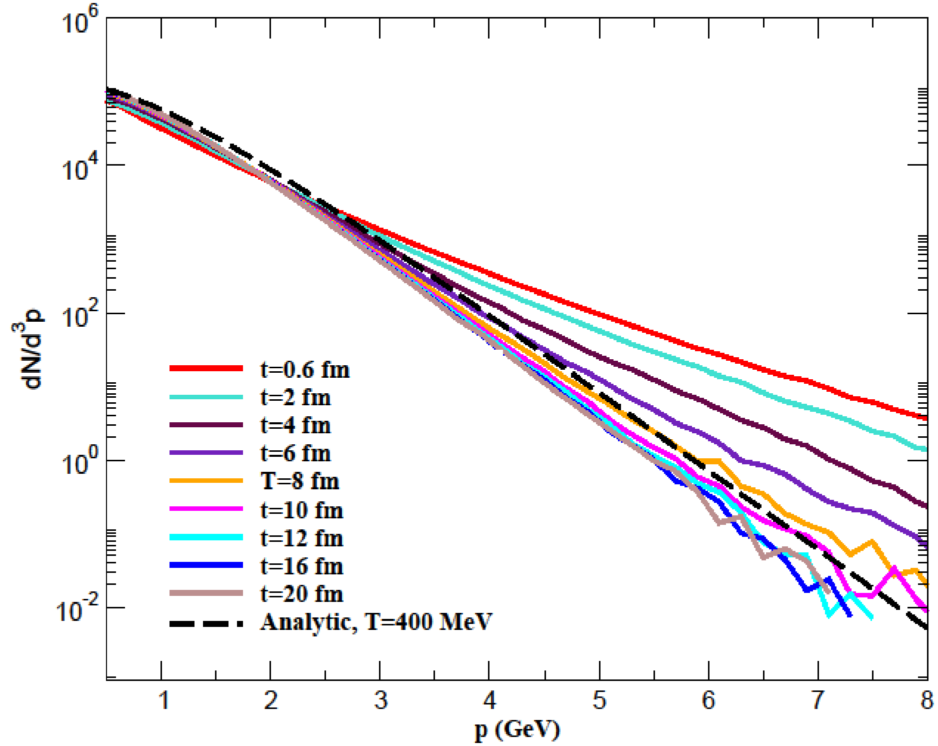
Bulk composed only by gluon in Thermal equilibrium at T= 400 MeV.



Case:2

Diffusion coefficient: D(p)
Drag coefficient: A(p)=D(p)/TE

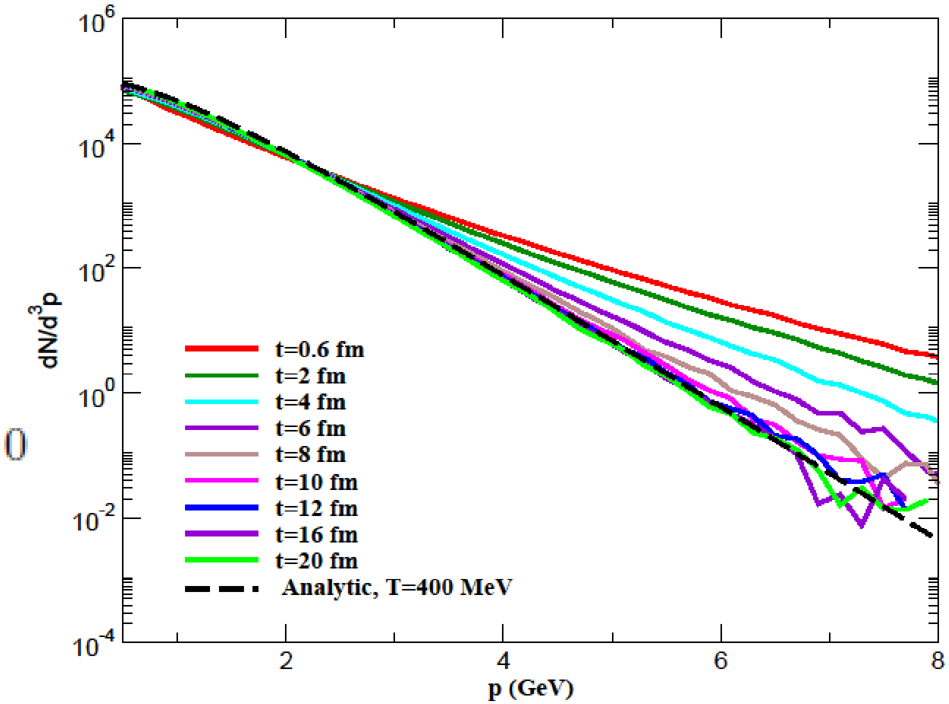




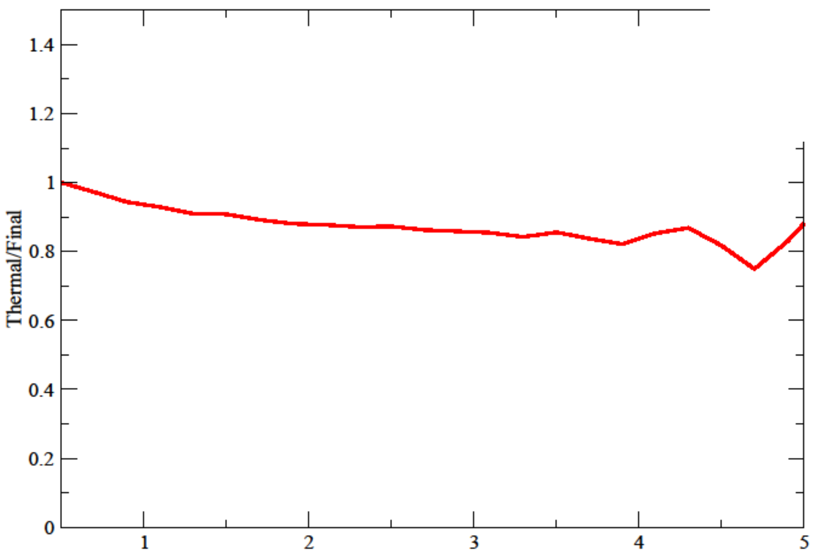
In this case we are away from thermalization around 50-60 %

Case:3

Diffusion coefficient: D(p)
Drag coefficient: From FDT with the derivative term



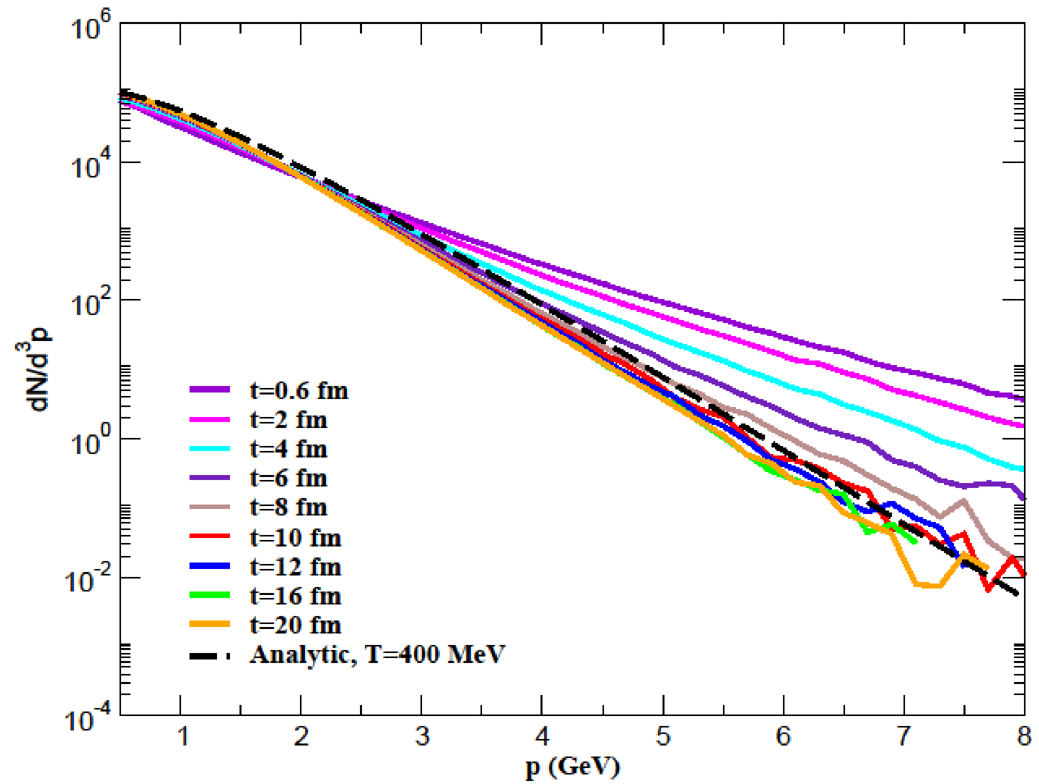
$$A(E)ET - D(E) + T(1-\xi)D'(E) = 0$$

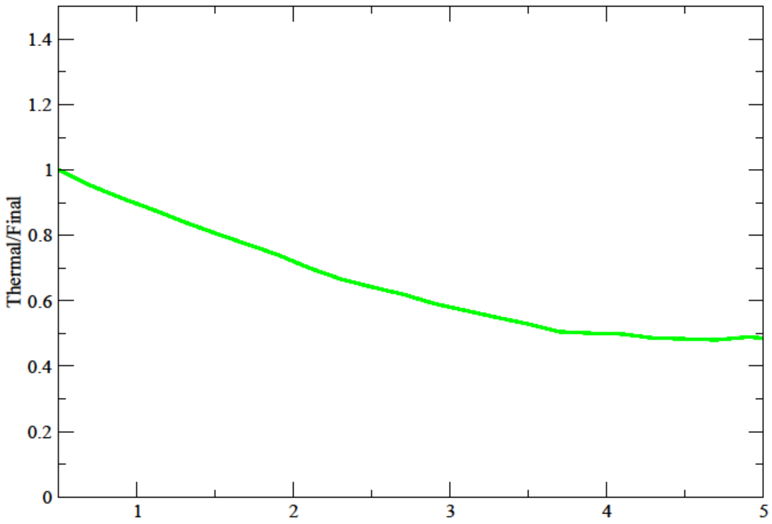


Implementation of the derivative term improve the results. But still we are around 10 % away from the thermal equilibrium.

