Van der Waals behaviour in Gravity and Gauge/Gravity duality

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- Definition of the system
  - AdS spacetime
  - Variable pressure
  - Modified metric
- Gauge/gravity duality picture
  - Hawking-Page $\rightarrow$ Confinement/Deconfinement transition in gauge theories based on a gauge group $\mathrm{SU}\,(\mathrm{N})$
- Phase transition and Van der Walls extension
- Results
- Conclusions

The Einstein equations with $\Lambda < 0 \rightarrow$ AdS space
Metric of the covering space of AdS:

$$ds^2 = -V(r)\,dt^2 + V^{-1}(r)\,ds^2 + r^2 d\Omega^2$$

$$V(r) = 1 + \frac{r^2}{b^2} \qquad \text{and} \qquad b = \left(-\frac{3}{\Lambda}\right)^{1/2}$$

A **Black Hole** in AdS space has a minimum Hawking temperature

$$\mathrm{T_{min}} = \frac{\sqrt{3}}{2\pi b} \qquad \text{when} \quad r_+ = r_{\mathrm{min}} = \frac{b}{\sqrt{3}}$$

- $\mathrm{T} < \mathrm{T_{min}}$ Free energy minimized by the *AdS* soliton solution
- $\mathrm{T} > \mathrm{T_{min}}$ Minimum energy solution is the BH solution
- $\mathrm{T} = \mathrm{T_{min}}$ Equal Free energies and a 1-order phase transition (HP)

Maldacena's gauge/gravity duality

Strings propagating in asymptotically AdS spacetimes $\leftrightarrow$ gauge theories defined on their conformal boundaries

Consequences

- Use the thermodynamic of Gravity (BH) to study the phase diagram of gauge theories
  - $N_c \gg 1$, $\lambda = g_{YM}^2 N_c \gg 1$: Classical Gravity is valid

Confinement/Deconfinement ph. trans. of the dual gauge theory:

- AdS-soliton geometry is a confining theory
- BH geometry describe a deconfined phase

$\rightarrow$ Order parameter: Wilson loop

Modifications

- Self-gravitating, anisotropic ($p_r \neq p_\perp$) fluid type matter in a fixed AdS background:

$$V(r) = 1 + \frac{r^2}{b^2} - \frac{\omega \mathrm{M}}{r}\gamma\left(\frac{3}{2}, \frac{r^2}{4\ell^2}\right) \qquad \omega = \frac{2\mathrm{G_N}}{\Gamma(3/2)}$$

1. In T^0_0 replace the Dirac delta function by a Gaussian distribution with variance ℓ (Classical limit: $r_+ \gg \ell$ $(N \gg 1, \lambda \gg 1)$)
2. NO singularity in AdS
3. ML framework $\Rightarrow$ effective description of QG spacetime

- Pressure

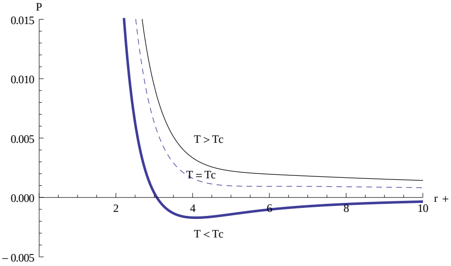
$$\frac{1}{b^2} = -\frac{\Lambda}{3} = \frac{8\pi \mathrm{P}}{3}$$

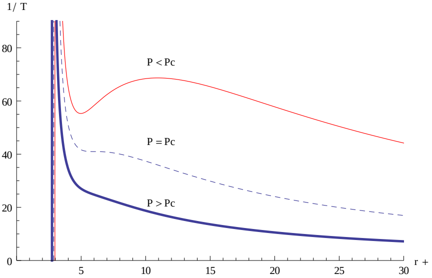
Using the definition of Hawking temperature and the pressure

Equation of state $\mathrm{P}(\mathrm{V},\mathrm{T})$

$$\mathrm{P} = \frac{3\gamma_{\frac{3}{2}}}{3\gamma_{\frac{3}{2}} - r_+\gamma'_{\frac{3}{2}}}\left[\frac{\mathrm{T_H}}{2r_+} - \frac{1}{8\pi r_+^2} + \frac{\gamma'_{\frac{3}{2}}}{8\pi r_+\gamma_{\frac{3}{2}}}\right], \qquad r_+ = \left(\frac{3\mathrm{V}}{4\pi}\right)^{1/3}$$

with $\gamma_{\frac{3}{2}} = \gamma\left(\frac{3}{2}, \frac{r_+^2}{4\ell^2}\right)$





