Impact of transverse coupling on the ATLAS luminosity calibration in the Gaussian beam limit

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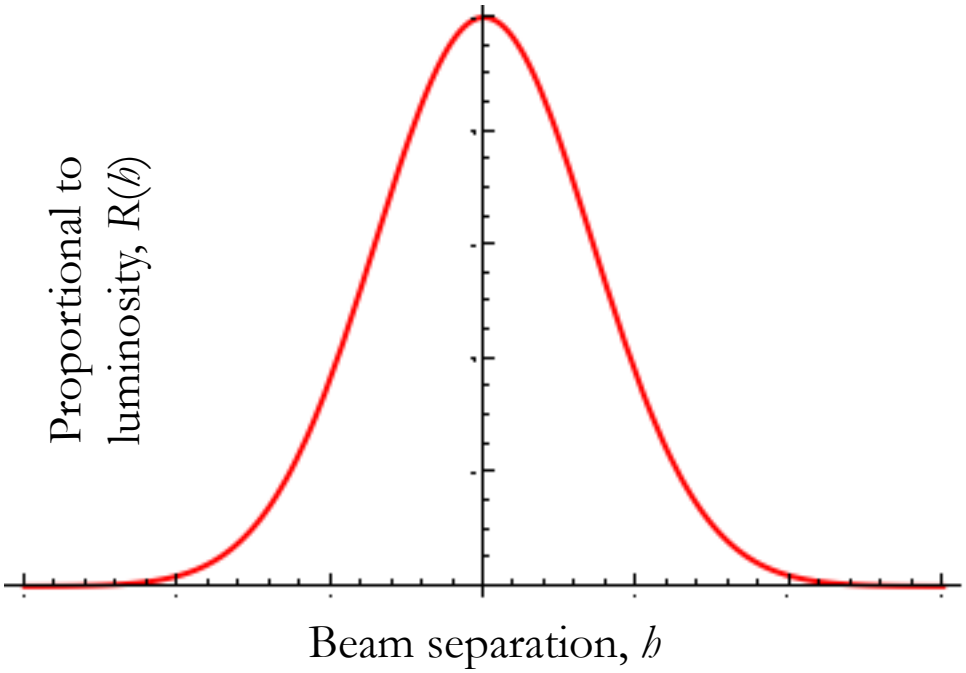
The van der Meer method



$$\mathcal{L} = \frac{R}{\sigma}$$

The vdM method allows one to calibrate the *absolute* luminosity scale by scanning the beams past one another.

S. van der Meer, *Calibration of the effective beam height in the ISR. oai:cds.cern.ch:296752*, Tech. Rep. CERN-ISR-PO-68-31, CERN, Geneva, 1968.

$$\Sigma_{x_i} = \frac{1}{\sqrt{2\pi}} \frac{\int dh\, R_{x_i}(h_{x_i})}{R(0)} \qquad \mathcal{L} = \frac{n_b\, f_r\, n_1\, n_2}{2\pi\, \Sigma_x\, \Sigma_y}$$

Implicitly assumes factorisation of x and y components! How does (linear) x-y correlation in the beam affect this method?

The single-Gaussian model

$$\rho_i(x,y,z,t) = \frac{\exp\left(-\frac{1}{2}\vec{x}\cdot\underline{\sigma}_i^{-1}\cdot\vec{x}\right)}{\sqrt{(2\pi)^3|\underline{\sigma}_i|}}$$

This is the single-Gaussian bunch parameterisation

where $\underline{\sigma}_i = \begin{pmatrix} \sigma_{x,i}^2 & \kappa_i\,\sigma_{x,i}\,\sigma_{y,i} & 0 \\ \kappa_i\,\sigma_{x,i}\,\sigma_{y,i} & \sigma_{y,i}^2 & 0 \\ 0 & 0 & \sigma_{z,i}^2 \end{pmatrix}$ and $\vec{x} = \begin{pmatrix} x \\ y \\ z \pm z_0(t) \end{pmatrix}$

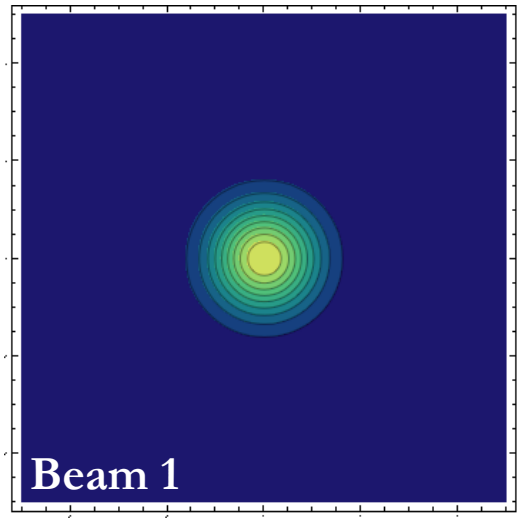
The luminosity in this model is proportional to

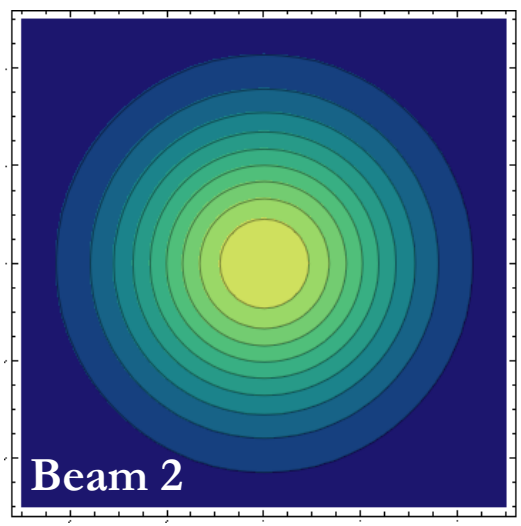
$$\mathcal{L}(x,y,z) \propto \int \rho_1(x,y,z,t)\,\rho_2(x,y,z,t)\,dt$$