Case: 4

Diffusion coefficient: D(p) pQCD
Drag coefficient: A(p) pQCD





In this case we are away from thermalization around 40-50 %.

More realistic value of drag and diffusion

↓

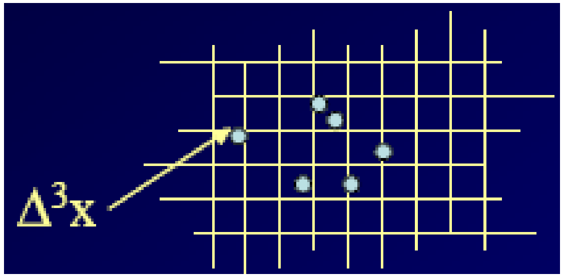
More we away from the thermalization !!!

Transport theory

$$p^{\mu}\partial_{\mu} f(x,p) = C_{22}$$

We consider two body collisions

$$C_{22} = \frac{1}{2E_1}\int \frac{d^3p_2}{(2\pi)^3 2E_2}\frac{1}{\nu}\int \frac{d^3p_1'}{(2\pi)^3 2E_1'}\frac{d^3p_2'}{(2\pi)^3 2E_2'} f_1' f_2' |\mathcal{M}_{1'2'\to 12}|^2 (2\pi)^4 \delta^{(4)}(p_1'+p_2'-p_1-p_2)$$
$$-\frac{1}{2E_1}\int \frac{d^3p_2}{(2\pi)^3 2E_2}\frac{1}{\nu}\int \frac{d^3p_1'}{(2\pi)^3 2E_1'}\frac{d^3p_2'}{(2\pi)^3 2E_2'} f_1 f_2 |\mathcal{M}_{12\to 1'2'}|^2 (2\pi)^4 \delta^{(4)}(p_1+p_2-p_1'-p_2')$$



$$\Delta t \to 0$$
$$\Delta^3 x \to 0$$

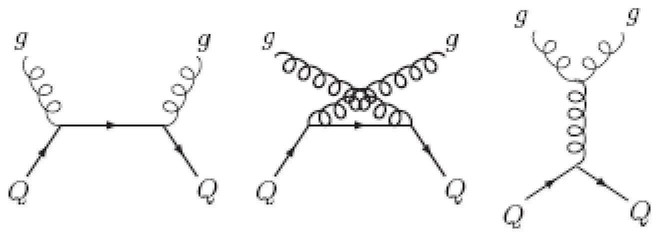
Exact solution

Collision integral is solved with a local stochastic sampling

$$P_{22} = \frac{\Delta N_{coll}^{2\to 2}}{\Delta N_1 \Delta N_2} = v_{rel}\sigma_{22}\frac{\Delta t}{\Delta^3 x}$$

Greco et al PLB670, 325 (08)]
[Z. Xhu, et al. PRC71(04)]

Cross Section gc -> gc



$$\sum|\mathcal{M}|^2 = \pi^2\alpha^2(Q^2)\left[\frac{32(s-M^2)(M^2-u)}{t^2} + \frac{64}{9}\frac{(s-M^2)(M^2-u)+2M^2(s+M^2)}{(s-M^2)^2}\right.$$

$$+\frac{64}{9}\frac{(s-M^2)(M^2-u)+2M^2(M^2+u)}{(M^2-u)^2} + \frac{16}{9}\frac{M^2(4M^2-t)}{(s-M^2)(M^2-u)}$$

$$\left.+16\frac{(s-M^2)(M^2-u)+M^2(s-u)}{t(s-M^2)} - 16\frac{(s-M^2)(M^2-u)-M^2(s-u)}{t(M^2-u)}\right].$$

The infrared singularity is regularized introducing a Debye-screaning-mass μ_D

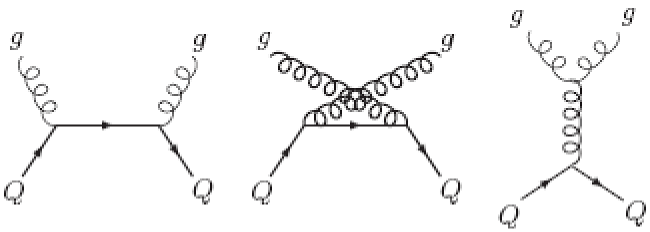
$$\frac{1}{t} \rightarrow \frac{1}{t-m_D}$$

$$m_D = \sqrt{4\pi\alpha_s}T$$

L. Combridge, Nucl. Phys. B151, 429 (1979)]

[B. Svetitsky, Phys. Rev. D 37, 2484 (1988)]

Cross Section gc -> gc



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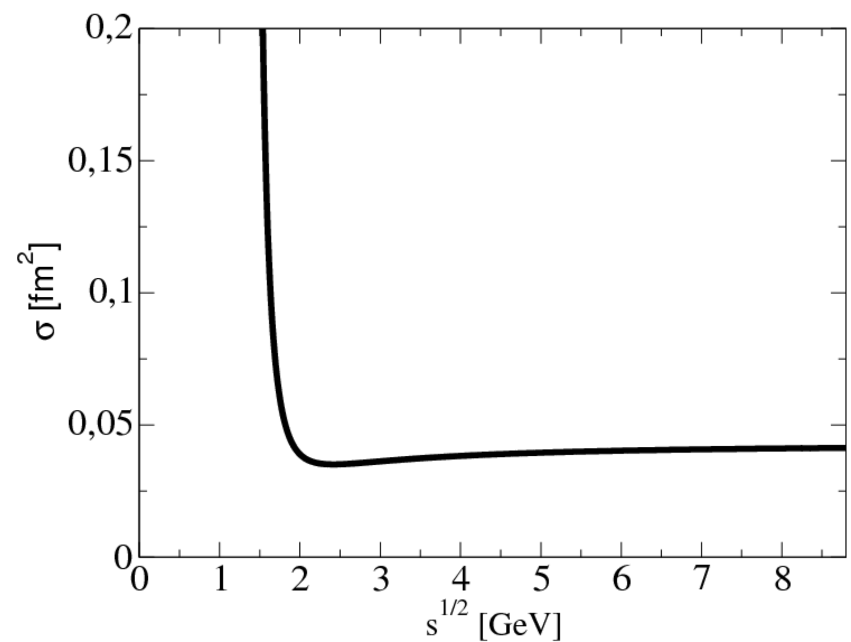
$$\hat{\sigma} = \frac{1}{16\pi(s-M^2)^2}\int_{-(s-M^2)^2/s}^{0} \mathrm{d}t \sum |\mathcal{M}|^2$$