Van der Waals

- VdW eq. is a modification of the ideal gas law, which approximates the behaviour of real fluids, taking into account:
  - the nonzero size of molecules
  - the attraction between molecules
- VdW eq. describes basic qualitative features of the liquid-gas phase transition
  - To get more information about the phase transition we study the Gibbs free energy $G = G(P, T)$

Action

$$S = \frac{1}{16\pi G}\int d^4x\,[\mathcal{L}_{\rm grav} + \mathcal{L}_{\rm matt}]$$

- $\mathcal{L}_{\rm grav}$ is the Einstein-Hilbert Lagrangian with a negative cosmological constant Λ
- $\mathcal{L}_{\rm matt}$ is the Lagrangian for matter fields.

G: Background subtraction technique to regularize the action

$$\mathrm{I_G} = \frac{\beta_0}{4G_N b^2}\left[\left(b^2 r_+ + r_+^3\right)\frac{\Gamma\left(3/2\right)}{\gamma_{\frac{3}{2}}} - 2r_+^3\right], \quad r_+ \gg \ell : \; \frac{\Gamma\left(3/2\right)}{\gamma_{\frac{3}{2}}} \to 1$$

Action

$$S = \frac{1}{16\pi G}\int d^4x\,[\mathcal{L}_{\text{grav}} + \mathcal{L}_{\text{matt}}]$$

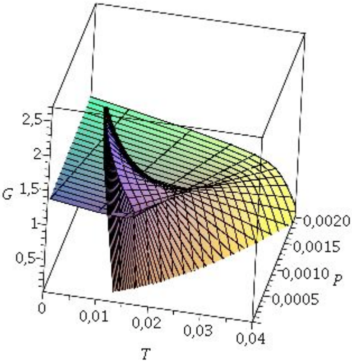
M: An anisotropic spherically symmetric matter distribution

$$\mathrm{I}_m = \frac{\beta_0}{4G_N b^2}\left(r_+ b^2 + r_+^3\right)\frac{\Gamma\left(\frac{3}{2}\right)}{\pi\ell\gamma_{\frac{3}{2}}}\left[\frac{r_+ e^{-\frac{r_+^2}{4\ell^2}}}{\ell}\left(\frac{r_+^2}{4\ell^2}+1\right)+\sqrt{\pi}-\gamma_{\frac{1}{2}}\right]$$

In this case is absolutely important to note that in the classical limit $\Rightarrow I_m \to 0$, giving the usual vacuum action independent from $T_{\mu\nu}$.

Gibbs free energy

$$G = \frac{(b^2 r_+ + r_+^3)}{4 G_{\mathrm{N}} b^2} \frac{\Gamma\left(\frac{3}{2}\right)}{\gamma_{\frac{3}{2}}} \left[1 + \frac{r_+ e^{-\frac{r_+^2}{4\ell^2}}}{\pi \ell^2} \left(\frac{r_+^2}{4\ell^2} + 1 \right) + \frac{1}{\sqrt{\pi}\ell} - \frac{\gamma_{\frac{1}{2}}}{\pi \ell} \right] - 2 r_+^3$$

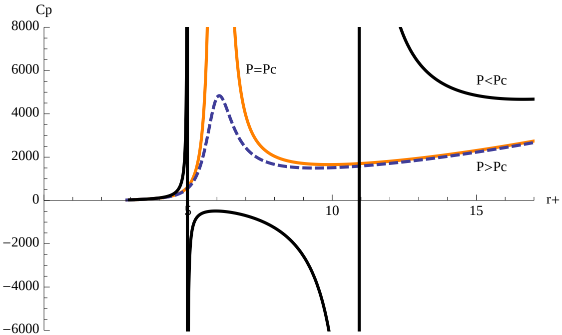


Swallowtail behaviour First-order phase

transition which occurs
at the intersection of G surfaces
(coexistence line)

- Analogy with Van der Waals fluid (liquid-gas$\rightarrow$ large-small BH)
- Positive Free Energy

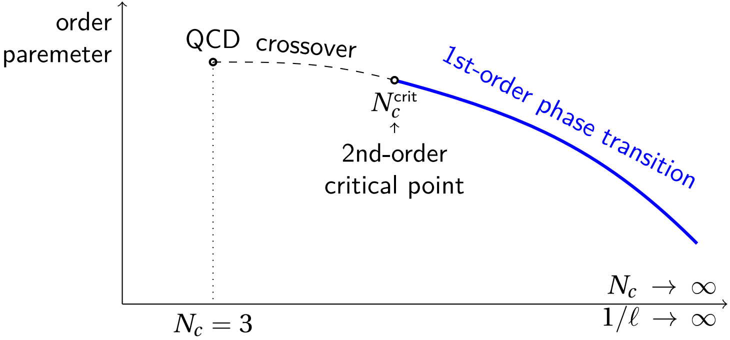
Analysis



$\mathrm{P} < \mathrm{P}_c$ Similar HP
$\mathrm{P} = \mathrm{P}_c$ Asymptote$\rightarrow$ large/small BH transition
$\mathrm{P} > \mathrm{P}_c$ The HP transition becomes a cross-over (dashed line)