$$= \rho_1(x,y)\,\rho_2(x,y)\int \rho_1(z,t)\,\rho_2(z,t)\,dt$$

In the absence of explicit beam crossing angle

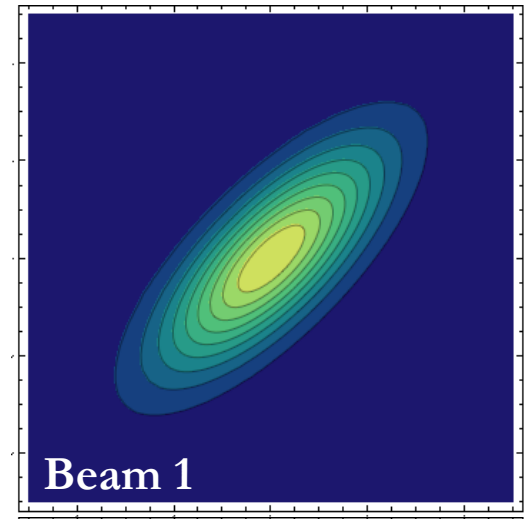
Two illuminating examples

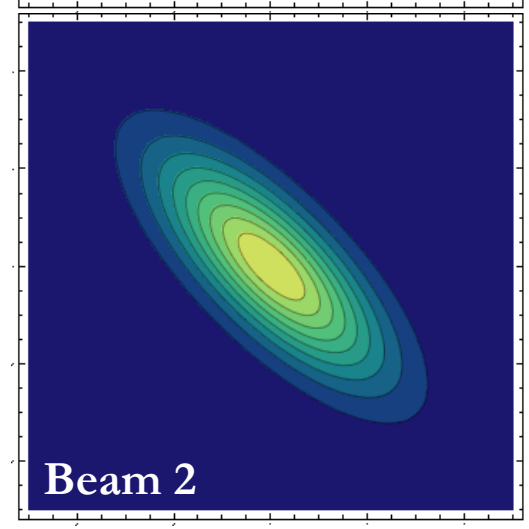


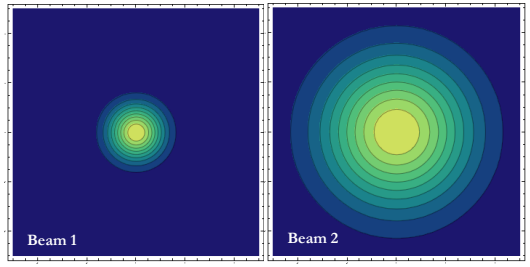


Different beam sizes

x-y correlation



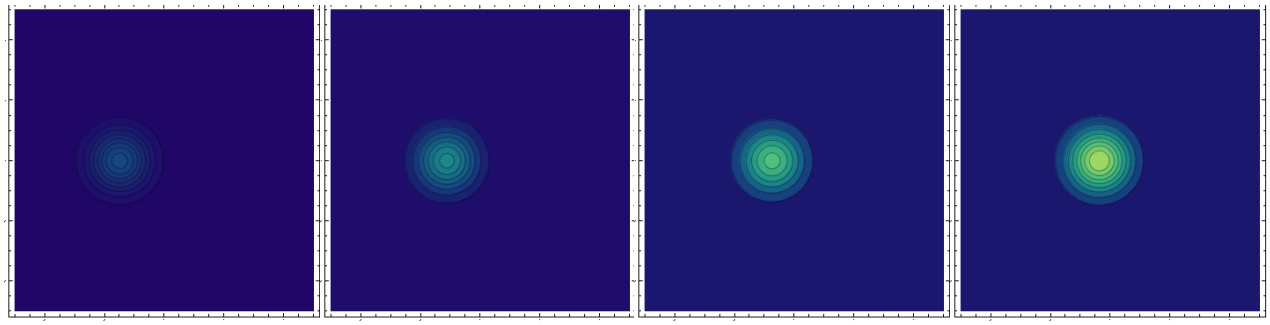




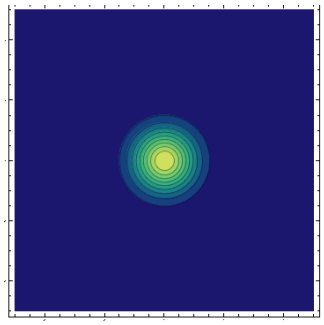
Scanning beams of different size

Luminous region follows the bulk of the **narrow** beam as it scans the wider beam

