Dark energy as a quantum condensate

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Main categories of dark energy

- ⋆ **Cosmological Constant**

- ⋆ **Deviation from Einstein Gravity at large distances and/or small curvature**

- ⋆ **Quintessence - a light scalar or vector field**

• : Based on: (astro-ph/0002400), *GCN Rep.* 11, (2005) 189 (hep-ph/0412314), Mod.Phys.Lett. A, 22, (2007), 1569 (astro-ph/0702519), Phys.Rev.D 81 (2010) 103526 (arXiv:1003.2996), JMPL 27 (2012), 1250154 (arXiv:1205.3304).

Cosmological constant

- ⋆ Oldest and yet the most consistent model with observation. [Komatsu, *et al.*10, Parcival, *et al.*09, SNLS Coll. 05, Hichen *et al.*09, . . .]
- ⋆ **But the Cosmological Constant is weird !**
  - **Density Problem:** According to its **bare** definition in Quantum Field Theory (QFT):

$$\rho_\Lambda = \int d^3k \sqrt{k^2 + m^2} \rightarrow \infty$$

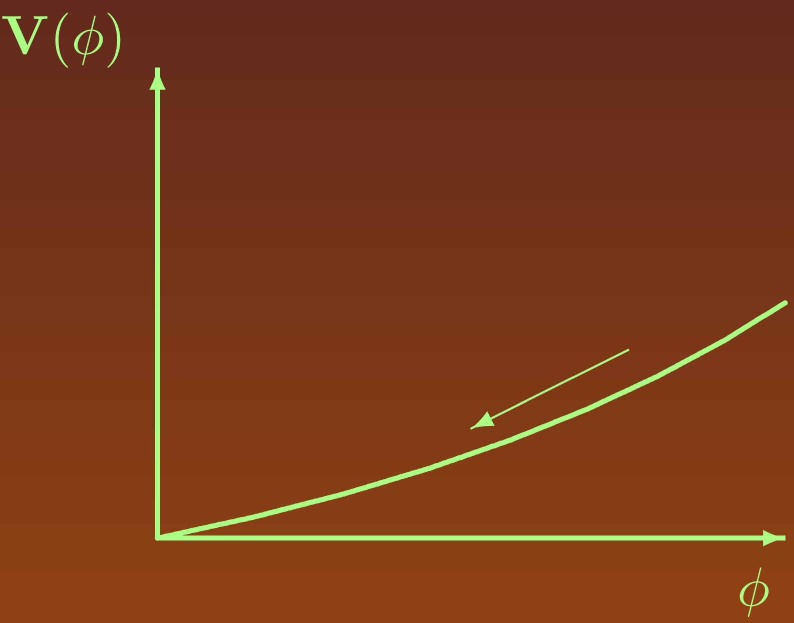
  - Even after introducing a UV cutoff Λ_{cut}, $E_\Lambda \sim 10^{123} \rho_{obs}$ (for $\Lambda_{cut} = M_P$) or $E_\Lambda \sim 10^{42} \rho_{obs}$ (for $\Lambda_{cut} = M_{QCD}$).
  - **Coincidence Problem:** Its density with respect to matter and radiation densities must be fine-tuned by tens of orders of magnitude such that galaxies could form before it becomes dominant and rips out the spacetime in a new inflation era.
- ⋆ An important unresolved issue is the fact that in in contrast to CMB anisotropies, there is no theoretical limit on the deviation of dark energy from a cosmological constant.

Cosmological constant and fundamental physics

- In the context of string theory, the vacuum energy of effective field theories obtained from compactified string theory has a huge landscape of values. [Vafa 05, Kumar 06 (rev.)]
- **Despite many attempts no natural explanation or model is found to give a significant probability to a very small but non-zero value in the landscape of vacua.**
- **The most natural value for vacuum energy density is zero and dark energy must have another origin.**

Quintessence

- ⋆ Generic name given to all models in which dark energy is due to the condensation of a scalar field including phantom and varying neutrino mass models
- ⋆ Similar to inflation, the scalar field roles down the potential but extremely slowly. [Wetterich 88, Peebles & Rata 88]



Under certain conditions for the potential *Tracking* solutions with necessary behaviour at late times without (or almost) fine-tuning of the initial conditions exist. [Wetterich 88, Peebles & Rata 88]

$$V(\phi) = e^{-\alpha\phi} \text{ or } \phi^{-n}$$

In SUGRA & string theory more sophisticated $V(\phi)$ potentials with tracing solutions are possible. [Brax & Martin 99]

Equation of State: $\mathbf{w} = \frac{\mathbf{P}}{\rho} = \frac{\frac{1}{2}\dot{\phi}^2 - \mathbf{V}(\phi)}{\frac{1}{2}\dot{\phi}^2 + \mathbf{V}(\phi)} > -1$

