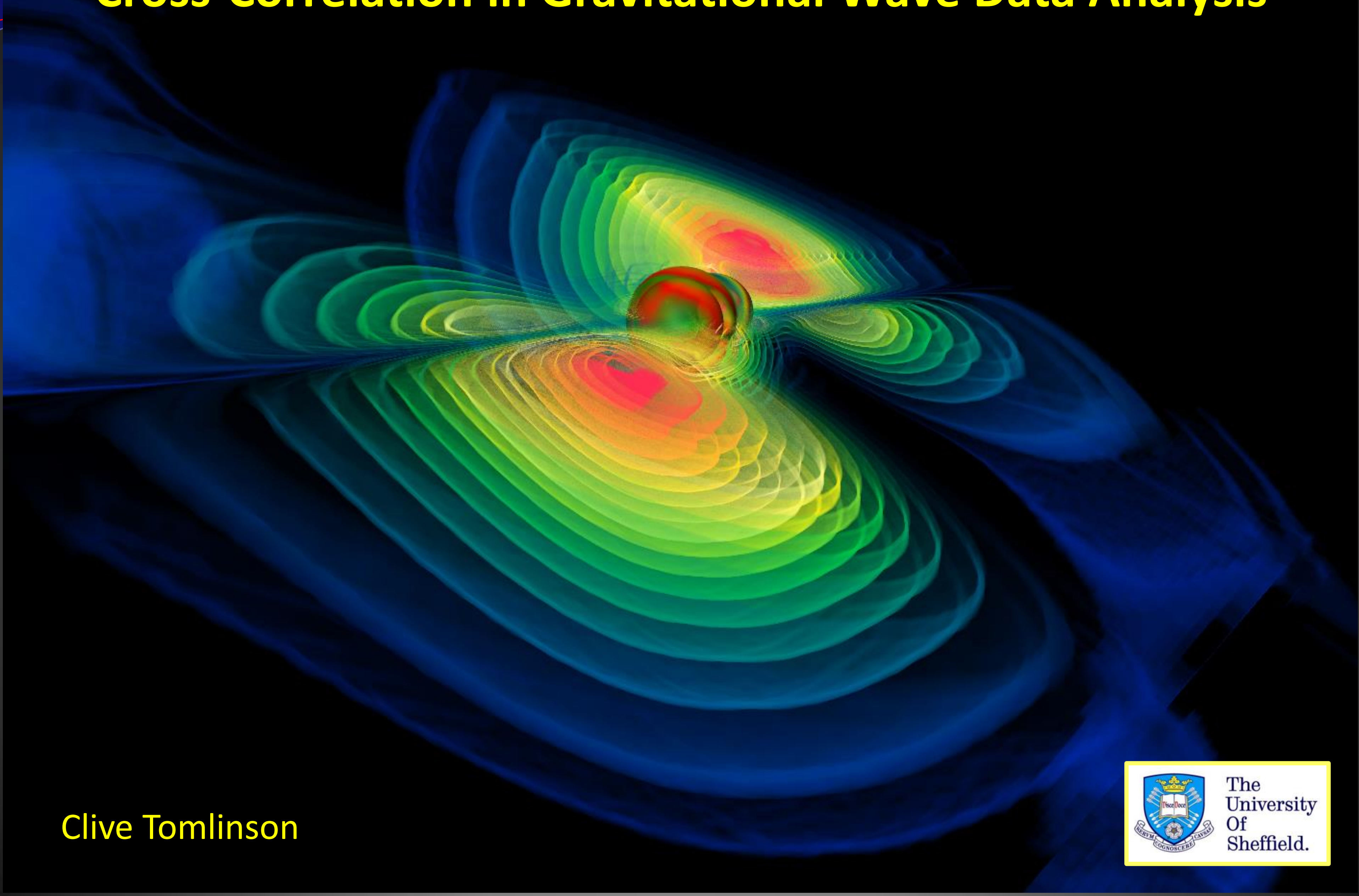
Cross-Correlation in Gravitational Wave Data Analysis



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Gravitational Waves

- Einstein's General Theory Of Relativity predicts the existence of gravitational waves (1916). Yet to be directly detected.

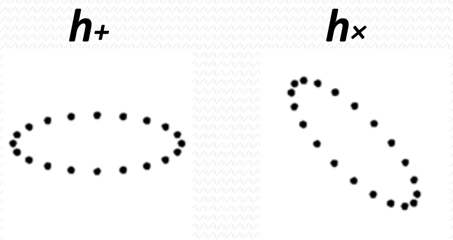
- Cause a time varying curvature of space-time, propagating at the speed of light.

