The spin evolution of the pulsars with non-rigid core

D P Barsukov, **O A Goglichidze**, A I Tsygan

Ioffe Physical-Technical Institute

Erevan, September 18-21, 2013

The long-term rotation evolution

Rigid star

$$I_{ns}\dot{\vec{\Omega}} = \vec{K}$$

Three scalar equations

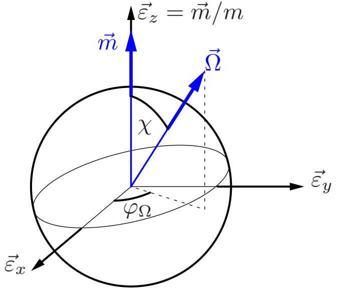
$$I_{ns}\dot{\Omega} = K_\Omega + K_m \cos\chi$$

$$I_{ns}\Omega\dot{\chi} = -K_m \sin\chi$$

$$I_{ns}\Omega\dot{\varphi}_\Omega = -K_\perp$$

where

$$\vec{K} = K_\Omega \vec{e}_\Omega + K_m \vec{e}_m + K_\perp [\vec{e}_\Omega \times \vec{e}_m]$$



Angular momentum loses mechanisms:

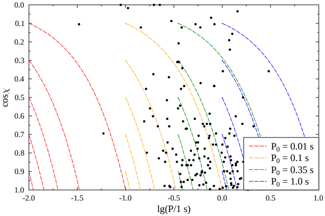
1. **Magneto-dipolar radiation** $\vec{K} = \frac{2\Omega^2}{3c^3}\vec{m} \times [\vec{m} \times \vec{\Omega}]$
2. **Current loses** $\vec{K} = -\alpha(\chi, \varphi_\Omega)\frac{2\Omega^3 m}{3c^3}\vec{m}$.
3. **Field moment of inertia** $\vec{K} = \frac{a}{R_{ns}c^2}(\vec{m}\cdot\vec{\Omega})[\vec{\Omega} \times \vec{m}]$
   Here, a is a coefficient ~ 1 depending on the fields configuration.

Inclination angle evolution

Vacuum approximation

$$\Omega \cos \chi = const$$

All pulsars should rapidly evolve to coaxial state.



Current loses

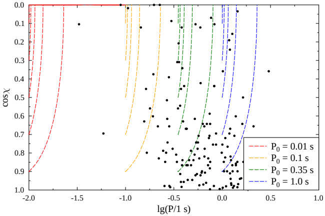
Magneto-dipolar radiation absents

V. S. Beskin, A. V. Gurevich, I. N. Istomin, *Soviet Journal of Experimental and Theoretical Physics* **58**, 235 (1983)

$$j \approx j_{GJ}$$

$$\Omega \sin \chi = const$$

All pulsars should rapidly evolve to orthogonal state.



Observational data: J. M. Rankin, *ApJS* **85**, 145 (1993)

Inclination angle evolution

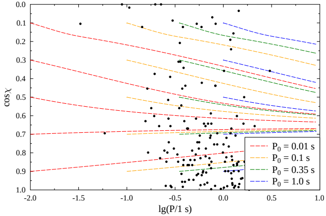
Magneto-dipolar radiation + current loses + small scale field

D. P. Barsukov, P. I. Polyakova, A. I. Tsygan, *Astronomy Reports* **53**, 1146 (2009),

D. P. Barsukov, O. A. Goglichidze, A. I. Tsygan, *Astronomy Reports* **57**, 26 (2013)

$$\vec{K} = \vec{K}_{dip} + \vec{K}_{cur}$$

There is an equilibrium angle for the range of non-dipolar parameters B_{nondip}/B_{dip}



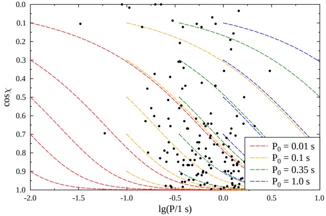
MHD simulations

A. Philippov, A. Tchekhovskoy, J. G. Li, [in preparation]

$$\vec{K} = -\frac{2\Omega^3 m^2}{3c^3}\left[(k_0 + k_1 \sin^2\chi + k_2 \cos^2\chi)\vec{e}_\Omega - -k_2 \cos\chi \vec{e}_m\right] + \vec{K}_\perp$$