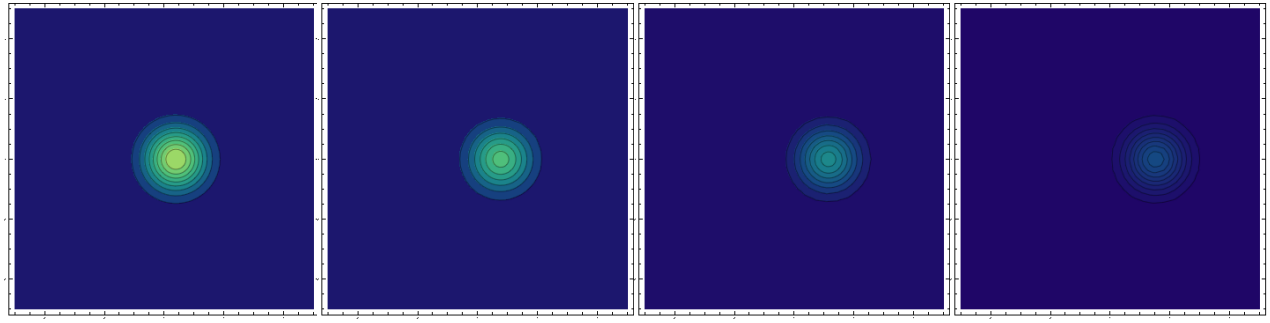
Higher negative separation

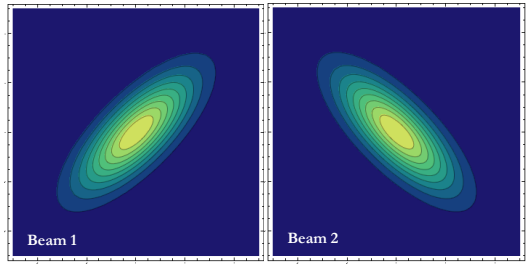


Centred



Higher positive separation



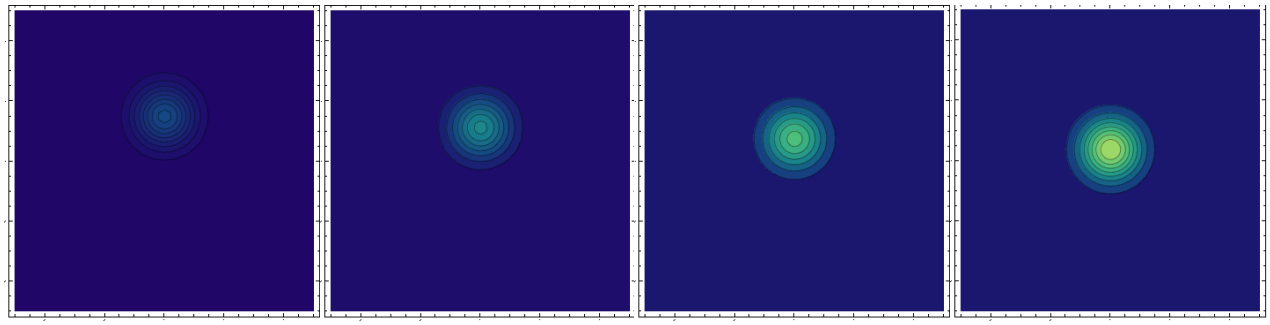


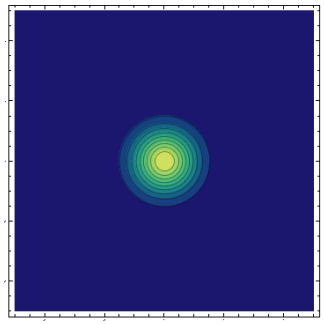
Scanning x-y correlated beams

Movements of luminous region is **transverse** to scan direction

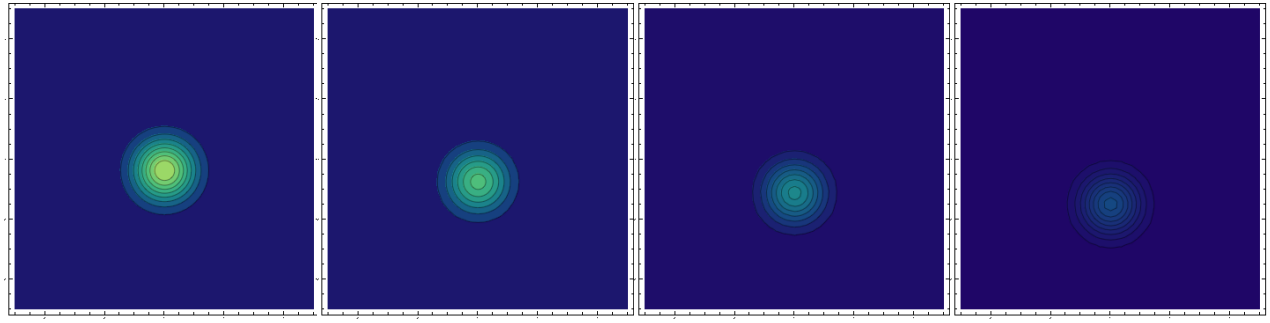
Higher negative separation

Centred





Higher positive separation

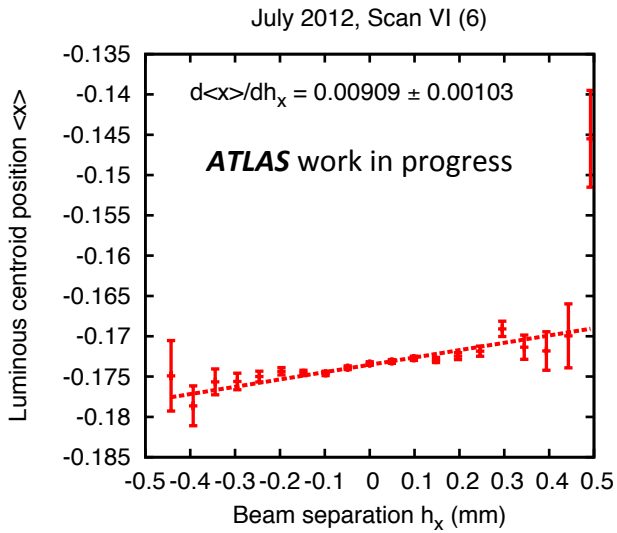


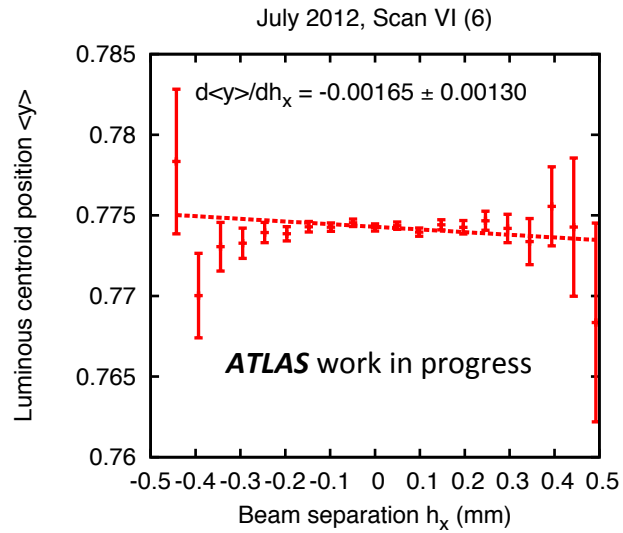
Movements of the luminous centroid

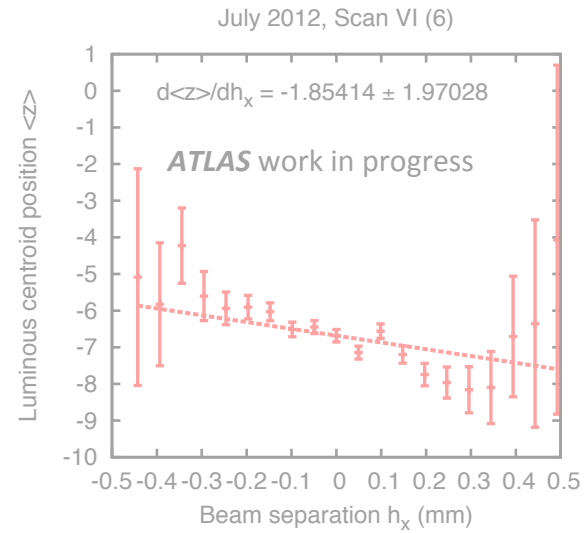
Observed phenomenon	Cause
$\frac{d\langle z\rangle}{dh_{x,y}}$	Beam crossing angle in x-z or y-z plane; α_{xz} and α_{yz}, respectively.
$\frac{d\langle x\rangle}{dh_x} \neq \frac{d\langle y\rangle}{dh_y} \neq 0$	Different beam sizes in transverse plane.
$\frac{d\langle y\rangle}{dh_x} \sim \frac{d\langle x\rangle}{dh_y} \neq 0$	x-y correlation within each beam.