Issues with self-interacting quintessence models

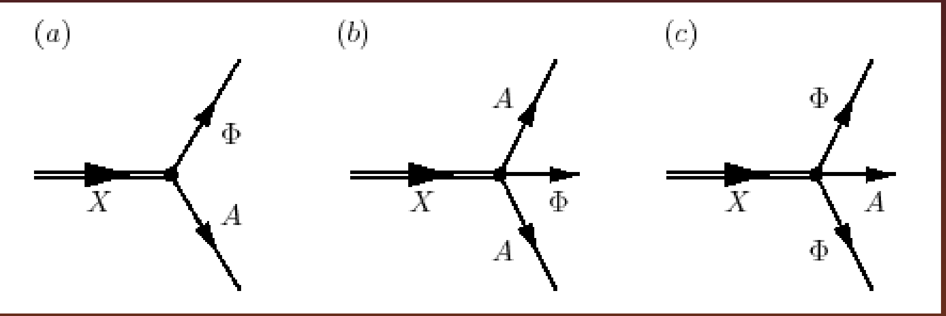
- ⋆ Simplest quintessence models - with only self-interaction - do not solve the **coincidence problem:**
  - Dilaton as quintessence field could solve this problem [Witterich 08], but it interacts with all type of matter and constraints on modified gravity restrict it.
- ⋆ It is difficult to find an isolated field that interacts only with itself in particle physics.
- ⋆ • **Polynomial or exponential potentials with negative power/exponent do not have an interpretation as self-scattering of particles and are not renormalizable.**
  - As the quintessence field must have a very weak self-interaction and interaction with other particles, its *effective potential* should be apriori close to its fundamental and perturbative interactions.
  - A complicated potential cannot be explained easily.

A simple quintessence model with decaying dark matter

- **We consider a slowly decaying dark matter:** [HZ 04, 05]
  - Fields: 3 scalar fields (2 of these fields can be fermions)
  - A heavy scalar field **X**(dark matter or inflaton)
  - Quintessence field **Φ**
  - Another field **A** (can be an effective field)
  - **Self-interaction is assumed to be ϕ^4**
- This model is motivated by top-down models for the origin of Ultra High Energy Cosmic Rays. [Hill 83, Hill, *et al.*87, Berezinsky & Mikhailov 99, HZ 99, 00]
- Some arguments are raised against these models [Hague *et al.*06, Auger Coll. 08], nonetheless they are not yet completely ruled out.
- Even if it turns up that UHECRs have astronomical origin, dark matter or one of its constituents can be meta-stable, e.g. [Datta *et al.*04, Ellis *et al.*06]

Simplest interactions

- Simplest interactions between the contents:



- Lagrangian:

$$\mathcal{L}_\Phi = \int d^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\mu\Phi - \frac{1}{2}m_\Phi^2\Phi^2 - \frac{\lambda}{n}\Phi^n\right]$$

$$\mathcal{L}_X = \int d^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu X\partial_\mu X - \frac{1}{2}m_X^2 X^2\right]$$

$$\mathcal{L}_A = \int d^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu A\partial_\mu A - \frac{1}{2}m_A^2 A^2 - \frac{\lambda'}{n'}A^{n'}\right]$$

$$\mathcal{L}_{int} = \int d^4x\sqrt{-g}\begin{cases} g\Phi XA, & \text{For (a)} \\ g\Phi XA^2, & \text{For (b)} \\ g\Phi^2 XA, & \text{For (c)} \end{cases}$$

Classical approximation

⋆ **Dynamics equations:**

$$\dot{\phi}[\ddot{\phi} + 3H\dot{\phi} + {m_q}^2\phi + \lambda\phi^3] = -2g\dot{\phi}\phi\left(\frac{2\rho_x}{{m_x}^2}\right) + \Gamma_q\rho_x$$

$$\dot{\rho_x} + 3H\rho_x = -(\Gamma_q + \Gamma_J)\rho_x - \pi^4 g^2\left(\frac{{\rho_x}^2}{{m_x}^3} - \frac{{\rho_q}'^2}{{m_q}^3}\right) \quad , \quad \dot{\rho_J} + 3H(\rho_J + P_J) = \Gamma_J\rho_x$$

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho_x + \rho_J + \rho_q) \quad , \quad \rho_q = \frac{1}{2}{m_q}^2\dot{\phi}^2 + \frac{1}{2}{m_q}^2\phi^2 + \frac{\lambda}{4}\phi^4$$

⋆ Small field regime approximation solution:

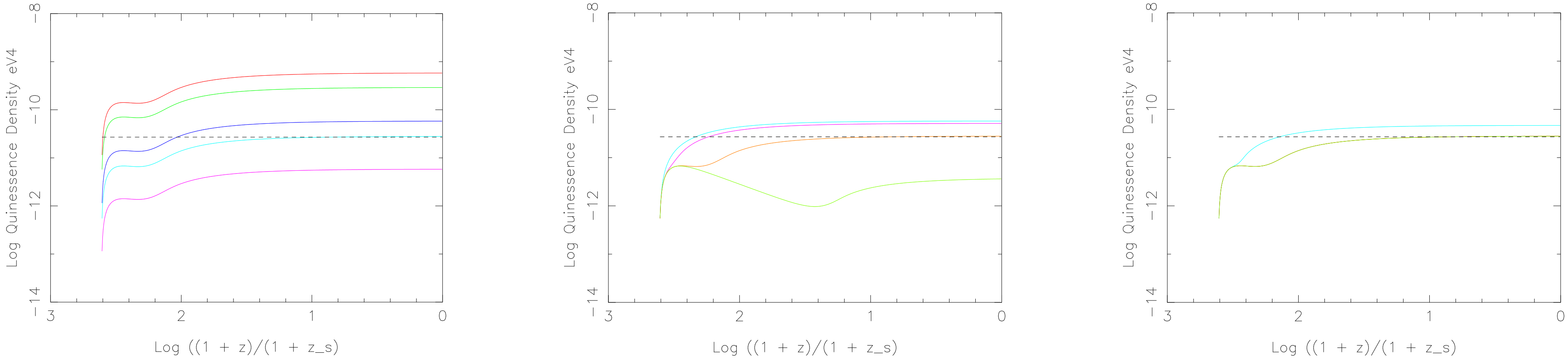
$$\frac{1}{2}\dot{\phi}^2(t) \equiv K_q(t) = \left(\frac{a(t_0)}{a(t)}\right)^6\left[K_q(t_0) + \Gamma_q\rho_x(t_0)\int_{t_0}^{t} dt\,\frac{a^3}{a(t_0)}e^{-\Gamma(t-t_0)}\right]$$

⋆ Quasi-static regime approximation:

$$V_q(\phi) = V_q(\phi(t'_0)) + \Gamma_q\rho_x(t'_0)\int_{t'_0}^{t} dt\left(\frac{a(t'_0)}{a(t)}\right)^3 e^{-\Gamma(t-t'_0)}$$

Results of classical approximation

- **Analytical approximation and numerical calculations show that in this model since very early time quintessence field behaves very similar to a Cosmological Constant.**
- This behaviour can be obtained for a large part of the parameter space. No fine-tuning in necessary.
- **This model shows that a constant dark energy is not necessarily the energy of the vacuum.**



$\Gamma_0 = \Gamma_q/\Gamma = 10^{-16}$, $5\Gamma_0$, $10\Gamma_0$, $50\Gamma_0$, $100\Gamma_0$; $m_q = 10^{-6}$ eV, $\lambda = 10^{-20}$.

$m_q = 10^{-3}$ eV, $m_q = 10^{-5}$ eV, $m_q = 10^{-6}$ eV, $m_q = 10^{-8}$ eV, $\lambda = 10^{-20}$;

$\lambda = 10^{-10}$, $\lambda = 10^{-15}$, $\lambda = 10^{-20}$, and $\lambda = 10^{-25}$, $m_q = 10^{-6}$ eV.

[HZ 03, 04, 05]