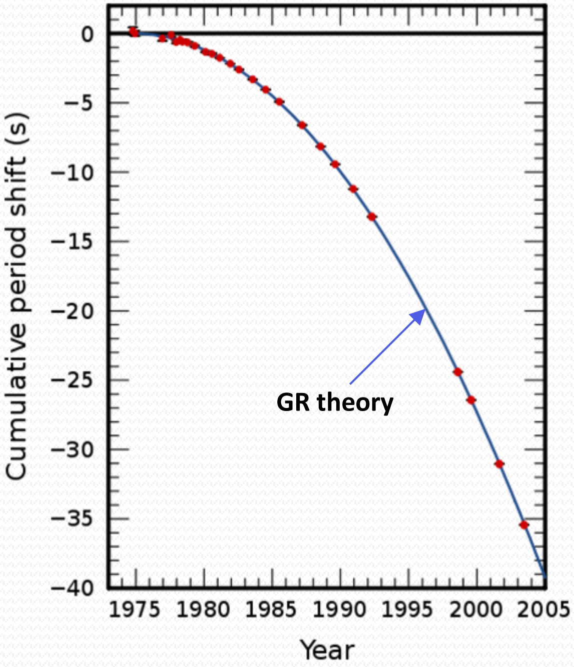
- Sources have non-zero quadrupole moment and large mass-energy flux.

- GW radiation carries energy away from emitting system/object.

- Induce an extremely small spatial strain, $h \approx 10^{-21}$ → large scale detector.

- Best indirect evidence of GWs from observation of binary pulsar PSR 1913+16.





Observation of binary pulsar PSR 1913+16 by Taylor & Weisberg

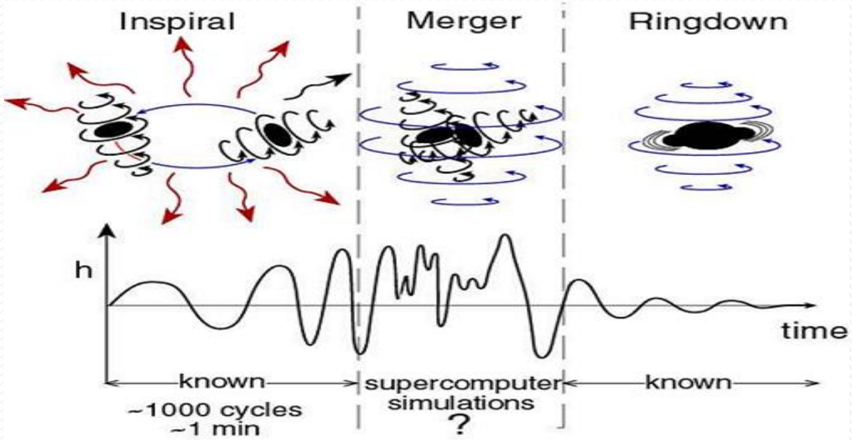
Electromagnetic Waves	Gravitational Waves
• Light interacts strongly with matter and may be absorbed/dispersed or in some cases never detected.	• GWs interact very weakly with matter. The Universe is virtually transparent to GWs .
• Easy to detect – eyes, astronomy.	• Very difficult to detect – not detected.
• Light frequencies emitted dependent upon the composition and processes occurring in outermost layers of source.	• GW frequencies depend upon bulk internal and external dynamics of the system.
• Frequency range: EM spectrum.	• Frequency range: Audio.
• Detailed image formation.	• Direct probe of internal motion.

In a sense we can see the Universe with EM waves and listen with GWs.

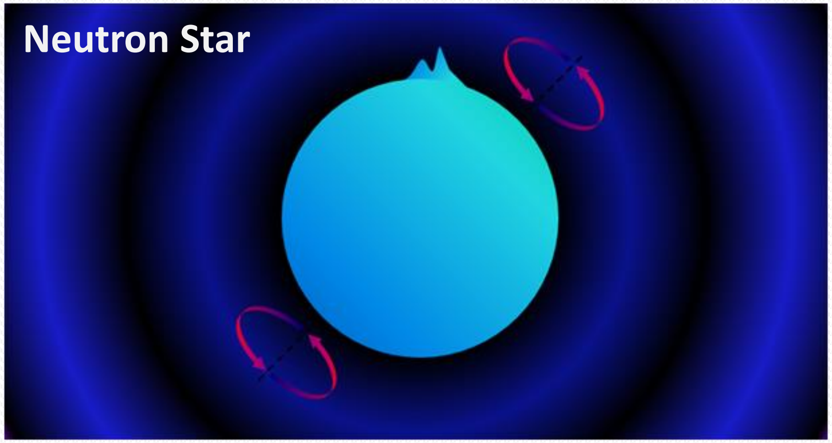
If detected GWs will offer a new window on the Universe.