$$\langle \vec{x} \rangle = h_x \frac{d\langle \vec{x} \rangle}{dh_x} + h_y \frac{d\langle \vec{x} \rangle}{dh_y}$$

July 2012 scan (6) data



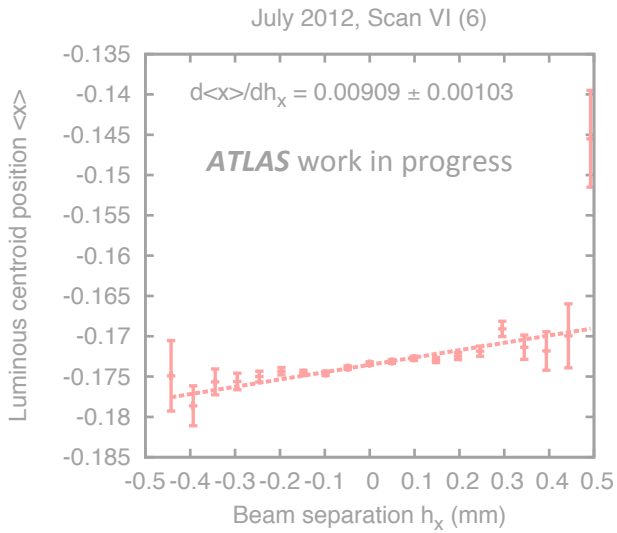


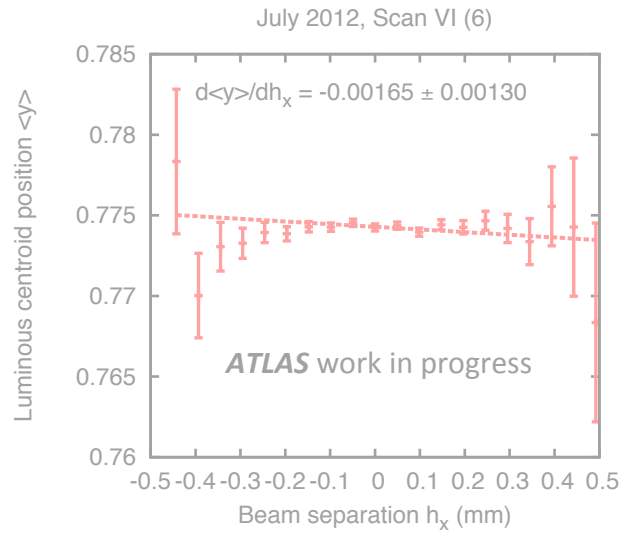


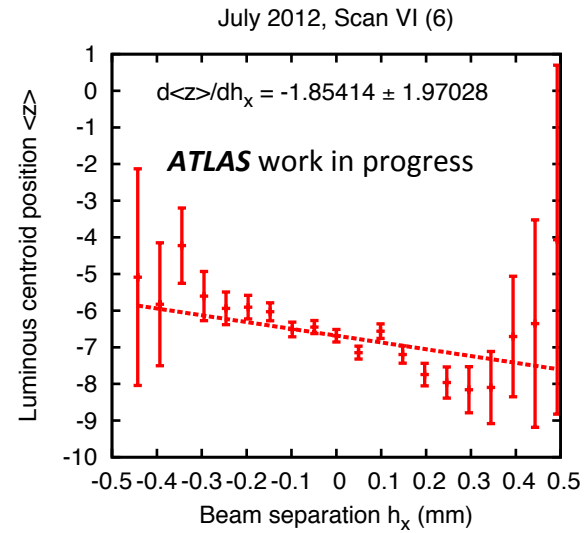
$$\frac{d\langle x\rangle}{dh_x} = \frac{\sigma_{x,1}^2 - \sigma_{x,2}^2}{2\left(\sigma_{x,1}^2 + \sigma_{x,2}^2\right)} + \text{h.o.t.}$$

$$\frac{d\langle x\rangle}{dh_y} = \frac{\sigma_{x,1}\,\sigma_{x,2}\left(\kappa_2\,\sigma_{x,1}\,\sigma_{y,2} - \kappa_1\,\sigma_{x,2}\,\sigma_{y,1}\right)}{\left(\sigma_{x,1}^2 + \sigma_{x,2}^2\right)\left(\sigma_{y,1}^2 + \sigma_{y,2}^2\right)} + \text{h.o.t.}$$

