Yerevan, 18. 9. 2013

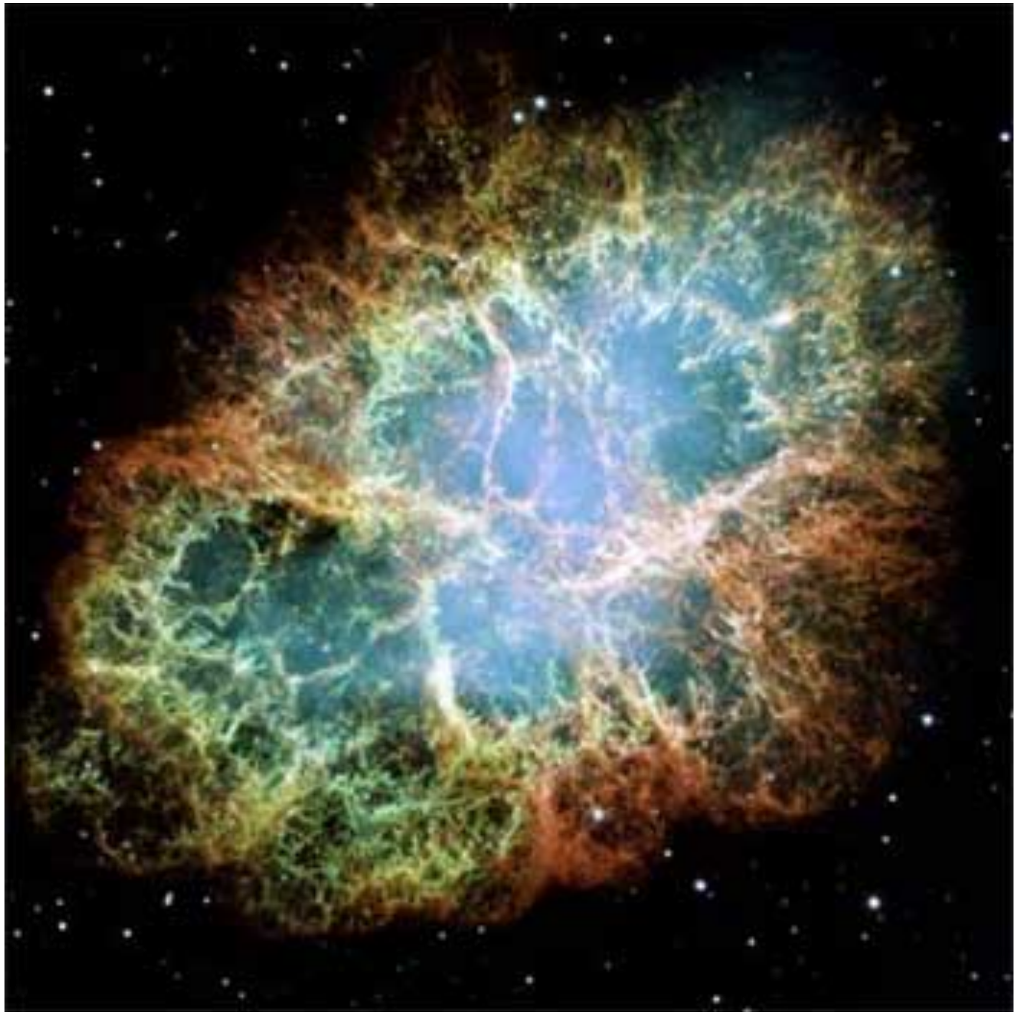
The Modern Physics of Compact Stars and Relativistic Gravity

Correlations in Nuclear Matter and the Symmetry Energy

Gerd Röpke, Rostock

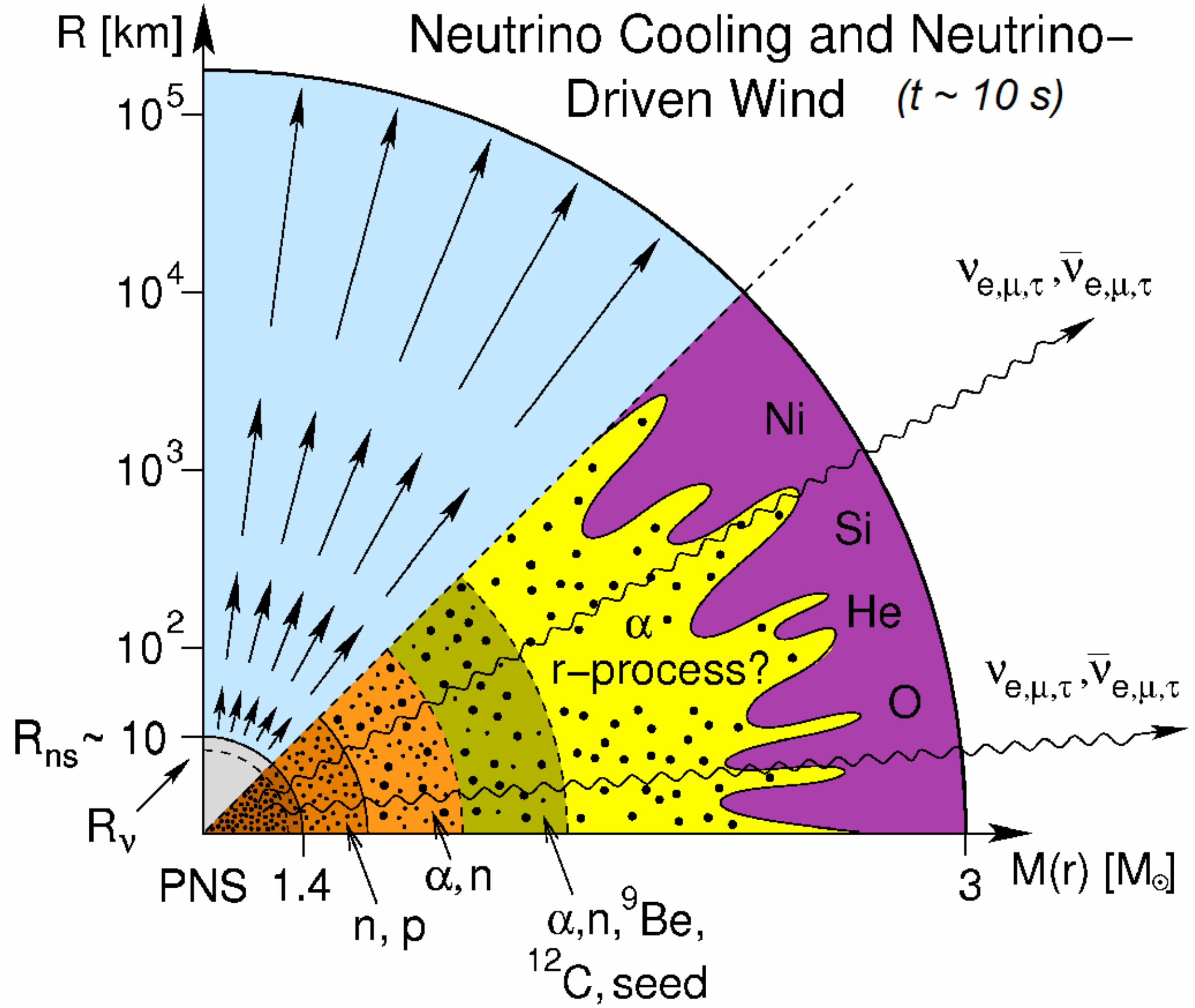
Supernova

Crab nebula, 1054 China, PSR 0531+21



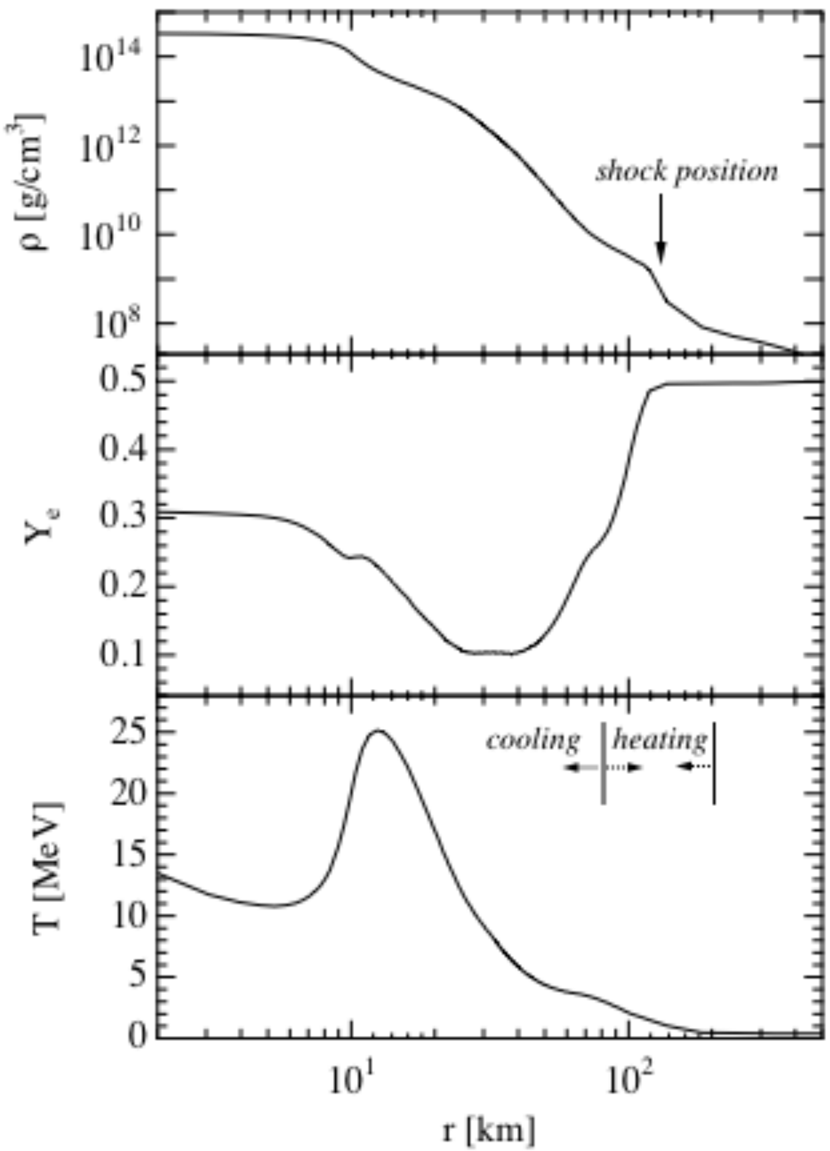
M1, the Crab Nebula. Courtesy of NASA/ESA

Supernova explosion



T.Janka

Core-collapse supernovae



Density.

electron fraction, and

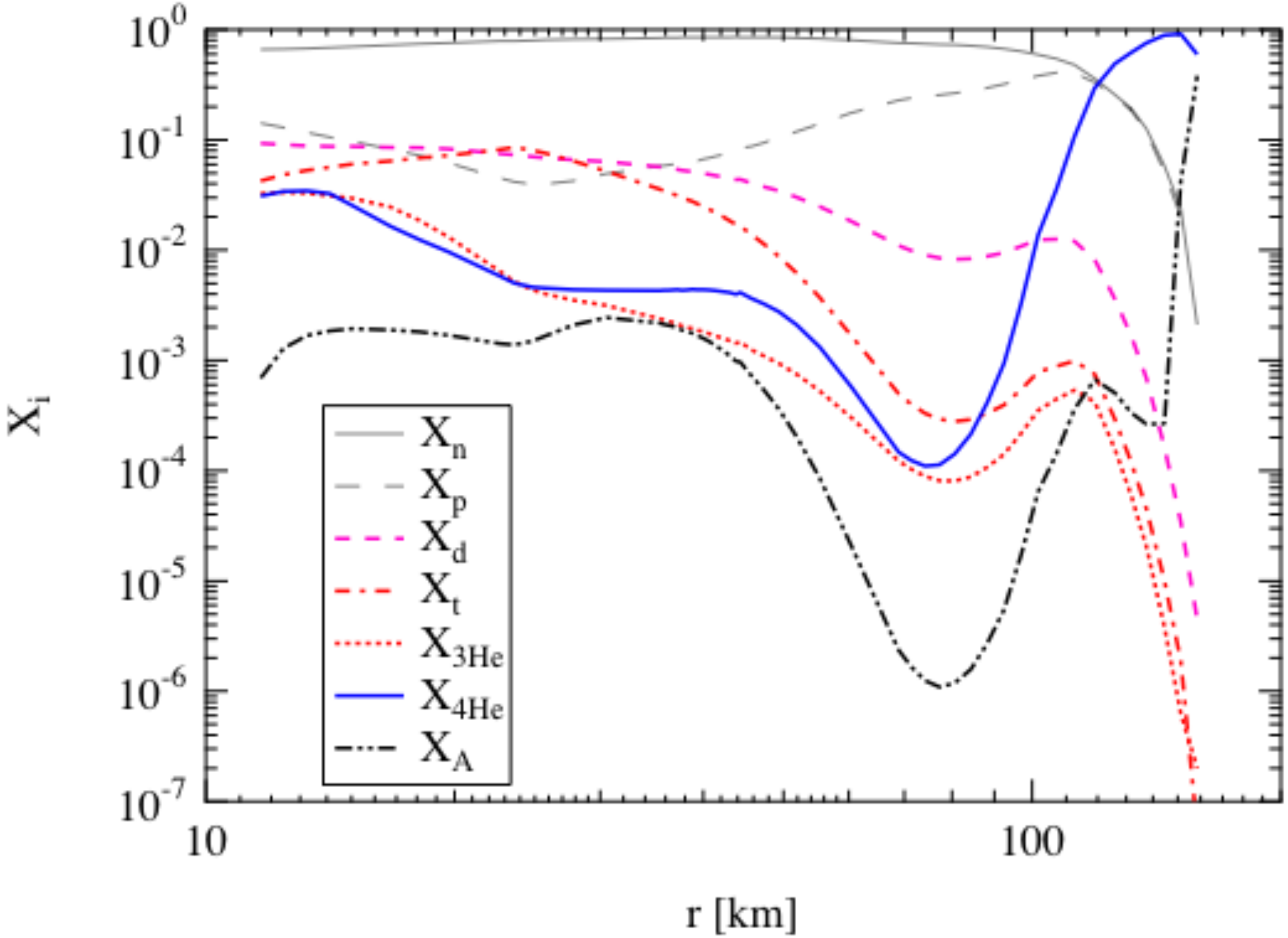
temperature profile

of a 15 solar mass supernova
at 150 ms after core bounce
as function of the radius.

Influence of cluster formation
on neutrino emission
in the cooling region and
on neutrino absorption
in the heating region ?

K.Sumiyoshi et al.,
Astrophys.J. **629**, 922 (2005)

Composition of supernova core



K.Sumiyoshi,
G. R.,
PRC **77**,
055804 (2008)

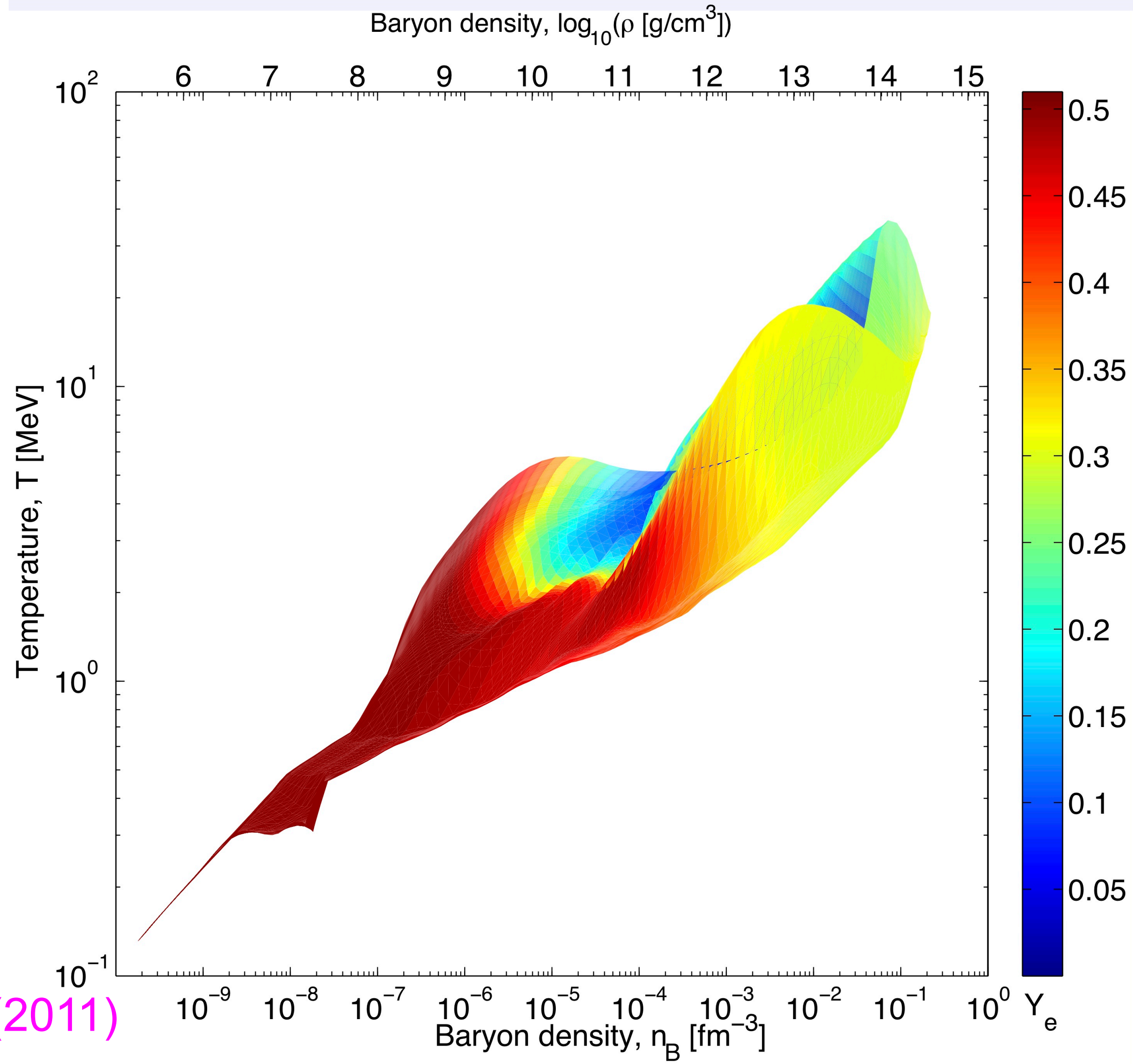
Mass fraction X of light clusters for a post-bounce supernova core

Nuclear matter phase diagram

Core collapse supernovae

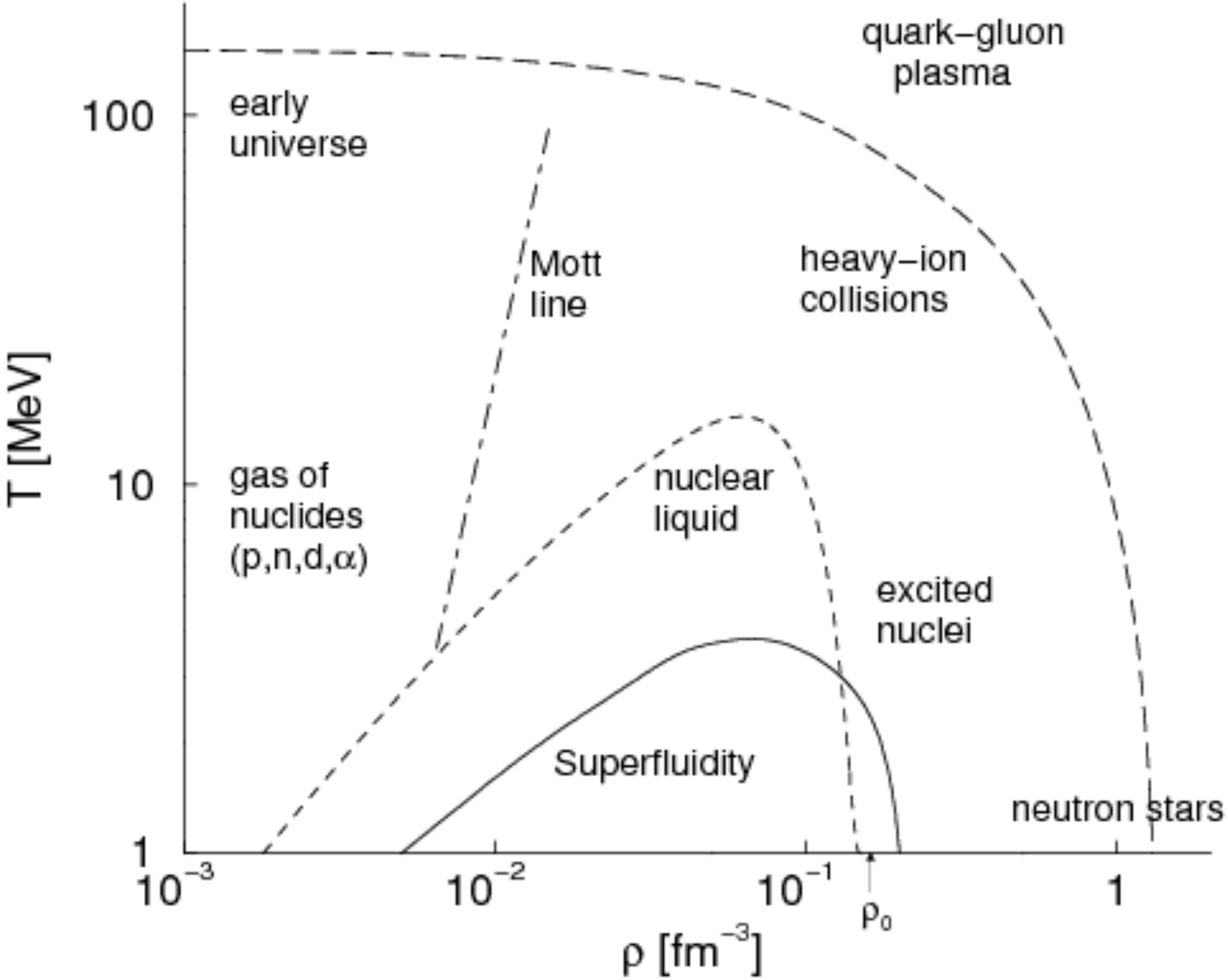
Relevant Parameters:

- **density**:
  $10^{-9} \lesssim \varrho/\varrho_{\rm sat} \lesssim 10$
  with nuclear saturation density
  $\varrho_{\rm sat} \approx 2.5 \cdot 10^{14}$ g/cm^3
  ($n_{\rm sat} = \varrho_{\rm sat}/m_n \approx 0.15$ fm^{-3})
- **temperature**:
  0 MeV $\leq k_B T \lesssim 50$ MeV
  ($\hat{=}\ 5.8 \cdot 10^{11}$ K)
- **electron fraction**:
  $0 \leq Y_e \lesssim 0.6$