Expected Sources of GWs

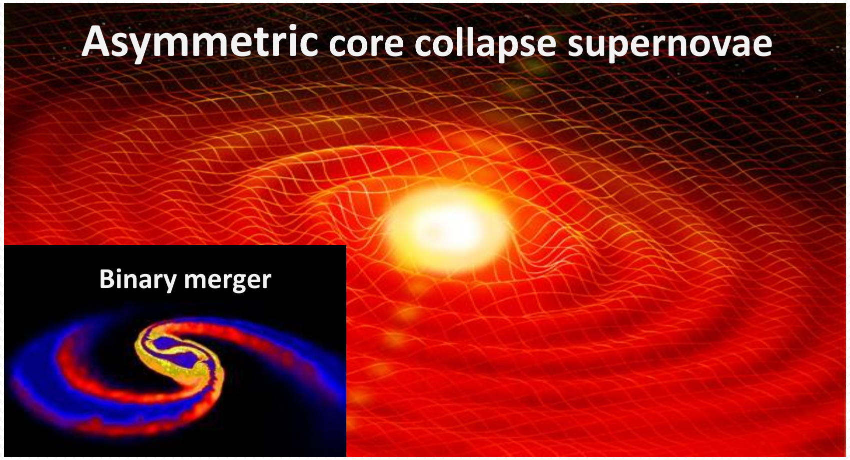
Compact Binary System



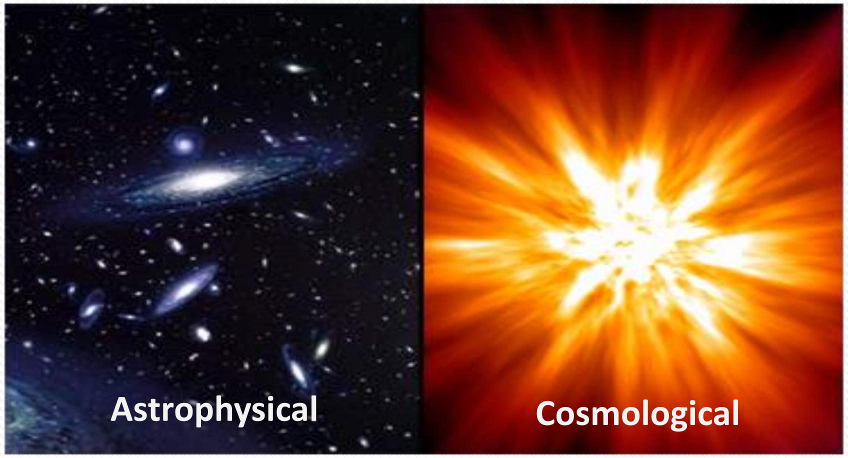
Continuous



Burst



Stochastic

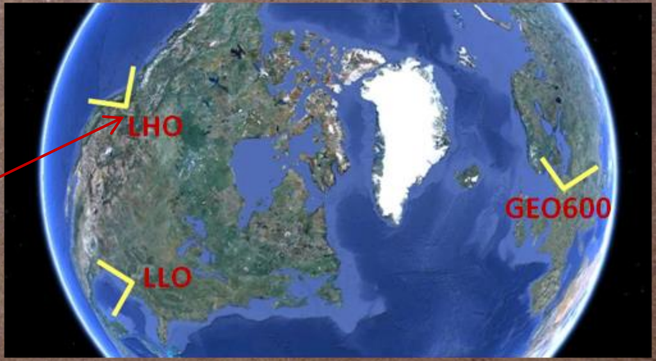


Interferometric Gravitational Wave Detection

- Detection principle essentially Michelson Interferometric length sensing.
- Several Km-scale interferometric detectors built. LIGO (USA), VIRGO (ITALY), GEO600 (Germany), TAMA300 (Japan).
- Network of three detectors permits GW source direction estimation which can be passed to robotic telescopes to search for coincident electromagnetic events.
- Motivates the need for real time analysis.