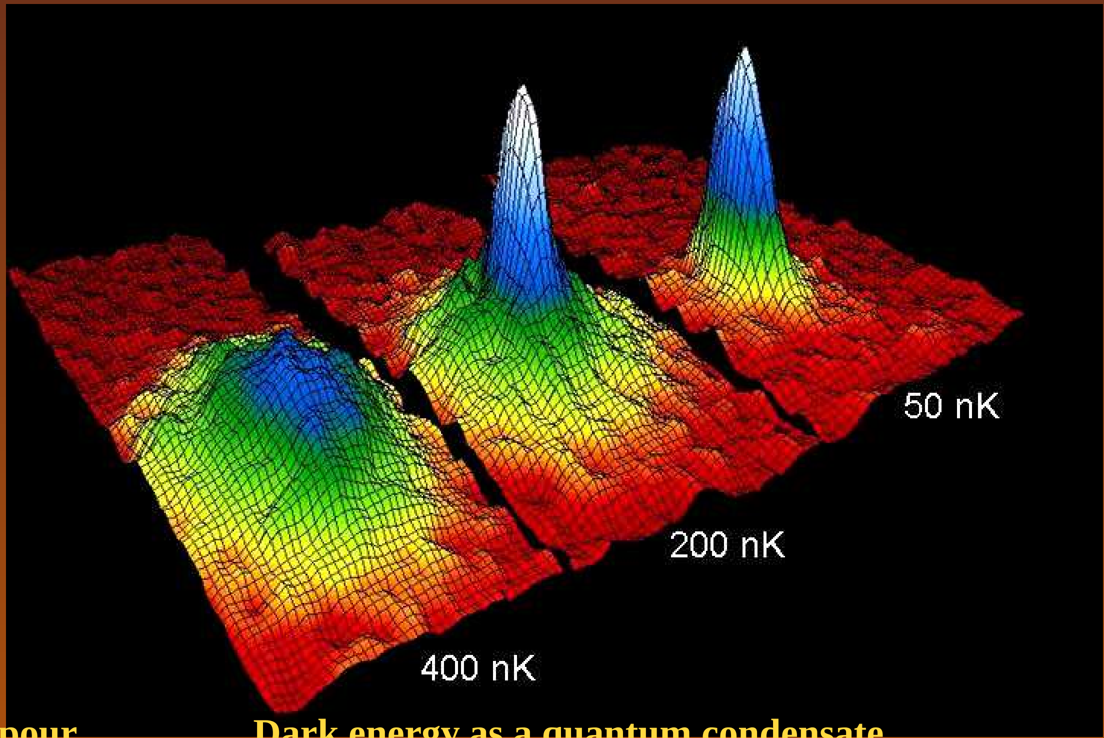
Issues with a classical treatment of quintessence models

⋆ **Classical treatment is not sufficient because it presumes the presence of a classically behaving condensate:**

- A classical scalar field is assumed to be $\mathbf{N}_\phi \rightarrow \infty$ limit of a quantum scalar field.
- For a very weakly interacting field the validity of such assumption in an expanding universe is not guaranteed.
- Decoherence of particles does not necessarily mean that they behave like a classical scalar field **- a condensate.**
- Most probably $\mathbf{\Phi}$ particles are initially relativistic and due to their very weak interaction they lose their energy only due to the expansion of the Universe.
- Due to their small mass $\mathbf{\Phi}$ particles must be relativistic today and $\mathbf{\Phi}$ cannot contribute in dark energy.
- These issues are studied for inflation [Starobinsky 86, Collins & Holman 05] and for Higgs [Sardanashvili & Subbotin 84, Consoli & Costanzo 04] but eceptionally for quintessence.

Quantum field theory of quintessence models

- It is easier to satisfy necessary conditions for classical scalar fields acting at short distances **e.g. for Higgs**.
- **We must study the formation and evolution of the quintessence condensate as a quantum field.** [HZ 10]
- A condensate in quantum field theory is a many particle analogue of a Bose-Einstein Condensate (BEC) in in condensed matter.



Decomposition

- ⋆ We decompose $\mathbf{\Phi}(\mathbf{x})$ to classical (condensate) and quantum components:

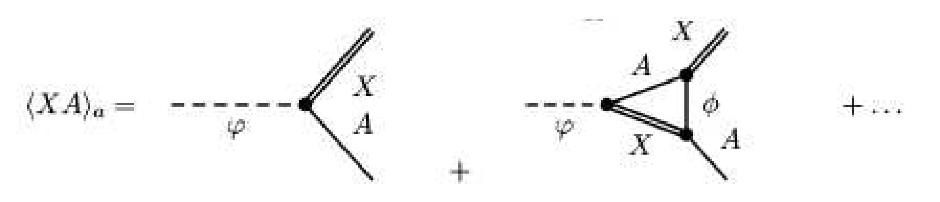
$$\mathbf{\Phi}(\mathbf{x}) = \varphi(\mathbf{x})\mathbf{I} + \phi(\mathbf{x}) \qquad \langle \mathbf{\Phi} \rangle \equiv \langle \mathbf{\Psi} | \mathbf{\Phi} | \mathbf{\Psi} \rangle = \varphi(\mathbf{x}) \qquad \langle \phi \rangle \equiv \langle \mathbf{\Psi} | \phi | \mathbf{\Psi} \rangle = \mathbf{0}$$

- ⋆ We assume that only $\mathbf{\Phi}(\mathbf{x})$ forms a condensate $\Longrightarrow \langle \mathbf{X} \rangle = \mathbf{0}$
- ⋆ Field equation for the condensate for model (a):