T. Fischer et al., ApJS 194, 39 (2011)

Symmetric nuclear matter: Phase diagram

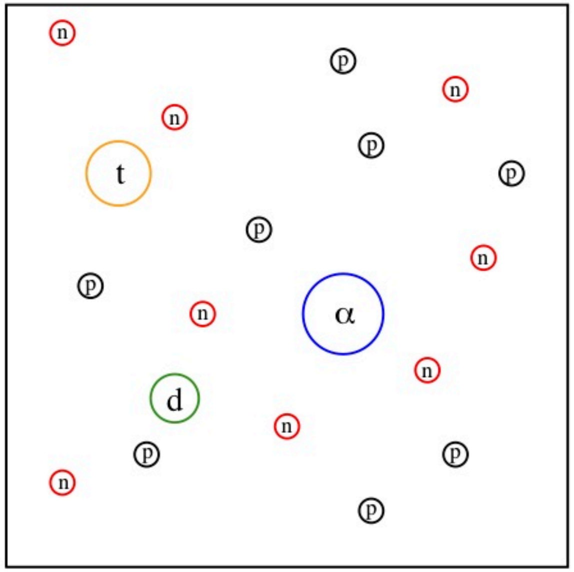


Nuclear statistical equilibrium (NSE)

Chemical picture:

Ideal mixture of reacting components

Mass action law



Ideal mixture of reacting nuclides

$$n_p(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} Z_A f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

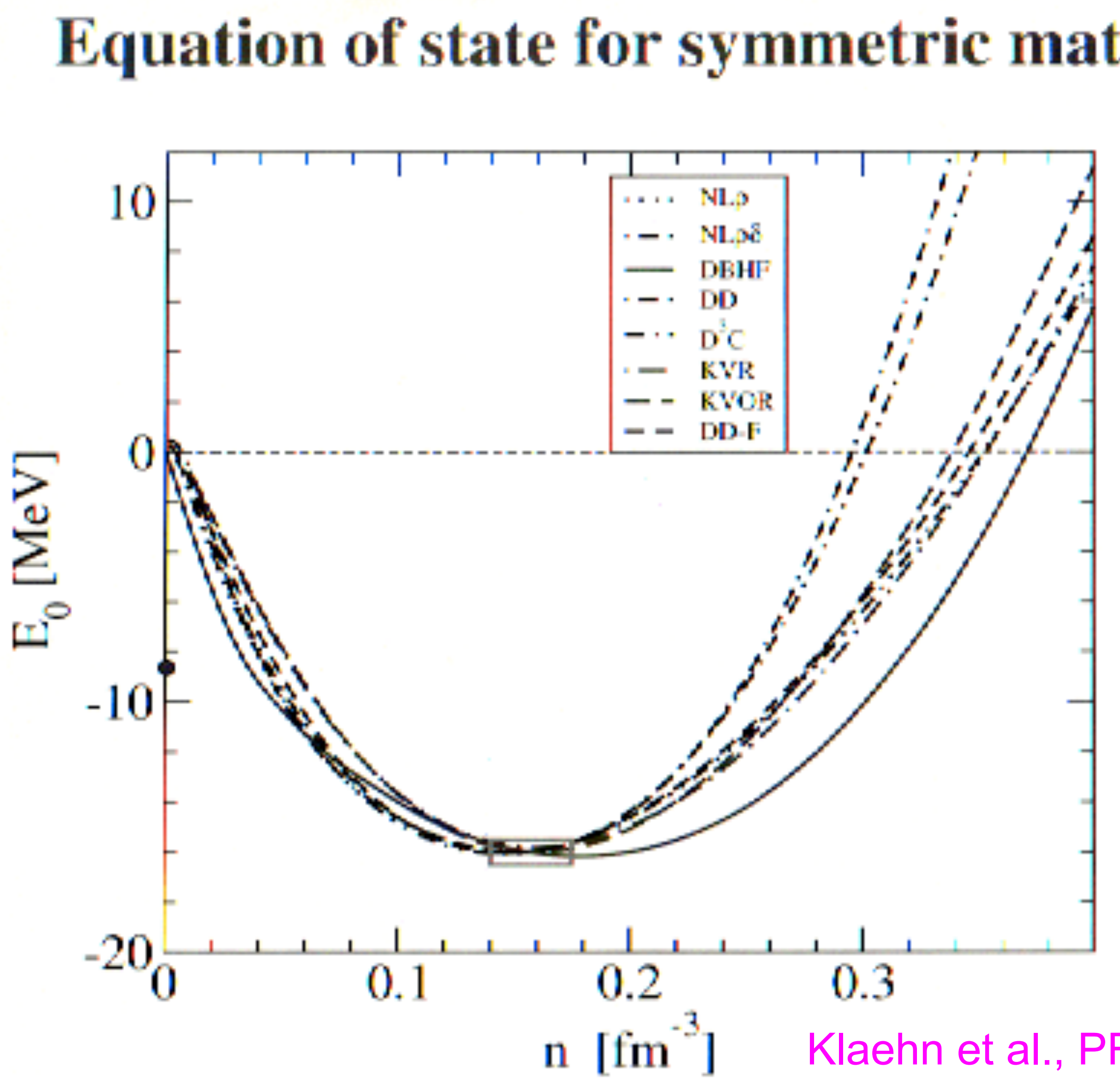
$$n_n(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} (A-Z_A) f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

$$f_{A(z)} = \frac{1}{\exp(z/T) - (-1)^A}$$

mass number *A*,
charge Z_A,
energy $E_{A,\nu,K}$,
ν internal quantum number,
K: center of mass momentum

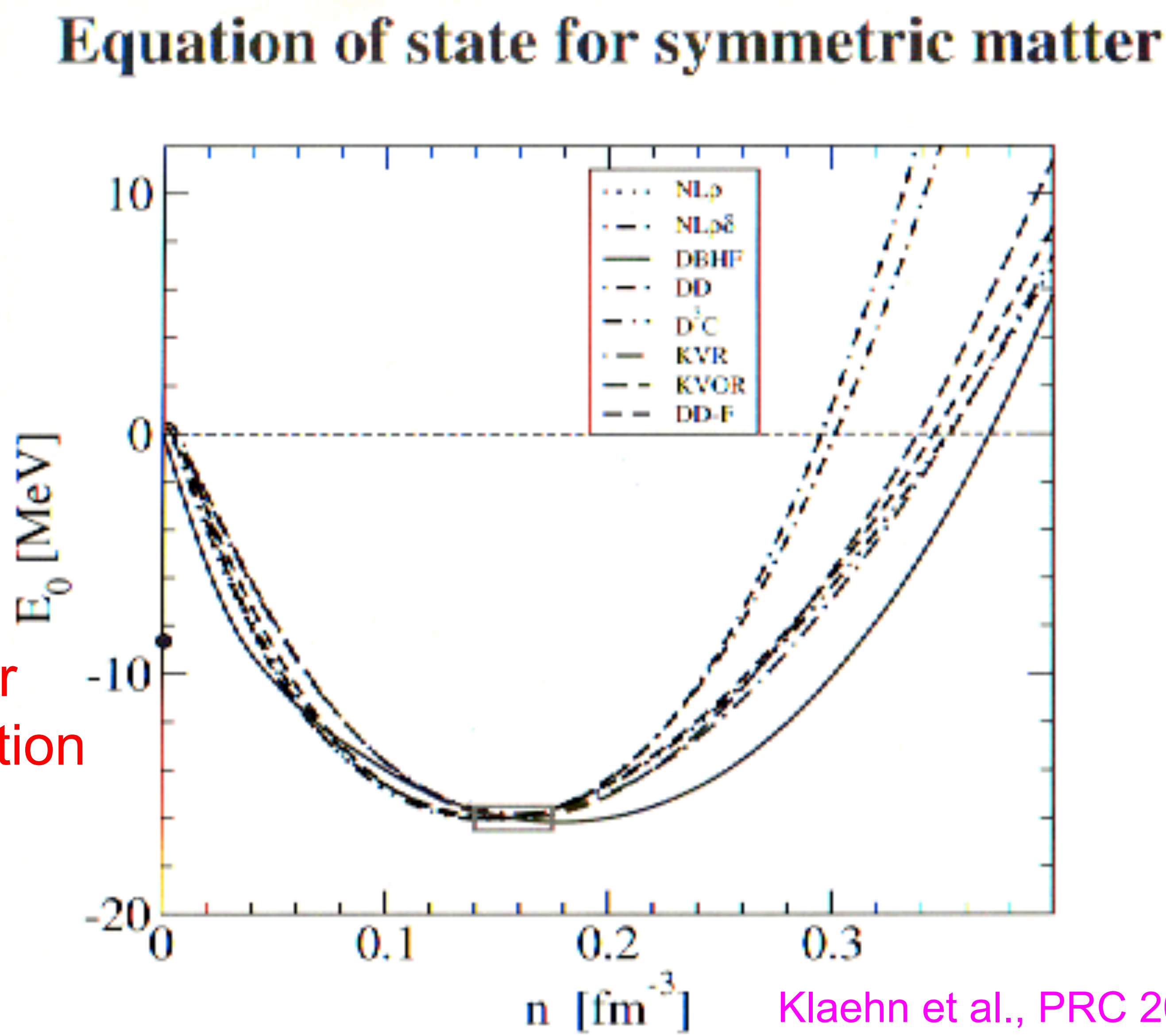
Nuclear Statistical Equilibrium (NSE)

Quasiparticle approximation for nuclear matter



Klaehn et al., PRC 2006

Quasiparticle approximation for nuclear matter



But:
cluster
formation

Incorrect
low-density
limit

Klaehn et al., PRC 2006

Outline

- Nuclear systems: Quasiparticle approach

  Brueckner, HFB; Skyrme, Relativistic Mean Field (RMF)
- Account of correlations in warm dense matter:

  two-particle (deuteron, pairing),

  four-particle (alpha-like) correlations, light elements
- Low-density regions: Nuclear Statistical Equilibrium (NSE)

  Hoyle-like states in light expanded nuclei,

  surface of nuclei, neck emission, alpha matter…
- Quantum statistical approach ($n < 0.15$ fm^{-3}, $T < 20$ MeV)

  Equation of state, Beth-Uhlenbeck formula

  disappearance of clusters at high densities, Pauli blocking
- Experimental signatures

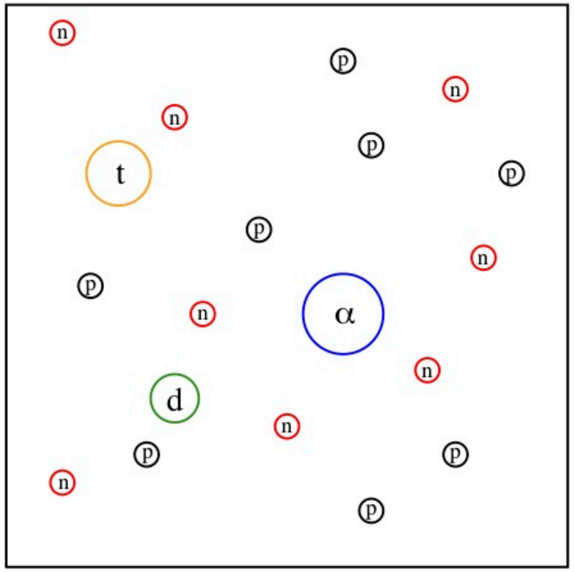
  Heavy Ion Collisions (HIC), Symmetry energy, SN explosions, …

Nuclear statistical equilibrium (NSE)

Chemical picture:

Ideal mixture of reacting components

Mass action law



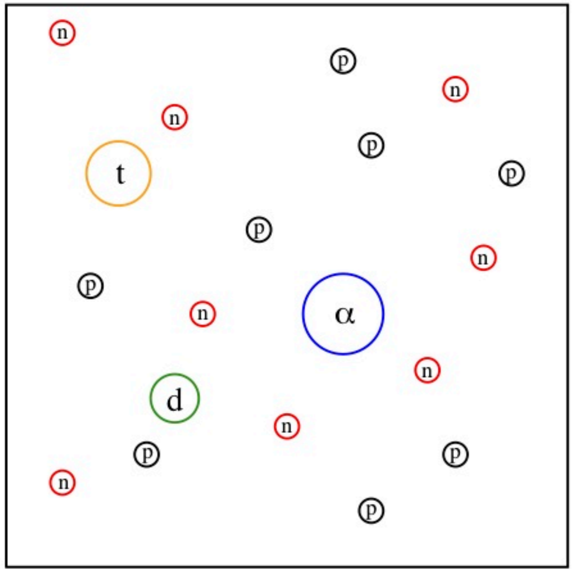
Interaction between the components
internal structure: Pauli principle

Nuclear statistical equilibrium (NSE)

Chemical picture:

Ideal mixture of reacting components

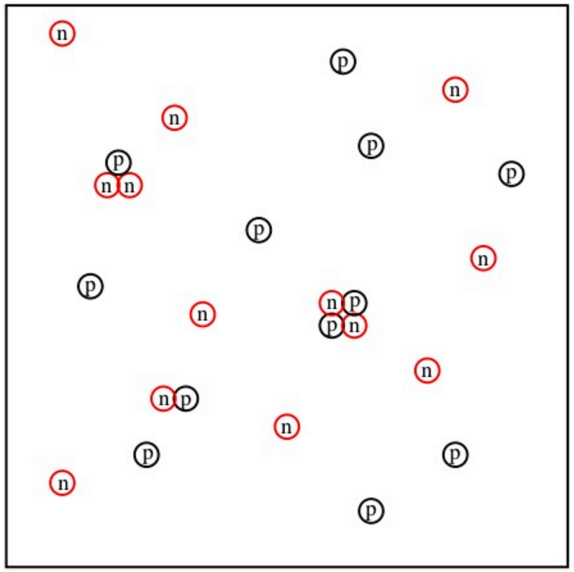
Mass action law



Interaction between the components
internal structure: Pauli principle

Physical picture:

"elementary" constituents
and their interaction



Quantum statistical (QS) approach,
quasiparticle concept, virial expansion

Many-particle theory

- equilibrium correlation functions

  e.g. equation of state $n(\beta, \mu) = \frac{1}{\Omega_0} \sum_1 \langle a_1^\dagger a_1 \rangle$

  density matrix $\langle a_1^\dagger a_{1'} \rangle = \int \frac{d\omega}{2\pi} \, e^{-i\omega t} f_1(\omega) A(1, 1', \omega)$

- Spectral function

$$A(1, 1', \omega) = \mathrm{Im}\left[G(1, 1', \omega + i\eta) - G(1, 1', \omega - i\eta)\right]$$

- Matsubara Green function

$$G(1, 1', iz_\nu), \qquad z_\nu = \frac{\pi\nu}{\beta} + \mu, \quad \nu = \pm 1, \pm 3, \cdots$$

$$1 \equiv \{\mathbf{p}_1, \sigma_1, c_1\}, \quad f_1(\omega) = \frac{1}{e^{\beta(\omega - \mu)} + 1}, \quad \Omega_0 - \text{volume}$$

Many-particle theory

- Dyson equation and self energy (homogeneous system)

$$G(1, iz_\nu) = \frac{1}{iz_\nu - E(1) - \Sigma(1, iz_\nu)}$$

- Evaluation of $\Sigma(1, iz_\nu)$:
  perturbation expansion, diagram representation

$$A(1,\omega) = \frac{2\mathrm{Im}\,\Sigma(1, \omega + i0)}{[\omega - E(1) - \mathrm{Re}\,\Sigma(1,\omega)]^2 + [\mathrm{Im}\,\Sigma(1, \omega + i0)]^2}$$

approximation for self energy $\longrightarrow$ approximation for equilibrium correlation functions

alternatively: simulations, path integral methods

Different approximations

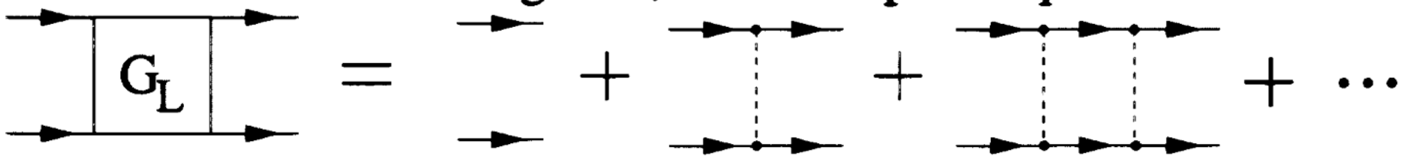
- Expansion for small Im $\Sigma(1,\omega+i\eta)$

$$A(1,\omega) \approx \frac{2\pi\delta(\omega - E^{\rm quasi}(1))}{1-\frac{\rm d}{{\rm d}z}{\rm Re}\ \Sigma(1,z)|_{z=E^{\rm quasi}-\mu_1}}$$
$$-2{\rm Im}\ \Sigma(1,\omega+i\eta)\frac{\rm d}{{\rm d}\omega}\frac{P}{\omega+\mu_1-E^{\rm quasi}(1)}$$

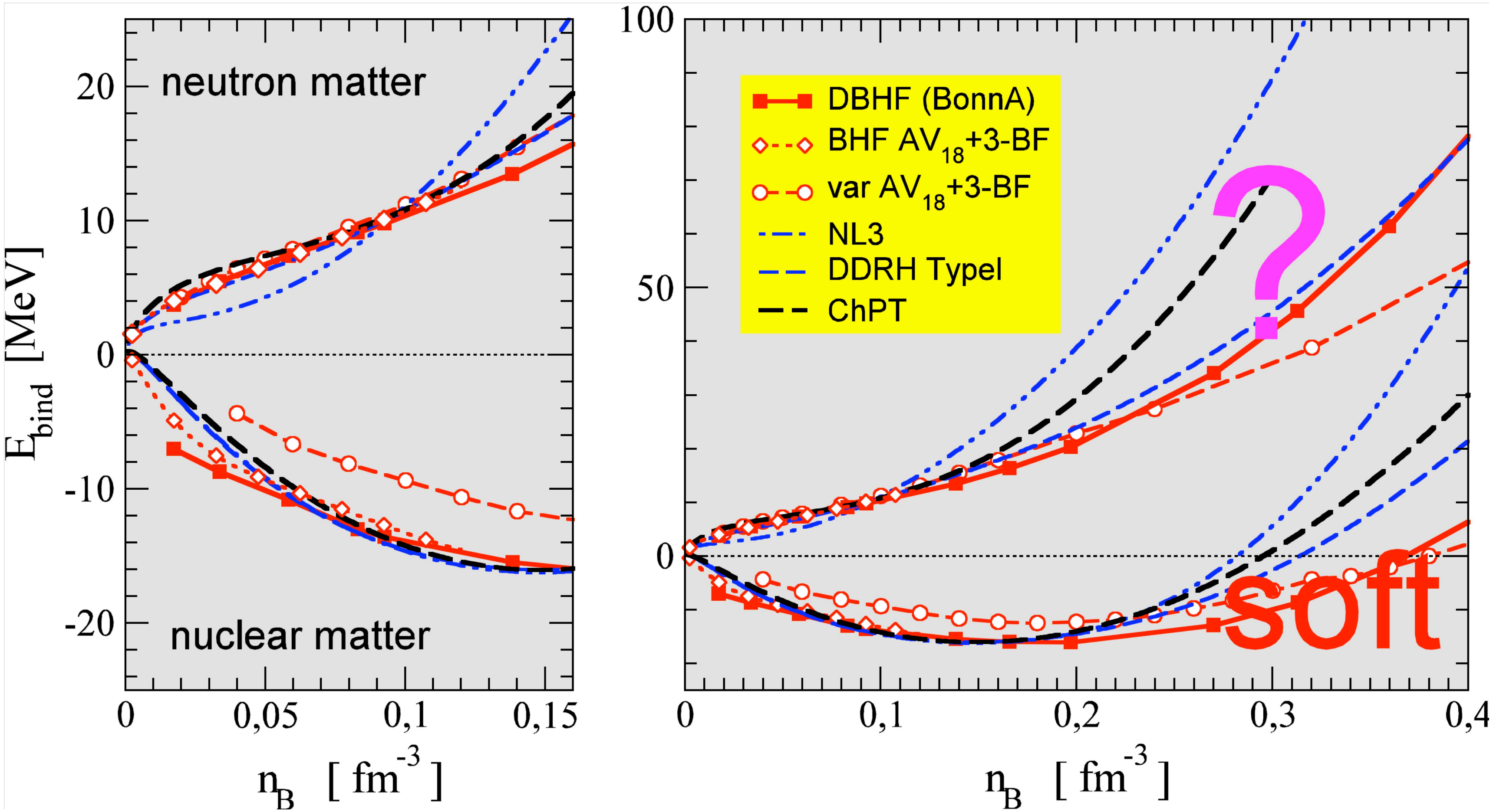
quasiparticle energy $E^{\rm quasi}(1) = E(1) + {\rm Re}\ \Sigma(1,\omega)|_{\omega=E^{\rm quasi}}$

- chemical picture: bound states $\hat{=}$ new species

summation of ladder diagrams, Bethe-Salpeter equation

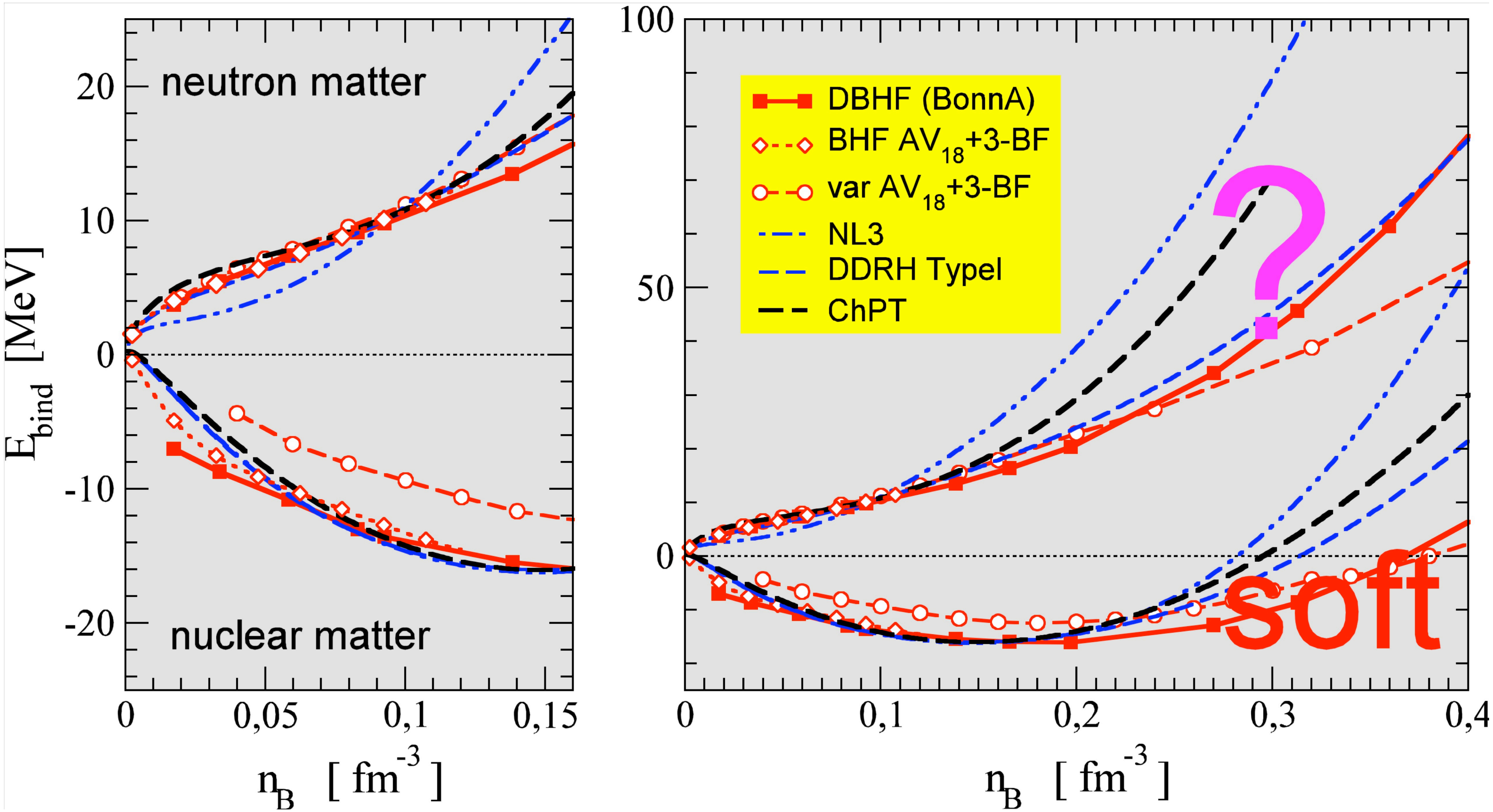


Quasiparticle picture: RMF and DBHF



C. Fuchs et al.;
J.Margueron et al., Phys.Rev.C **76**,034309 (2007)

Quasiparticle picture: RMF and DBHF



But: cluster formation

Incorrect low-density limit

C. Fuchs et al.;
J.Margueron et al., Phys.Rev.C **76**,034309 (2007)

Different approximations

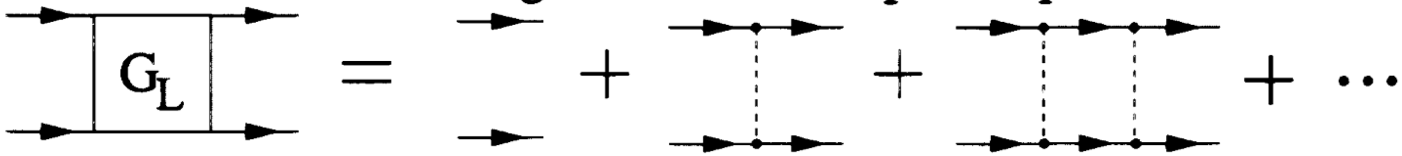
- **Expansion for small Im $\Sigma(1, \omega + i\eta)$**

$$A(1,\omega) \approx \frac{2\pi\delta(\omega - E^{\mathrm{quasi}}(1))}{1 - \frac{\mathrm{d}}{\mathrm{d}z}\mathrm{Re}\ \Sigma(1,z)|_{z=E^{\mathrm{quasi}}-\mu_1}} - 2\mathrm{Im}\ \Sigma(1,\omega+i\eta)\frac{\mathrm{d}}{\mathrm{d}\omega}\frac{P}{\omega+\mu_1-E^{\mathrm{quasi}}(1)}$$

quasiparticle energy $E^{\mathrm{quasi}}(1) = E(1) + \mathrm{Re}\ \Sigma(1,\omega)|_{\omega=E^{\mathrm{quasi}}}$