$h = \Delta L/L$

LIGO Hanford 4Km detector



Gravitational Wave Detector Noise

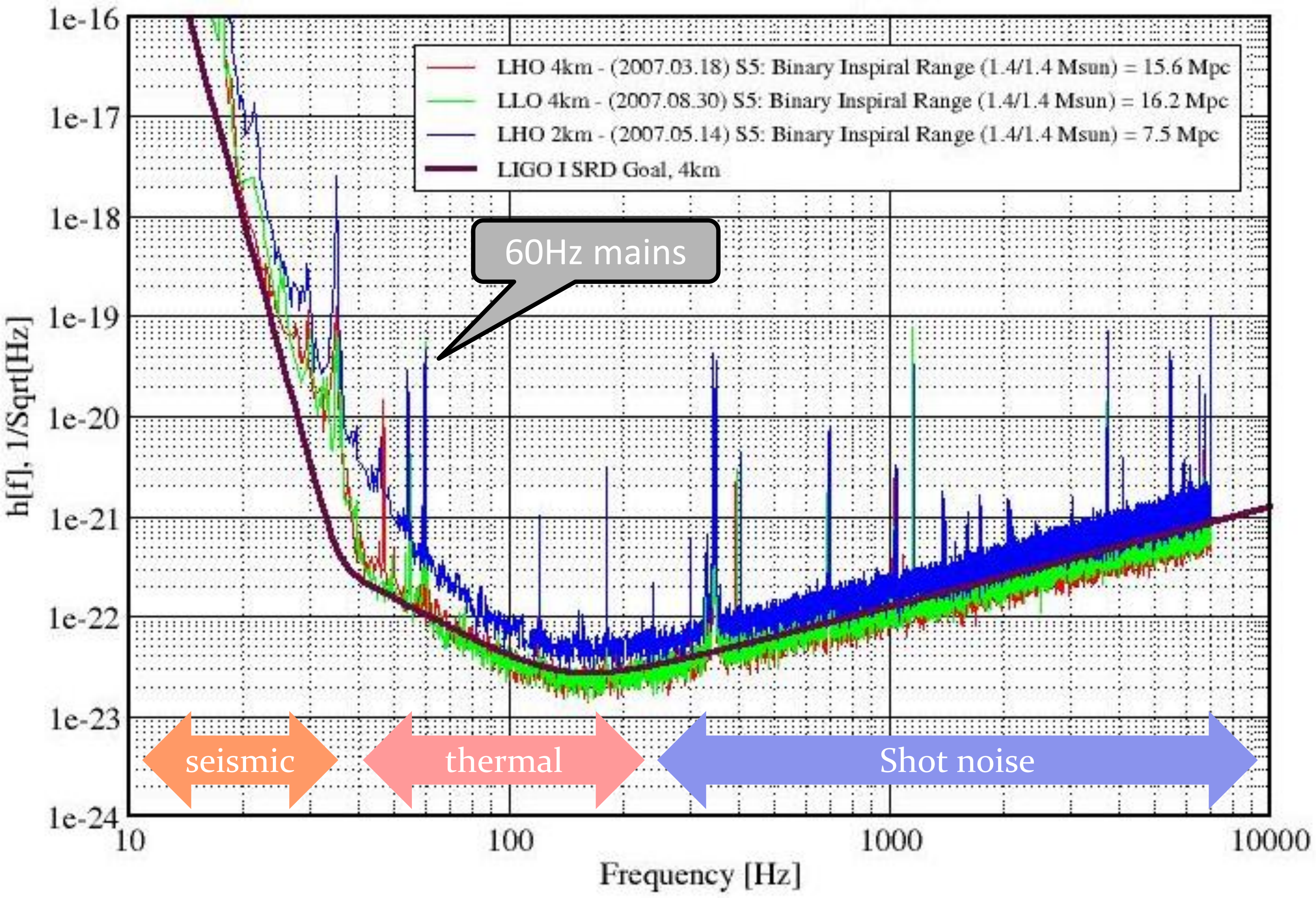
GW detection is hindered by the presence of many sources of noise.

Fundamental noise – intrinsic randomness of the physical detection principle.
- **Laser power shot noise – high frequency (higher laser power).**
- **Thermal noise – mid frequency (ALIGO-cooling).**

Technical noise – experimental design.
- **Power line harmonics (e.g. 60Hz USA) (Signal processing).**
- **Thermal resonances of mirror suspensions (Signal processing).**
- **Scattering/absorption of laser light by particles (Operate in high vacuum).**
- **Stray light (Sealed light paths and light baffles).**

External noise – environmental disturbance of the experiment.
- **Seismic activity (Pendulum suspension of mirrors).**
- **Anthropogenic – vehicular activity, pedestrian (Monitoring).**
- **Gravity gradient noise – local changes in density underground.**

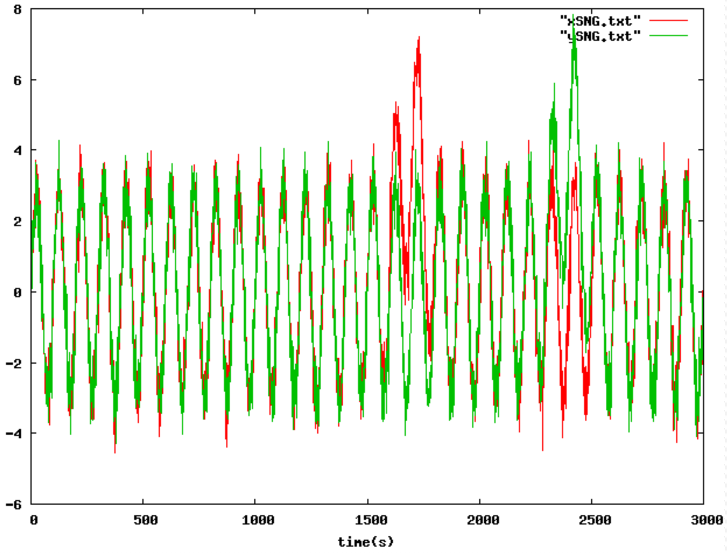
Strain Sensitivity of LIGO Interferometers

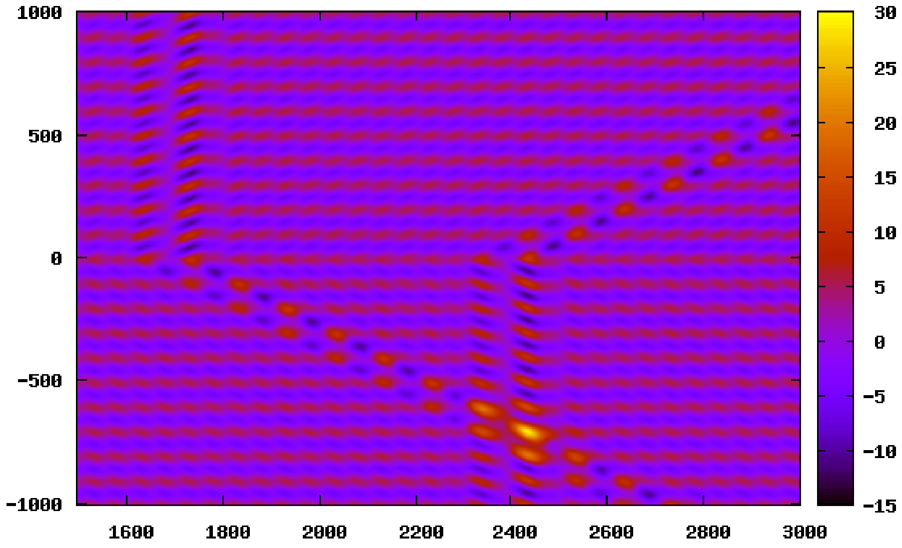


GW Data Analysis

Noise severely impairs GW signal detection algorithms and so raw strain data requires two main signal processing steps.