$k_0 = 1, \quad k_1 = 1.2, \quad k_2 = 1$

Large current $j \gg j_{GJ}$



Observational data: J. M. Rankin, *ApJS* **85**, 145 (1993)

The core hydrodynamics

The system of hydrodynamical equations

$$\partial_t \vec{v} + (\vec{v} \cdot \nabla)\vec{v} + \nabla P/\rho + \nabla \Phi = \vec{f}_{int} + \vec{f}_{EM} + \vec{f}_{visc}$$

$$\partial_t \rho + \mathrm{div}(\rho \vec{v}) = 0,$$

$$\Phi(\vec{r}) = -G \int \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} d^3 r',$$

Co-rotating frame of reference $(\vec{v} = [\vec{\Omega} \times \vec{r}] + \vec{u})$

$$\partial_t^* \vec{u} + \underline{2[\vec{\Omega} \times \vec{u}]} + (\vec{u} \cdot \nabla)\vec{u} + \nabla P/\rho + \nabla \left(\Phi - 1/2[\vec{\Omega} \times \vec{r}]^2\right) = -\underline{\underline{[\dot{\vec{\Omega}} \times \vec{r}]}} + \vec{f}_{int} + \vec{f}_{EM} + \vec{f}_{visc}$$

$$\partial_t^* \rho + \mathrm{div}(\rho \vec{u}) = 0$$

The core hydrodynamics

The system of hydrodynamical equations

$$\partial_t \vec{v} + (\vec{v}\cdot\nabla)\vec{v} + \nabla P/\rho + \nabla\Phi = \vec{f}_{int} + \vec{f}_{EM} + \vec{f}_{visc}$$

$$\partial_t \rho + \mathrm{div}(\rho\vec{v}) = 0,$$

$$\Phi(\vec{r}) = -G\int \frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|} d^3 r',$$

Co-rotating frame of reference $(\vec{v} = [\vec{\Omega}\times\vec{r}] + \vec{u})$

$$\partial_t^* \vec{u} + \underline{2[\vec{\Omega}\times\vec{u}]} + \cancel{(\vec{u}\cdot\nabla)\vec{u}} + \nabla P/\rho + \nabla\left(\Phi - 1/2[\vec{\Omega}\times\vec{r}]^2\right) = -\underline{\underline{[\dot{\vec{\Omega}}\times\vec{r}]}} + \vec{f}_{int} + \vec{f}_{EM} + \vec{f}_{visc}$$

$$\partial_t^* \rho + \mathrm{div}(\rho\vec{u}) = 0$$

1. Small perturbations

The core hydrodynamics

The system of hydrodynamical equations

$$\partial_t \vec{v} + (\vec{v}\cdot\nabla)\vec{v} + \nabla P/\rho + \nabla\Phi = \vec{f}_{int} + \vec{f}_{EM} + \vec{f}_{visc}$$

$$\partial_t \rho + \mathrm{div}(\rho\vec{v}) = 0,$$

$$\Phi(\vec{r}) = -G\int \frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|} d^3 r',$$

Co-rotating frame of reference $(\vec{v} = [\vec{\Omega}\times\vec{r}] + \vec{u})$

$$\partial_t^* \vec{u} + \underline{2[\vec{\Omega}\times\vec{u}]} + \cancel{(\vec{u}\cdot\nabla)\vec{u}} + \nabla P/\rho + \nabla\left(\Phi - 1/2[\vec{\Omega}\times\vec{r}]^2\right) = -\underline{\underline{[\dot{\vec{\Omega}}\times\vec{r}]}} + \vec{f}_{int} + \vec{f}_{EM} + \vec{f}_{visc}$$

$$\partial_t^* \rho + \mathrm{div}(\rho\vec{u}) = 0$$

1. Small perturbations
2. Isothermality and strong degeneracy of core matter

$$\nabla P = \sum_\beta \rho_\beta \nabla\mu_\beta + \cancel{S\nabla T}$$

The core hydrodynamics

The system of hydrodynamical equations

$$\partial_t \vec{v} + (\vec{v}\cdot\nabla)\vec{v} + \nabla P/\rho + \nabla\Phi = \vec{f}_{int} + \vec{f}_{EM} + \vec{f}_{visc}$$

$$\partial_t \rho + \mathrm{div}(\rho\vec{v}) = 0,$$

$$\Phi(\vec{r}) = -G\int \frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|} d^3 r',$$