- **chemical picture: bound states $\hat{=}$ new species**

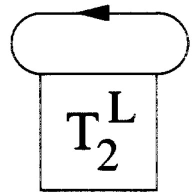
summation of ladder diagrams, Bethe-Salpeter equation



Different approximations

low density limit:

$$G_2^L(12,1'2',i\lambda) = \sum_{n\mathbf{P}} \Psi_{n\mathbf{P}}(12)\frac{1}{i\omega_\lambda - E_{n\mathbf{P}}}\Psi^*_{n\mathbf{P}}(12)$$

$\Sigma =$ 

$$n(\beta,\mu) = \sum_1 f_1(E^{\text{quasi}}(1)) + \sum_{2,n\mathbf{P}}^{\text{bound}} g_{12}(E_{n\mathbf{P}})$$

$$+\sum_{2,n\mathbf{P}} \int_0^\infty \mathrm{d}k\; \delta_{\mathbf{k},\mathbf{p}_1-\mathbf{p}_2} g_{12}(E^{\text{quasi}}(1) + E^{\text{quasi}}(2))2\sin^2\delta_n(k)\frac{1}{\pi}\frac{\mathrm{d}}{\mathrm{d}k}\delta_n(k)$$

- generalized Beth-Uhlenbeck formula
  correct low density/low temperature limit:
  mixture of free particles and bound clusters

Effective wave equation for the deuteron in matter

In-medium two-particle wave equation in mean-field approximation

$$\left(\frac{p_1^2}{2m_1}+\Delta_1+\frac{p_2^2}{2m_2}+\Delta_2\right)\Psi_{d,P}(p_1,p_2)+\sum_{p_1',p_2'}(1-f_{p_1}-f_{p_2})V(p_1,p_2;p_1',p_2')\Psi_{d,P}(p_1',p_2')$$

$$=E_{d,P}\Psi_{d,P}(p_1,p_2)$$

Add self-energy

Pauli-blocking

Thouless criterion

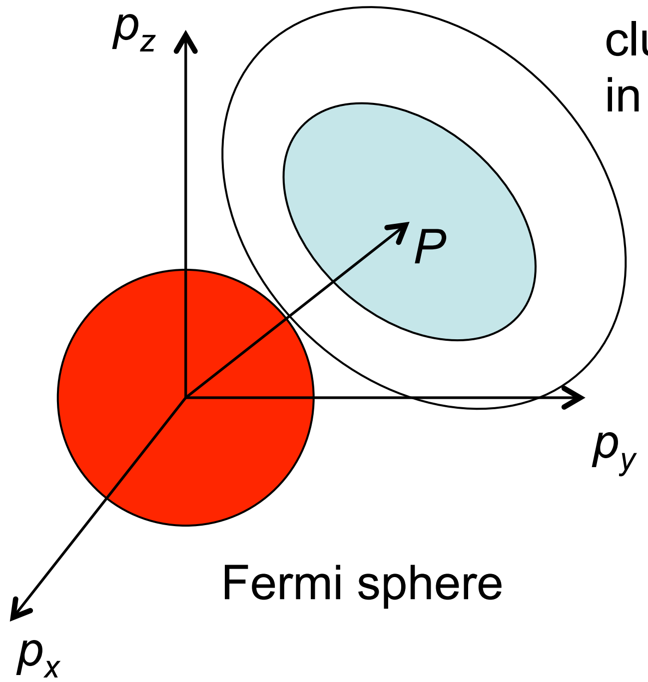
$$E_d(T,\mu)=2\mu$$

Fermi distribution function

$$f_p=\left[e^{(p^2/2m-\mu)/k_BT}+1\right]^{-1}$$

BEC-BCS crossover:
Alm et al.,1993

Pauli blocking – phase space occupation



cluster wave function (deuteron, alpha,…)
in momentum space

P - center of mass momentum

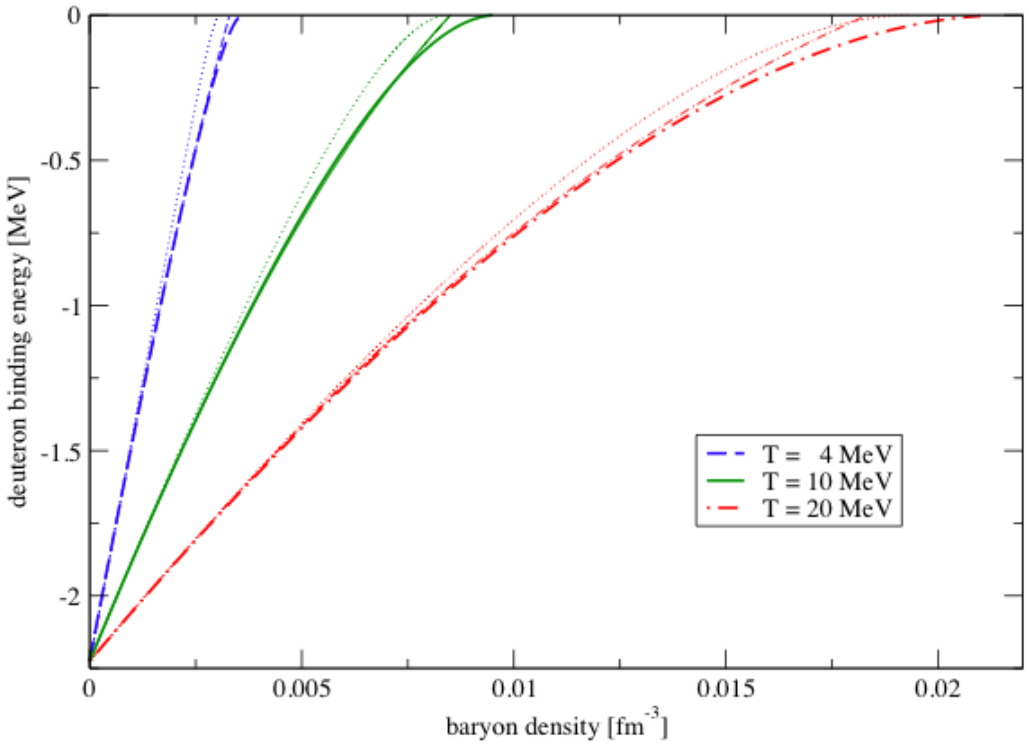
The Fermi sphere is forbidden,
deformation of the cluster wave function
in dependence on the c.o.m. momentum P

momentum space

The deformation is maximal at $P = 0$.
It leads to the weakening of the interaction
(disintegration of the bound state).

Shift of the deuteron binding energy

Dependence on nucleon density, various temperatures,
zero center of mass momentum

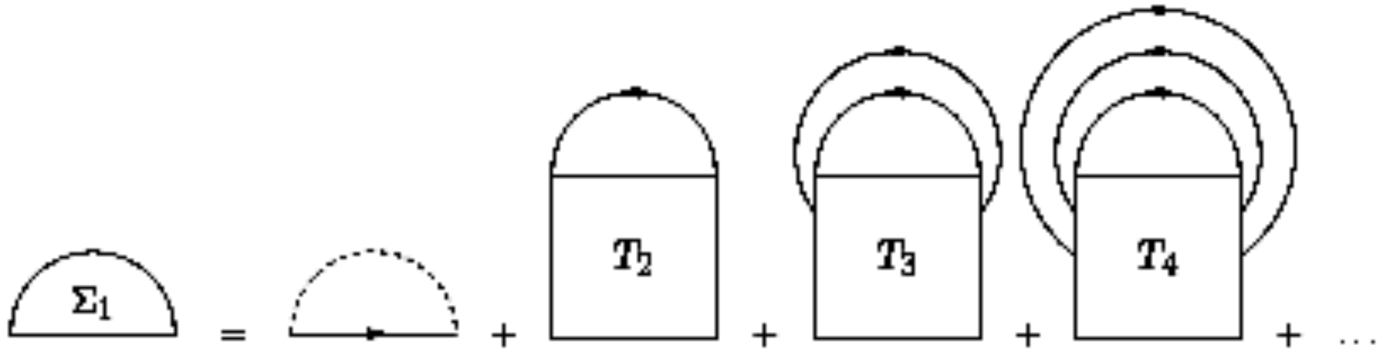


thin lines:

fit formula

G.R., NP A **867**, 66 (2011)

Cluster decomposition of the self-energy



T-matrices: bound states, scattering states
Including clusters like new components
chemical picture,
mass action law, nuclear statistical equilibrium (NSE)

Few-particle Schrödinger equation in a dense medium

4-particle Schrödinger equation with medium effects

$$\left(\left[E^{HF}(p_1)+E^{HF}(p_2)+E^{HF}(p_3)+E^{HF}(p_4)\right]\right)\Psi_{n,P}(p_1,p_2,p_3,p_4)$$

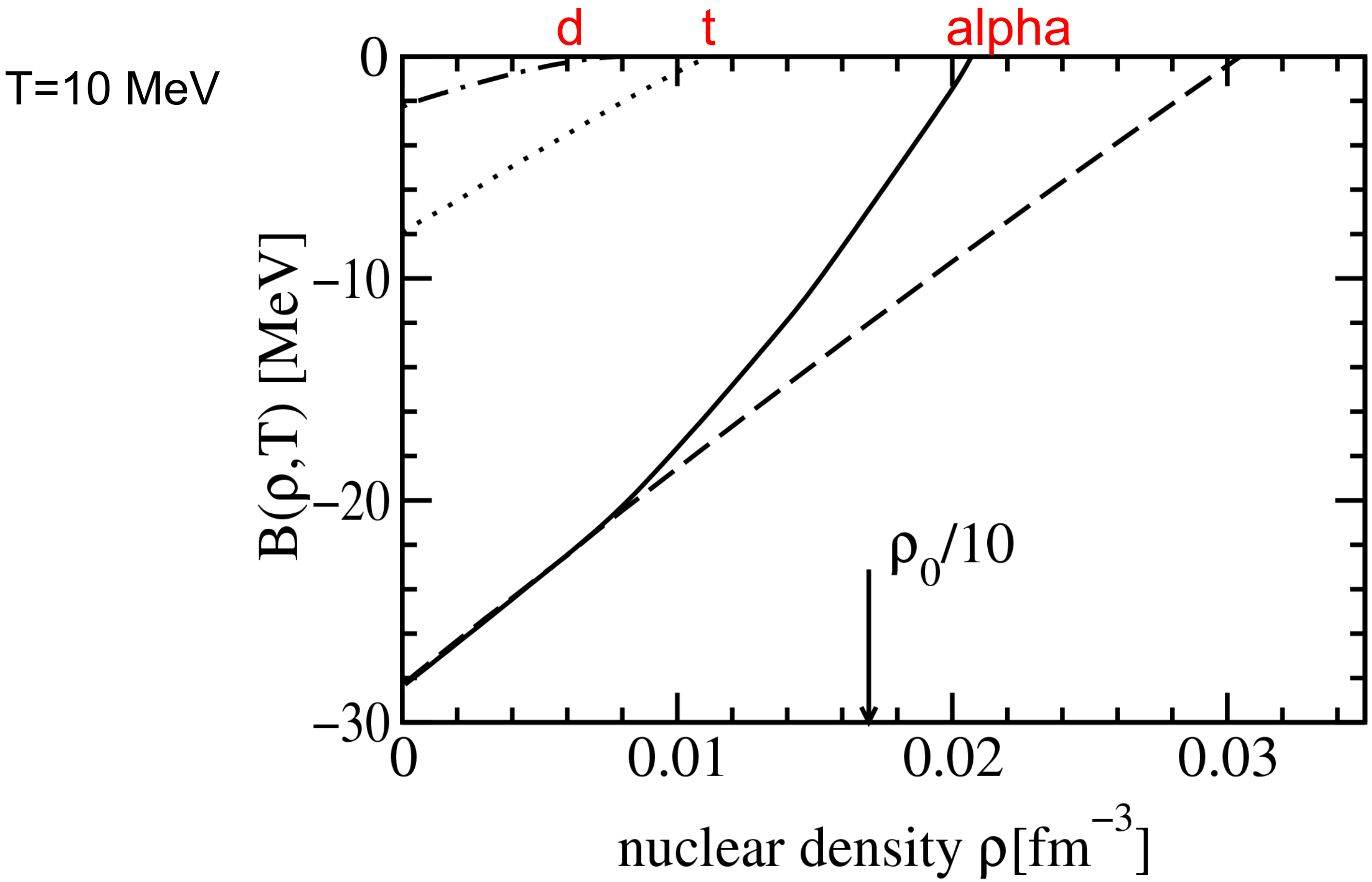
$$+\sum_{p_1',p_2'}(1-f_{p_1}-f_{p_2})V(p_1,p_2;p_1',p_2')\Psi_{n,P}(p_1',p_2',p_3,p_4)$$

$$+\left\{permutations\right\}$$

$$=E_{n,P}\Psi_{n,P}(p_1,p_2,p_3,p_4)$$

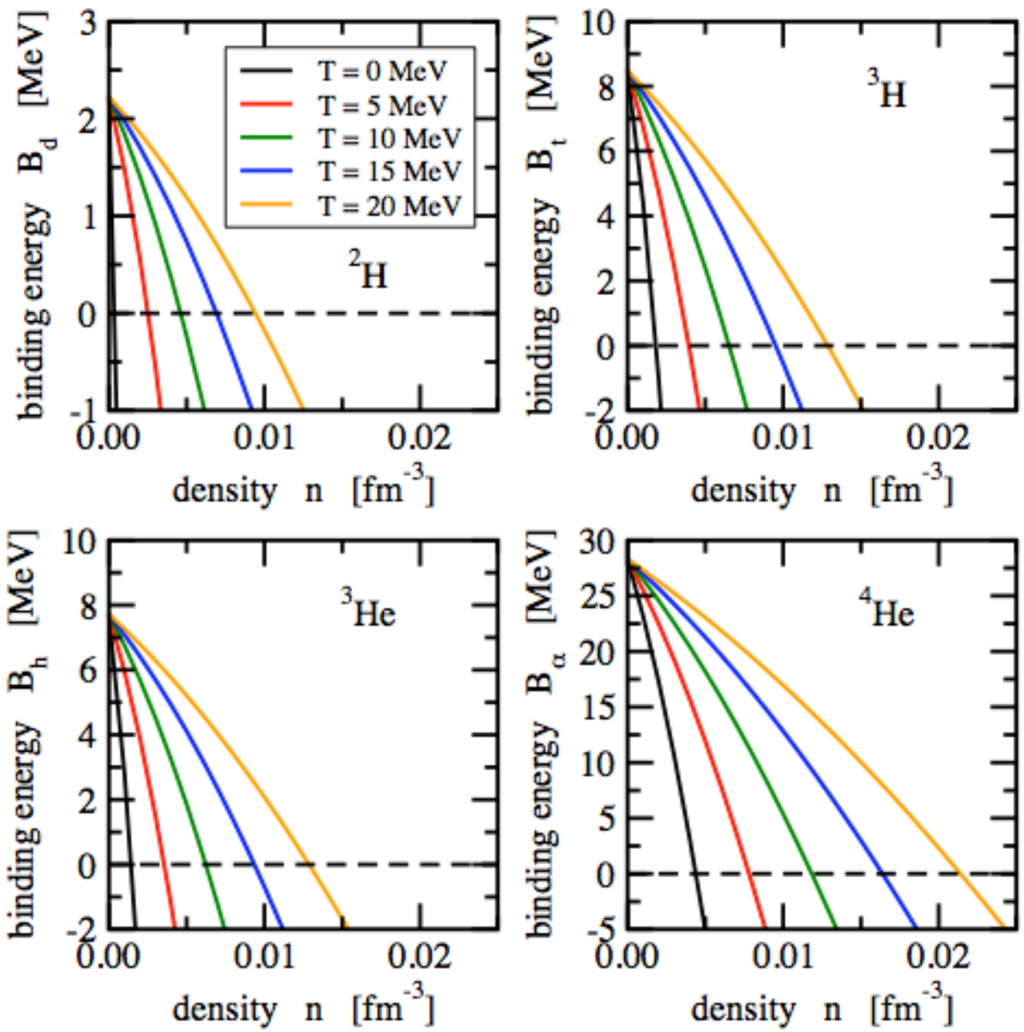
In-medium shift of binding energies of clusters

Solution of the Faddeev-Yakubovski equation with Pauli blocking



M. Beyer et al., PLB **488**, 247 (00), A. Sedrakian et al., Ann. Phys; PRC **73**, 035803 (06)

Shift of Binding Energies of Light Clusters



Symmetric matter

G.R., PRC **79**, 014002 (2009)
S. Typel et al.,
PRC **81**, 015803 (2010)

Composition of dense nuclear matter

$$n_p(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} Z_A f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

$$n_n(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} (A-Z_A) f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

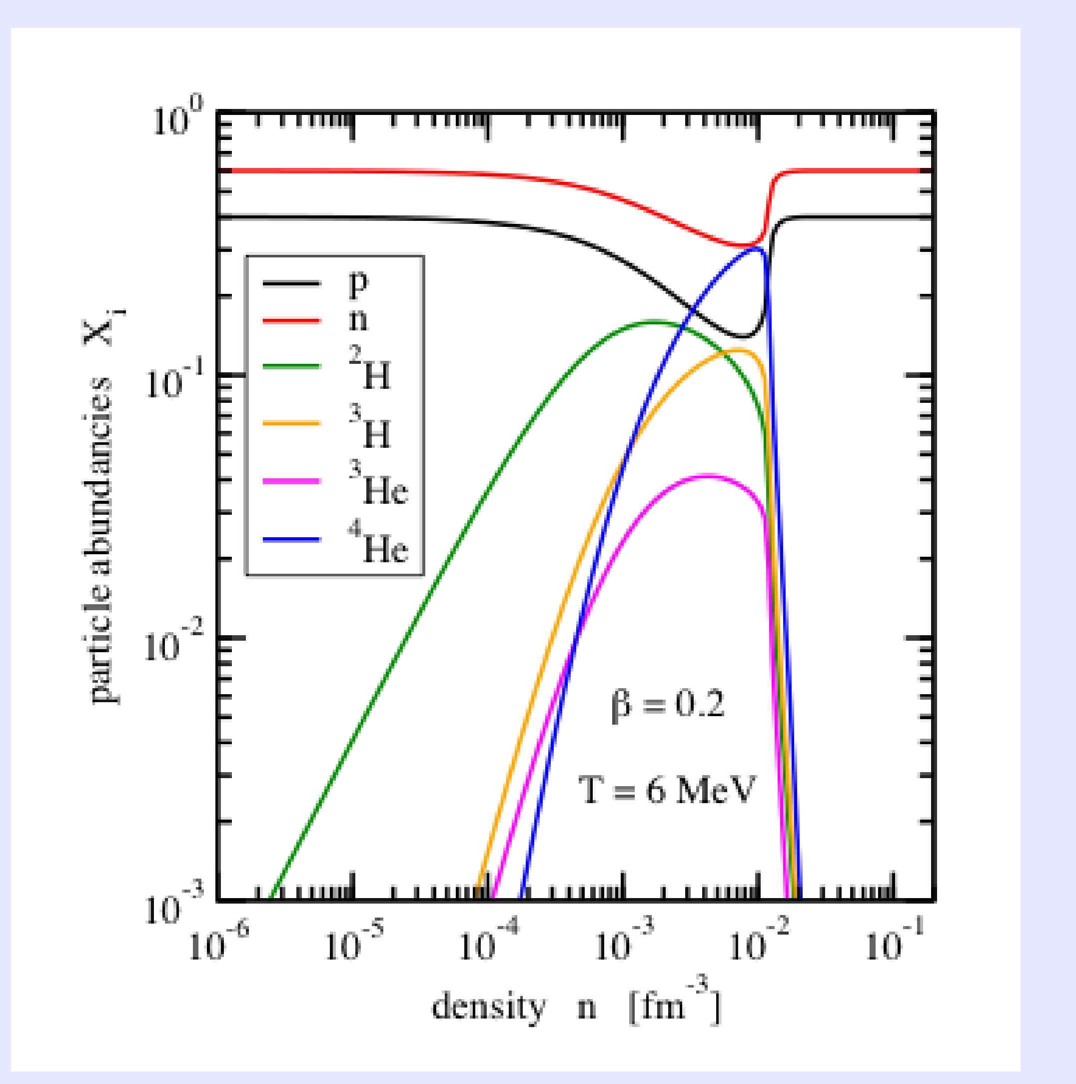
mass number A,
charge Z_A,
energy $E_{A,\nu,K}$,
ν: internal quantum number,

$$f_{A(z)} = \frac{1}{\exp(z/T) - (-1)^A}$$

- Inclusion of excited states and continuum correlations

- Medium effects:
  self-energy and Pauli blocking shifts of binding energies,
  Coulomb corrections due to screening (Wigner-Seitz,Debye)