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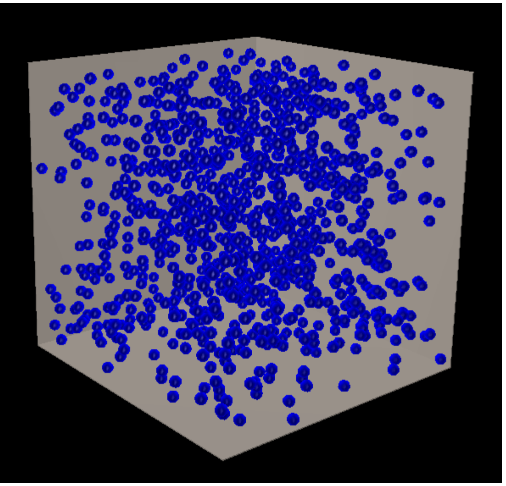
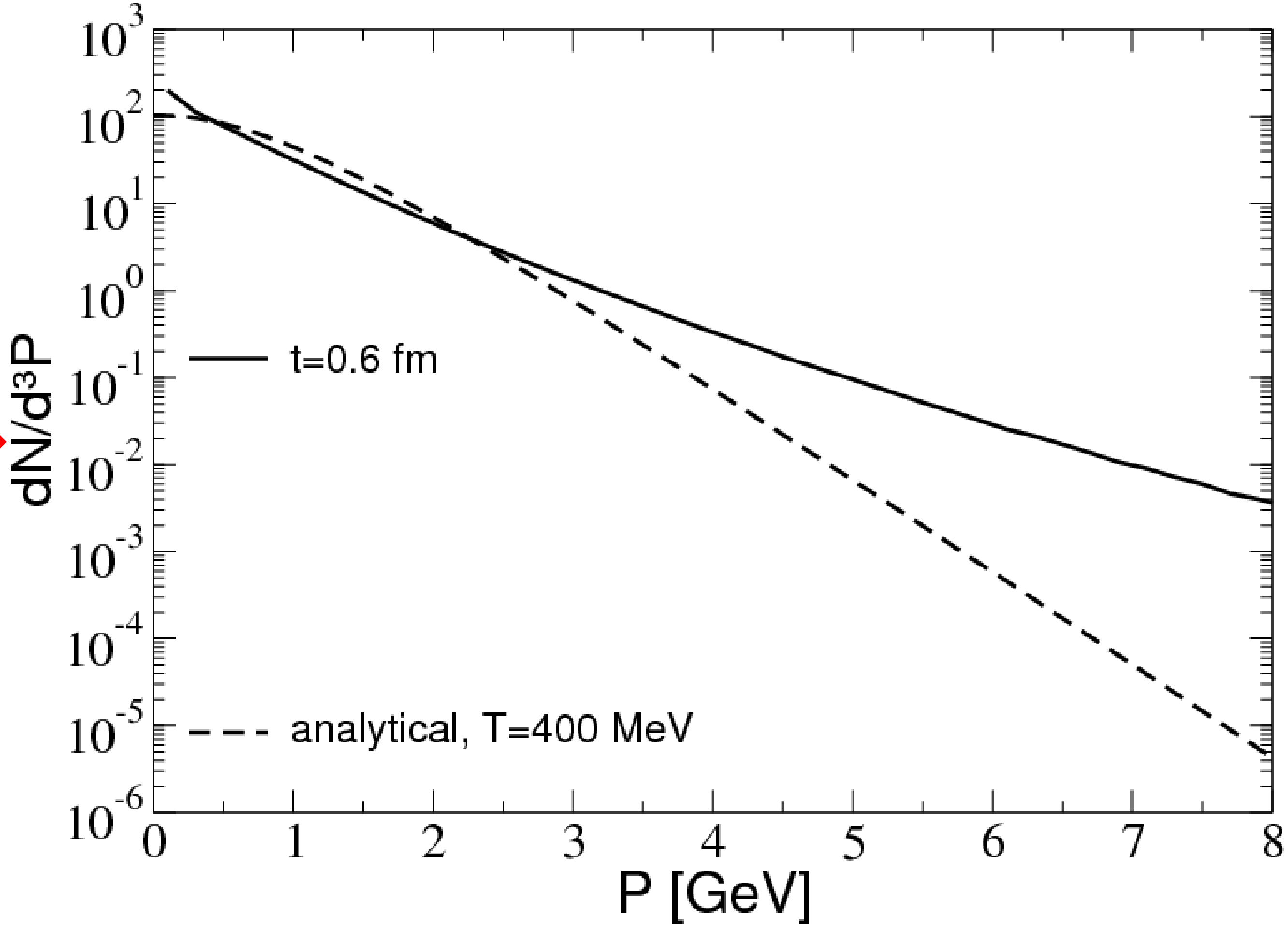
Charm evolution in a static medium



Simulations in which a particle ensemble in a box evolves dynamically

Bulk composed only by gluons in thermal equilibrium at T=400 MeV

C and $\bar{C}$ initially are distributed: uniformily in r-space, while in p-space



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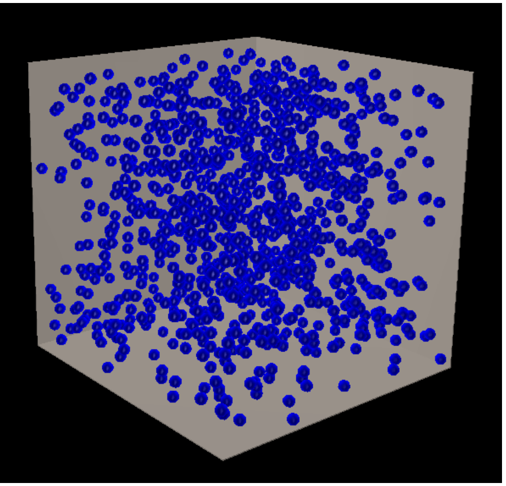

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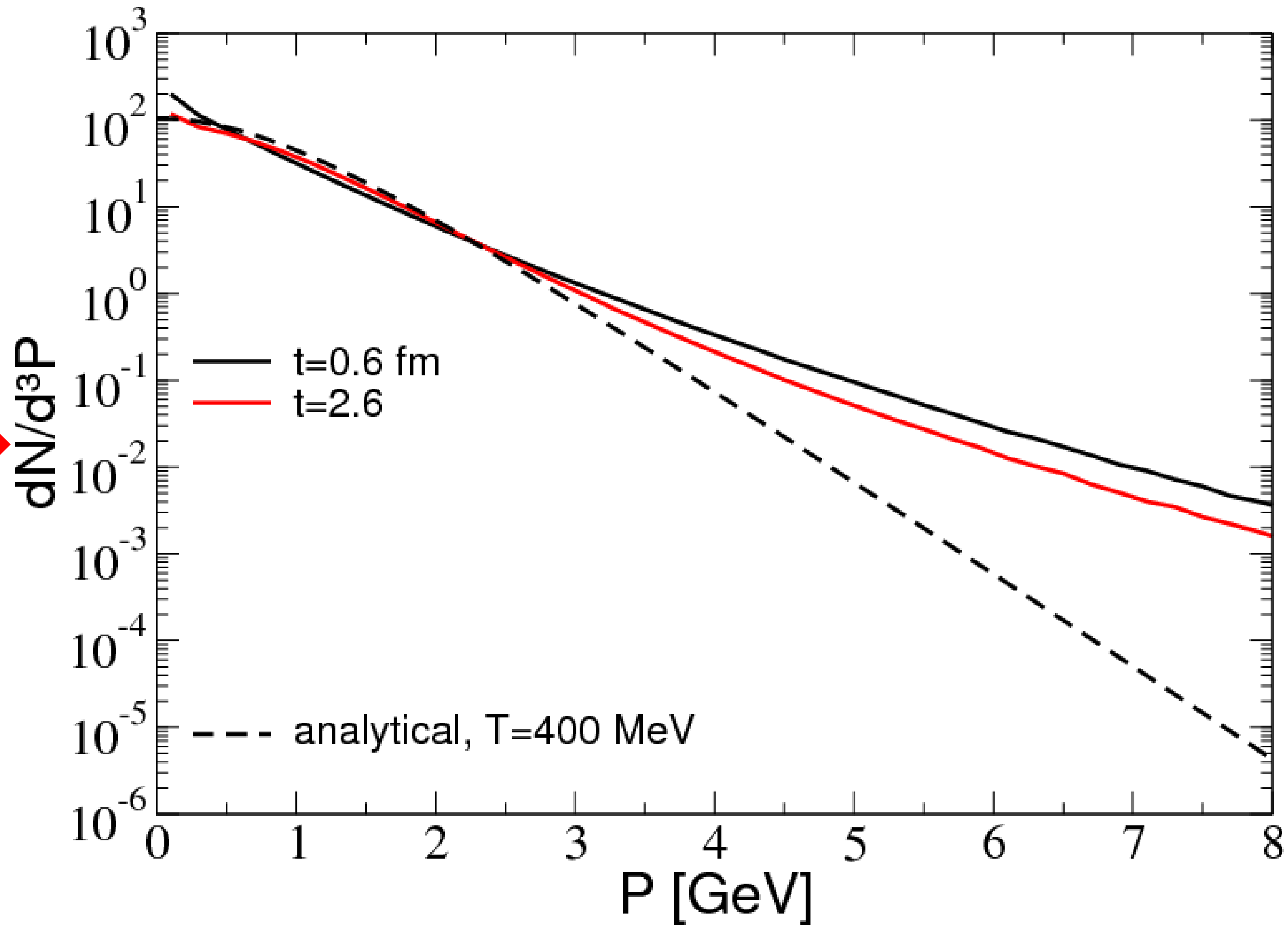

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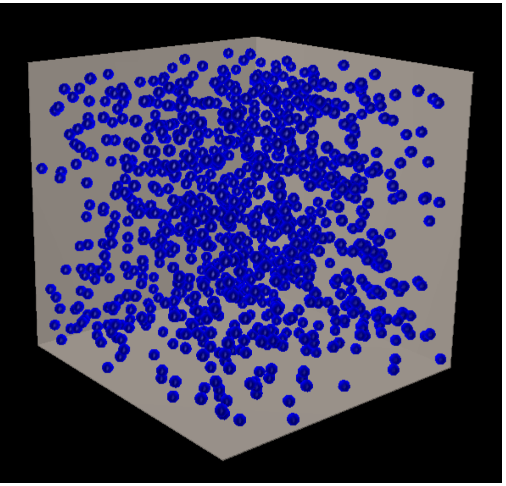