Co-rotating frame of reference $(\vec{v} = [\vec{\Omega}\times\vec{r}] + \vec{u})$

$$\partial_t^* \vec{u} + \underline{2[\vec{\Omega}\times\vec{u}]} + \cancel{(\vec{u}\cdot\nabla)\vec{u}} + \nabla P/\rho + \nabla\left(\Phi - 1/2[\vec{\Omega}\times\vec{r}]^2\right) = -\underline{\underline{[\dot{\vec{\Omega}}\times\vec{r}]}} + \vec{f}_{int} + \vec{f}_{EM} + \vec{f}_{visc}$$

$$\partial_t^* \rho + \mathrm{div}(\rho\vec{u}) = 0$$

1. Small perturbations
2. Isothermality and strong degeneracy of core matter

$$\nabla P = \sum_\beta \rho_\beta \nabla\mu_\beta + \cancel{S\nabla T}$$

3. Quasi-stationarity $(\tau_{rel} \ll \tau_{ev})$.

The core hydrodynamics

The system of hydrodynamical equations

$$\partial_t \vec{v} + (\vec{v}\cdot\nabla)\vec{v} + \nabla P/\rho + \nabla\Phi = \vec{f}_{int} + \vec{f}_{EM} + \vec{f}_{visc}$$

$$\partial_t \rho + \mathrm{div}(\rho\vec{v}) = 0,$$

$$\Phi(\vec{r}) = -G\int \frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|}d^3r',$$

Co-rotating frame of reference $(\vec{v} = [\vec{\Omega}\times\vec{r}] + \vec{u})$

$$\cancel{\partial_t^* \vec{u}} + \underline{2[\vec{\Omega}\times\vec{u}]} + \cancel{(\vec{u}\cdot\nabla)\vec{u}} + \nabla P/\rho + \nabla\left(\Phi - 1/2[\vec{\Omega}\times\vec{r}]^2\right) = -\underline{\underline{[\dot{\vec{\Omega}}\times\vec{r}]}} + \vec{f}_{int} + \vec{f}_{EM} + \vec{f}_{visc}$$

$$\cancel{\partial_t^* \rho} + \mathrm{div}(\rho\vec{u}) = 0$$

1. Small perturbations
2. Isothermality and strong degeneracy of core matter

$$\nabla P = \sum_\beta \rho_\beta \nabla\mu_\beta + \cancel{S\nabla T}$$

3. Quasi-stationarity $(\tau_{rel} \ll \tau_{ev})$.

Spin evolution

$$I_{crust}\dot{\vec{\Omega}} = \vec{K} + \vec{N}$$

$$2[\vec{\Omega} \times \vec{u}] + \sum_{\beta} \frac{\rho_{\beta}^{(0)}}{\rho^{(0)}} \nabla \mu_{\beta}^{(1)} + \nabla \Phi^{(1)} = -[\dot{\vec{\Omega}} \times \vec{r}] + \vec{f}_{int}^{(1)} + \vec{f}_{EM}^{(1)} + \vec{f}_{visc}^{(1)}$$

$$\mathrm{div}(\rho^{(0)} \vec{u}) = 0$$

+boundary conditions

Interaction torque

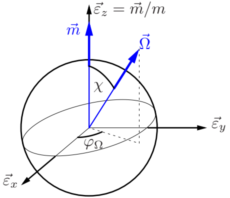
Quasi-stationarity $\Rightarrow \vec{N} = -S_1 I_{core} \vec{e}_z (\vec{e}_\Omega \cdot \dot{\vec{\Omega}}) - S_2 I_{core} \vec{e}_\Omega \times [\vec{e}_\Omega \times \dot{\vec{\Omega}}] + S_3 I_{core} [\vec{e}_\Omega \times \dot{\vec{\Omega}}]$
$\Rightarrow S_1 = 1$

Spin evolution

$$\dot{\Omega} = \frac{K_\Omega + K_m \cos\chi}{I_{core} + I_{crust}},$$

$$\dot{\chi} = -\frac{1}{\Omega} \frac{(I_{crust} - S_2 I_{core}) K_m - S_3 I_{core} K_\perp}{(I_{crust} - S_2 I_{core})^2 + S_3^2 I_{core}^2} \sin\chi,$$

$$\dot{\varphi}_\Omega = -\frac{1}{\Omega} \frac{(I_{crust} - S_2 I_{core}) K_\perp + S_3 I_{core} K_m}{(I_{crust} - S_2 I_{core})^2 + S_3^2 I_{core}^2}.$$

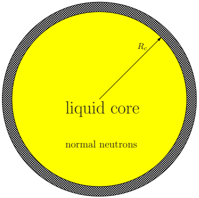


Model 1: Weak coupling

- There is no magnetic field inside the core;
- The crust-core interaction occurs through the friction force;
- npe-matter.