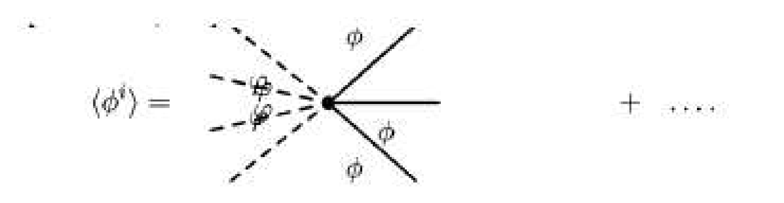
$$\frac{\mathbf{1}}{\sqrt{-\mathbf{g}}}\partial_\mu(\sqrt{-\mathbf{g}}\mathbf{g}^{\mu\nu}\partial_\nu\varphi) + \mathbf{m}^2_{\mathbf{\Phi}}\varphi + \frac{\lambda}{\mathbf{n}}\sum_{\mathbf{i}=\mathbf{0}}^{\mathbf{n}-\mathbf{1}}(\mathbf{i}+\mathbf{1})\binom{\mathbf{n}}{\mathbf{i}+\mathbf{1}}\varphi^{\mathbf{i}}\langle\phi^{\mathbf{n}-\mathbf{i}-\mathbf{1}}\rangle - \mathbf{g}\langle\mathbf{X}\mathbf{A}\rangle = \mathbf{0}$$

- ⋆ **Solution of this equation is the main goal.**
- ⋆ **We must use non-equilibrium QFT methods such as Schwinger-Keldysh in-in method to calculate expectation values which depend on the condensate field.**

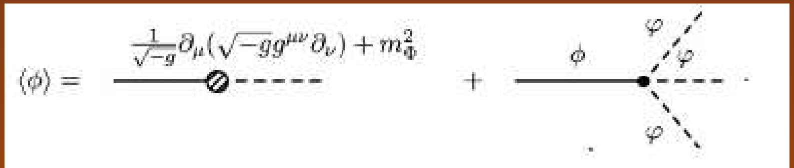
Expectation values



$$\langle \mathbf{XA} \rangle_{\mathbf{a}} = -\mathbf{i}\mathbf{g} \int \sqrt{-\mathbf{g}}\mathbf{d}^4\mathbf{y}\varphi(\mathbf{y}) \left[\mathbf{G}^{>}_{\mathbf{A}}(\mathbf{x},\mathbf{y})\mathbf{G}^{>}_{\mathbf{X}}(\mathbf{x},\mathbf{y}) - \mathbf{G}^{<}_{\mathbf{A}}(\mathbf{x},\mathbf{y})\mathbf{G}^{<}_{\mathbf{X}}(\mathbf{x},\mathbf{y}) \right]$$



$$\langle \phi^{\mathbf{i}} \rangle = -\mathbf{i}\lambda \int \sqrt{-\mathbf{g}}\mathbf{d}^4\mathbf{y}\varphi^{\mathbf{n}-\mathbf{i}}(\mathbf{y}) \left[[\mathbf{G}^{>}_{\phi}(\mathbf{x},\mathbf{y})]^{\mathbf{i}} - [\mathbf{G}^{<}_{\phi}(\mathbf{x},\mathbf{y})]^{\mathbf{i}} \right]$$



Sum of these two diagrams are null - consistent with the definition $\langle \phi \rangle = 0$

Coherent states

- A general description for a condensate state does not exist.
- Special case: [Matsumoto & Moroi 08]

$$|\Psi_C\rangle \equiv e^{-|C|^2} e^{C a_0^\dagger}|0\rangle = e^{-|C|^2} \sum_{i=0}^{\infty} \frac{C^i(x)}{i!} (a_0^\dagger)^i |0\rangle$$

$$a_0|\Psi_C\rangle = C|\Psi_C\rangle$$

$$\chi(x) \equiv a\langle\Psi_C|\Phi|\Psi_C\rangle = C\mathcal{U}_0(x) + C^*\mathcal{U}_0^*(x)$$

- As χ is a real field the argument of C is arbitrary and therefore we assume that C is real:

$$C = \frac{\mathcal{U}_0(x) + \mathcal{U}_0^*(x)}{\chi(x)}$$

- **This condensate state is a generalization of Bose-Einstein condensate in quantum field theory with infinite number of entangled particles.**
- **This means that dark energy is the largest and the oldest coherent state ever existed in the Universe.**