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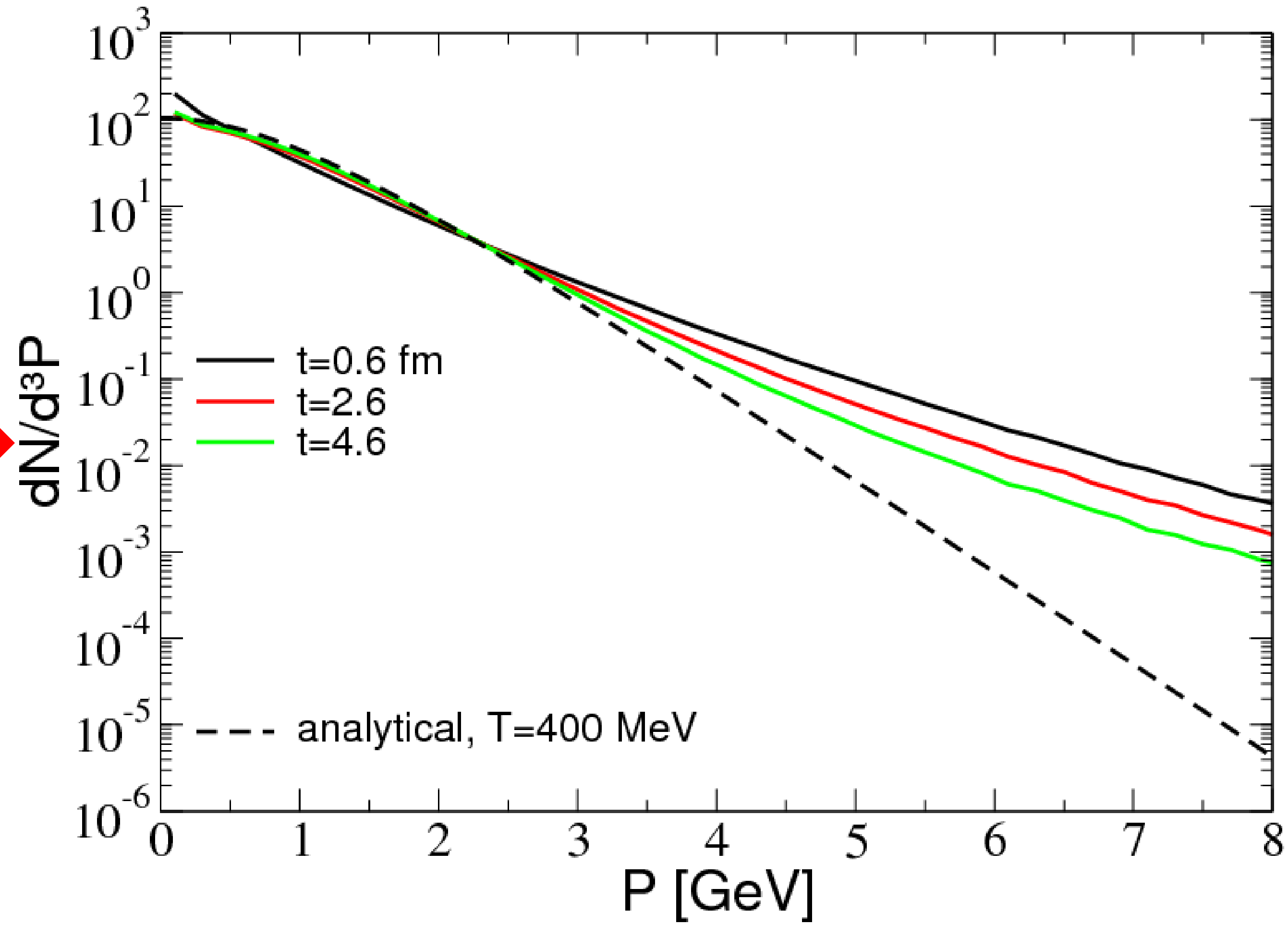

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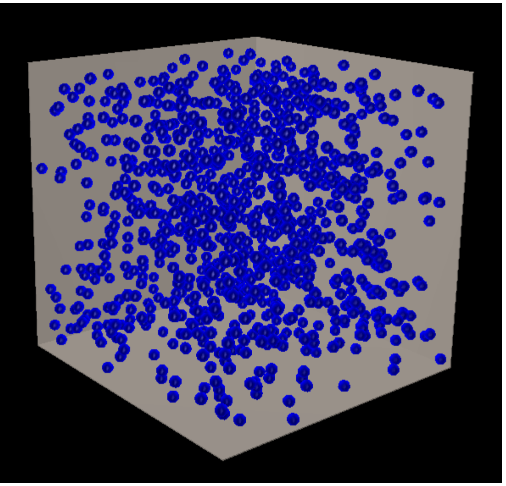

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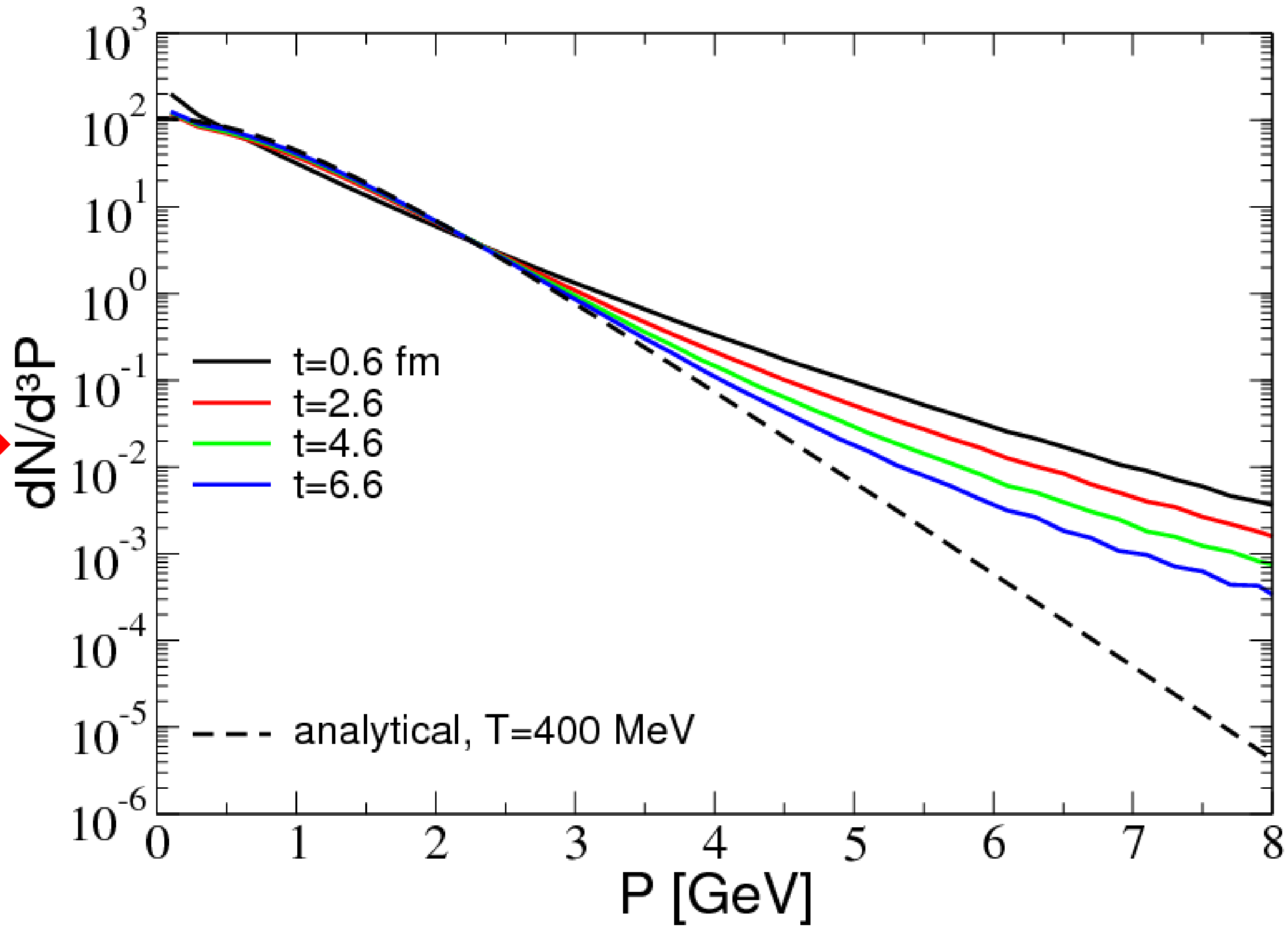

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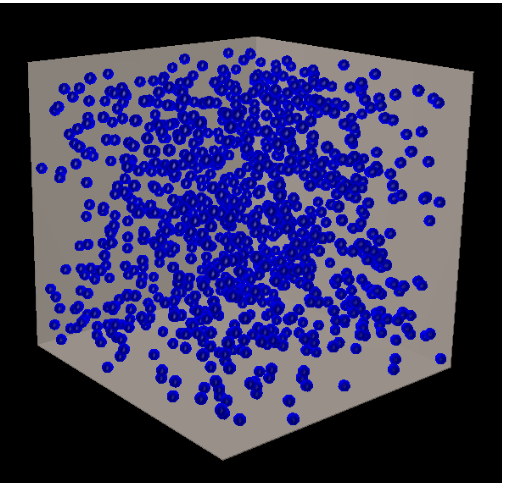

