Gravitational collapse in alternative gravity

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Why gravitational collapse?

Leads to formation of ***compact objects, black holes, singularities***, all presenting ***astrophysical interest*** and within reach of modern ***observations***

The theory of ***gravity*** can be ***tested outside the weak-field regime***

Quantum effects in the semi-classical approximation, like Hawking radiation, in principle could be also ***tested***

Gravitational collapse in General Relativity

The gravitational collapse of a pressureless fluid (dust) in general relativity (***Oppenheimer–Snyder collapse***) results in a black hole:

Friedmann- Lemaître-Robertson-Walker (FLRW) metric inside:

$$ds^2_{FLRW} = -d\tau^2 + a^2(\tau)\left[d\chi^2 + \chi^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right]$$

scale factor

Friedmann: $$\frac{\dot{a}^2}{a^2} = \frac{8\pi G\rho}{3},$$

Raychaudhuri: $$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho .$$

energy density

Schwarzschild metric outside

Israel junction conditions imposed at some comoving χ_0

metric:

$$r_0 = a(\tau)\chi_0$$

$$\dot{t}_0 = \left(1 - \frac{2GM}{a(\tau)\chi_0}\right)^{-1}$$

extrinsic curvature:

$$a(\tau)\dot{a}^2(\tau) = \frac{2GM}{\chi_0^3}$$

$$\ddot{a}(\tau) = -\frac{GM}{a^2(\tau)\chi_0^3}$$

$$a^{3/2} = a_0^{3/2} - \left(\frac{9GM}{2\chi_0^3}\right)^{1/2}\tau$$

$a = 0$ reached at

$$\tau_1 = \left(2\chi_0^3 a_0^3/9GM\right)^{1/2}$$

singularity is formed

Critical phenomena in the gravitational collapse

In contrast with dust, collapsing massless scalar or Yang-Mills fields show a ***critical behaviour*** (Choptuik): ***black hole formation occurs only above a treshold mass***

Famous theorems / conjectures on BHs

Singularity theorems: (Penrose, Hawking) the singularity at the center of the BH is ***not due to the highly symmetric setup***

Cosmic censorship conjecture: each singularity remains hidden below a horizon (when asymptotic flatness & dominant energy condition obeyed)

Why alternative gravity theories?

• motivated by string / M-theory; or the need to either obey Mach's principle or to replace dark matter and dark energy by geometric effects

• as a rule, they imply (at least one) additional distance scale

• in most of the cases, the basic dynamical equation can be rewritten in the form of an effective Einstein equation

Examples:

- vacuum $f(R)$ theories as Einstein equations with a curvature fluid, which in the spherically symmetric case violates all energy conditions ⟹ leads to accelerated expansion

- brane-worlds governed by an effective Einstein equation, with new sources: the "electric part" of the 5D Weyl curvature and a quadratic contribution of the energy-momentum tensor

f(R) gravity = special scalar-tensor theory

f(R) gravity

$$\mathcal{A} = \int d^4x \sqrt{-g}\,[f(R) + \mathcal{L}_m]$$

$$G_{\mu\nu} = \frac{1}{f'(R)}\left\{\frac{1}{2}g_{\mu\nu}\,[f(R) - Rf'(R)] + f'(R)_{;\mu\nu} - g_{\mu\nu}\Box f'(R)\right\} + \frac{T^{(m)}_{\mu\nu}}{f'(R)}$$

Equivalent to a Brans-Dicke theory with df/dR being the scalar field and **ω=0** (no kinetic term for the scalar)

The simplest choice for $f(R)$ is a power law like $f(R) \propto R^n$

The Jebsen-Birkhoff theorem holds, however a weak-field approximation of a spherically symmetric BH solution, other than GR solutions, is known:

Metric functions:

$$A(r) = \frac{1}{B(r)} = 1 + \frac{2\Phi(r)}{c^2}$$

potential:

$$\Phi(r;\sigma,r_c) = -\frac{Gm}{2r}\left[1 + \left(\frac{r}{r_c}\right)^{\sigma}\right],$$

$$\sigma = \frac{12n^2 - 7n - 1 - \sqrt{36n^4 + 12n^3 - 83n^2 + 50n + 1}}{6n^2 - 4n + 2}.$$

GR limit for n=1 or **σ=0**

No unique spherically symmetric, static BH solution!!!