Normal neutrons

- Two components
  1. Neutron component (ρ_n, μ_n)
  2. Charged component ($\rho_c = \rho_p + \rho_e$, $\mu_c = \frac{\rho_p \mu_p + \rho_e \mu_e}{\rho_p + \rho_e}$)
- One velocity field $\vec{u}$.



$$2[\vec{\Omega} \times \vec{u}] + \nabla h + \frac{\rho_c^{(0)}}{\rho^{(0)}} \nabla(\mu_c^{(1)} - \mu_n^{(1)}) = -[\dot{\vec{\Omega}} \times \vec{r}] + \nu^{(0)} \Delta \vec{u},$$

$$\mathrm{div}\left(\rho_n^{(0)} \vec{u}\right) = \lambda_n^{(0)} \left(\mu_c^{(1)} - \mu_n^{(1)}\right),$$

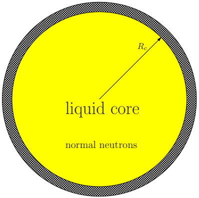
$$\mathrm{div}\left(\rho_c^{(0)} \vec{u}\right) = -\lambda_n^{(0)} \left(\mu_c^{(1)} - \mu_n^{(1)}\right),$$

$$[\vec{u}]_{r=R_c} = 0$$

Here, $h = \mu_n^{(1)} + \Phi^{(1)}$.

Model 1: Weak coupling

- There is no magnetic field inside the core;
- The crust-core interaction occurs through the friction force;
- npe-matter.



Normal neutrons

- Two components
  1. Neutron component (ρ_n, μ_n)
  2. Charged component ($\rho_c = \rho_p + \rho_e$, $\mu_c = \frac{\rho_p \mu_p + \rho_e \mu_e}{\rho_p + \rho_e}$)
- One velocity field $\vec{u}$.

$$2[\vec{\Omega} \times \vec{u}] + \nabla h + \frac{\rho_c^{(0)}}{\rho^{(0)}} \nabla(\mu_c^{(1)} - \mu_n^{(1)}) = -[\dot{\vec{\Omega}} \times \vec{r}] + \nu^{(0)} \Delta \vec{u},$$

$$\mathrm{div}\left(\rho_n^{(0)} \vec{u}\right) = \lambda_n^{(0)} \left(\mu_c^{(1)} - \mu_n^{(1)}\right), \quad \approx 0$$

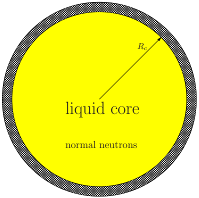
$$\mathrm{div}\left(\rho_c^{(0)} \vec{u}\right) = -\lambda_n^{(0)} \left(\mu_c^{(1)} - \mu_n^{(1)}\right), \quad \approx 0$$

$$[\vec{u}]_{r=R_c} = 0$$

Here, $h = \mu_n^{(1)} + \Phi^{(1)}$.

Model 1: Weak coupling

- There is no magnetic field inside the core;
- The crust-core interaction occurs through the friction force;
- npe-matter.



Normal neutrons

- Two components
  1. Neutron component ($\rho_n,\ \mu_n$)
  2. Charged component ($\rho_c = \rho_p + \rho_e,\ \mu_c = \frac{\rho_p\mu_p+\rho_e\mu_e}{\rho_p+\rho_e}$)
- One velocity field $\vec{u}$.

$$2[\vec{\Omega}\times\vec{u}] + \nabla h + \frac{\rho_c^{(0)}}{\rho^{(0)}}\nabla(\mu_c^{(1)} - \mu_n^{(1)}) = -[\dot{\vec{\Omega}}\times\vec{r}] + \nu^{(0)}\Delta\vec{u},$$

$$u_r \frac{d}{dr}\frac{\rho_n^{(0)}}{\rho^{(0)}} \approx 0$$

$$\mathrm{div}\left(\rho_n^{(0)}\vec{u}\right) = \lambda_n^{(0)}\left(\mu_c^{(1)} - \mu_n^{(1)}\right), \quad \approx 0$$

$$\mathrm{div}\left(\rho_c^{(0)}\vec{u}\right) = -\lambda_n^{(0)}\left(\mu_c^{(1)} - \mu_n^{(1)}\right), \quad \approx 0$$

$$[\vec{u}]_{r=R_c} = 0$$

$\Rightarrow$

Radial flows are strongly damped by composition gradient ($u_r \ll u_\theta,\ u_\phi$)
Until

$$\frac{\Omega\lambda_n^{(0)}}{\rho^{(0)}}\left[\frac{dy^{(0)}}{dr}\right]^{-2} \ll 1$$