Generalized condensate state

- ⋆ Definition of a condensate state $\langle \Psi | \Phi | \Psi \rangle \neq 0$ does not restrict particles to be in the same energy state.
- ⋆ The following state is also a coherent state according to this definition: [HZ 10]

$$|\Psi_k\rangle \equiv A_k e^{C_k a_k^\dagger} |0\rangle = A_k \sum_{i=0}^{N} \frac{C_k^i}{i!} (a_k^\dagger)^i |0\rangle$$

$$a_k |\Psi_k\rangle_N = C_k |\Psi_k\rangle_{(N-1)}$$

- ⋆ For $N \to \infty$:

$$|\Psi_{GC}\rangle \equiv \sum_k A_k e^{C_k a_k^\dagger} |0\rangle = \sum_k A_k \sum_{i=0}^{N\to\infty} \frac{C_k^i}{i!} (a_k^\dagger)^i |0\rangle$$

$$\chi(x, \eta) \equiv a(\eta) \langle \Psi_{GC} | \Phi | \Psi_{GC} \rangle = \sum_k C'_k \mathcal{U}_k(x) + C'^*_k \mathcal{U}^*_k(x) \quad C'_k \equiv A_k C_k$$

- ⋆ **Physical interpretation:**
  - **Multiple condensates each at a given energy state.**
  - **Each condensate state behaves like a single free** *particle*

Total quantum state

- ⋆ To calculate Green's functions we need to know the quantum state of particles.
- ⋆ **In this approximation complete state of the Universe is defined by *particles* $|\mathbf{\Psi_f}\rangle, \ \mathbf{\Psi_f} \equiv \mathbf{X}, \ \mathbf{A}, \ \phi$ which perturbatively can be considered as *free* and the condensate:**

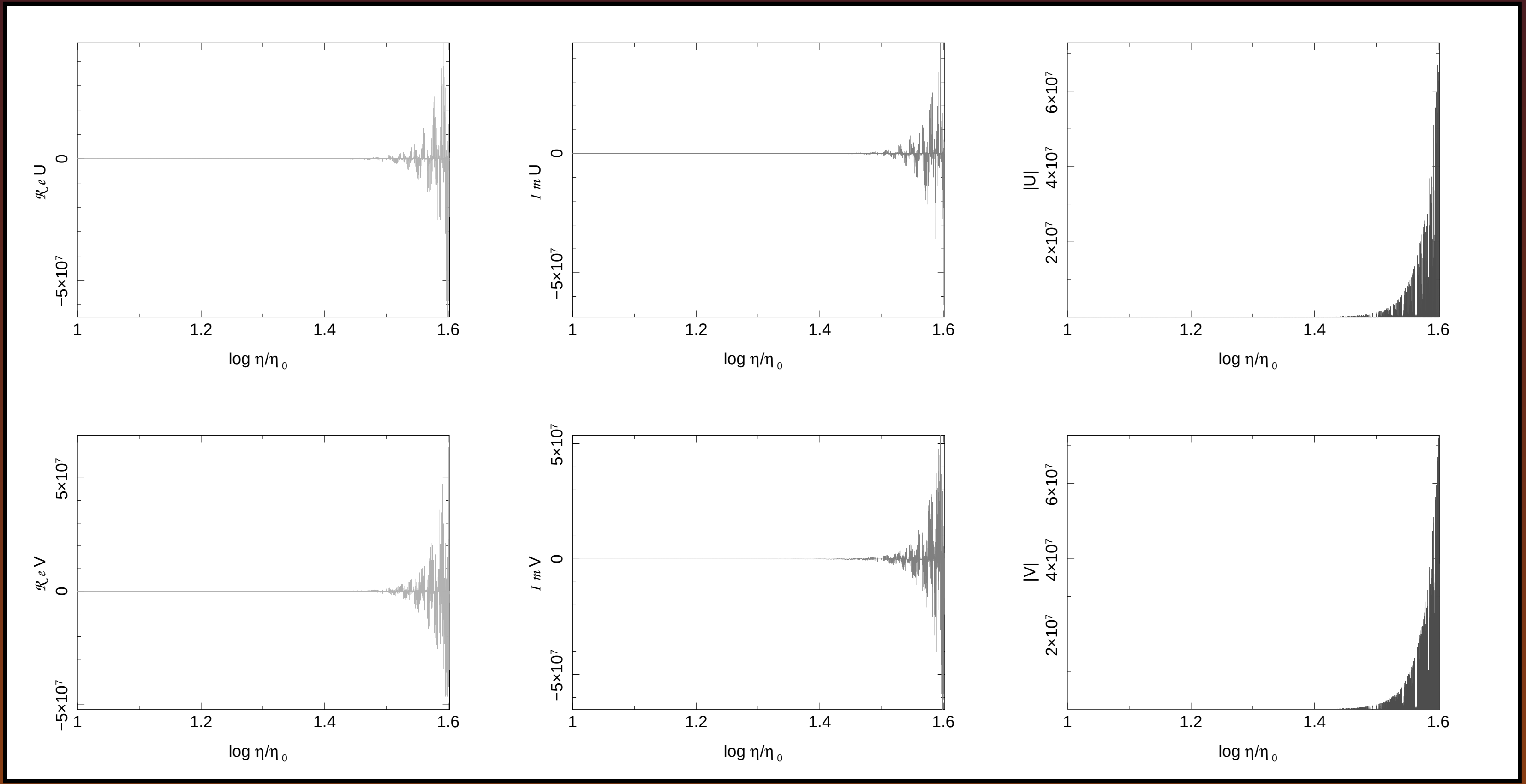
$$|\mathbf{\Psi}\rangle \equiv |\mathbf{\Psi_f}, \varphi\rangle \xrightarrow[\text{interaction}]{\text{weak}} \approx |\mathbf{\Psi_f}\rangle \otimes |\varphi\rangle$$