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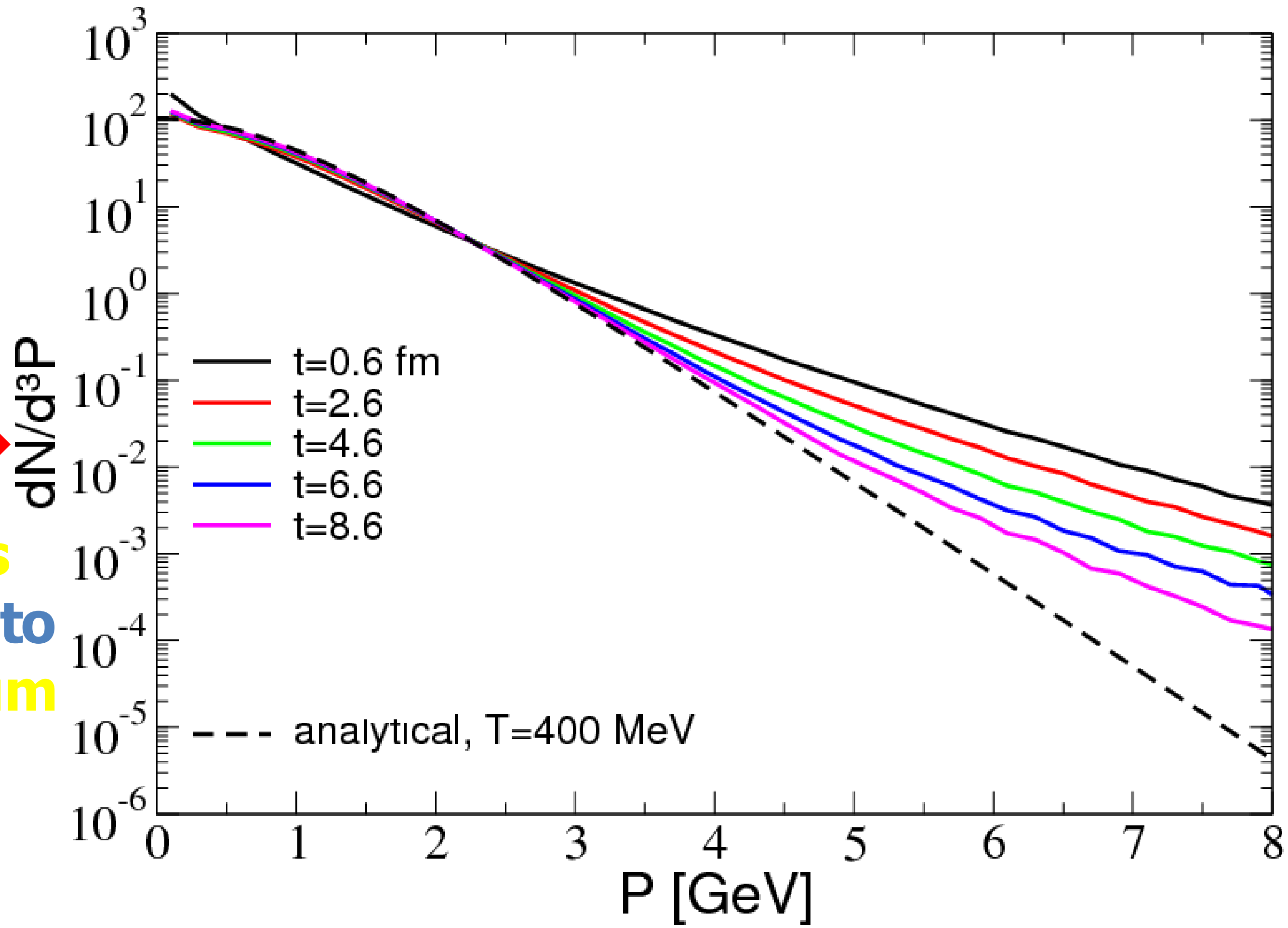
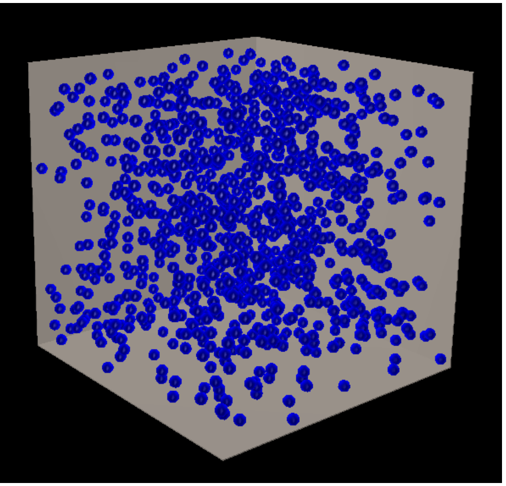


Due to collisions charm approaches to thermal equilibrium with the bulk

Charm evolution in a static medium



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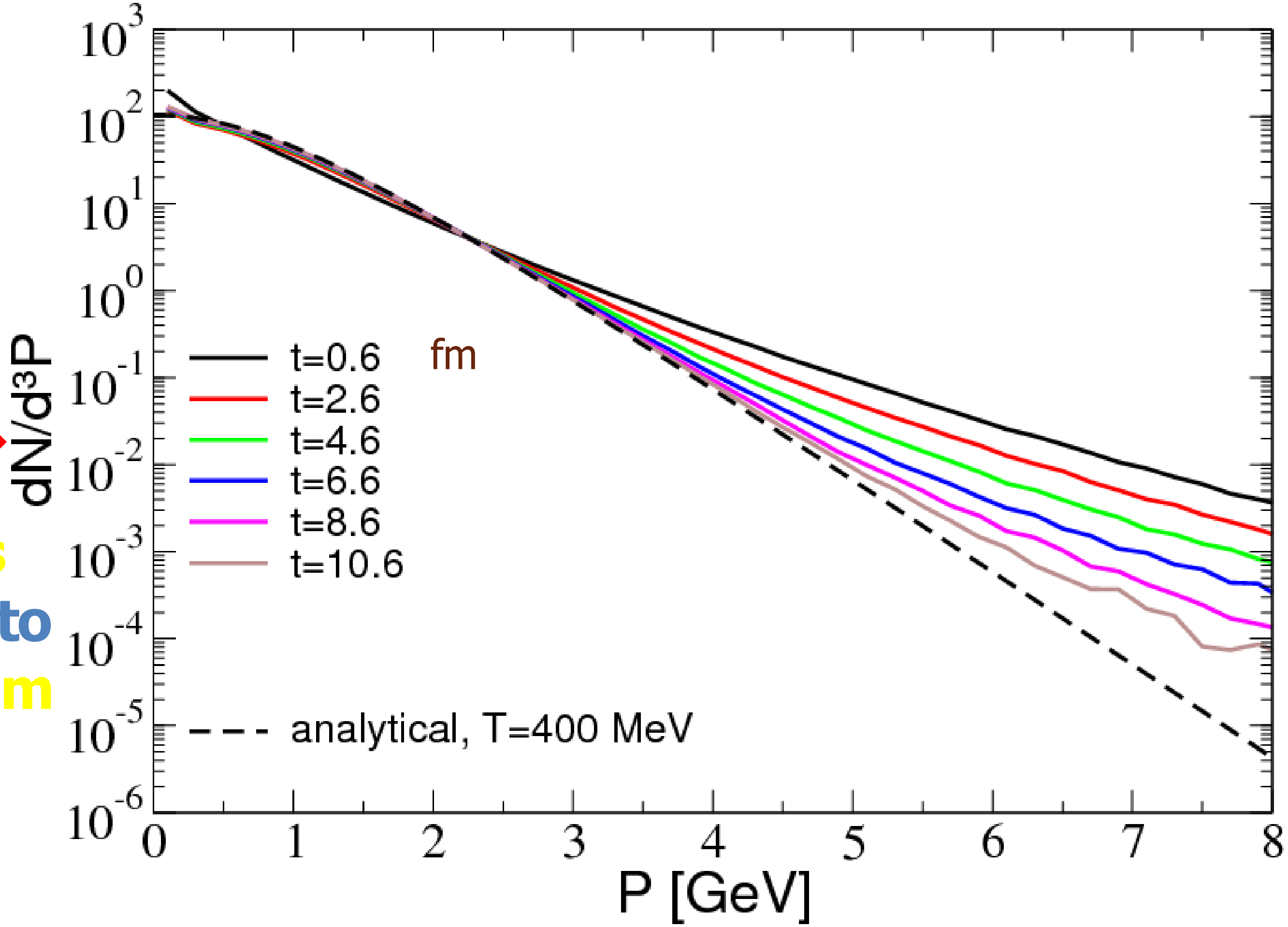

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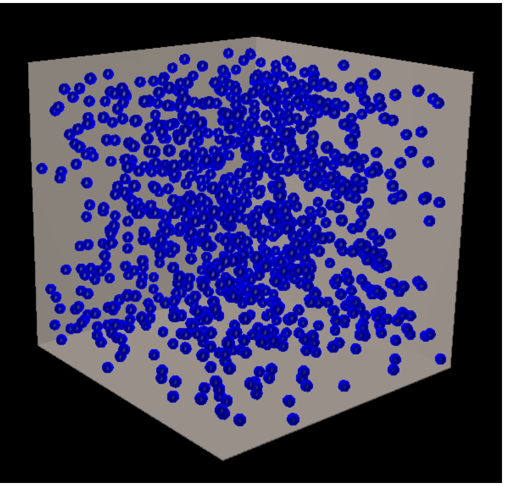