July 2012 scan (6) data



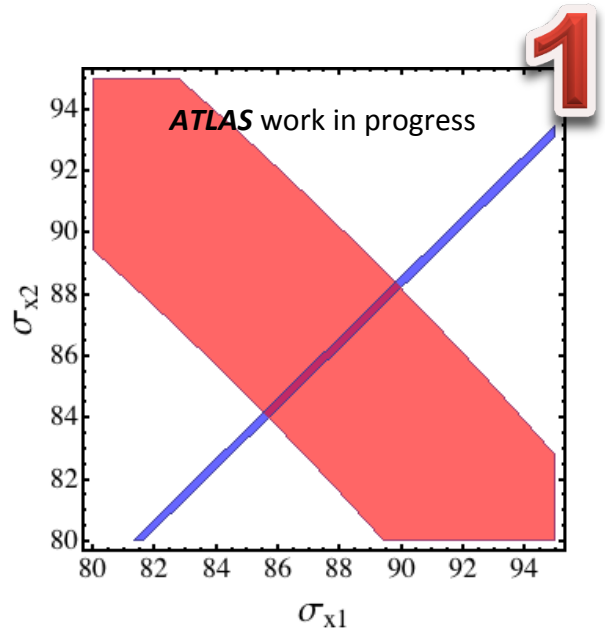


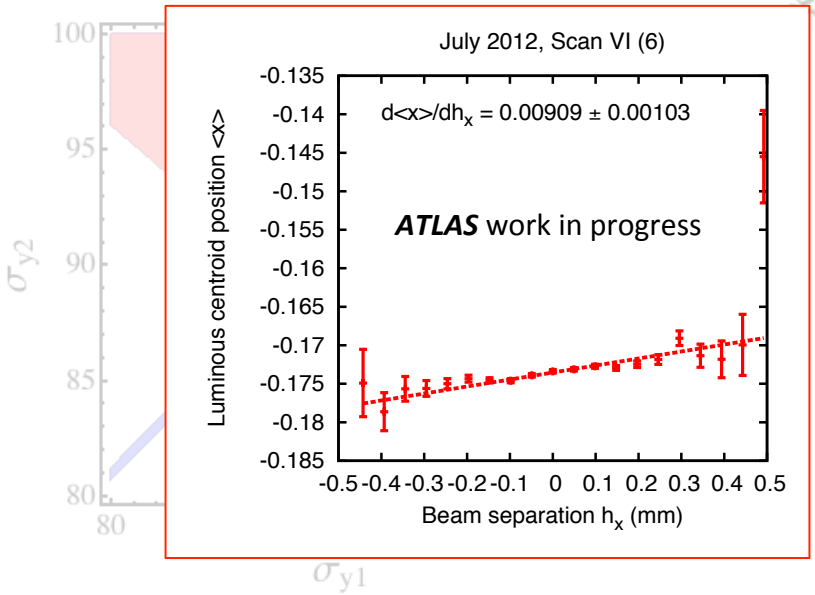


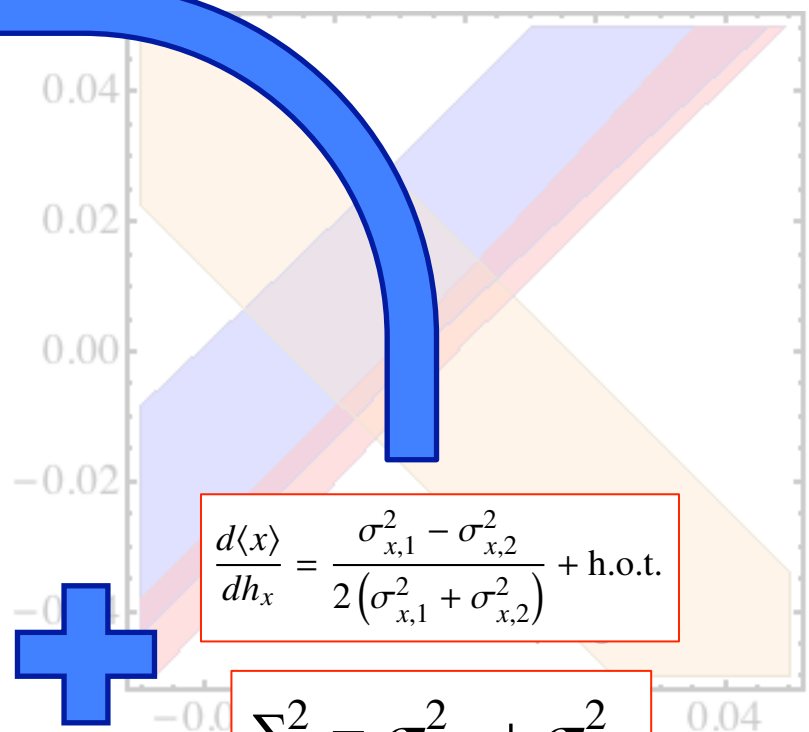
$$\frac{d\langle z\rangle}{dh_x} = \frac{\alpha_{xz}\,\sigma_z^2}{2\left(\sigma_{x,1}^2 + \sigma_{x,2}^2\right)} + \ldots$$

Constraints

(July 2012 scan 6)





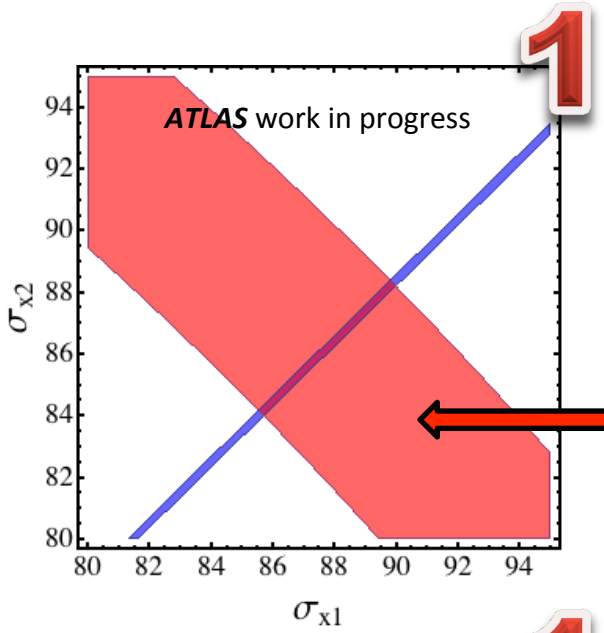


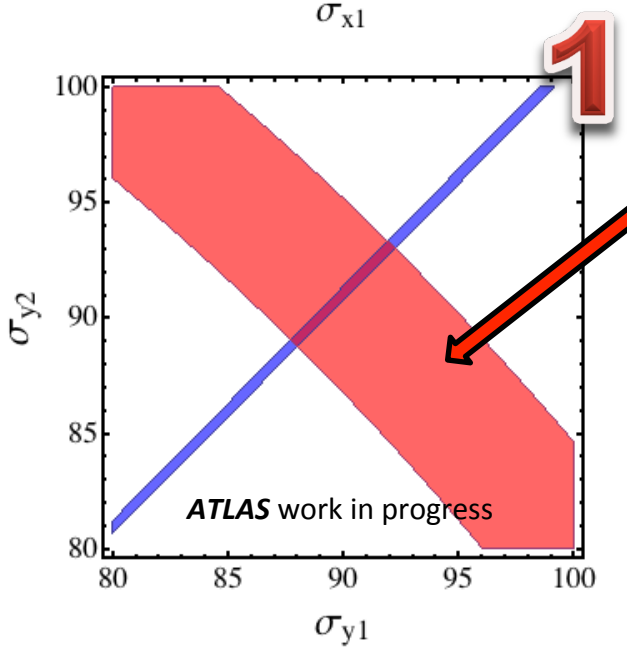
$$\frac{d\langle x\rangle}{dh_x} = \frac{\sigma_{x,1}^2 - \sigma_{x,2}^2}{2\left(\sigma_{x,1}^2 + \sigma_{x,2}^2\right)} + \text{h.o.t.}$$