Gravitational collapse in f(R) theories

- numerical study of the spherically symmetric Oppenheimer-Snyder collapse of dust for Brans-Dicke theory showed significant deviations from GR

- the apparent horizon area decreases, violating the second law of black hole thermodynamics

- this is due to the violation of the null energy condition by the Brans-Dicke scalar field

- end result: a Schwarzschild black hole

Scheel, Shapiro, Teukolsky, *Phys. Rev. D* **51**, 4208 (1995).
Scheel, Shapiro, Teukolsky, *Phys. Rev. D* **51**, 4236 (1995).

Brane-words: the effective Einstein equation

$$G_{ab} = -\Lambda g_{ab} + \kappa^2 T_{ab} + \tilde{\kappa}^4 S_{ab} - \mathcal{E}_{ab} + \bar{L}^{TF}_{ab} + \mathcal{P}_{ab}$$

sources of Einstein gravity ($-\Lambda g_{ab}$, $\kappa^2 T_{ab}$)

T^2 term ($\tilde{\kappa}^4 S_{ab}$)

electric part of the bulk Weyl-curvature (Kaluza-Klein energy density) ($\mathcal{E}_{ab}$)

asymmetric embedding ($\bar{L}^{TF}_{ab}$)

bulk sources ($\mathcal{P}_{ab}$)

Gravitational collapse on the brane

- collapsing dust sphere

Bruni, Germani, Maartens, Phys. Rev. Lett. (2001)

→ static exterior and KK energy density: bounce / black hole / naked singularity

→ no static exterior if no KK energy density !

Gravitational collapse on the brane, revisited

Gergely: J. Cosmol. Astropart. Phys. JCAP **07** (02), 027 (2007)

Stellar model: perfect fluid, Friedmann metric, boundary in free fall (but no p=0 there !), no KK energy density

Static exterior: Schwarzschild

→ same junction conditions as for the Swiss-cheese universes

Evolution of the scale factor of the star

$$a^{3/2} = a_0^{3/2} - \left(\frac{9m}{2\chi_0^3}\right)^{1/2} \tau$$

Relation between Schwarzschild mass m and

physical mass M

(the integral of the energy density over the volume of the star)

$$m = M\left(1 + \frac{\rho}{2\lambda}\right)$$

energy density

$$m = \frac{4\pi a^3 \chi_0^3 \rho}{3}\left(1 + \frac{\rho}{2\lambda}\right)$$

Evolution of the collapsing fluid

EOS: $$\frac{p_\pm}{\lambda} = \frac{1}{2}\left(1 - \frac{\rho_\pm}{\lambda}\right) - \frac{1}{2}\left(1 + \frac{\rho_\pm}{\lambda}\right)^{-1}$$

Energy density: $$\frac{\rho_\pm}{\lambda} = -1 \pm \sqrt{1 + \frac{3m}{2\pi\lambda\chi_0^3\left[a_0^{3/2} - \left(\frac{9m}{2\chi_0^3}\right)^{1/2}\tau\right]^2}}$$