- **Whitening – equalise the power spectrum.**
- **Line removal – subtract narrow band noise.**

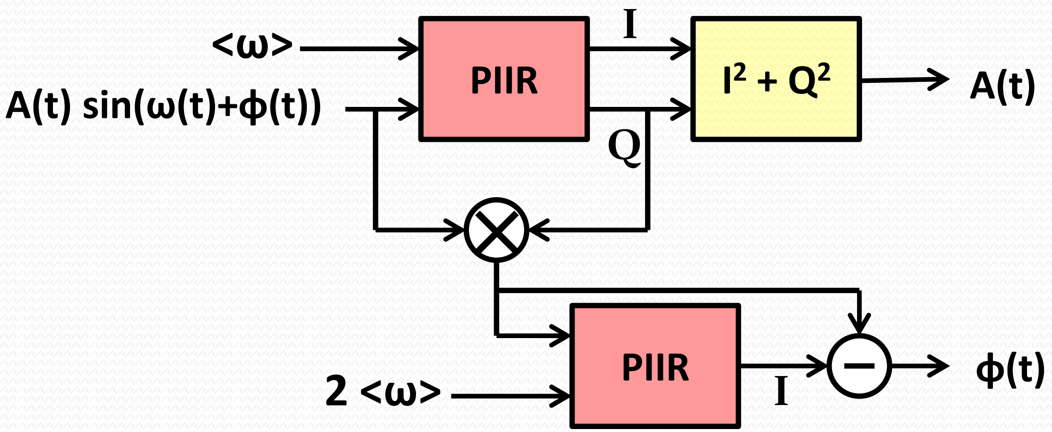




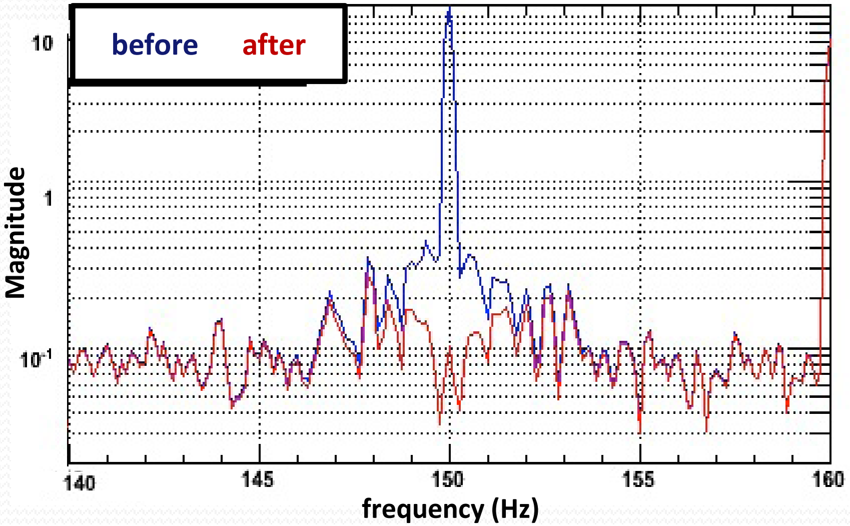
- **We have developed PIIR, a line removal and monitoring tool.**

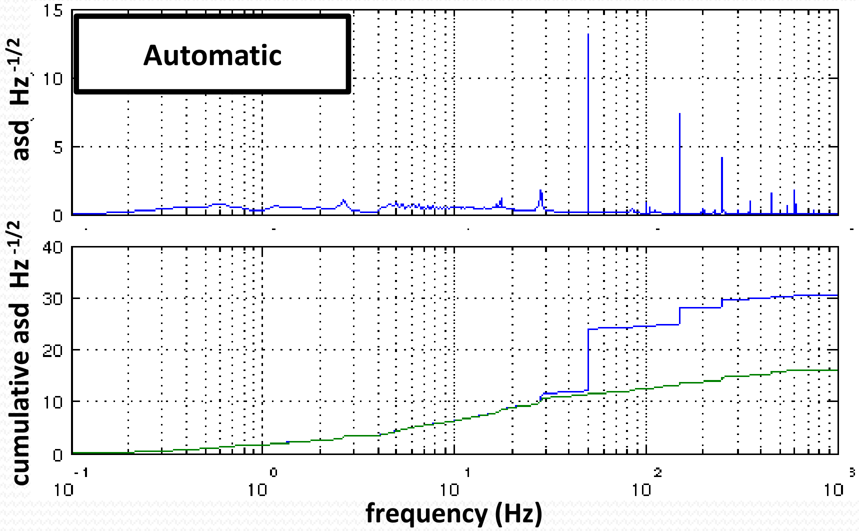
Line Subtraction at GEO600

- **Lines may vary slightly in frequency, amplitude and phase.**
- **PIIR filter is a data driven oscillator which locks onto line frequency and phase.**