Light Cluster Abundances



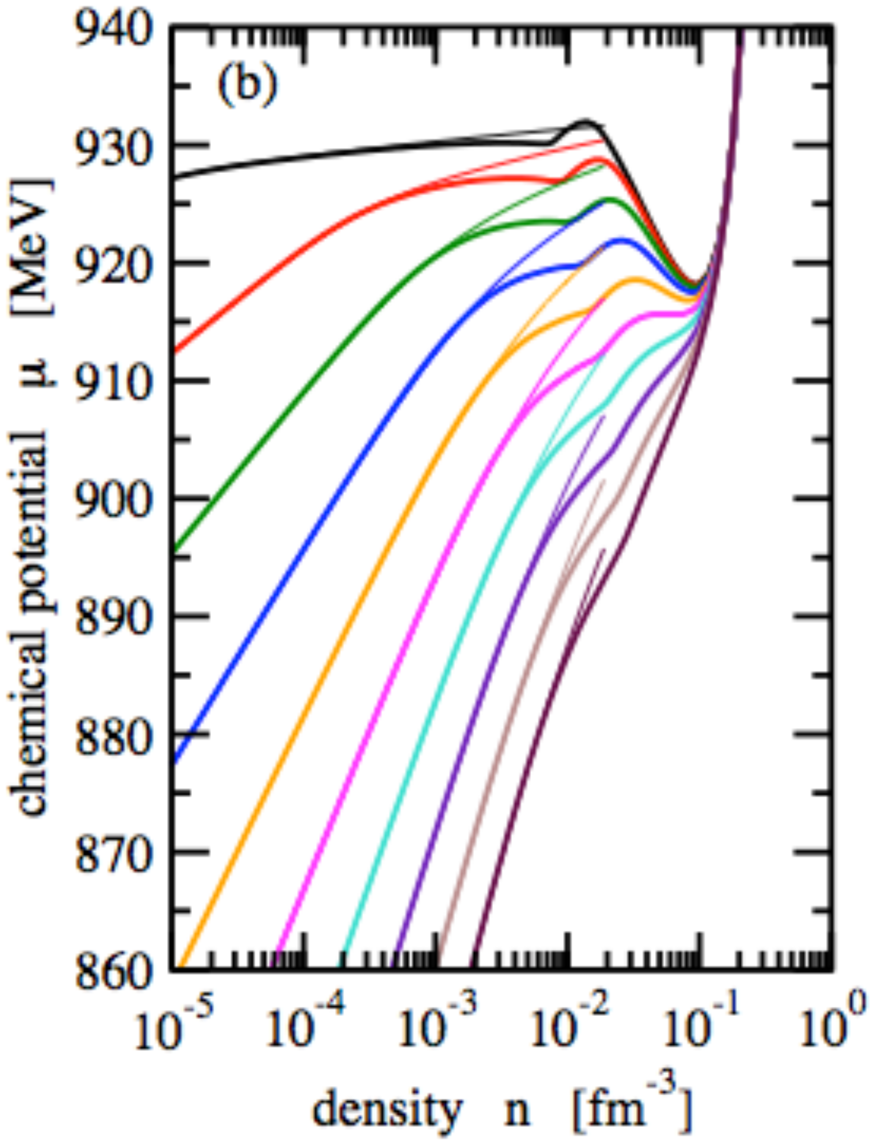
S. Typel et al.,
PRC **81,** 015803 (2010)

Chemical potential of symmetric matter

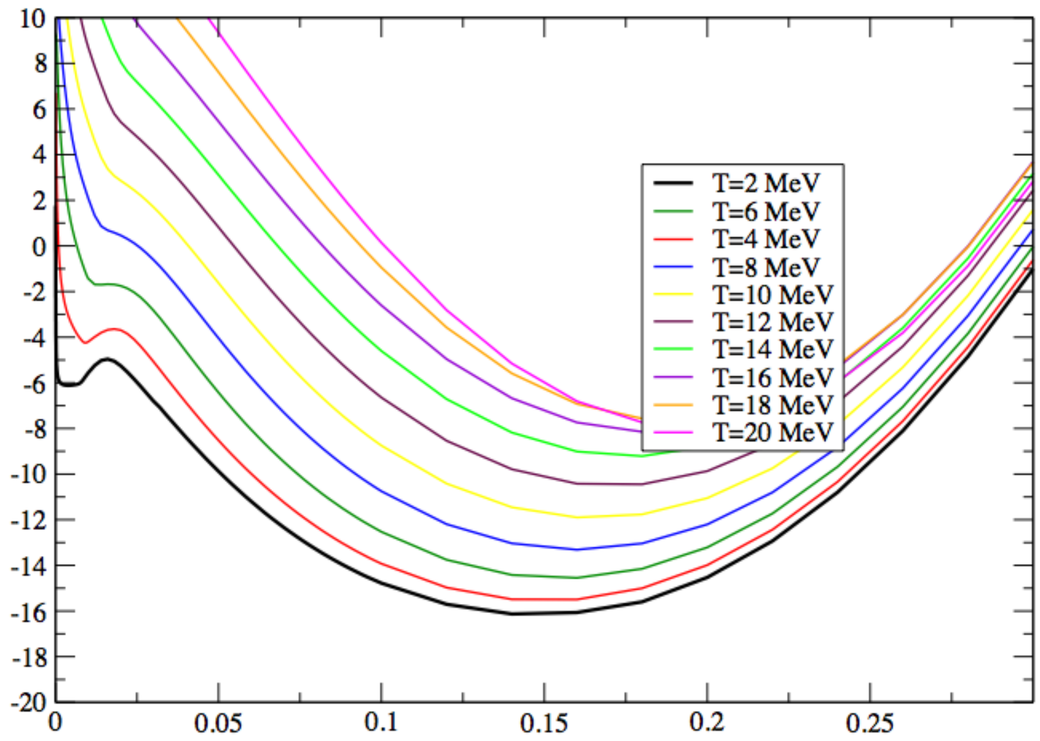
Isotherms

T[MeV]
2
4
6
8
10
12
14
16
18
20

thin lines: NSE



Internal energy per nucleon



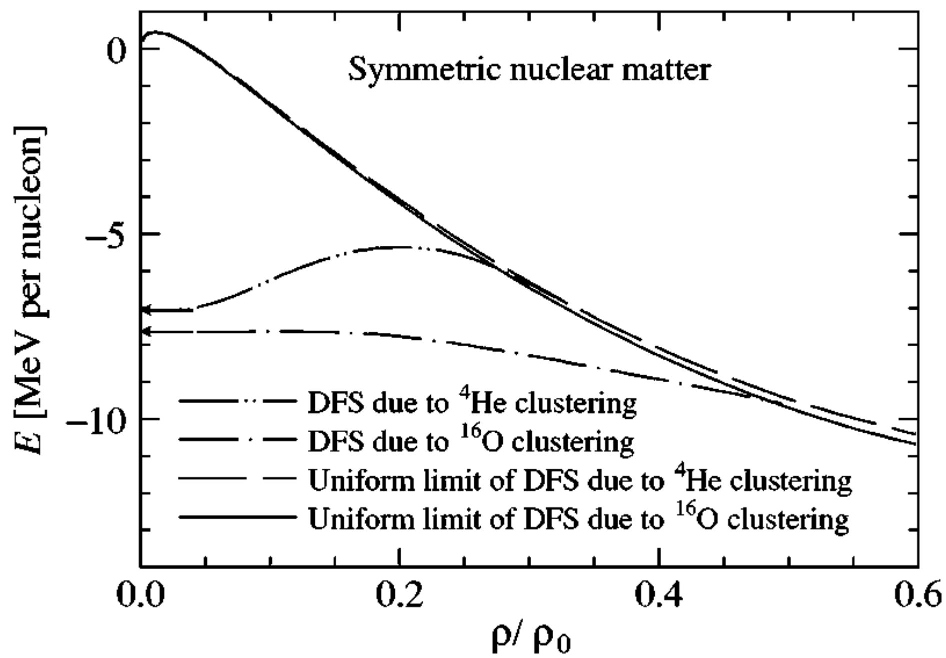
Quantum statistical approach:

Cluster ?

Condensate?

EOS for symmetric matter - low density region?

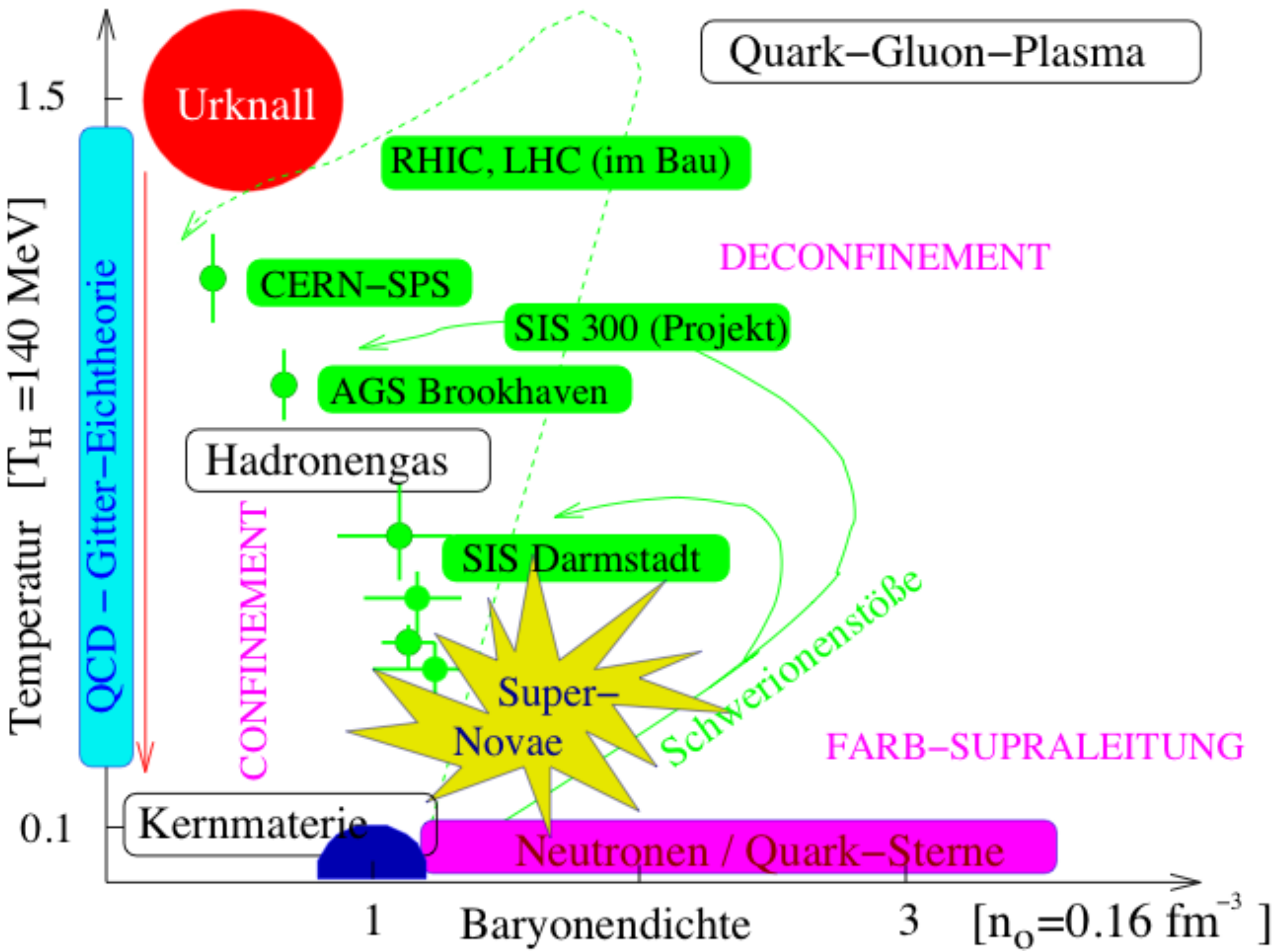
Clustering phenomena in nuclear matter below the saturation density



- FIG. 8. Energy curves of DFSs due to a and 16O clustering in
- the symmetric nuclear matter by the use of the BB sB4d force. The
- density of matter is normalized by the saturation density of the
- uniform matter with the Fermi sphere, r0=0.206 fm−3. The presentation
- of the curves is similar to that in Fig. 4.

Hiroki Takemoto et al., PR C **69**, 035802 (2004)

Phase diagram nuclear matter



Application to Heavy Ion Reactions

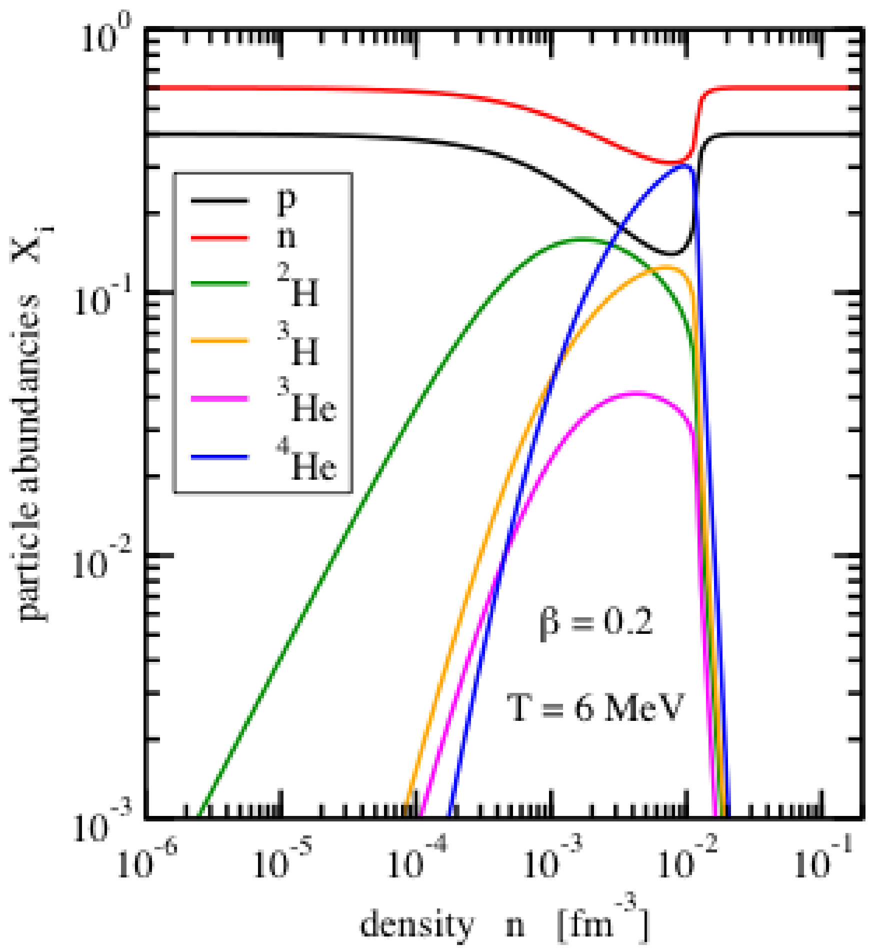
- Test the EOS
  (NSE, virial,… at low densities,
  Skyrme, DBHF, RMF,… near saturation)
- Unifying quantum statistical approach, medium effects, Mott effect
- Symmetry energy
- Bose enhancement?

Nimrod @ TAMU,
40Ar + 112,124Sn,
64Zn + 112,124Sn; 47 A MeV

Yields of p, (n), d, t, ^{3}He, ^{4}He,…

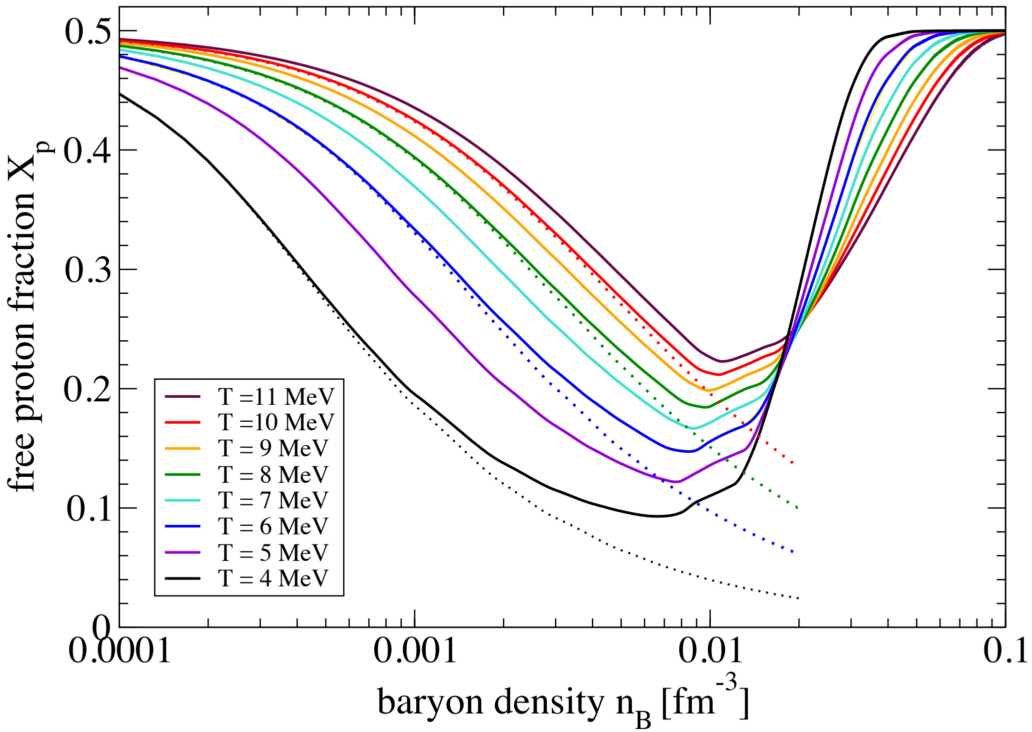
Open questions: freeze-out model or dynamical transport models?
Identification of the source?

Light Cluster Abundances



S. Typel et al.,
PRC **81,** 015803 (2010)

Pauli blocking in symmetric matter



Free proton fraction as function of density and temperature in symmetric matter.
QS calculations (solid lines) are compared with the NSE results (dotted lines).
Mott effect in the region $n_{saturation}/5$.

EOS at low densities from HIC

PRL **108,** 172701 (2012) PHYSICAL REVIEW LETTERS week ending 27 APRIL 2012

Laboratory Tests of Low Density Astrophysical Nuclear Equations of State

L. Qin,[1] K. Hagel,[1] R. Wada,[2,1] J. B. Natowitz,[1] S. Shlomo,[1] A. Bonasera,[1,3] G. Röpke,[4] S. Typel,[5] Z. Chen,[6] M. Huang,[6] J. Wang,[6] H. Zheng,[1] S. Kowalski,[7] M. Barbui,[1] M. R. D. Rodrigues,[1] K. Schmidt,[1] D. Fabris,[8] M. Lunardon,[8] S. Moretto,[8] G. Nebbia,[8] S. Pesente,[8] V. Rizzi,[8] G. Viesti,[8] M. Cinausero,[9] G. Prete,[9] T. Keutgen,[10] Y. El Masri,[10] Z. Majka,[11] and Y. G. Ma[12]

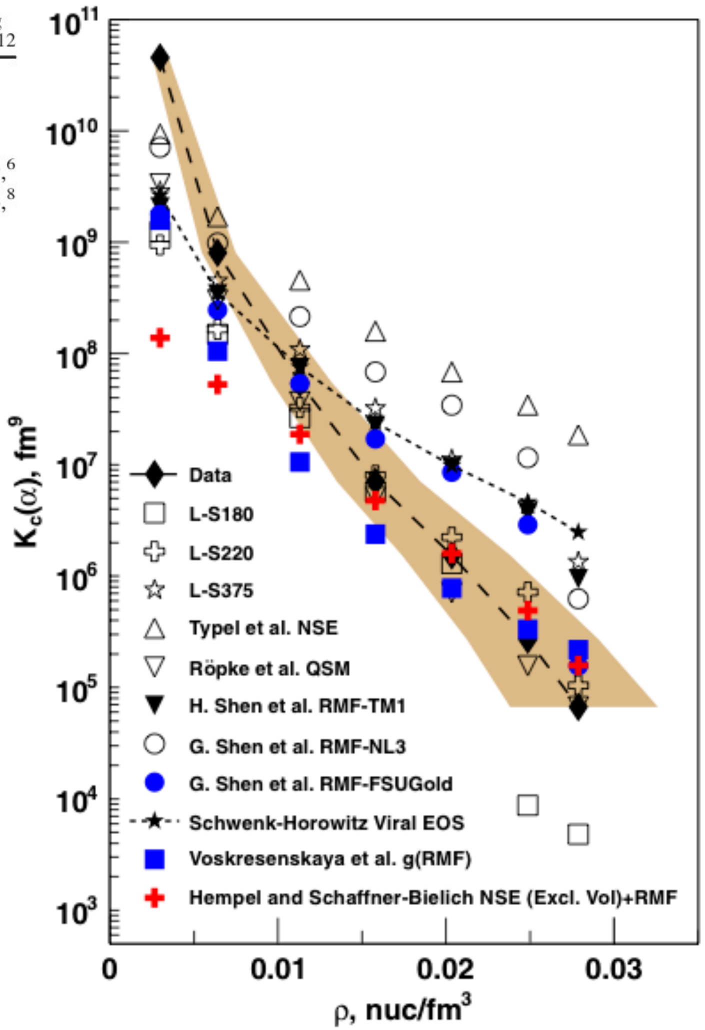
Yields of clusters from HIC: p, n, d, t, h, α

chemical constants

$$K_c(A,Z) = \rho_{(A,Z)}/[(\rho_p)^Z(\rho_n)^N]$$

Bose enhancement?