- ⋆ Quantum state of free particles $|\mathbf{\Psi_f}\rangle$ (including ϕ):

$$|\mathbf{\Psi_f}\rangle = \sum_{\mathbf{p_j}} \bigotimes_{\mathbf{i,j}} \mathbf{f^i}(\mathbf{x}, \{\mathbf{p_j}\}, \varphi)|\mathbf{p_j^i}\rangle \qquad \langle\mathbf{\Psi_f}|\phi|\mathbf{\Psi_f}\rangle = \mathbf{0}$$

- ⋆ After decoherence of free particles their classical distributions $\mathbf{f_i}$ in general depend on coordinate, momentum, and the condensate φ.
- ⋆ In a homogeneous universe coordinate-dependence is neglected.
- ⋆ When Boltzmann equations are solved, evolution of $\mathbf{f_i}, \ \mathbf{i} = \ \mathbf{X}, \ \mathbf{A}, \ \phi$ can be determined.

Radiation domination epoch



⋆ **Due to similarity of evolution equation of quintessence field with inflation, the growth of quintessence condensate in this regime is very similar to exponential particle production during preheating after inflation.** [Kofman, *et al.*97]

Self-interaction, back reaction, and other issues

- ⋆ It can be shown that during radiation domination epoch self-interaction initially helps the exponential growth of the condensate.
- ⋆ At later times the back reaction of self-interaction slows down the growth rate of the condensate and prevents its over production.
- ⋆ **The generalized coherent state description of the condensate shows that Φ particles do not need to lose all their momentum $\mathbf{k}$ to join.**
- ⋆ When the number of free Φ particles and the amplitude of the condensate increase, the probability of self-interaction as well as interaction with $\mathbf{X}$ and $\mathbf{A}$ increases.
- ⋆ **This process leads to slow down or even** *evaporation* **of condensate.**
- ⋆ **Only a consistent solution of condensate equation along with Boltzmann equations can provide a quantitative picture of these processes.**
- ⋆ **Such a study - possible only through numerical calculation - is an indispensable test for quintessence models.**

Matter domination epoch

- An exact solution (neglecting interactions) exists only for $m_\phi = 0$ or $k = 0$.
- **Asymptotic solution without self-interaction:**

$$\chi_{\mathbf{k}}(\eta) \xrightarrow[\lambda=0]{\frac{\eta}{\eta_0}\gg 1} \sqrt{\frac{2}{\pi\beta'_{\Phi}}}\frac{\eta_0}{\eta}\left(1 - \frac{3\mathbf{k}^2\eta_0}{2\mathbf{m}_{\Phi}^2\eta} + \mathcal{Y}_{\mathbf{k}}(\eta)\right)\left\{\mathbf{c}'^{(\mathbf{a})}_{\mathbf{k}} \sin\left(\beta'\frac{\eta^3}{\eta_0^3}(1 - \frac{3\mathbf{k}^2\eta_0}{2\mathbf{m}_{\Phi}^2\eta}) + \mathcal{Y}_{\mathbf{k}}(\eta)\right) + \mathbf{d}'^{(\mathbf{a})}_{\mathbf{k}} \cos\left(\beta'\frac{\eta^3}{\eta_0^3}(1 - \frac{3\mathbf{k}^2\eta_0}{2\mathbf{m}_{\Phi}^2\eta}) + \mathcal{Y}_{\mathbf{k}}(\eta)\right)\right\}$$

$$\mathcal{Y}_{\mathbf{k}}(\eta) = \frac{\mathbf{i}\mathbf{g}^2}{4(2\pi)^3\pi\sqrt{\beta'_{\mathbf{A}}\beta'_{\mathbf{X}}}}\left\{\sum_\alpha \mathbf{C}_\alpha(\mathbf{k},\bar{\mathbf{x}})\gamma(-2, \mathbf{i}\alpha\frac{\eta^3}{\eta_0^3}) + \sum_\alpha \mathbf{C}'_\alpha(\mathbf{k},\bar{\mathbf{x}})\gamma(-\frac{1}{3}, \mathbf{i}\alpha\frac{\eta^3}{\eta_0^3})\right\}$$