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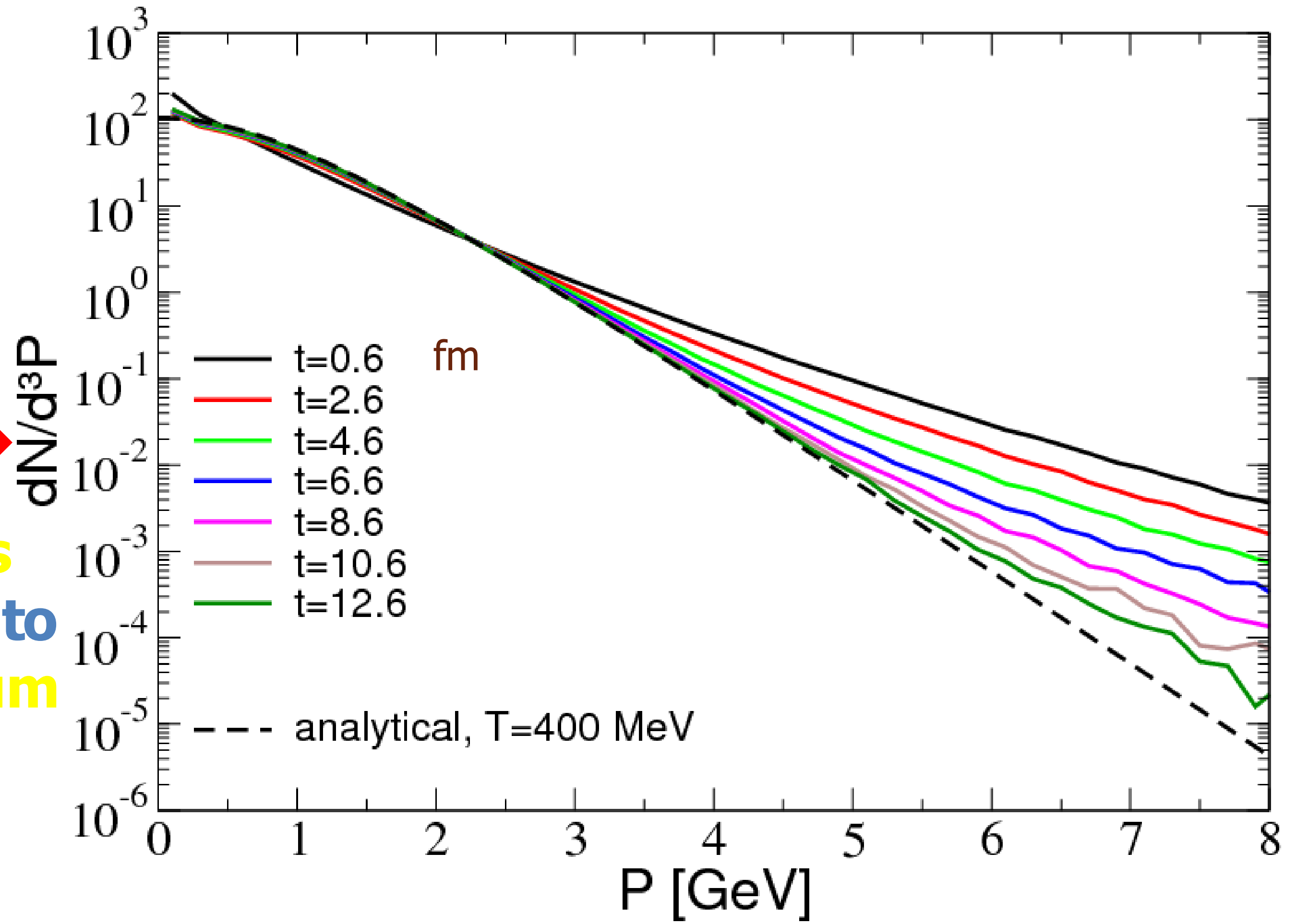

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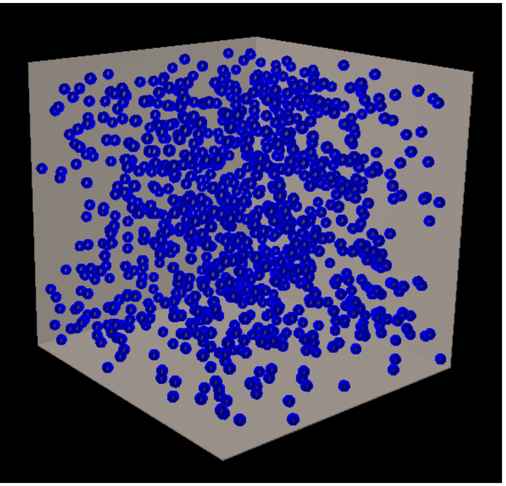

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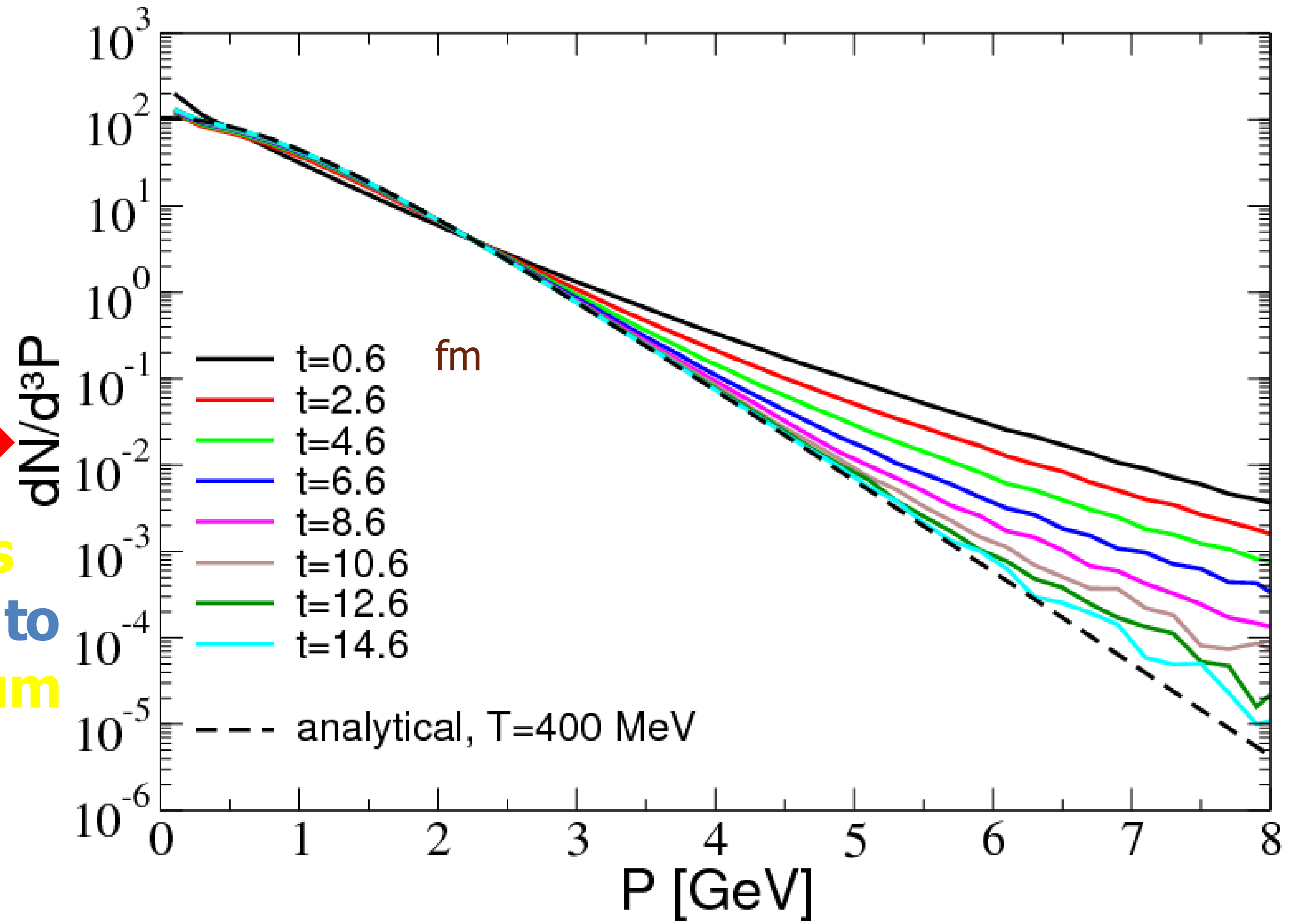

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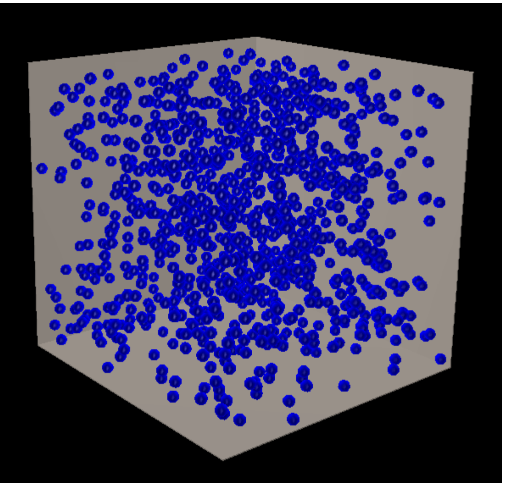