Symmetry energy

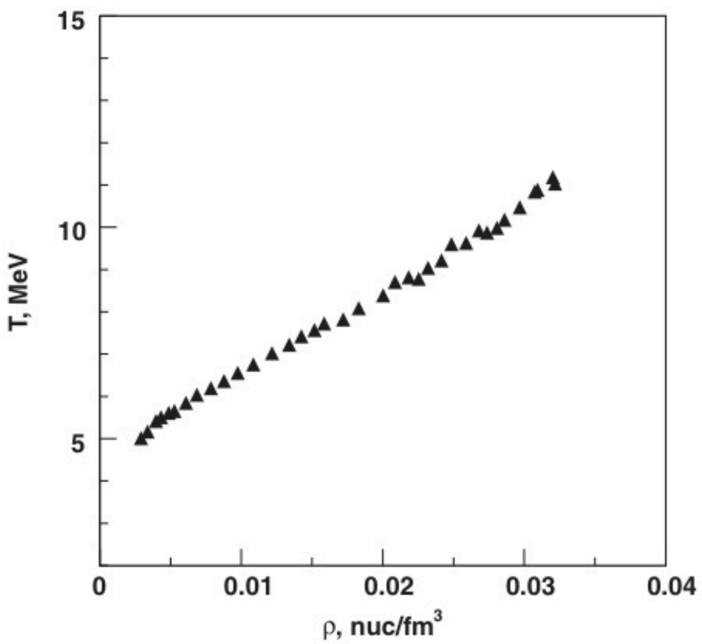


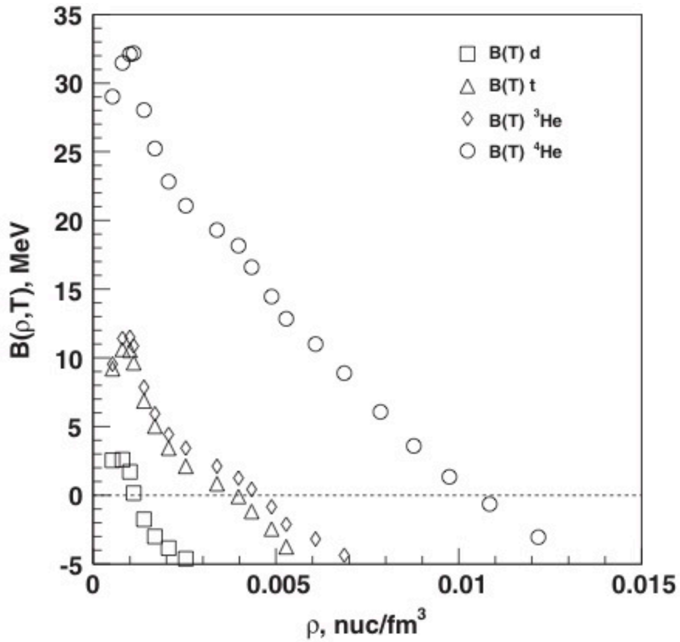
Cluster yields in HIC

PRL 108, 062702 (2012) PHYSICAL REVIEW LETTERS week ending 10 FEBRUARY 2012

Experimental Determination of In-Medium Cluster Binding Energies and Mott Points in Nuclear Matter

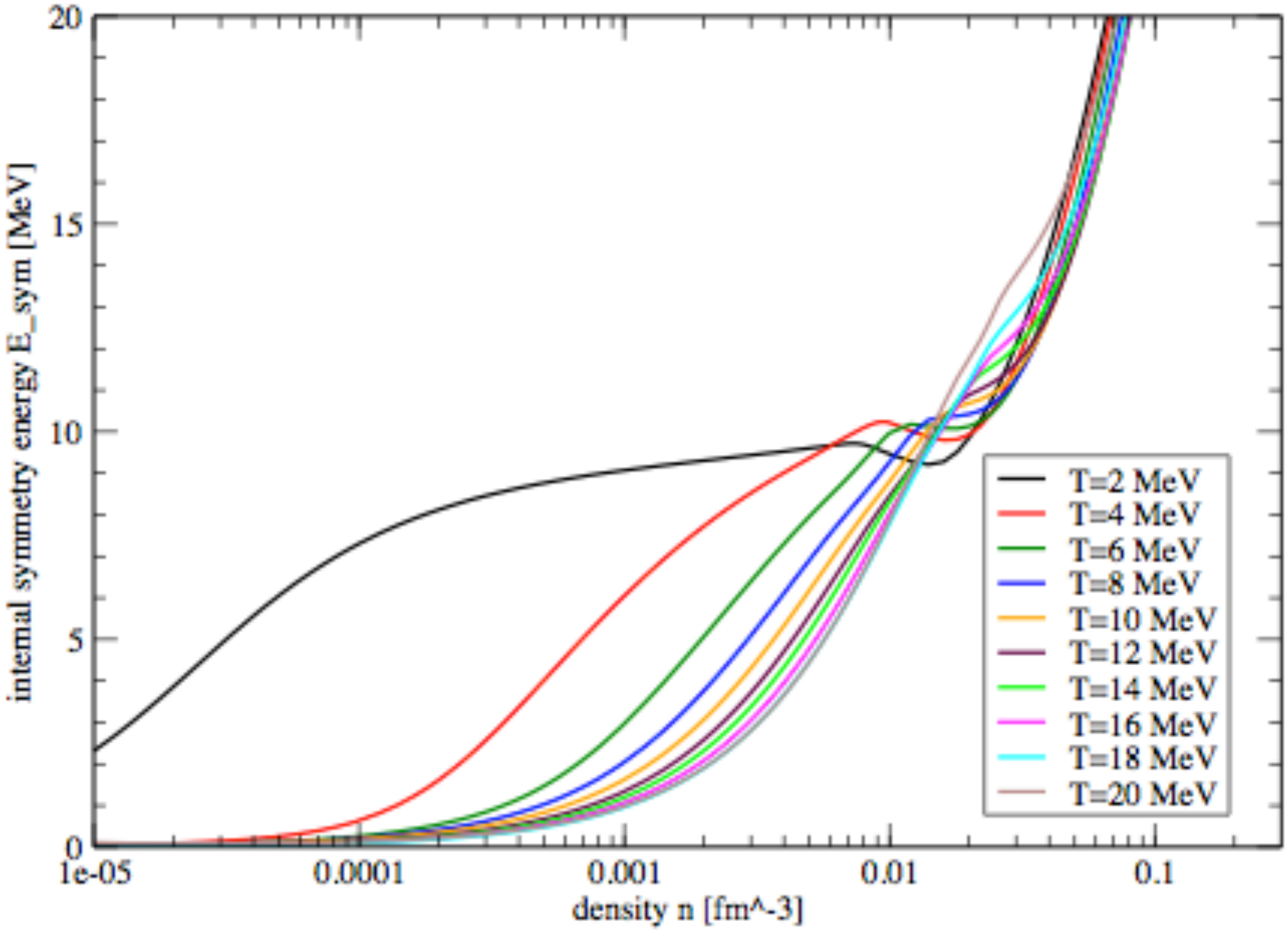
K. Hagel,[1] R. Wada,[2,1] L. Qin,[1] J. B. Natowitz,[1] S. Shlomo,[1] A. Bonasera,[1,3] G. Röpke,[4] S. Typel,[5] Z. Chen,[2] M. Huang,[2] J. Wang,[2] H. Zheng,[1] S. Kowalski,[6] C. Bottosso,[1] M. Barbui,[1] M. R. D. Rodrigues,[1] K. Schmidt,[1] D. Fabris,[7] M. Lunardon,[7] S. Moretto,[7] G. Nebbia,[7] S. Pesente,[7] V. Rizzi,[7] G. Viesti,[7] M. Cinausero,[8] G. Prete,[8] T. Keutgen,[9] Y. El Masri,[9] and Z. Majka[10]





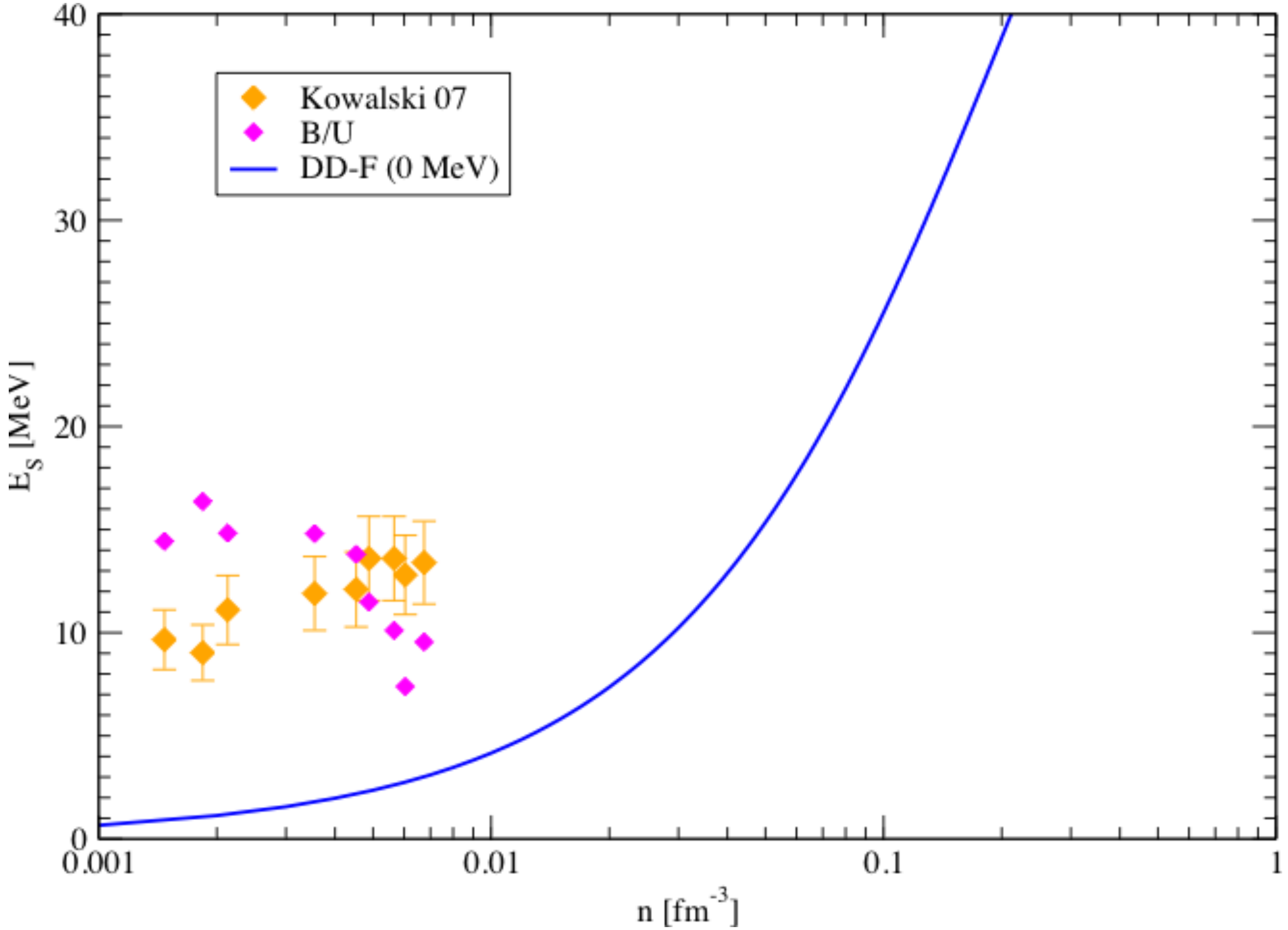
in-medium binding energies

Internal symmetry energy



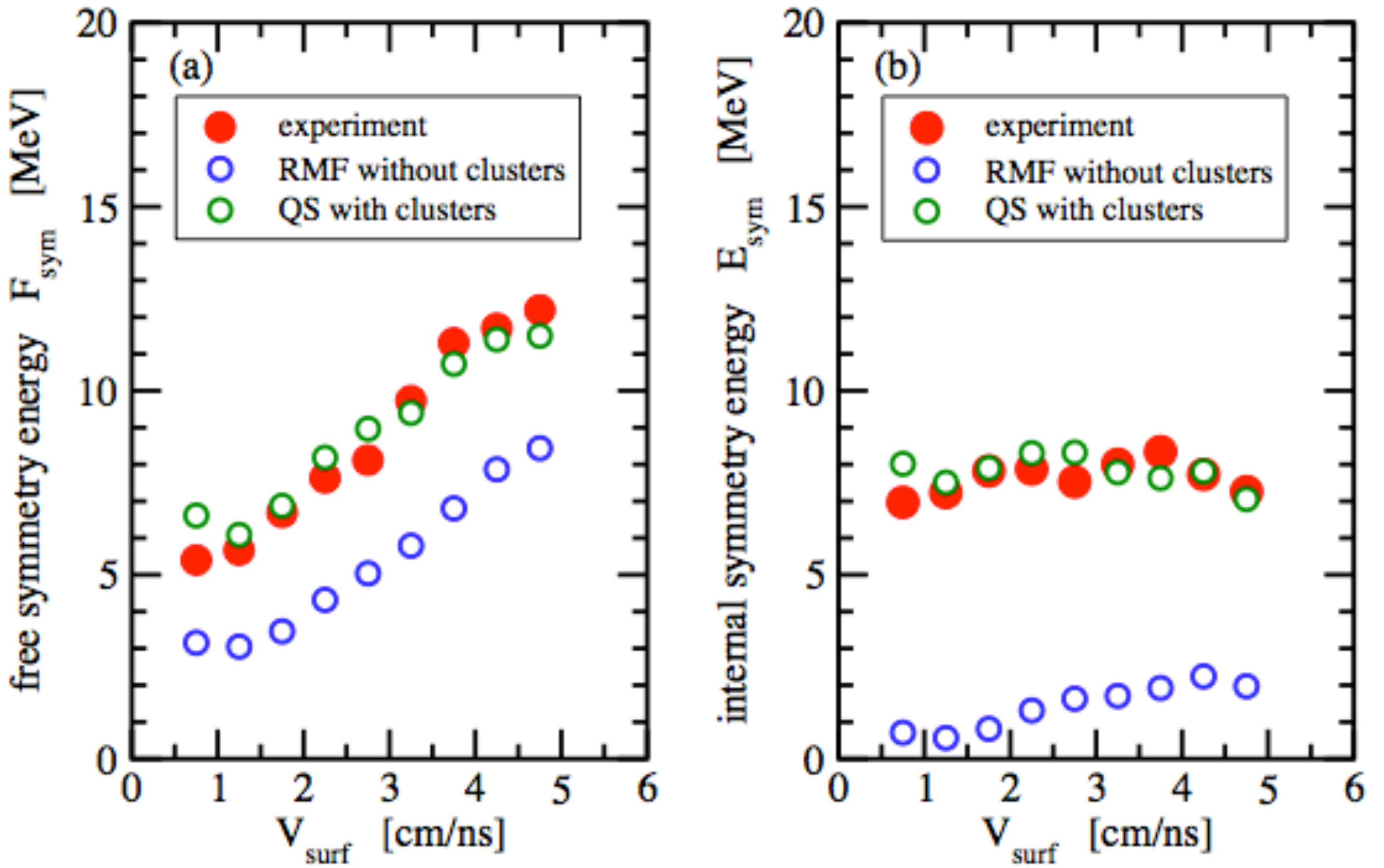
Symmetry energy

Heavy-ion collisions, spectra of emitted clusters,
temperature (3 - 10 MeV), free energy



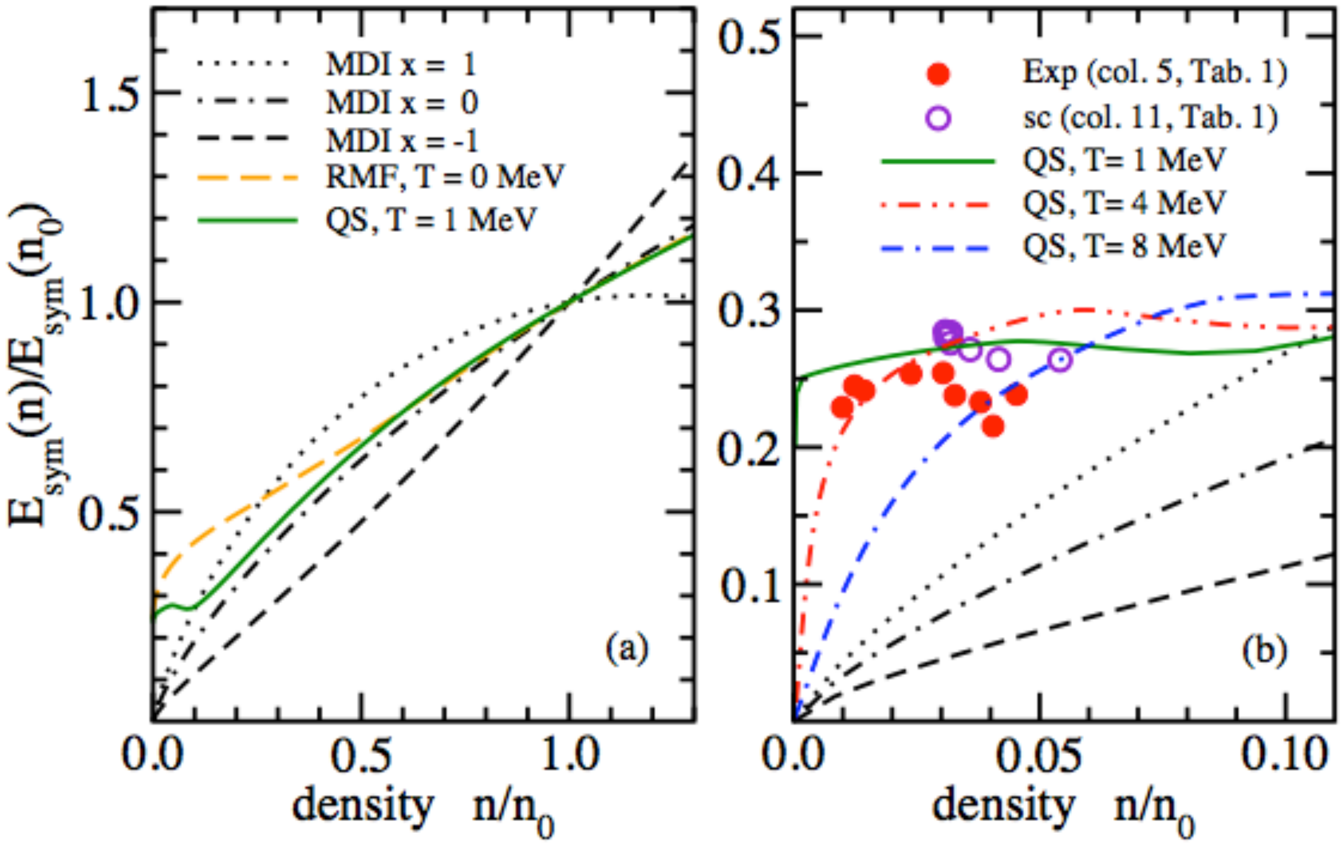
S. Kowalski et al., PRC **75**, 014601 (2007)

Symmetry energy, comparison experiment with theories



J.Natowitz et al., PRL 2010

Symmetry Energy



Scaled internal symmetry energy as a function of the scaled total density.
MDI: Chen et al., QS: quantum statistical, Exp: experiment at TAMU

J.Natowitz et al. PRL, May 2010

Problems:

1. Inclusion of larger Clusters in the equation of state
2. HFB and cluster formation/quartetting
3. EoS at low temperatures/low densities
4. Inclusion of larger nuclei
5. Influence of the symmetry energy
   on beta equilibrium

- Transition from α−matter to nucleon matter
  Is there a region of metastability?

Problems:

1. Inclusion of larger Clusters in the equation of state
2. HFB and cluster formation/quartetting
3. EoS at low temperatures/low densities
4. Inclusion of larger nuclei
5. Influence of the symmetry energy
   on beta equilibrium:
   Diploma thesis work, Armen Sedrakian, 1987?

Astrophysical Applications

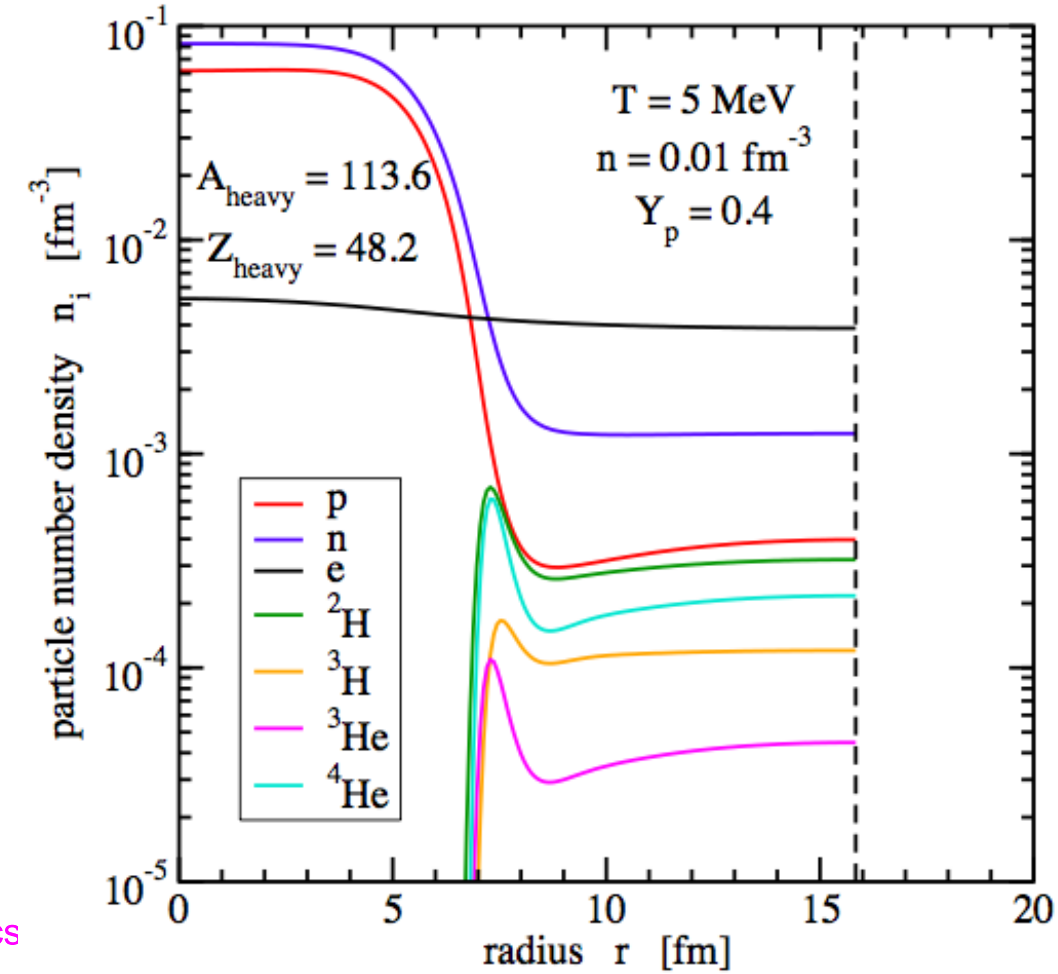
- Supernova explosions
- Neutrino transport
- Neutron star structure
- Equation of state (EOS)
- Composition
- Transport properties (cross sections)

α cluster in astrophysics

Crust of neutron stars

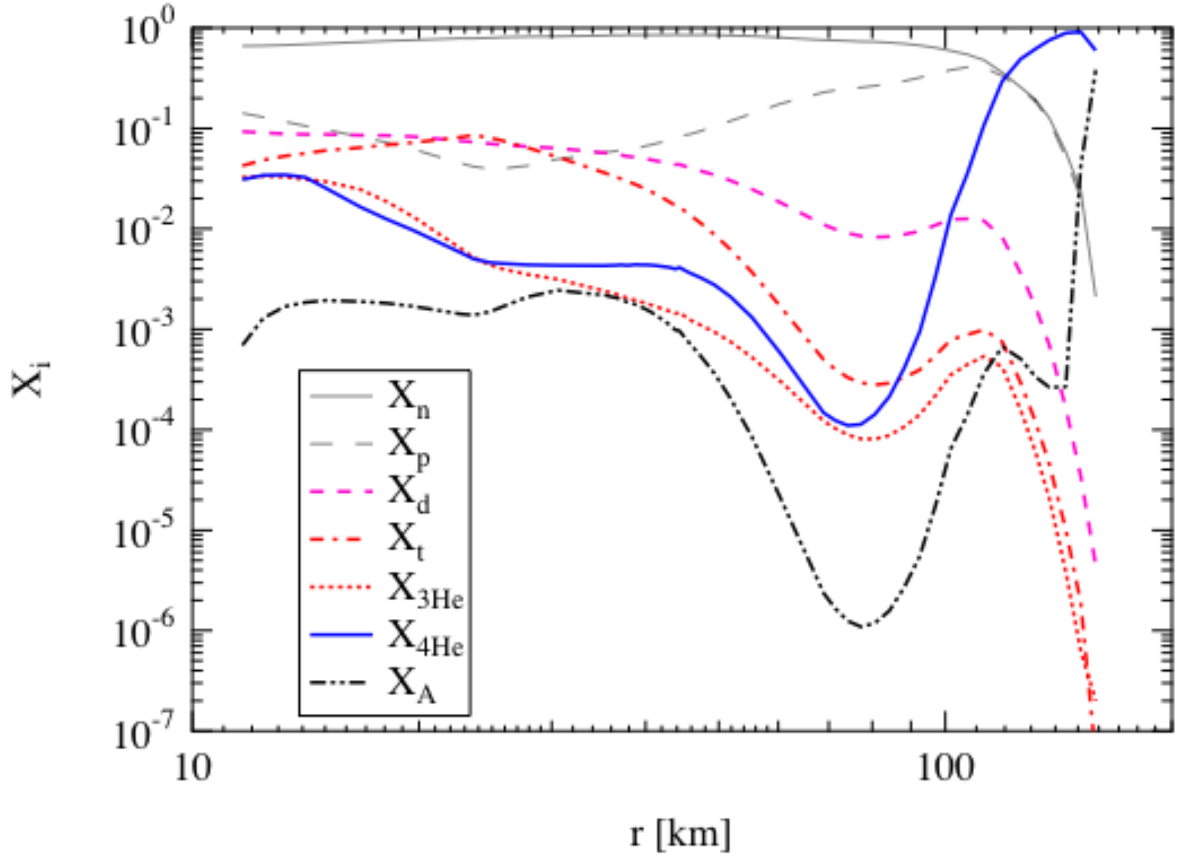
Protons in droplets
(heavy nuclei)

α–cluster outside,
at the surface,
condensate?