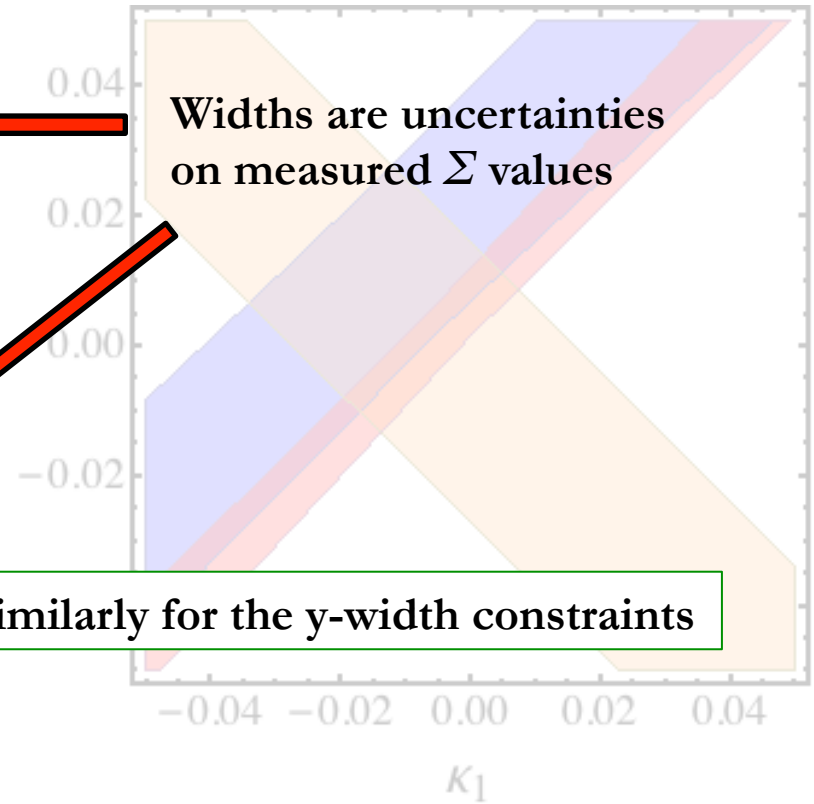
$$\Sigma_j^2 = \sigma_{j,1}^2 + \sigma_{j,2}^2$$

Constraints

(July 2012 scan 6)







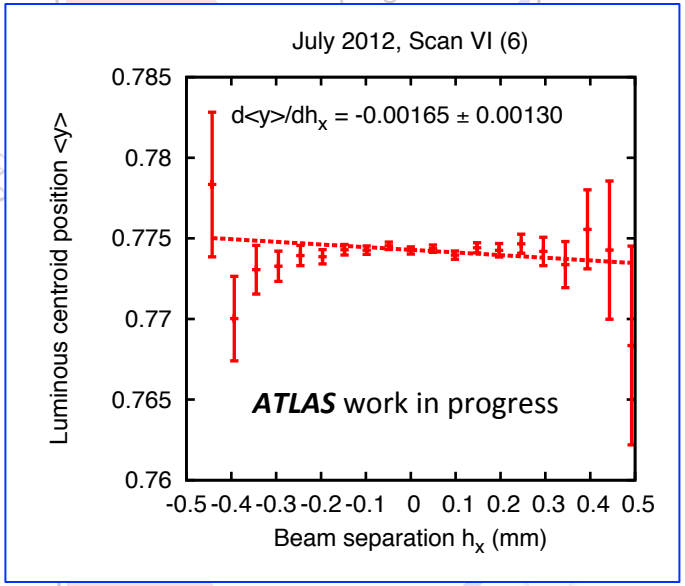
Widths are uncertainties on measured Σ values

Similarly for the y-width constraints

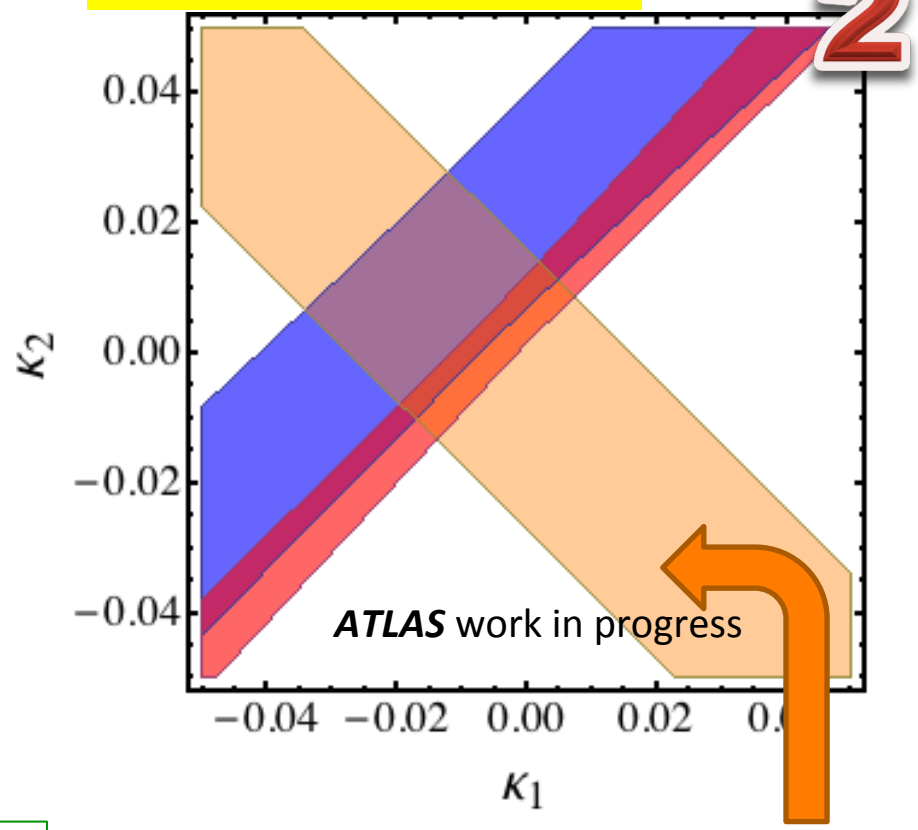
Constraints

(July 2012 scan 6)

κ is correlation coefficient



$$\frac{d\langle x\rangle}{dh_y} = \frac{\sigma_{x,1}\,\sigma_{x,2}\left(\kappa_2\,\sigma_{x,1}\,\sigma_{y,2} - \kappa_1\,\sigma_{x,2}\,\sigma_{y,1}\right)}{\left(\sigma_{x,1}^2 + \sigma_{x,2}^2\right)\left(\sigma_{y,1}^2 + \sigma_{y,2}^2\right)} + \text{h.o.t.}$$