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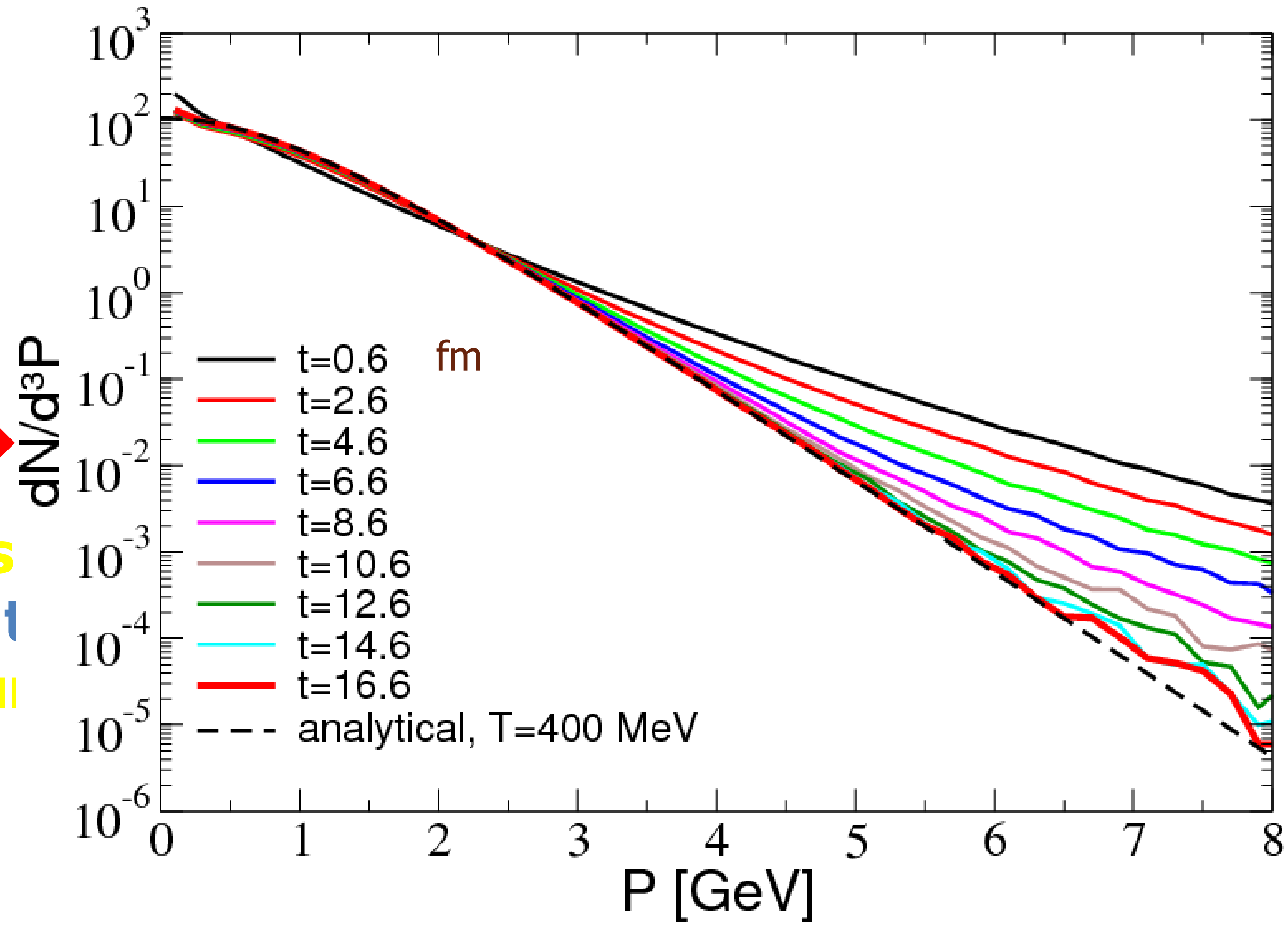
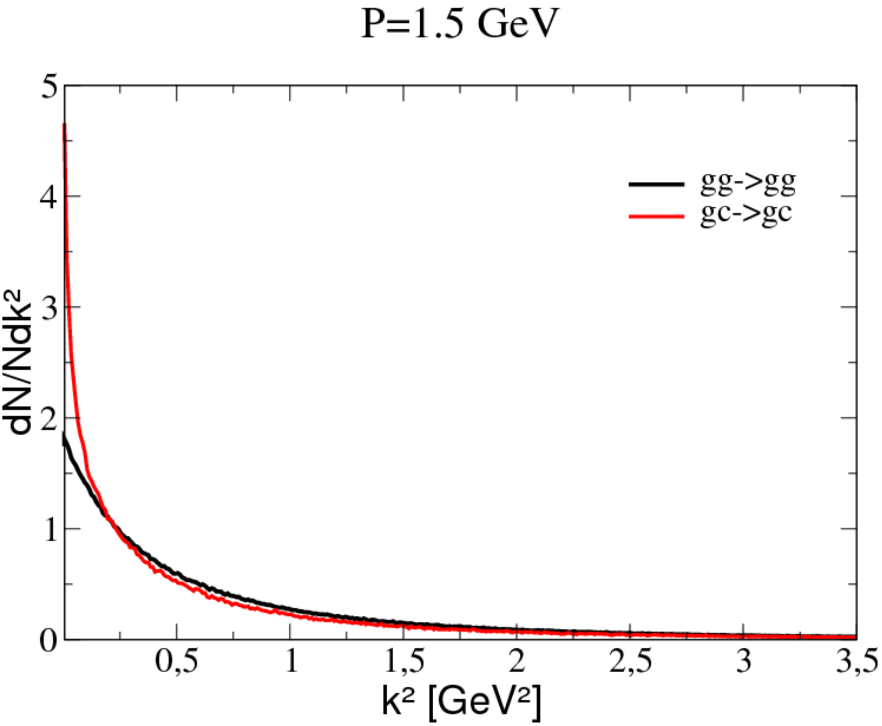
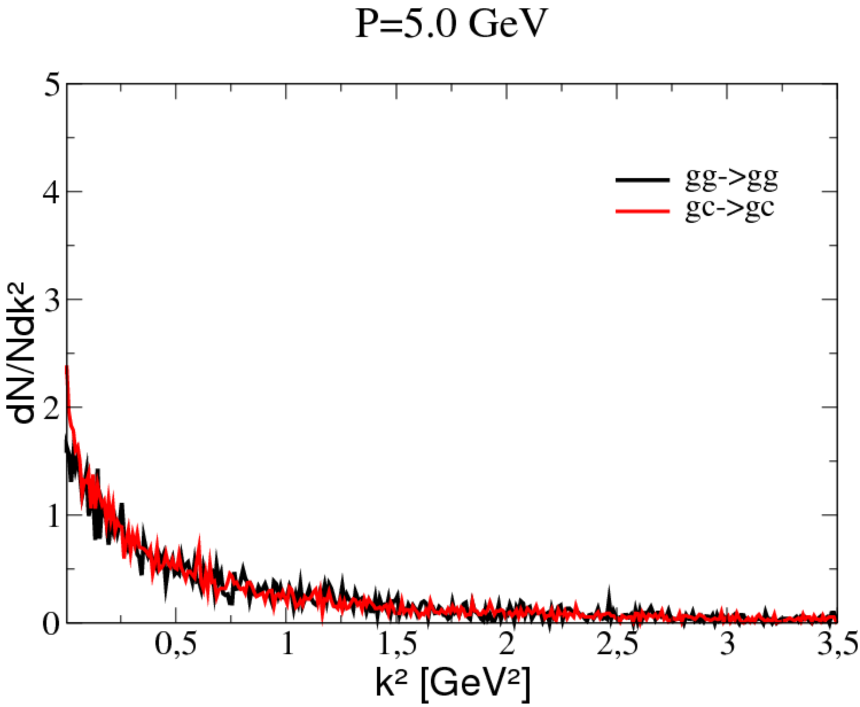


Due to collisions charm approaches t thermal equilibriu with the bulk

Momentum transfer

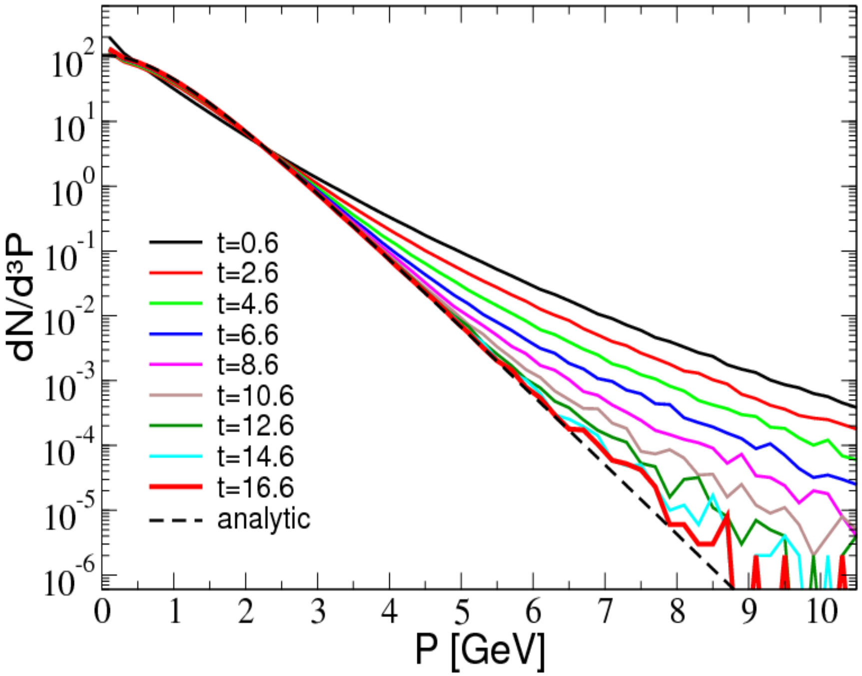
Distribution of the squared momenta transfer k^2 for fixed momentum P of the charm