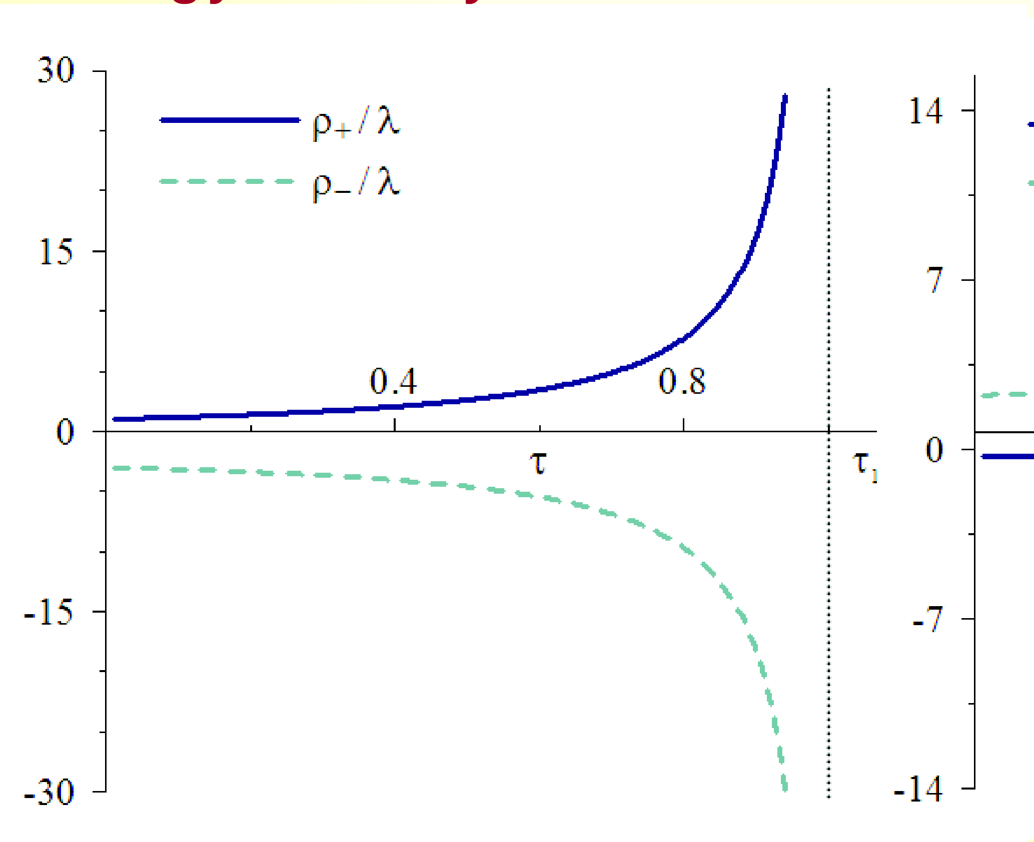
Pressure:

$$\frac{p_\pm}{\lambda} = 1 \mp \frac{1}{2}\sqrt{1 + \frac{3m}{2\pi\lambda\chi_0^3\left[a_0^{3/2} - \left(\frac{9m}{2\chi_0^3}\right)^{1/2}\tau\right]^2}} \mp \frac{1}{2\sqrt{1 + \frac{3m}{2\pi\lambda\chi_0^3\left[a_0^{3/2} - \left(\frac{9m}{2\chi_0^3}\right)^{1/2}\tau\right]^2}}}$$