S. Typel, Proc. Int. Workshop XII Hadron Physics

Composition of supernova core



Mass fraction X
of light clusters
for a post-bounce
supernova core

K.Sumiyoshi,
G. R.,
PRC **77**,
055804 (2008)

S. Heckel, P. P. Schneider and A. Sedrakian,
Light nuclei in supernova envelopes: a quasiparticle gas model
Phys. Rev. C **80**, 015805 (2009).

In-medium modification of transport properties of dense matter

- D. Blaschke, G. Röpke, H. Schulz, A.D. Sedrakian,
  Nuclear in-medium effect on the thermal conductivity and viscosity of neutron star matter
  PL B 338, **111** (1994)

- D. Blaschke, G. Röpke, H. Schulz, A.D. Sedrakian, D. Voskresensky,
  Nuclear in-medium effects and neutrino emissivity of neutron stars.
  M. N. R. A. S. **273**, 596 (1995)

Summary

- Correlations (cluster formation, quantum condensates) are essential in low-density matter. They are suppressed with increasing density (Pauli blocking).

- The low-density limit of the nuclear matter EoS can be rigorously treated. The Beth-Uhlenbeck virial expansion is a benchmark. Larger nuclei and pasta structures must be treated in future works.

- An extended quasiparticle approach can be given for single nucleon states and nuclei. In a first approximation, self- energy and Pauli blocking is included. An interpolation between low and high densities is possible.

- Compared with the standard quasiparticle approach, significant changes arise in the low-density limit due to clustering. Examples are Bose-Einstein condensation (quartetting), and the behavior of the symmetry energy.

- Finite Systems (nuclei): Clusters in low-density regions, quantum condensates. Possibly preformed clusters at surface.

Thanks

to D. Blaschke, C. Fuchs, Y. Funaki, H. Horiuchi,
J. Natowitz, T. Klaehn, Z. Ren, S. Shlomo, P. Schuck,
A. Sedrakian, K. Sumiyoshi, A. Tohsaki, S. Typel,
H. Wolter, Z. Xu, T. Yamada, B. Zhou
for collaboration

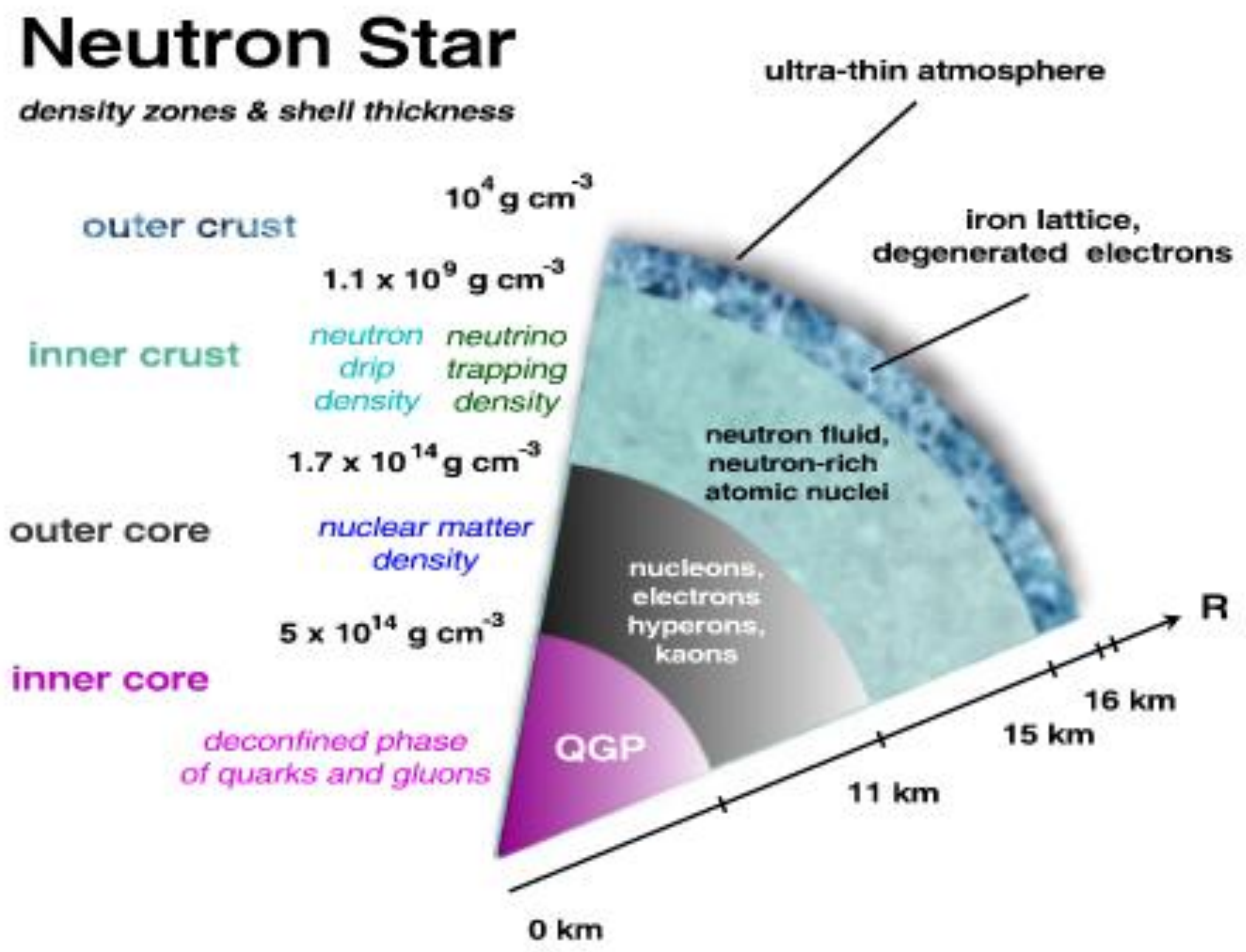
to you
for attention

D.G.

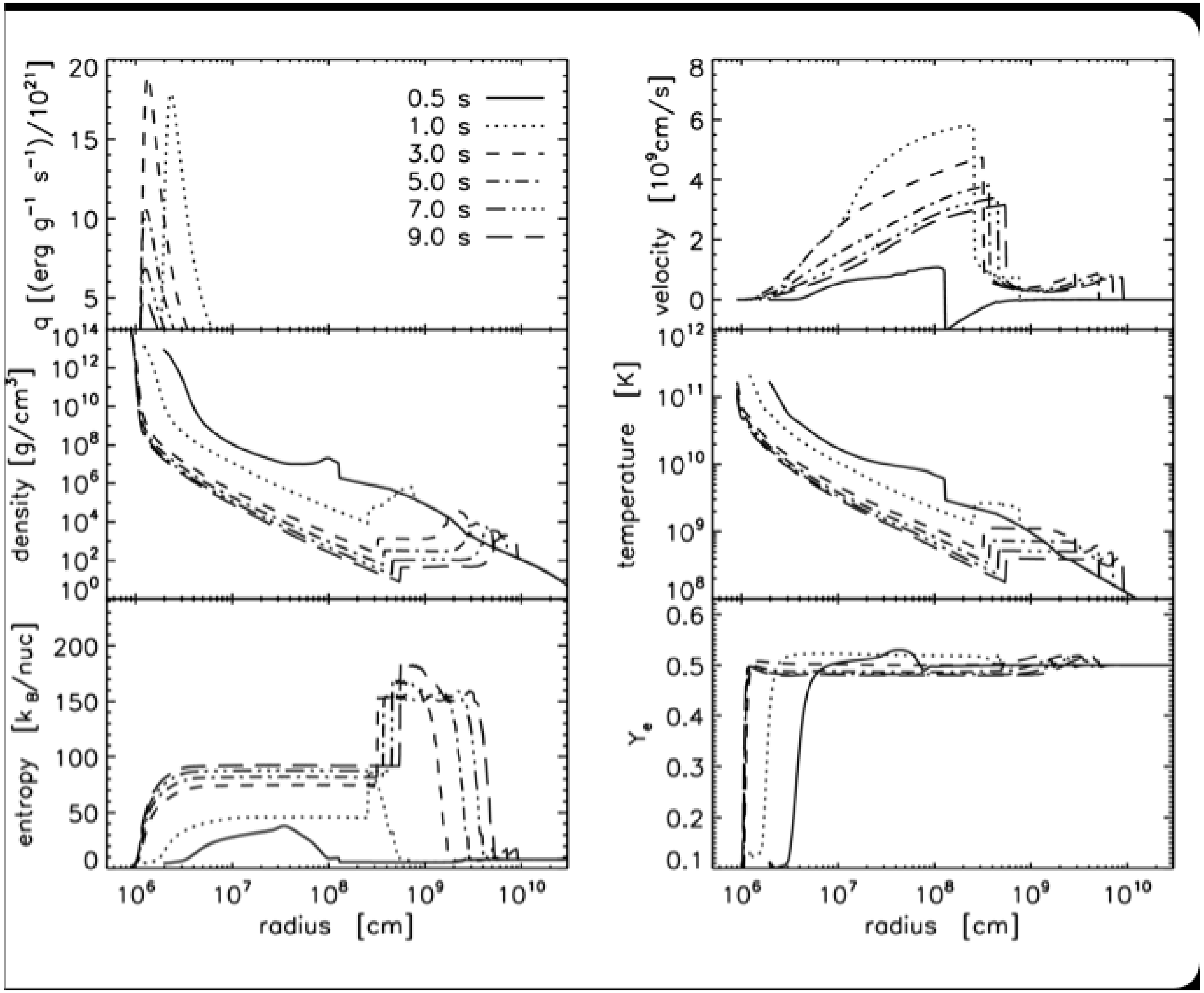
Supernova

Crab nebula, 1054 China, PSR 0531+21

Structure of a Neutron star



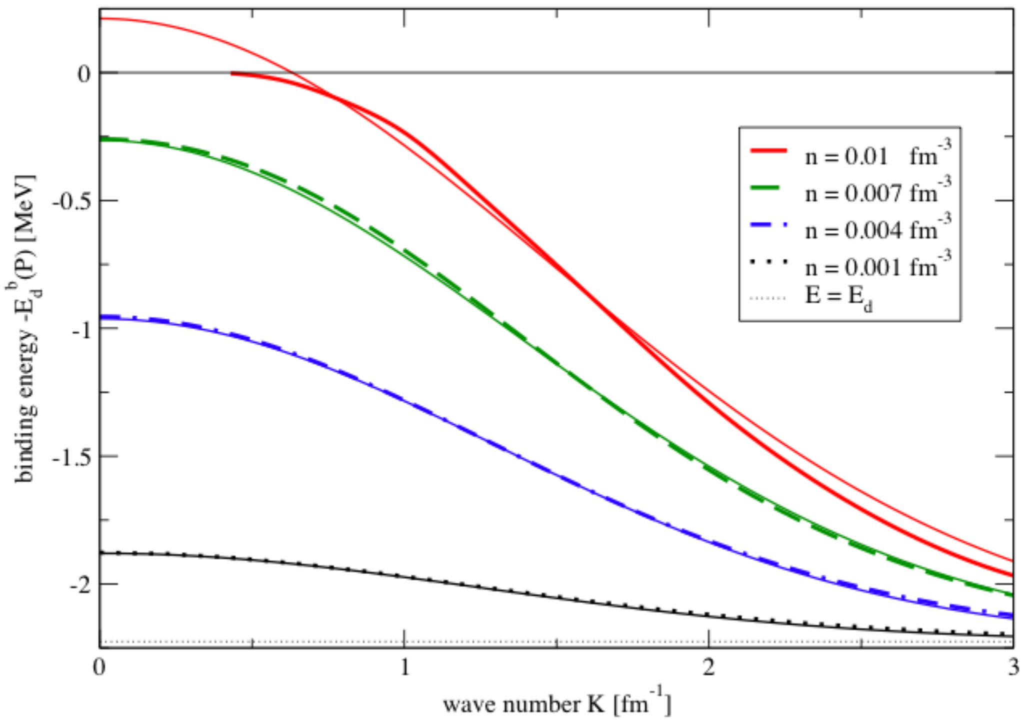
Supernova collapse:
spherically symmetric simulations



A. Arcones et al.
Neutrino driven winds,
PRC **78**, 015806 (08)

Shift of the deuteron binding energy

Dependence on center of mass momentum,
various densities, T=10 MeV



thin lines:

fit formula

G.R., NP A **867**, 66 (2011)

Deuteron quasiparticle properties

$$E_d^{\rm qu}(P) = E_d^{\rm free} + \Delta E_d + \frac{\hbar}{2m_d^*}P^2 + O(P^4)$$

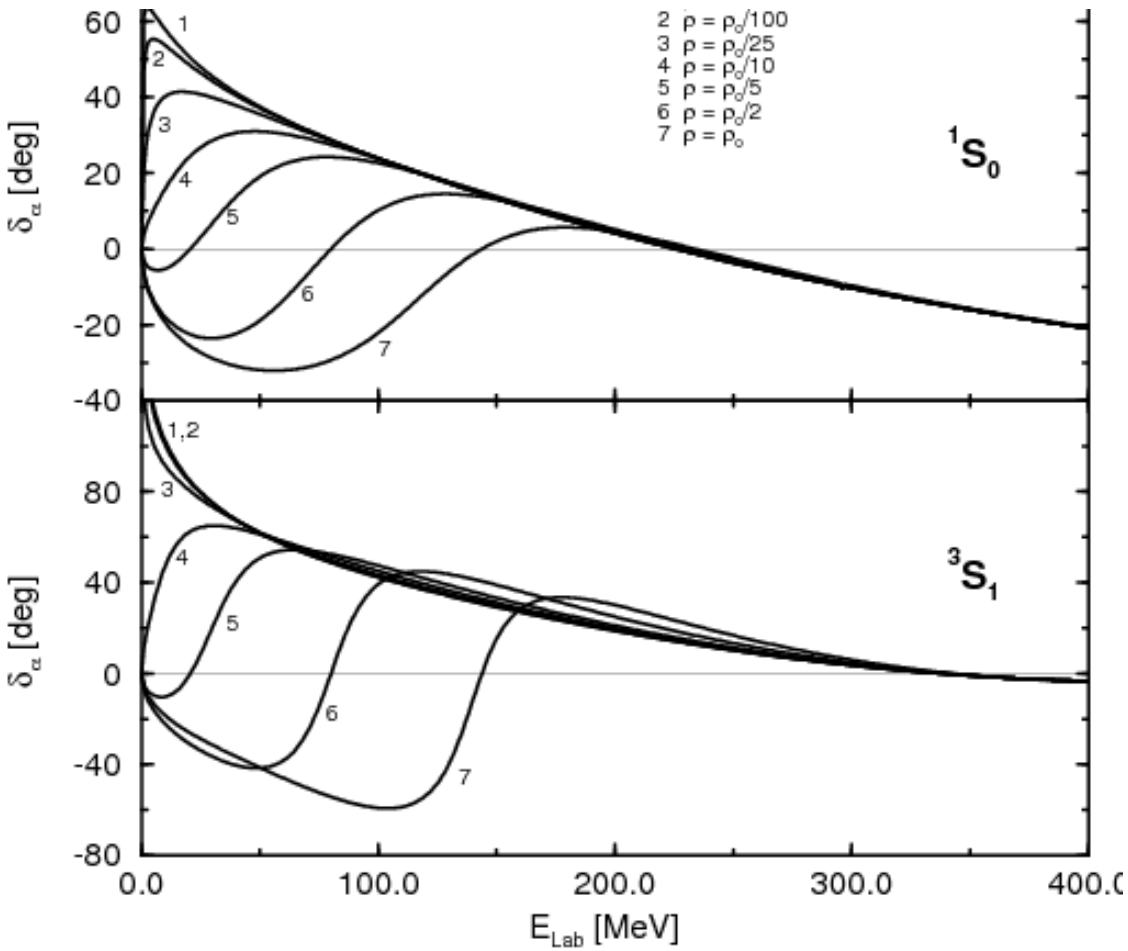
$$E_d^{\rm free} = -2.225\,{\rm MeV}$$

$$\Delta E_d^{\rm Pauli}(T, n_B, \alpha) = \delta E_d^{(0)}(T, \alpha) n_B + O(n_B^2)$$

$$\frac{m_d^*}{m_d}(T, n_B, \alpha) = 1 + \delta m_d^{(0)}(T, \alpha) n_B + O(n_B^2)$$

T [MeV]	delta E [MeV fm^3]	delta m* [fm^3]
10	364.3	21.3
4	712.9	87.1

Scattering phase shifts in matter



Application to Heavy Ion Reactions

- Test the EOS
  (NSE, virial,… at low densities,
  Skyrme, DBHF, RMF,… near saturation)
- Unifying quantum statistical approach, medium effects, Mott effect
- Symmetry energy
- Bose enhancement?

Nimrod @ TAMU,
40Ar + 112,124Sn,
64Zn + 112,124Sn; 47 A MeV

Open questions: freeze-out model or dynamical transport models?
Identification of the source? - yields of p, (n), d, t, 3He, 4He,…

Mott points from cluster yields

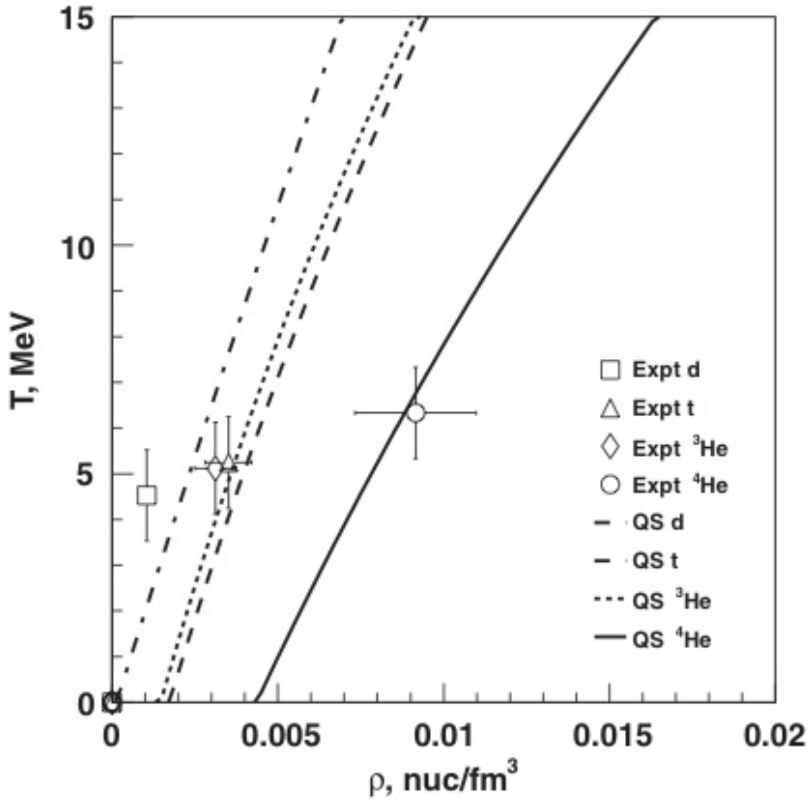
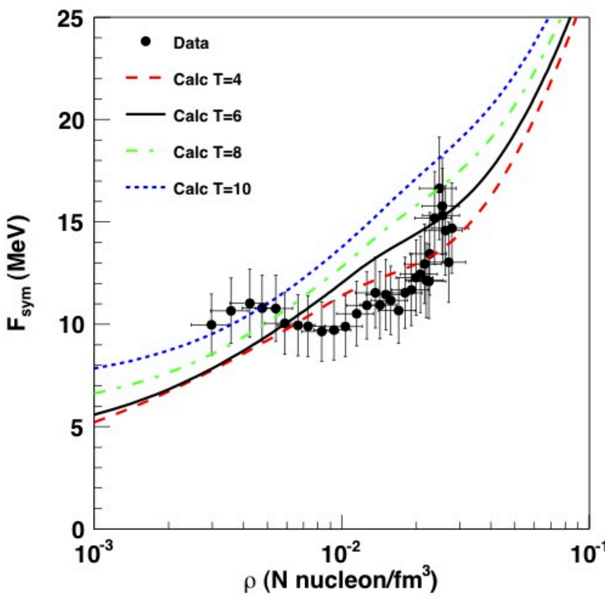
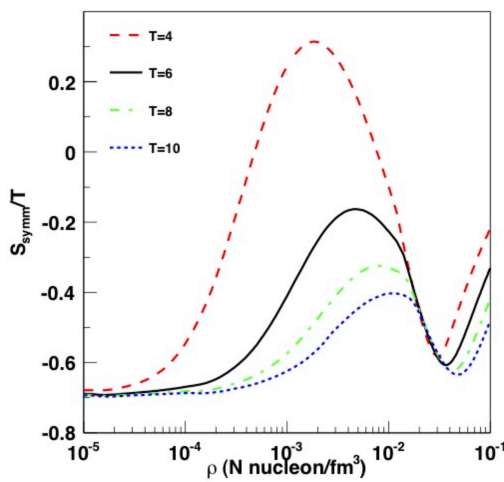


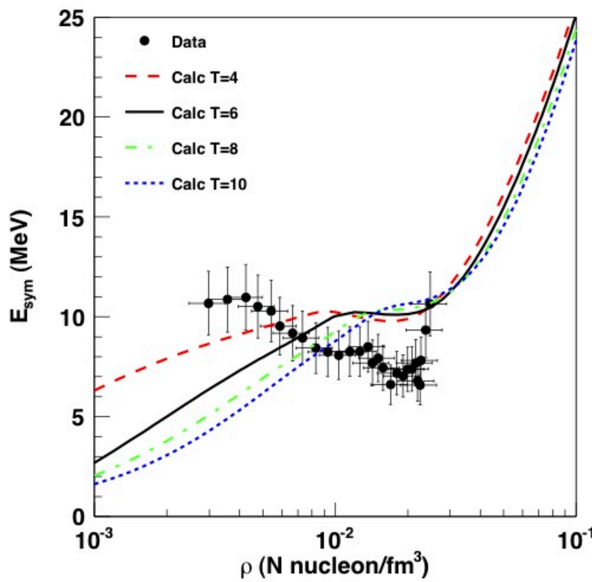
FIG. 3. Comparison of experimentally derived Mott point densities and temperatures with theoretical values. Symbols represent the experimental data. Estimated errors on the temperatures are 10% and on the densities 20%. Lines show polynomial fits to the Mott points presented in Ref. [1].