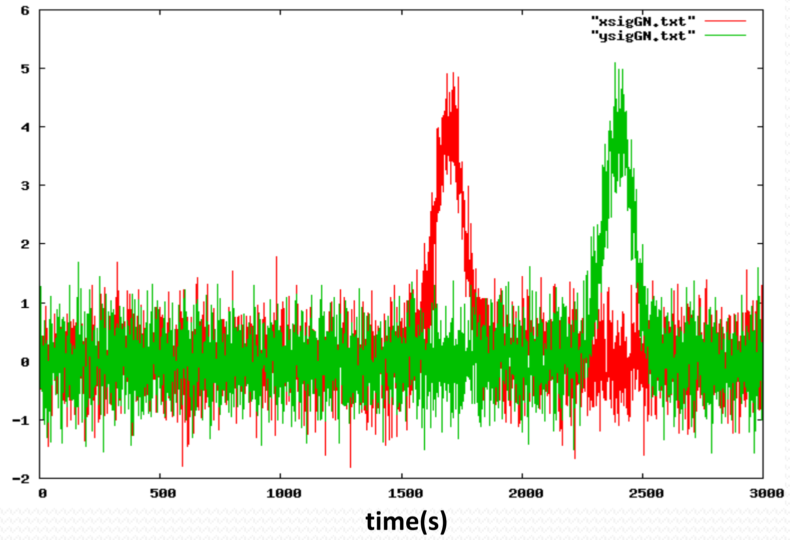
- **Successful implementation at GEO600.**

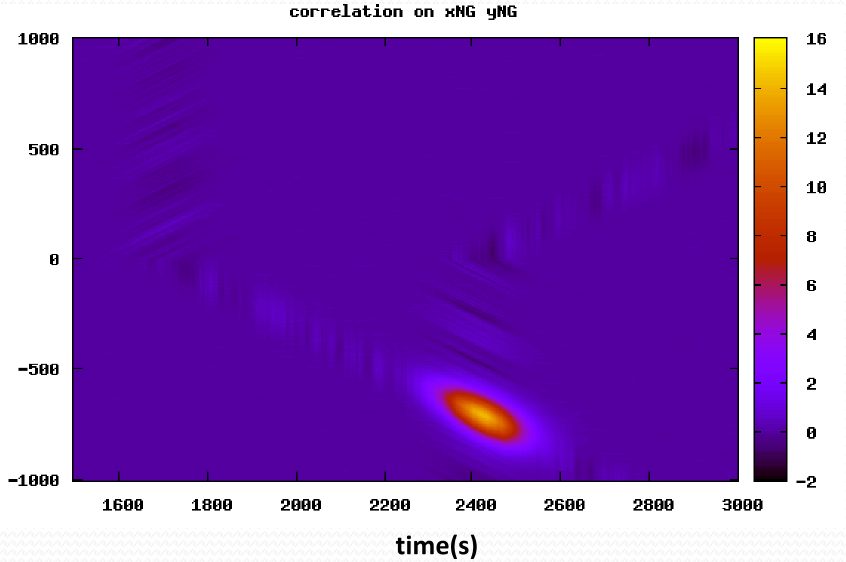




Signal Detection using Cross-Correlation

- **Modelled waveforms – Matched Template**
  - **Cross-correlate detector output with waveform.**
  - **Optimal if signal is *known*.**
  - **Large template banks.**
- **Unmodelled**
  - **Excess power or CC multiple detector output.**
- **Current approach to CC**
  - **FFT blocks of data $\longrightarrow$ FFT^{-1} gives CC.**
  - **FFT *is* fast, $N\log_2 N$ cf. time domain CC N^2.**
- **Problems with this method**
  - **edge effects of windowing FFT.**
  - **compromise time resolution.**
- **We have developed a rapid time domain estimator of CC and propose a comparison with methods currently in use.**





Time Domain Cross-Correlation Estimation

CC approximation (RTCC)

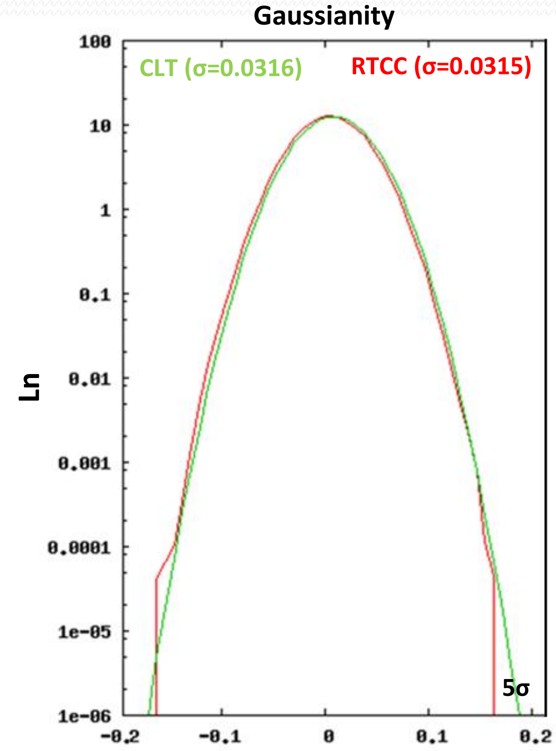
$$C_n^{\,l} = (1-w)\, C_{n-1}^{\,l} + w x_n y_{n-l}$$