Here, $h = \mu_n^{(1)} + \Phi^{(1)}$.

Model 1: Weak coupling

Core rotation

Vector $\dot{\vec{\Omega}}$ can be expanded as $\dot{\vec{\Omega}} = \dot{\vec{\Omega}}_{||} + \dot{\vec{\Omega}}_{\perp}$, where $\dot{\vec{\Omega}}_{||} = \vec{e}_{\Omega}(\vec{e}_{\Omega} \cdot \dot{\vec{\Omega}})$, $\dot{\vec{\Omega}}_{\perp} = \dot{\vec{\Omega}} - \dot{\vec{\Omega}}_{||}$.
$\Rightarrow \ \vec{u} = \vec{u}_{||} + \vec{u}_{\perp}$

1. **"Parallel" flow**

$$\vec{u}_{||} = -\vec{e}_{\phi} \frac{1}{10} \frac{\dot{\Omega}_{||}}{\Omega} E^{-1} \left(1 - \frac{r^2}{R_c^2}\right) r \sin\theta .$$

   Here, $E = \nu^{(0)}/\Omega R_s^2$ is the Ekman number. For neutron star matter $E \ll 1$.

2. **"Perpendicular" flow**

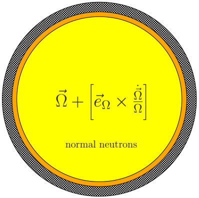
$$\vec{u}_{\perp} \approx \frac{\dot{\Omega}_{\perp}}{\Omega} Re\left\{ U(r)(\vec{e}_{\phi}\cos\theta - i\vec{e}_{\theta})e^{i\phi} \right\}, \quad U(r) = iR_c\left(\frac{r}{R_c} - \frac{j_1(kr)}{j_1(kR_c)}\right),$$

$$j_1(x) = \frac{1}{x}\left(\frac{\sin(x)}{x} - \cos(x)\right), \quad k = \frac{1+i}{\sqrt{2}} \frac{E^{-1/2}}{R_c}$$

$$\vec{u}_{\perp} = \left[\vec{e}_{\Omega} \times \frac{\dot{\vec{\Omega}}}{\Omega}\right] \times \vec{r}.$$

In almost the entire volume of the star core the flow has the form of a rigid rotation

Adjustment occurs only in the thin layer $\sim E^{1/2} R_c$ near the crust-core boundary.



Model 1: Weak coupling

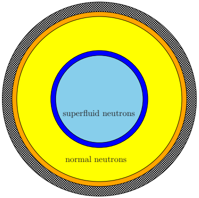
Interaction torque

$$\vec{N} = \oint_{r=R_c} \pi^{ij}[\vec{r} \times \vec{e}_i](\vec{e}_j \cdot \vec{e}_r)dS = -S_1 I_{core}\vec{e}_z(\vec{e}_\Omega \cdot \dot{\vec{\Omega}}) - S_2 I_{core}\vec{e}_\Omega \times [\vec{e}_\Omega \times \dot{\vec{\Omega}}] + S_3 I_{core}[\vec{e}_\Omega \times \dot{\vec{\Omega}}]$$

$$\vec{u}_{||} \;\Rightarrow\; S_1 = 1, \qquad \vec{u}_\perp \;\Rightarrow\; S_2 = S_3 = \frac{8\pi R_c^5}{3 I_{core}} E^{1/2}$$

Neutron superfluidity

- Additional velocity field $\vec{u}_s$. Radial flows can exist (Ekman pumping);
- "Parallel" flow always gives $S_1 = 1$;
- "Perpendicular" flow doesn't feel the presence of superfluid core.