K. Hagel et al., PRL **108**, 062702 (2012)

Free symmetry energy







symmetry entropy

Internal symmetry energy

R. Wada et al., Phys. Rev. C 85, 064618 (2012).

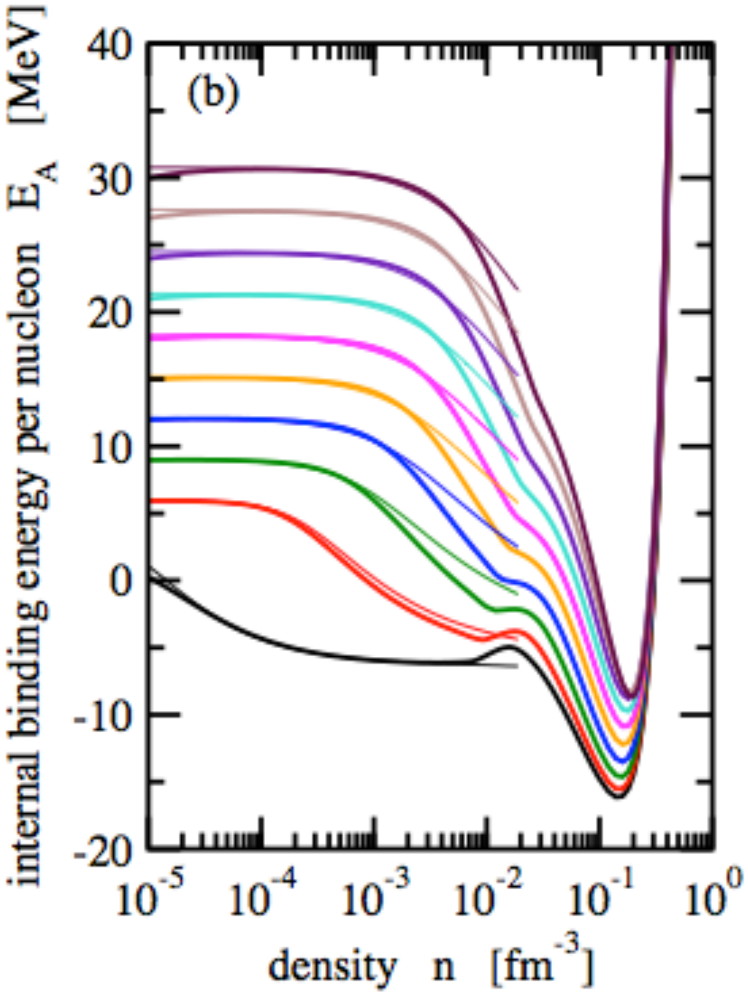
Internal energy per nucleon

Isotherms

T[MeV]

20
18
16
14
12
10
8
6
4
2

thin lines: NSE



S. Typel et al., PRC **81**, 015803 (2010)

Composition of dense nuclear matter

$$n_p(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} Z_A f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

$$n_n(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} (A-Z_A) f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

mass number A
charge Z_A
energy $E_{A,\nu,K}$
ν: internal quantum number

$$f_{A(z)} = \frac{1}{\exp(z/T) - (-1)^A}$$

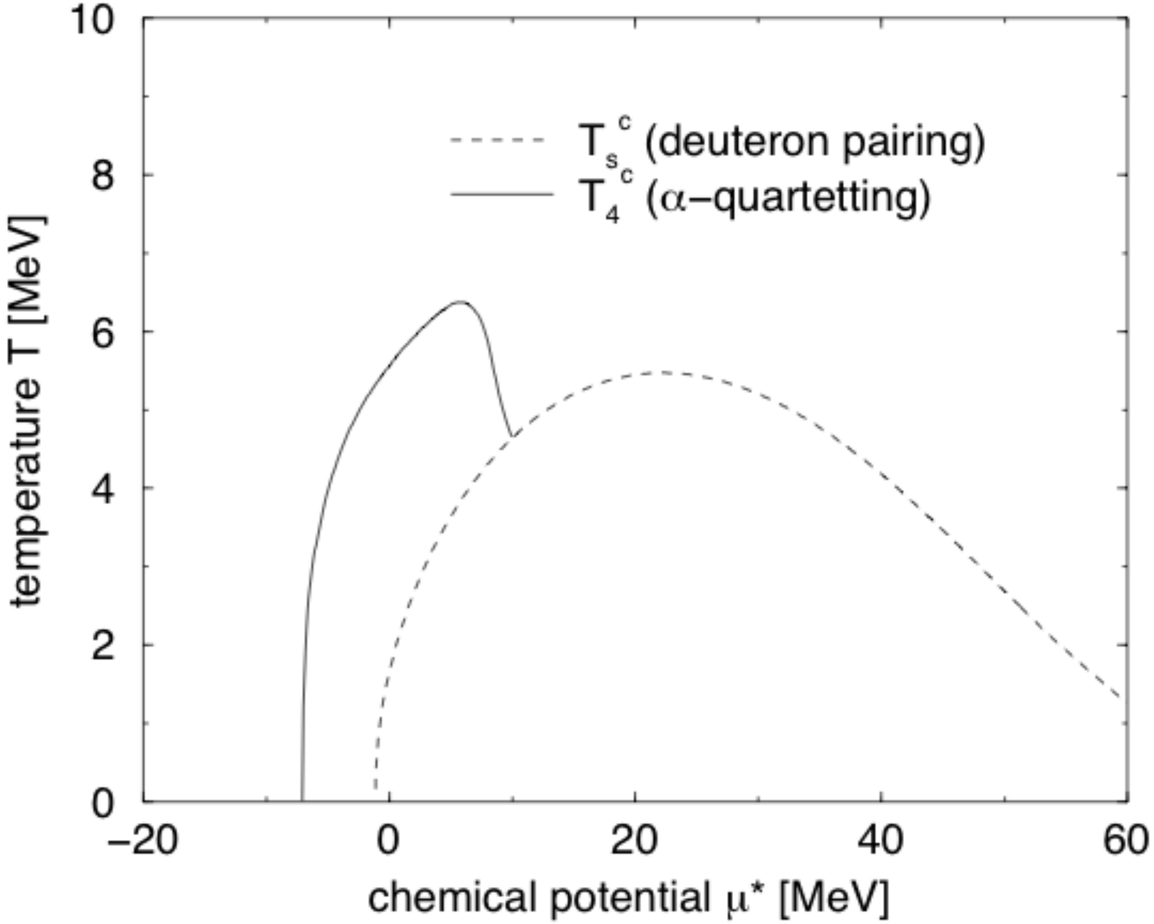
- Inclusion of excited states and continuum correlations

- Medium effects:
  self-energy and Pauli blocking shifts of binding energies,
  Coulomb corrections due to screening (Wigner-Seitz,Debye)

- Bose-Einstein condensation

α-cluster-condensation

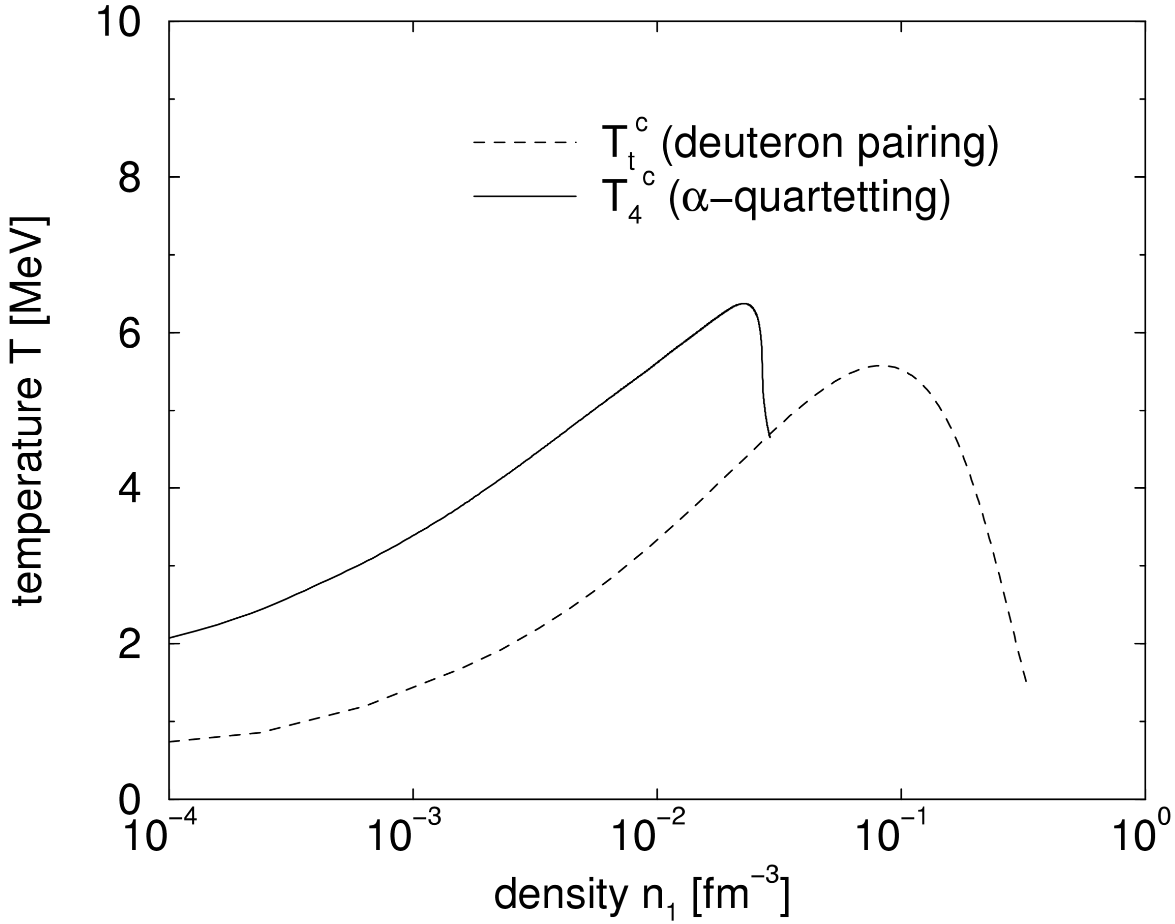
(quartetting)



G.Röpke, A.Schnell, P.Schuck, and P.Nozieres, PRL 80, 3177 (1998)

α-cluster-condensation

(quartetting)



G.Röpke, A.Schnell, P.Schuck, and P.Nozieres, PRL 80, 3177 (1998)

Effective wave equation for the deuteron in matter

In-medium two-particle wave equation in mean-field approximation

$$\left(\frac{p_1^2}{2m_1}+\Delta_1+\frac{p_2^2}{2m_2}+\Delta_2\right)\Psi_{d,P}(p_1,p_2)+\sum_{p_1',p_2'}(1-f_{p_1}-f_{p_2})V(p_1,p_2;p_1',p_2')\Psi_{d,P}(p_1',p_2')$$

$$=E_{d,P}\Psi_{d,P}(p_1,p_2)$$

Add self-energy

Pauli-blocking

Thouless criterion

$$E_d(T,\mu)=2\mu$$

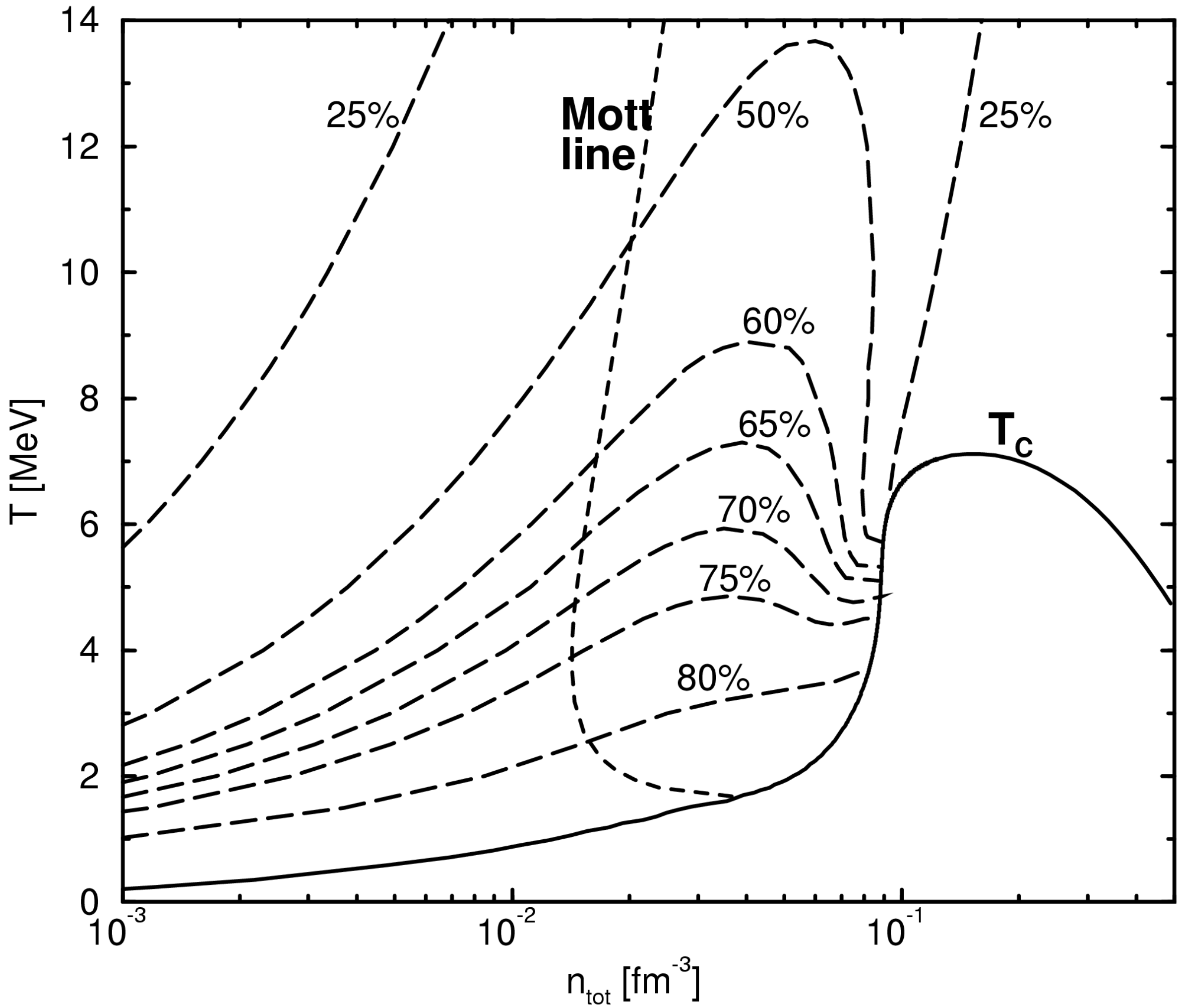
Fermi distribution function

$$f_p=\left[e^{(p^2/2m-\mu)/k_BT}+1\right]^{-1}$$

BEC-BCS crossover:
Alm et al.,1993

Composition of symmetric nuclear matter

Fraction of correlated matter (virial expansion, Generalized Beth-Uhlenbeck approach, contribution of bound states, of scattering states, phase shifts)



H. Stein et al.,
Z. Phys. **A351**, 259 (1995)

Self-conjugate 4n nuclei

^{12}C:

0^+ state at 0.39 MeV above the 3α threshold energy:
α cluster interact predominantly in relative S waves, gaslike structure

α-particle condensation in low-density nuclear matter ($\rho \leq \rho_0/5$)

$n\alpha$ cluster condensed states
– a general feature in $N = Z$ nuclei?

Self-conjugate 4n nuclei

$n\alpha$ nuclei: ^{8}Be, ^{12}C, ^{16}O, ^{20}Ne, ^{24}Mg, ...

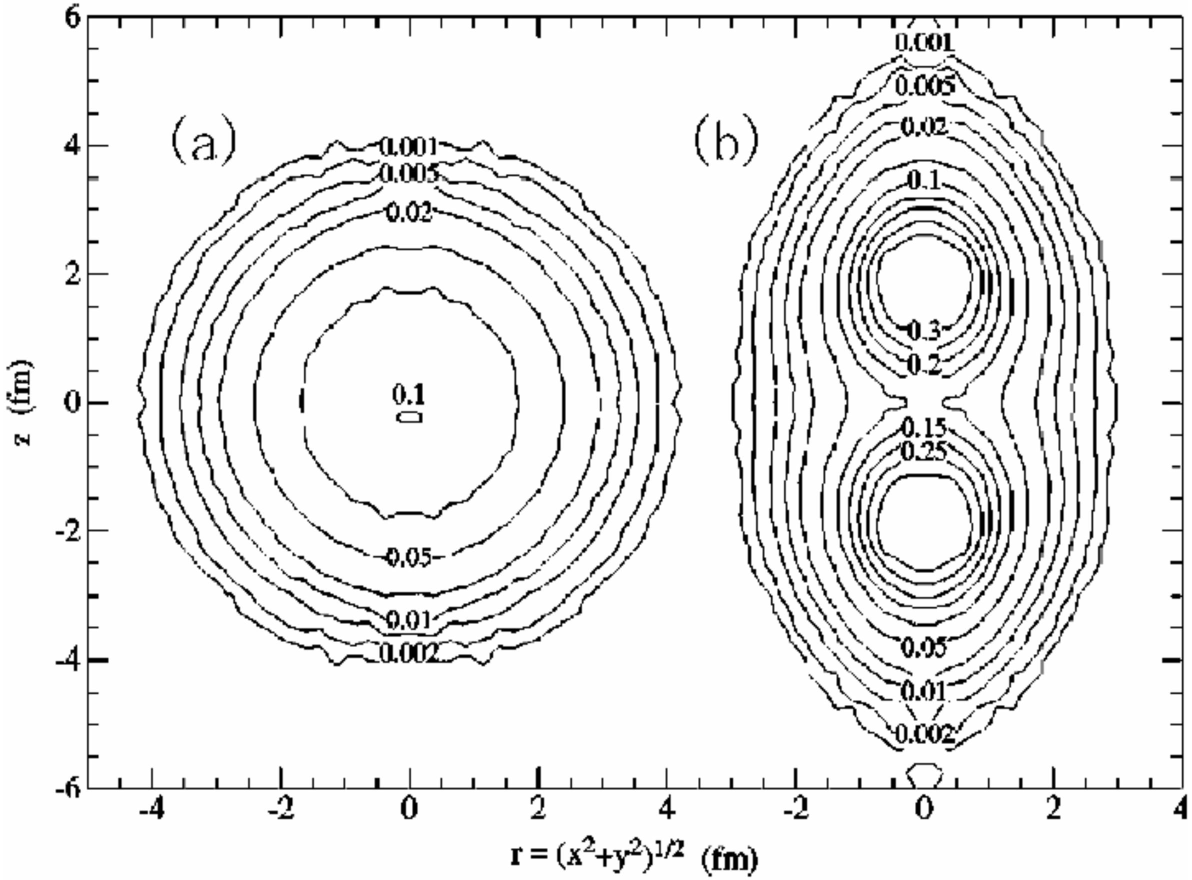
Single-particle shell model, or

Cluster type structures

ground state, excited states

$n\alpha$ break up at the threshold energy $E_{n\alpha}^{\text{thr}} = nE_\alpha$

Alpha cluster structure of Be 8



R.B. Wiringa et al.,
PRC **63**, 034605 (01)

Contours of constant density, plotted in cylindrical coordinates, for 8Be(0+) .
The left side is in the laboratory frame while the right side is in the intrinsic frame.

Results

		E_k (MeV)	E_{exp} (MeV)	$E_k - E_{n\alpha}^{thr}$ (MeV)	$(E - E_{n\alpha}^{thr})_{exp}$ (MeV)	$\sqrt{\langle r^2 \rangle}$ (fm)	$\sqrt{\langle r^2 \rangle}_{exp}$ (fm)
^{12}C	$k=1$	-85.9	-92.16 (0_1^+)	-3.4	-7.27	2.97	2.65
	$k=2$	-82.0	-84.51 (0_2^+)	+0.5	0.38	4.29	
	$E_{3\alpha}^{thr}$	-82.5	-84.89				
^{16}O	$k=1$	-124.8 $(-128.0)^*$	-127.62 (0_1^+)	-14.8 $(-18.0)^*$	-14.44	2.59	2.73
	$k=2$	-116.0	-116.36 (0_3^+)	-6.0	-3.18	3.16	
	$k=3$	-110.7	-113.62 (0_5^+)	-0.7	-0.44	3.97	
	$E_{4\alpha}^{thr}$	-110.0	-113.18				
^8Be				-0.17	+0.1		

Tabelle 1: Comparison of the generator coordinate method calculations with experimental values. $E_{n\alpha}^{thr} = nE_\alpha$ denotes the threshold energy for the decay into α-clusters, the values marked by * correspond to a refined mesh.