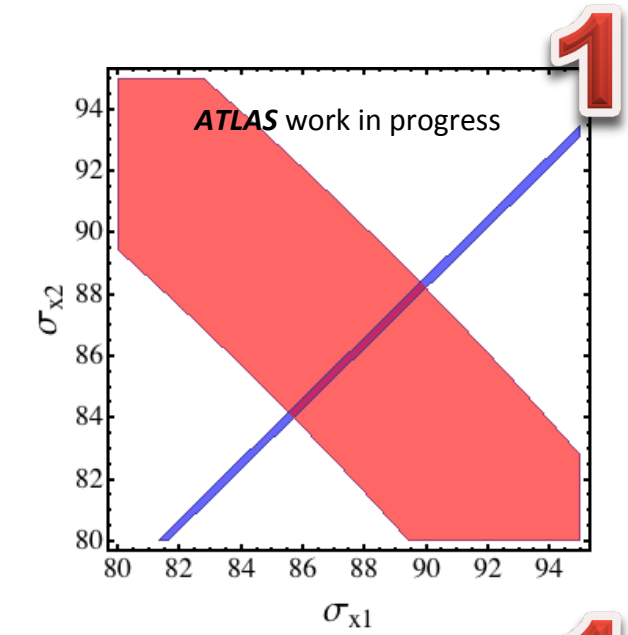
Similarly for the y position during x scan

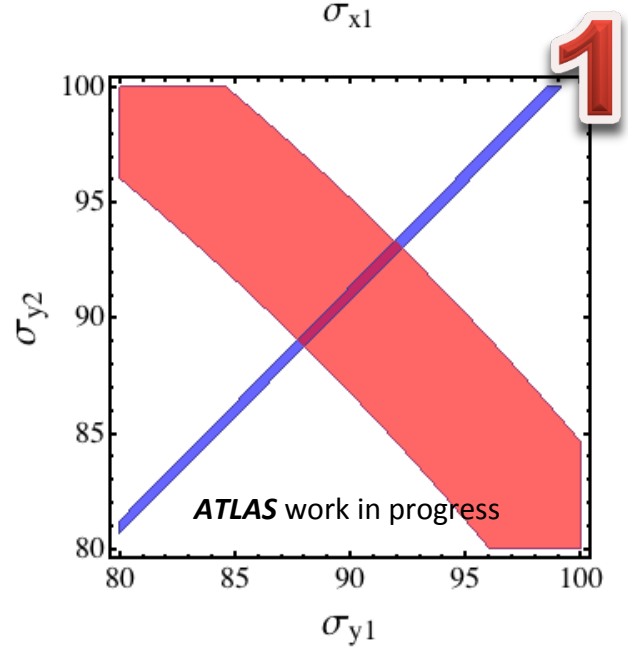
From direct measurement of luminous region correlation

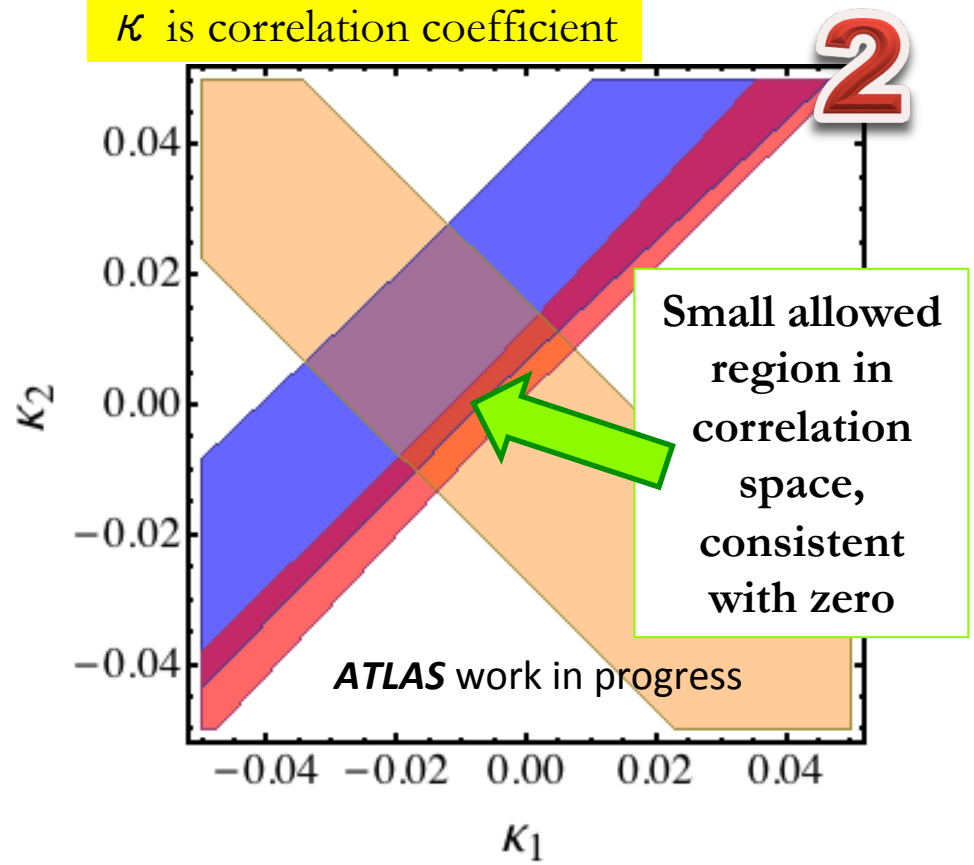
Constraints

(July 2012 scan 6)