- **At late times $\varphi_{\mathbf{k}} \propto \mathbf{t}_0/\mathbf{t}$, thus the condensate does not survive the expansion of the Universe.**
- With self-interaction and linearized equation, the evolution of condensate has similar asymptotic behaviour at late times.
- Linearization ignores quantum corrections because ϕ propagators include terms proportional to χ^{-2} which cannot be linearized.
- **We need to solve the full nonlinear equation to find out the behaviour of the condensate.**

Tracking solution conditions

- After various simplifications:

$$\chi'' + (\mathbf{k^2} + \mathbf{a^2 m_\Phi^2} - \frac{\mathbf{2}}{\eta^2})\chi + \frac{\mathbf{i}}{\mathbf{3}}\lambda^2 \mathbf{a^{4-n}} (\frac{\mathbf{2}}{\pi \beta'_\Phi})^{\mathbf{n-2}} \mathbf{e}^{\mathbf{i}\frac{(8-n)\pi}{6}} (\frac{\eta_0}{\eta})^{\mathbf{n-1}} \sum_{\alpha,\beta} \beta^{-\frac{8-n}{3}} \gamma(\frac{\mathbf{8-n}}{\mathbf{3}}, -\mathbf{i}\beta \frac{\eta^3}{\eta_0^3})$$

$$\mathbf{e}^{\mathbf{i}(\alpha+\beta)\frac{\eta^3}{\eta_0^3}} \times \sum_{\mathbf{i=1}}^{\mathbf{n-1}} \binom{\mathbf{n-1}}{\mathbf{i}} (\frac{\mathbf{2}}{\pi \beta'_\Phi})^{\mathbf{n-i}} \cos^{\mathbf{2(n-i)}}(\beta'_\Phi \frac{\eta^3}{\eta_0^3}) \, (\frac{\eta_0}{\eta})^{\mathbf{2(n-i)}} \chi^{-\mathbf{2(n-i)+1}}(\eta) + \ldots = \mathbf{0}$$

$$\alpha, \beta = \mathbf{j}\beta'_\Phi, \mathbf{j} = -(\mathbf{n-1}), \ldots, \mathbf{n-1}$$

- **The potential includes terms with negative power of χ that can lead to a tracking solution.**
- In quintessence models where the potential has constant coefficients the condition for the existence of tracking solution is: [Steinhardt *et al.*99]

$$\Gamma \equiv \mathbf{V"V/V'2} > \mathbf{1}$$

- For $\mathbf{V(\Phi)} \propto \mathbf{\Phi^n}$: $\Gamma = \mathbf{n(n-1)/n^2} < \mathbf{1}$ for $\mathbf{n} > \mathbf{0}$.
- $\chi^{-\mathbf{2}}$ terms in the propagators play the role of a potential with negative exponents.
- The condition for the existence of tracking solutions can be applied if coefficients of the evolution equation is time independent or vary slowly.

Tracking solution conditions

- Quantum correction terms of order i have negative component and roughly constant coefficient if:

$$\mathbf{17 - 6n + 2i \geqslant 0, \quad i < n - 1}$$

- **This condition is satisfied only for $\mathbf{n \leqslant 3}$.**
- For $n = 4$ also the decay rate of the condensate can be enough small to be consistent with observations.

- **These two cases are the only renormalizable scalar field models in 4-dim spacetimes.**

- The approximation presented here cannot be applied to dark energy dominant regime because we assumed that the expansion rate $\mathbf{a(t)}$ is governed by other type of matter and is decoupled from quintessence field.

- Such approximation is crucial for the analytical solution, otherwise all the equations are coupled.

A new vacuum state

- The generalized coherent state can be extended to define a mixture of coherent state with all momentum combination for particles:

$$|\Psi_G\rangle \equiv \sum_{k_1, k_2, \cdots} \left(\prod_{k_i} A_{k_i} \right) e^{\sum_i C_{k_i} a^{\dagger}_{k_i}} |0\rangle$$

- When $C_{k_i} \to 0$, $|\Psi_G\rangle$ is annihilated by all the annihilation operators **thus it is a vacuum state.**
- It is easily seen that a Lorentz transformation project this state to itself, thus it is independent of reference frame.
- The expectation value of number operator $\langle \Psi_G | \hat{N}_k | \Psi_G \rangle = 0$.
- **Using the number operator, the energy expectation value** $\langle \Psi_G | \omega_k \hat{N}_k | \Psi_G \rangle = 0$.
- The analogue of this calculation for the Fock space vacuum $|0\rangle$ is considered to be ad hoc because this vacuum is not reference independent.[Birrell & Davis 83]