M. Chernykh et al., PRL **98**, 032501 (07); Y. Funaki et al., PRL **101**, 082502 (08)

Excited light nuclei

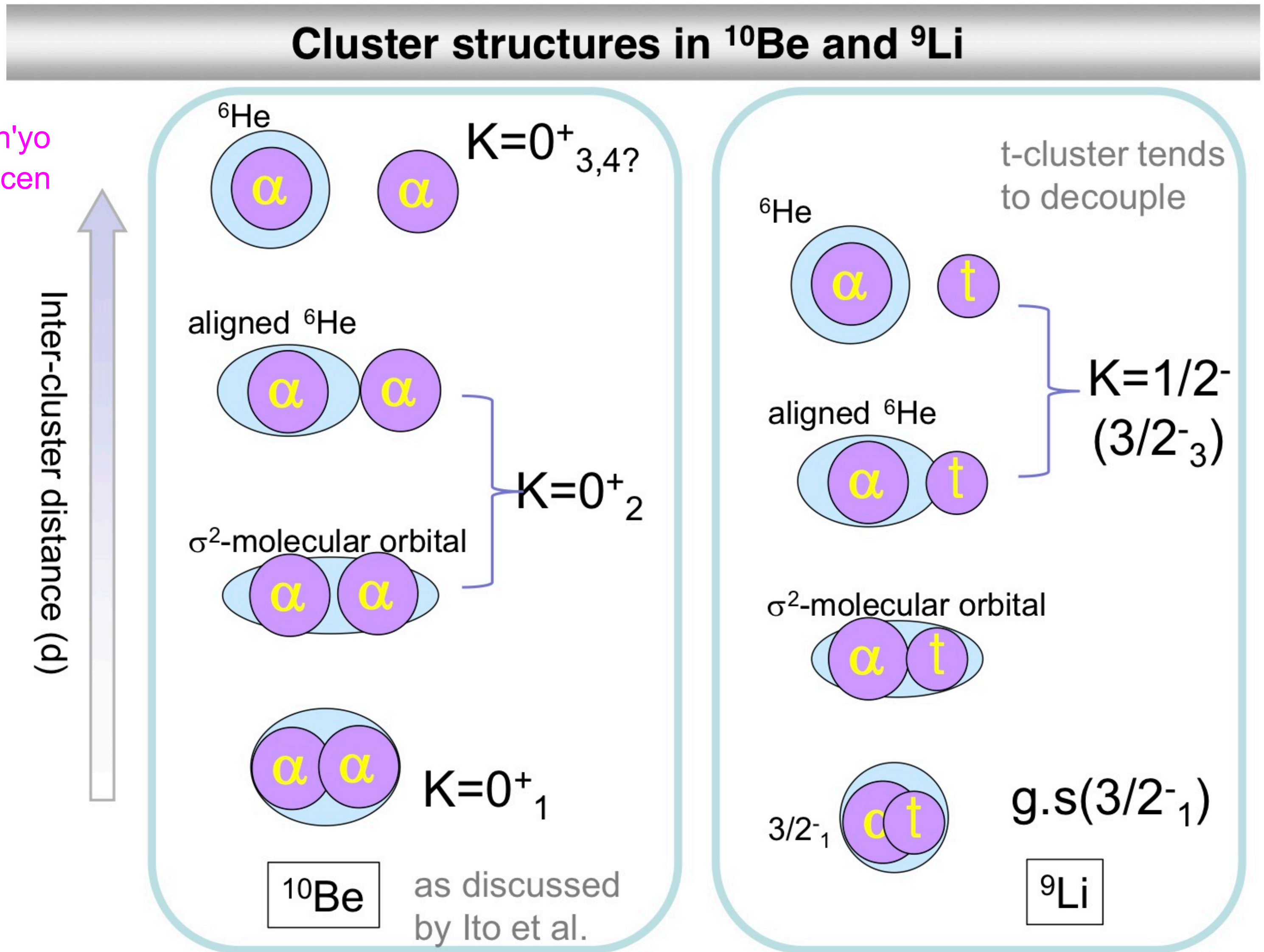
Yoshiko Kanada-En'yo
Cluster2012,Debrecen

decreasing
density

deuterons?

systematics
in
weakly bound
light
elements

light
clusters
in neutron
matter

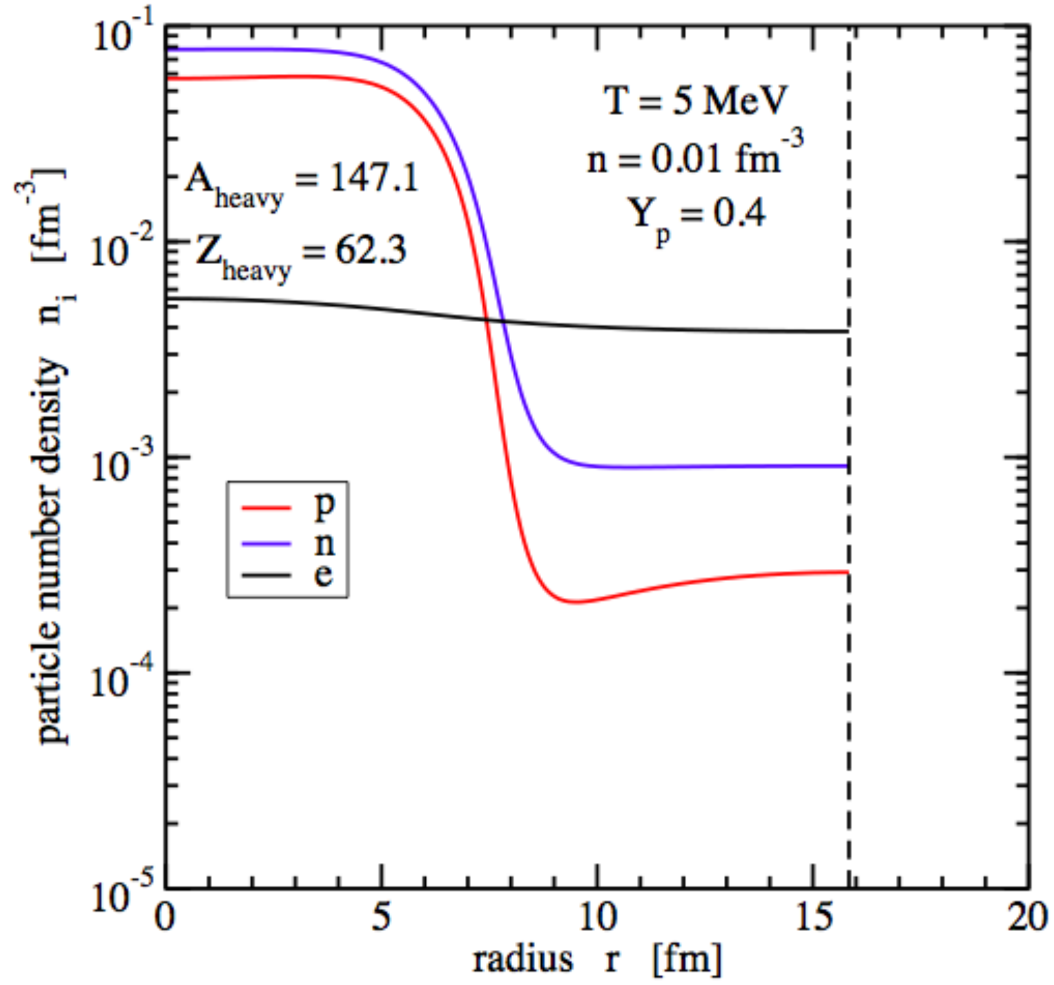


α cluster in astrophysics

Crust of neutron stars

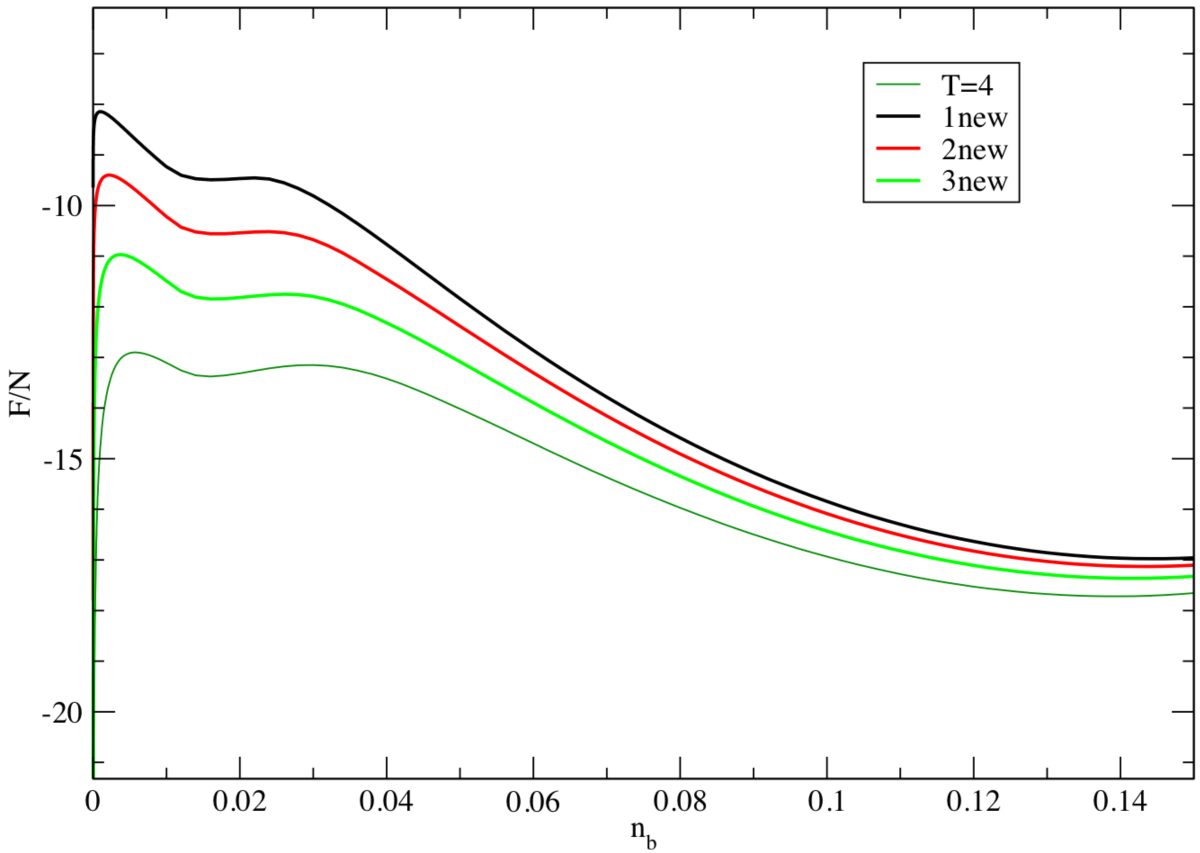
Protons in droplets
(heavy nuclei)

α–cluster outside,
at the surface,
condensate?



S. Typel, Proc. Int. Workshop XII Hadron Physics

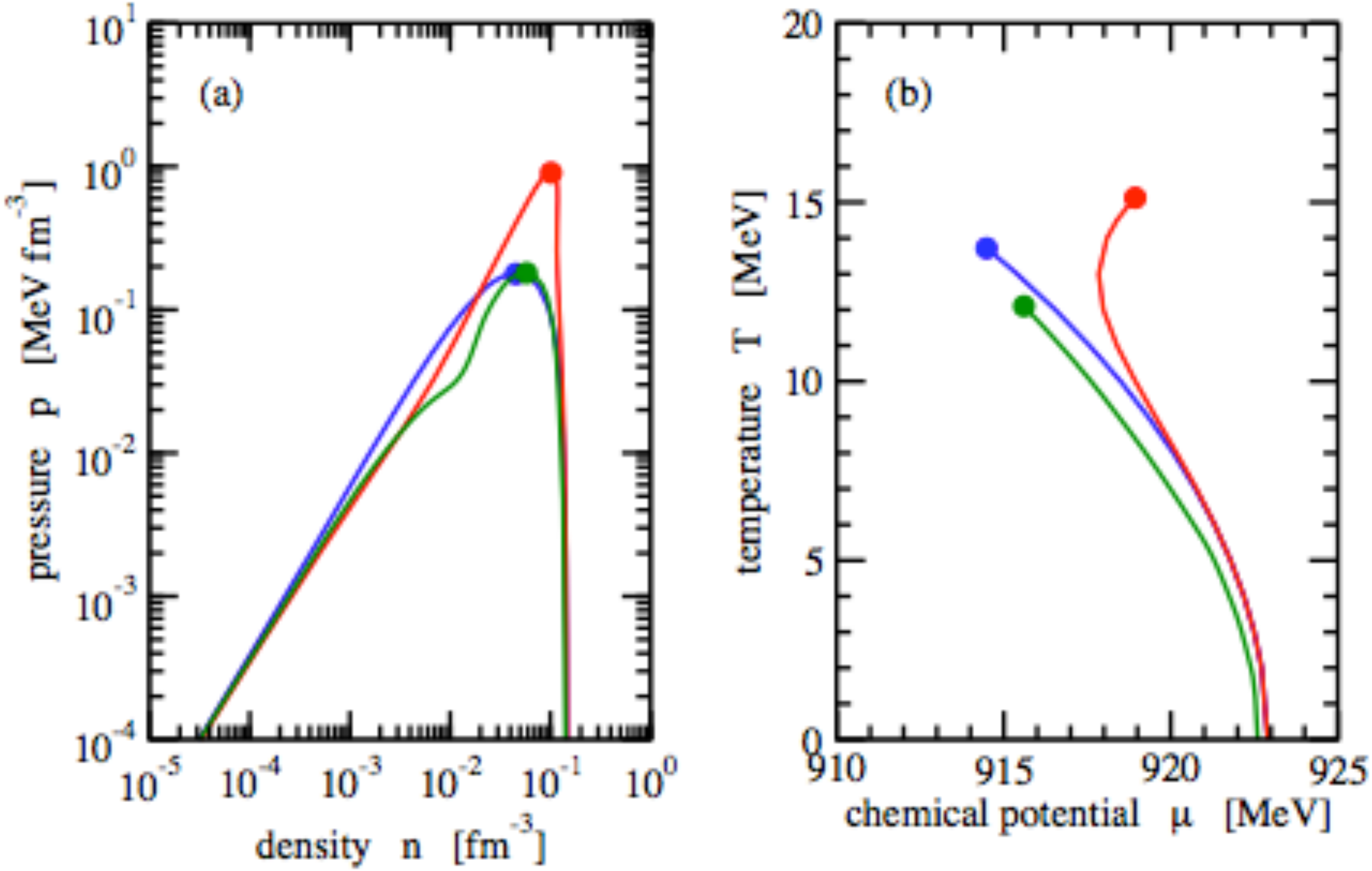
Free energy per nucleon



preliminary

correlated medium

Liquid-vapor phase transition

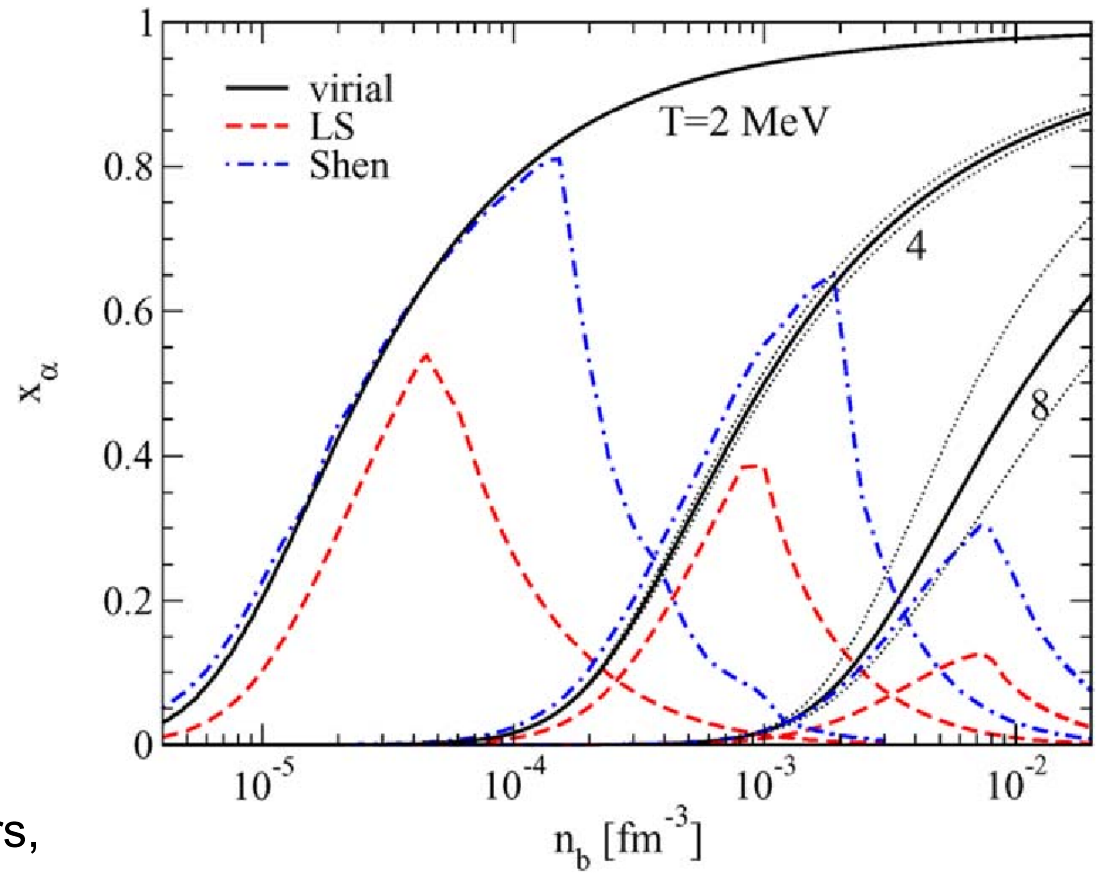


blue: no light cluster, green: with light clusters, QS, red: cluster-RMF

S. Typel et al., PRC **81**, 015803 (2010)

Alpha-particle fraction in the low-density limit

symmetric matter, T=2, 4, 8 MeV



LS, Shen:
higher clusters,
excluded volume

C.J.Horowitz, A.Schwenk, Nucl. Phys. A **776**, 55 (2006)

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Nuclear Physics A 897 (2013) 70–92
NUCLEAR PHYSICS A
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Cluster-virial expansion for nuclear matter within a quasiparticle statistical approach

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[d] *GSI Helmholtzzentrum für Schwerionenforschung GmbH, Theorie, Planckstraße 1, D-64291 Darmstadt, Germany*
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Abstract

Correlations in interacting many-particle systems can lead to the formation of clusters, in particular bound states and resonances. Systematic quantum statistical approaches allow to combine the nuclear statistical equilibrium description (law of mass action) with mean-field concepts. A chemical picture, which treats the clusters as distinct entities, serves as an intuitive concept to treat the low-density limit. Within a generalized Beth–Uhlenbeck approach, the quasiparticle-virial expansion is extended to include arbitrary clusters, where special attention must be paid to avoid inconsistencies such as double counting. Correlations are suppressed with increasing density due to Pauli blocking. The contribution of the continuum to the virial coefficients can be reduced by considering clusters explicitly and introducing quasiparticle energies. The cluster-virial expansion for nuclear matter joins known benchmarks at low densities with those near saturation density.

Cluster - mean field approximation

Cluster (A) interacting with a distribution of clusters (B) in the medium,
fully antisymmetrized

$$\sum_{1'\ldots A'} \{H_A^0(1\ldots A,1'\ldots A') + \sum_i \Delta_i^{A,mf}\delta_{k,k'} + \frac{1}{2}\sum_{i,j}\Delta V_{ij}^{A,mf}\delta_{l,l'} - E_{A\nu P}\delta_{k,k'}\}\psi_{A\nu P}(1'\ldots A') = 0$$

self-energy

$$\Delta_1^{A,mf}(1) = \sum_2 V(12,12)_{ex} f^*(2) + \sum_{B\nu P}\sum_{2\ldots B'} f_B(E_{B\nu P})\sum_i V_{1i}(1i,1'i')\psi^*_{B\nu P}(1\ldots B)\psi_{B\nu P}(1'\ldots B')$$

effective interaction

$$\Delta V_{12}^{A,mf} = -\frac{1}{2}[f^*(1) + f^*(2)]V(12,1'2') - \sum_{B\nu P}\sum_{2^*\ldots B''} f_B(E_{B\nu P})\sum_i V_{1i}\psi^*_{B\nu P}(22^*\ldots B^*)\psi_{B\nu P}(2'2''\ldots B'')$$

phase space occupation $\quad f^*(1) = f_1(1) + \sum_{B\nu P}\sum_{2\ldots B} f_B(E_{B\nu P})\,|\psi_{B\nu P}(1\ldots B)|^2$

Self-consistent RPA

Two-time cluster Matsubara Green's functions

$$G_{\alpha\beta}^{\tau-\tau'} = -\langle Tr A_\alpha(\tau) A_\beta^\dagger(\tau')\rangle$$
$$= -Tr\left[\rho_G T_\tau e^{\tau K} A_\alpha e^{-(\tau-\tau')K} A_\beta^\dagger e^{-\tau' K}\right]$$

Equation of motion method

$$-\frac{\partial}{\partial\tau} G_{\alpha\beta}^{\tau-\tau'} = \delta_{\tau-\tau'}\langle[A_\alpha, A_\beta^\dagger]\rangle - \langle Tr[A_\alpha, K]^\tau A_\beta^\dagger(\tau')\rangle$$
$$= \delta_{\tau-\tau'} N_{\alpha\beta} + \sum_\gamma \int d\tau_1' \mathcal{H}_{\alpha\gamma}^{\tau-\tau_1'} G_{\gamma\beta}^{\tau_1'-\tau'} \ .$$

Effective Hamiltonian is split into an instantaneous and a dynamic part

$$\mathcal{H}_{\alpha\beta}^{\tau-\tau'} = \sum_{\beta'}\left\{\delta_{\tau-\tau'}\langle[[A_\alpha, K], A_{\beta'}^\dagger]\rangle - \langle Tr[A_\alpha, K]^\tau [K, A_{\beta'}^\dagger]^{\tau'}\rangle_{irr}\right\} N_{\beta'\beta}^{-1}$$
$$\equiv \mathcal{H}_{\alpha\beta}^{(0)}\delta_{\tau-\tau'} + \mathcal{H}_{\alpha\beta}^{(r)\tau-\tau'} \ .$$

J.Dukelsky, G. Roepke, and P.Schuck, NPA **628**, 17 (1998)
P. Schuck, D.S. Delion, J.Dukelsky, and G. Roepke, in preparation

Deuteron quasiparticle properties

$$E_d^{\mathrm{qu}}(P) = E_d^{\mathrm{free}} + \Delta E_d + \frac{\hbar^2}{2m_d^*}P^2 + O(P^4)$$

$$\Delta E_d^{\mathrm{Pauli}}(T, n_B, \alpha) = \delta E_d^{(0)}(T,\alpha) n_B + O(n_B^2)$$

$$\frac{m_d^*}{m_d}(T, n_B, \alpha) = 1 + \delta m_d^{(0)}(T,\alpha) n_B + O(n_B^2)$$

T [MeV]	delta E [MeV fm^3]	delta m* [fm^3]
10	364.3	21.3
4	712.9	87.1

$$E_d^{\mathrm{free}} = -2.225\,\mathrm{MeV}$$

G.R., PRC **79**, 014002 (2009)

Nuclear matter properties

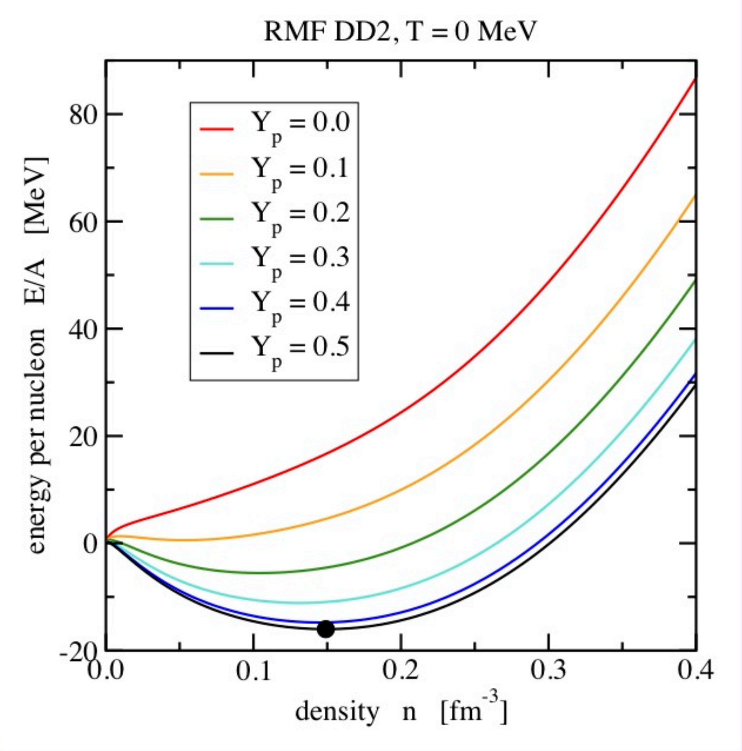
binding energy per nucleon near saturation:

$$\frac{E}{A}(n,\beta) = \frac{\varepsilon}{n} = a_V + \frac{K}{18}x^2 + \frac{K'}{162}x^3 + \beta^2\left(J + \frac{L}{3}x + \dots\right) + \dots$$

with $x = (n - n_{\mathrm{sat}})/n_{\mathrm{sat}}$, asymmetry $\beta = 1 - 2Y_p$ and

nuclear matter parameters

- n_{sat} saturation density
- a_V bulk energy
- K incompressibility
- K' skewness
- J bulk symmetry energy
- L symmetry energy slope



S. Typel, 2012