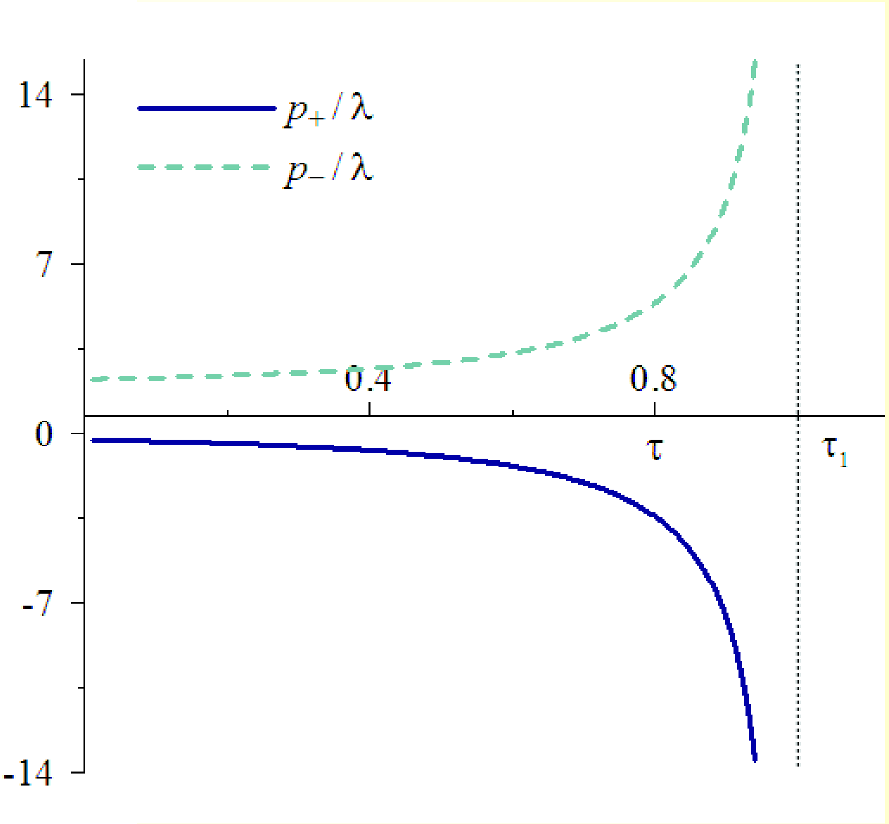
(2 branches allowed, one physical)

Evolution of the collapsing fluid II.

Energy density:



Pressure:



Rising tension in the collapsing star, black hole formation

- At the beginning the collapsing star is almost dust, has an infinitesimal tension

Low-energy EOS:
(initial stage of the collapse)

$$p_+ \approx -\frac{\rho_+^2}{2\lambda}$$

basically dust

polytrope with constant $K = -1/2\lambda$, and polytropic index $n = 1$

- As the collapse proceeds, the tension grows in the fluid due to the nonlinearity of the dynamical equations in the energy-momentum tensor

- In spite of it, the collapse continues (as the quadratic source term dominates), the horizon is reached, the black hole is formed

Below the horizon a dark energy ball is formed

• In the process of continued collapse below the horizon, the tension grows excessively, turning the fluid ball into dark energy

High-energy EOS:
(final stage of the collapse)

$$p_+ \approx -\frac{\rho_+}{2}$$

dark energy

• This transition occurs right below the horizon for astrophysical black holes and far beyond the horizon for intermediate mass and galactic black holes

With the astrophysical limit set on the brane tension, which in units $c = 1 = G$ is $\lambda_{\rm astro} = 1.5 \times 10^{-136}\ {\rm eV}^{-2}$ we obtain $A = 40$ (from $\lambda M_\odot^2 = 1.8662 \times 10^{-4}$) and

$$r_{\rm de} = 3.42 \mu^{-2/3} r_{\rm H} = \mu^{1/3} \times 7.6293 \times 10^{66}\ {\rm eV}. \quad (33)$$

The first expression shows that the bigger the black hole, the more the fluid has to collapse below the horizon before the dark energy condition is obeyed. For example, for astrophysical black holes with $\mu = 10, 100$ and galactic black holes with 10^4, 10^6, 10^8 the ratio $r_{\rm de}/r_{\rm H}$ is $0.737, 0.159$ and 7.37×10^{-3}, 3.42×10^{-4}, 1.59×10^{-5} respectively.

mass in Solar masses

Singularity formation

• Both the energy density and the tension further increase towards infinite values during continued collapse. The infinite tension could not stop the formation of the singularity

⇨ Gravitational collapse of perfect fluid (rather than dust) can occur in a static exterior (even without KK energy density)

→ dark energy is produced

→ but will not stop gravitational collapse, due to the T^2 terms

→ the end product is again Schwarzschild, with mass m

The tale of the two stories

- Gravitational collapse is modified by the dynamics of the alternative gravity theories in both cases
- during the collapse some processes interesting on their own occur (horizon contraction, dark energy production)
- the end product is the same as in general relativity !!
- ALTHOUGH the BHs produced by collapse may in principle have hair...

... remarkably they do NOT grow hair in either of these cases