$$w = 1 - e^{-1/N}$$

Discrete CC definition

$$C_n^{\,l} = \sum_{i=0}^{N-1} x_{n-i} y_{n-l-i}$$

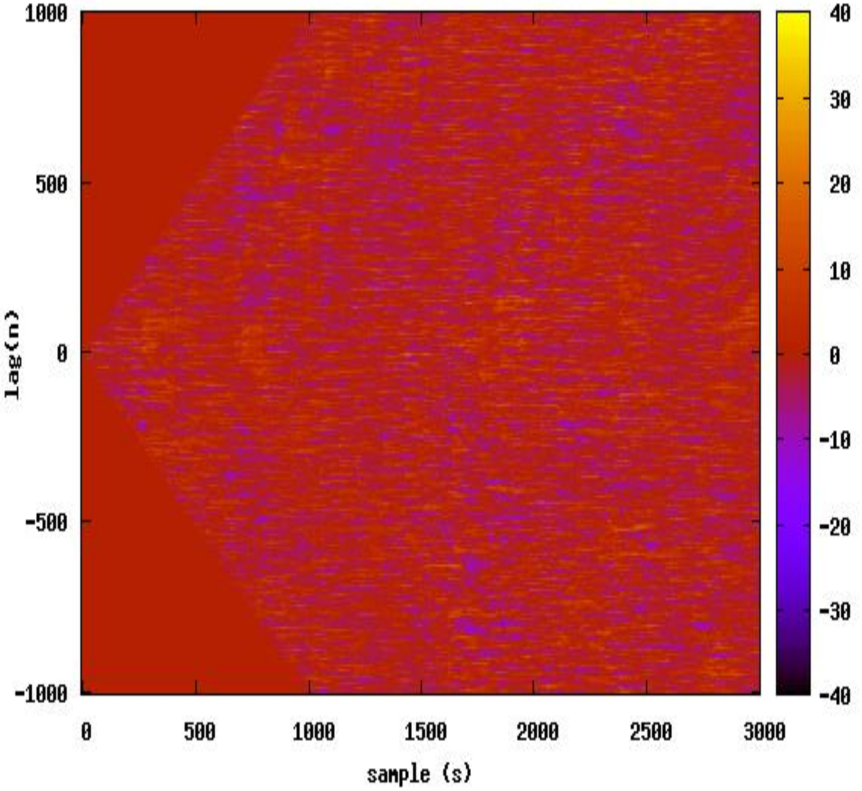
- **Performance**
  - **Computes CC faster than sampling rate 16384/s.**
  - **Symmetric treatment of input data.**
  - **Detect signals with low SNR .**
- **RTCC output characterisation**
  - **Demonstrated output of CC noise is Gaussian in applied use.**
  - **Suitable for event trigger generation.**



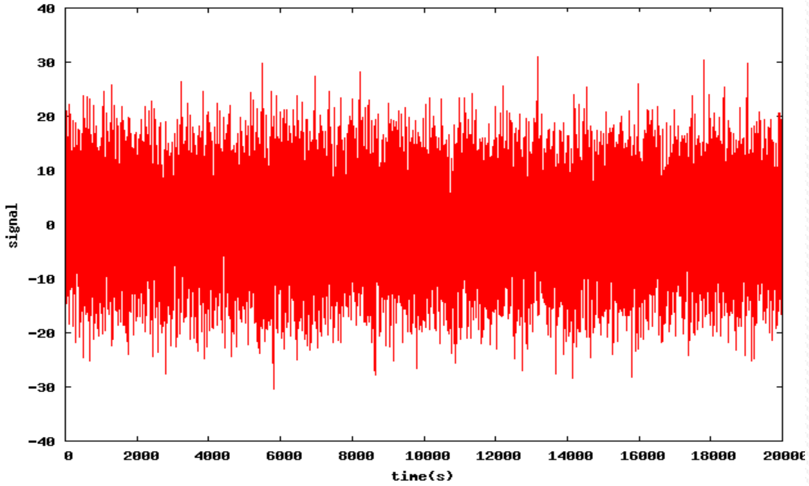
Sensitivity Demonstration

Sine wave unit amplitude buried in Gaussian noise (σ=8).