κ is correlation coefficient



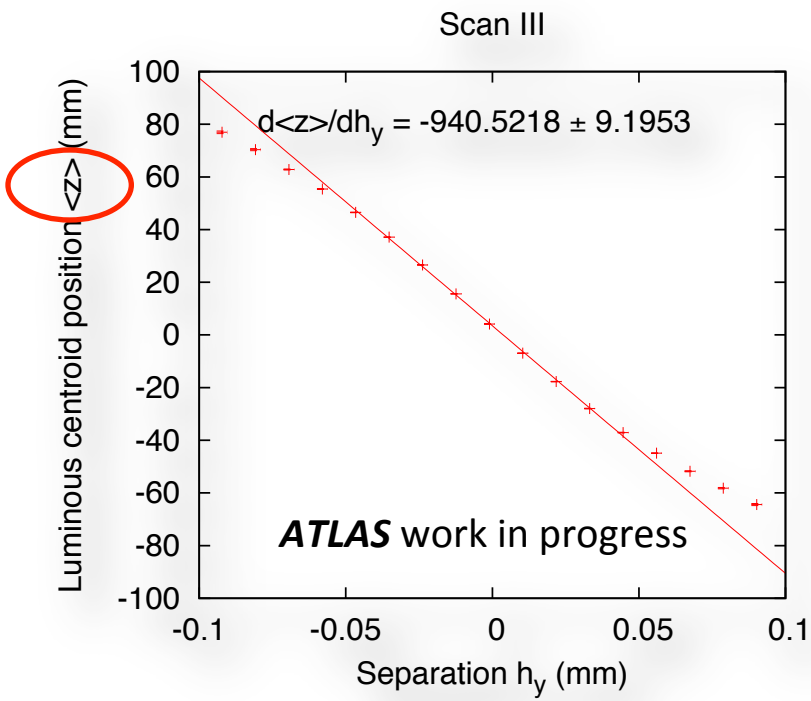




Uncertainties from constraints in (1) are propagated to band widths in (2)

Explicit beam crossing angle

…has been included in the model, essentially functioning as an *x*-*z* and a *y*-*z* correlation.



…vdM scan III (3) in April 2012

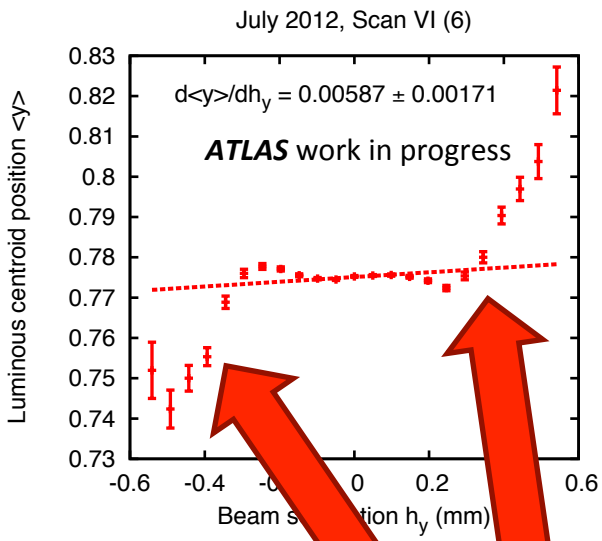
"Significant" crossing angle

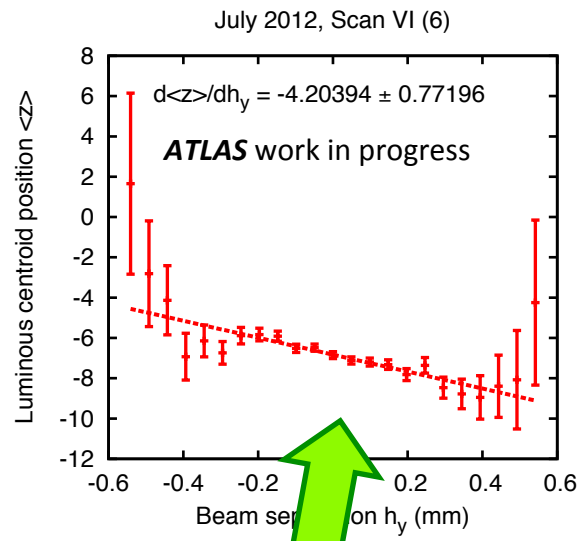
Conclusions

- Permits a fully-analytic approach. Soluble with series solution about small correlation.
- Systematic method for constraining all beam parameters without boot-strapping, iterating, etc.
- Arbitrary crossing-angle easily incorporated, retaining analyticity.
- Applied to vdM scans in *Oct '10*, *Mar '11*, *May '11*, *Apr '12* and *Jul '12*.
- Consistently suggests impact of (linear) transverse coupling on luminosity calibration is negligible.
- One of the systematic contributions that enters an unprecedented 1.8% luminosity error at ATLAS.
- On-going work: Further models have been explored in detail, which successfully account for many of the non-linear features observed in the data.

BACKUP SLIDES

Successes and limitations





Non-linear tails cannot be described using single-Gaussian model

Crossing angles are easily implemented in single-Gaussian model

Further models and work in progress

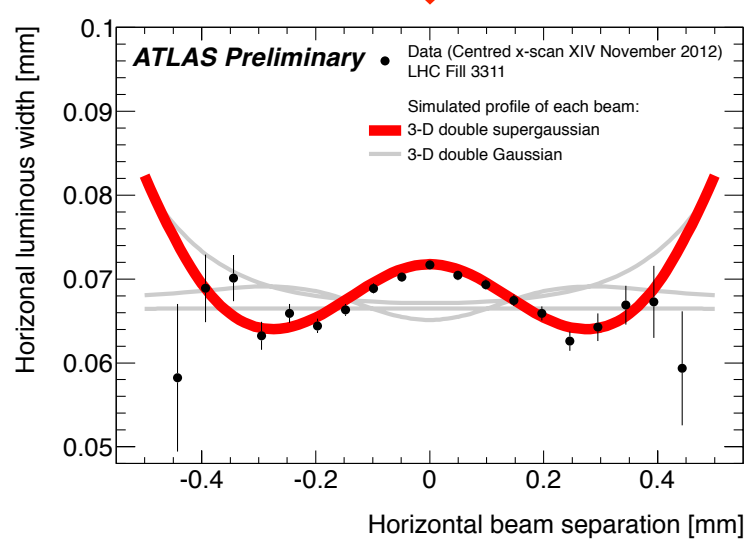
Double Gaussian

$$\rho(\vec{x};\ldots) = w\,\rho_{\mathrm{A}}(\vec{x};\ldots) + (1-w)\,\rho_{\mathrm{B}}(\vec{x};\ldots)$$

$$\ldots = \text{model parameters}$$

Super Gaussian

$$\rho(x;\ldots) = \mathcal{N}\exp\left[-\frac{1}{2}\left(\frac{|x|}{\sigma}\right)^{2+\epsilon}\right]$$



Further models have been explored in detail, which successfully account for many of the non-linear features observed in the data.

An analytical approach is not always possible, as with the single-Gaussian model. Work is based on solving these as far as possible.