Results

- Similarity with the Van der Walls system:
  - Significantly recalls the $P - V$ diagram of the Van der Walls theory,
  - The similarity with the Van der Walls system is even more evident when we compare the critical exponents.
- Same behavior of RN-AdS black holes, at least in four-dimensional spacetime
- New phase transition small BH / large BH



- Large N_c limit (Classical Gravity regime)$\rightarrow$ 1 order ph. transition
- At $N_c = 3 \rightarrow$ cross-over

A transition at finite $N_c \rightsquigarrow$ Study a QG regime
$\Rightarrow$ Effect of QG corrections is to smooth the singularities in GR
(no point-like particles because of a minimal length ℓ)

n-dimensions

$$V_n\left(r\right) = 1 + \frac{r^2}{b^2} - \frac{\omega_n M}{r^{n-2}}\gamma\left(\frac{n}{2}, \frac{r^2}{4\ell^2}\right) \quad \omega_n = \frac{16\pi G_{\mathrm{N}}}{\left(n-1\right)\Gamma\left(\frac{n}{2}\right)\mathrm{Vol}\left(\mathrm{S}^{n-1}\right)}$$

$$\mathrm{I_G} = \frac{n}{8\pi G_{\mathrm{N}}}\left[\left(\frac{r_+^2}{b^2}+1\right)\frac{r_+^{n-2}}{\gamma_{\frac{n}{2}}}\frac{b^2\,\Gamma\left(\frac{n}{2}\right)}{2n} - \frac{r_+^n}{n}\right]\beta_0\mathrm{Vol}\left(\mathrm{S}^{n-1}\right)$$

Anisotropic spherically symmetric matter distribution $T_{\mu\nu}$

$$\begin{aligned} I_m \quad = \quad & \frac{n}{8\pi G_{\mathrm{N}}}\left\{\frac{\left(n-1\right)\mathrm{Vol}\left(\mathrm{S}^{n-1}\right)}{2\pi^{3/2}n}\left[\frac{\left(n+1\right)\gamma_{\frac{n}{2}+1}}{2\gamma_{\frac{n}{2}}} - 1\right]\times \right. \\ & \left.\left(\frac{r_+^2}{b^2}+1\right)\left(2r_+\right)^{n-2}\Gamma\left(\frac{n}{2}\right)\right\}\beta_0\mathrm{Vol}\left(\mathrm{S}^{n-1}\right) \end{aligned}$$

Conclusions

- Extension of the Hawking-Page transition into a Van der Waals-like phase diagram
- This behaviour is valid in n-generic dimensions (e.g. $n = 5$)
- Application of Gauge/Gravity duality in semi-classical QG regime: possibility of investigating the dual phase diagram in gauge theory
  - In gauge theories based on a gauge group $SU(N)$:
    ℓ parameter $\longleftrightarrow 1/N$, $1/\lambda$ expansion
- Making confinement into a cross-over is equivalent to smooth BH singularities
- From the n-dimensional ML action$\rightarrow$ first step toward the calculation of the correlation functions

Thank you!

References: arXiv:1105.0188 (JHEP 1108:097,2011),
arXiv:1205.0559 (JHEP 1207:033,2012)

Gravitational action

Both the AdS and the black holes spacetime have infinite volume

"Background subtraction" technique to regularize the action

1. Define the volume of the spacetime
2. Upper cutoff R to perform the radial integration
   Regularized volume of the AdS and the black hole spacetime
3. Condition: At the boundary $(r = \mathrm{R})$ impose $\mathrm{T}_{\mathrm{AdS}} \equiv \mathrm{T}_{\mathrm{BH}}$
4. $R \to \infty$
5. Result

$$I_{\mathrm{G}} = \frac{\beta_0}{4 G_N b^2} \left[\left(b^2 r_+ + r_+^3 \right) \frac{\Gamma(3/2)}{\gamma_{\frac{3}{2}}} - 2 r_+^3 \right], \quad r_+ \gg \sqrt{\theta} : \frac{\Gamma(3/2)}{\gamma_{\frac{3}{2}}} \to 1$$