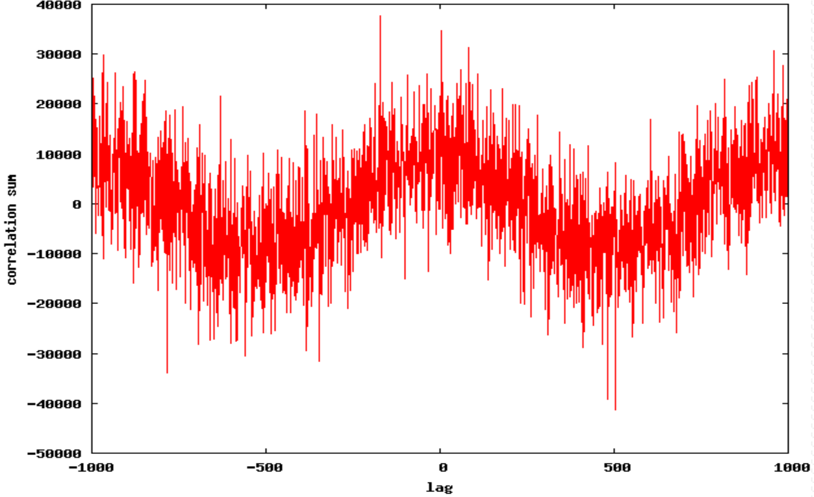
RTCC output



Sine + Noise

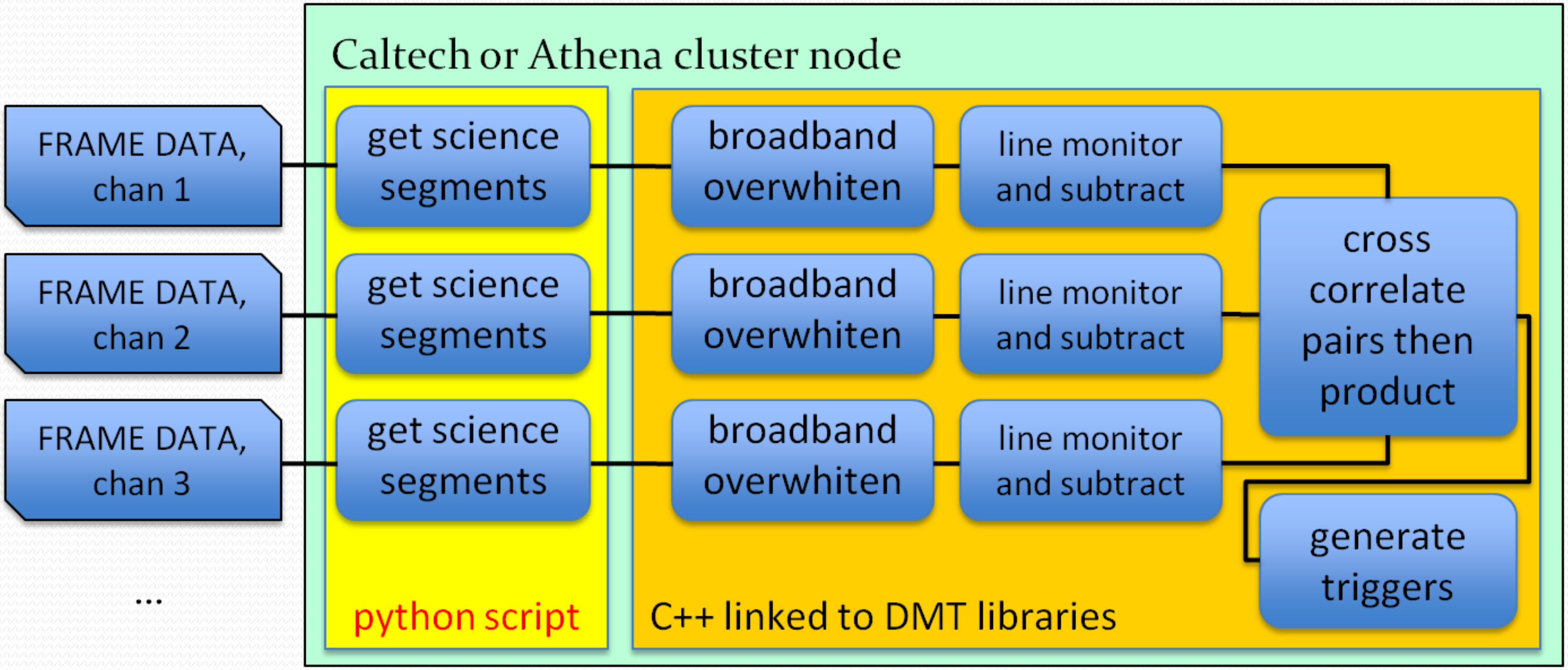


Integrated lags



Current Work – Offline/Online RTCC Implementation

- **Implement RTCC/PIIR into existing LIGO detector software (GDS/DMT).**
- **Testing sensitivity of RTCC on archived detector data (Frames) (Big Dog ?).**



Future Work

- Comparison of PIIR line tracking/removal with existing methods.
- Blind signal injection analysis.
- Contrast detection efficiency of existing event trigger algorithms with our CC estimator as input and/or triggers we develop.
- Blind signal injection analysis.
- Investigate potential of RTCC as a detector diagnostic/commissioning tool.
- Investigate the Frequentist and Bayesian approaches to data analysis in this field.

End