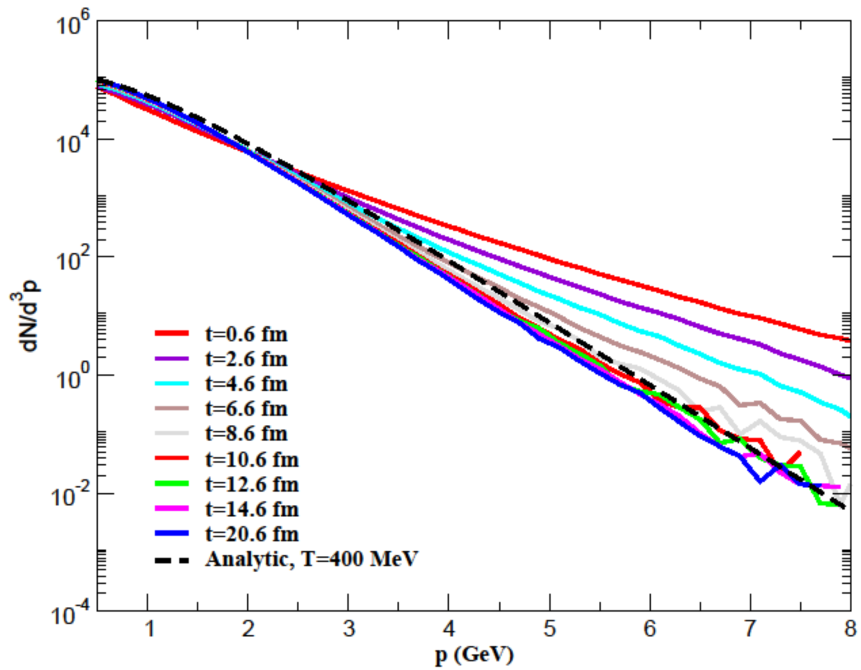




The momenta transfer of gg->gg and gc-> gc are not so different

Boltzmann vs Langevin

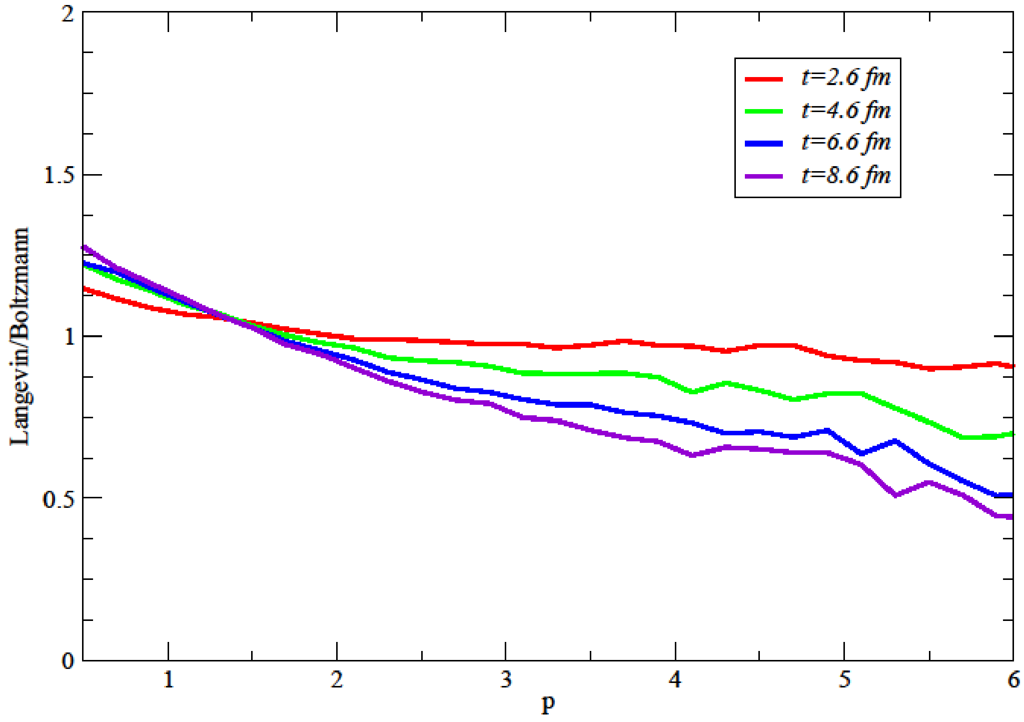




Both drag and diffusion from pQCD

Langeven approaches thermalisation in a faster rate.

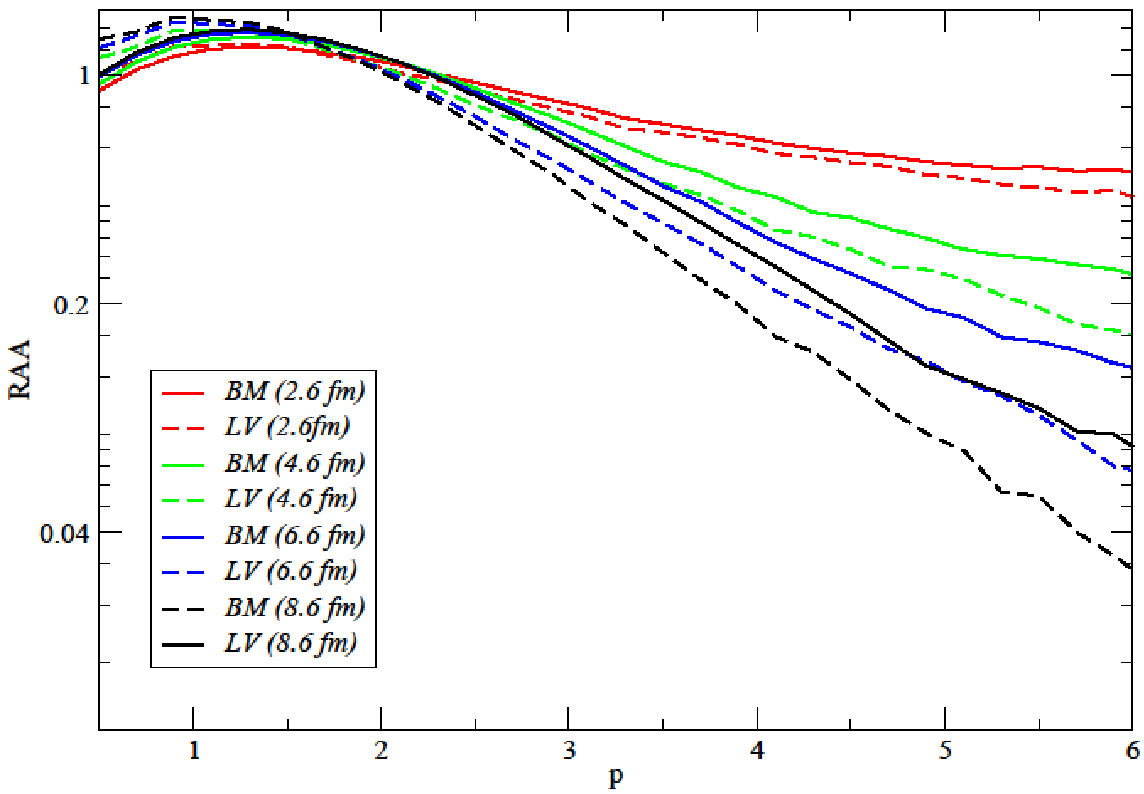
Ratio between Langevin and Boltzmann
At fixed time



A factor 2 difference

Nuclear Suppression: Langevin vs Boltzmann

$$R_{AA} = \frac{\left(\frac{dN}{d^3p}\right)_{output}}{\left(\frac{dN}{d^3p}\right)_{input}}$$



Suppression is more in Langevin approach than Boltzmann

Summary & Outlook ……

- Both Langevin and Boltzmann equation has been solved in a box for heavy quark propagating in a thermal bath composed of gluon at T= 400 MeV.

- In Langevin approach it is difficult to achieve thermalization criteria for realistic value of drag and diffusion coefficients.

- Boltzmann equation follow exact thermalization criteria.

- It is found that charm quark momentum transfer is not very differ from light quark momentum transfer.

- In Langevin case suppression is stronger than the Boltzmann case by a factor around 2 with increasing pT.

- It seems Langevin approach may not be really appropriate to heavy flavor dynamics.


Thank You . . . .