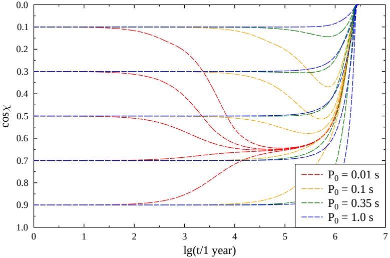


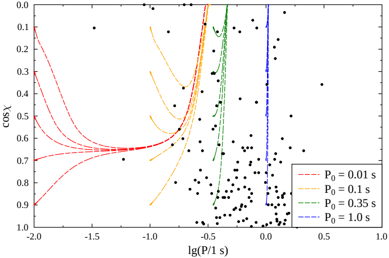
Inclination angle evolution

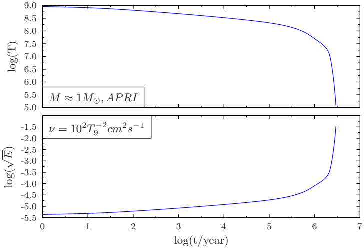
$$\dot{\chi} = -\frac{1}{\Omega}\frac{(I_{crust} - S_2 I_{core})K_m - S_3 I_{core} K_\perp}{(I_{crust} - S_2 I_{core})^2 + S_3^2 I_{core}^2}\sin\chi$$

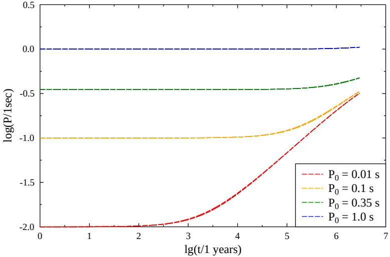
- $K_\perp \sim P$ – grows with P;
- $S_3 \sim \nu^{0.5} P^{0.5}$ – grows with P and with time.

Trajectories ($I_{crust}/I_{core} = 0.1$, $B_{nondip}/B_{dip} = 0.5$)





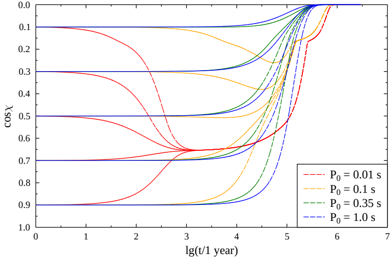


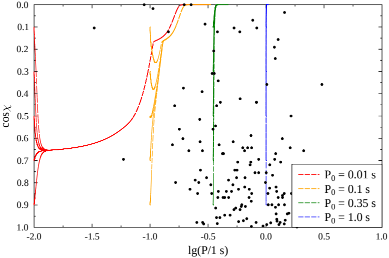


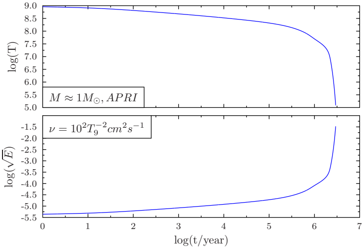
Observational data: J. M. Rankin, *ApJS* **85**, 145 (1993);

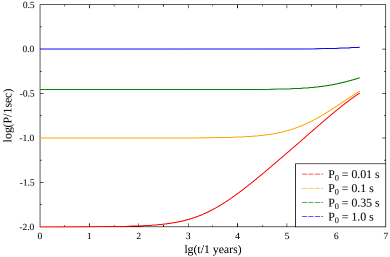
Cooling code: O. Y. Gnedin, D. G. Yakovlev, A. Y. Potekhin, *MNRAS* **324**, 725 (2001)

Trajectories ($I_{crust}/I_{core} = 0.01$, $B_{nondip}/B_{dip} = 0.5$)









Observational data: J. M. Rankin, *ApJS* **85**, 145 (1993);

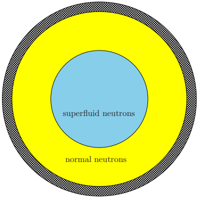
Cooling code: O. Y. Gnedin, D. G. Yakovlev, A. Y. Potekhin, *MNRAS* **324**, 725 (2001)

Model 2: Strong coupling

- Charged component is strongly coupled and rotates rigidly with the crust($\vec{u}_c \approx 0$);
- The is a neutron superfluid core interacting with the charged component through the mutual friction

$$\vec{f}_{int} = \beta' \vec{\omega}_s \times (\vec{u}_s - \vec{u}_c) + \beta \vec{e}_s \times [\vec{\omega}_n \times (\vec{u}_s - \vec{u}_c)] \,,$$

$$\vec{\omega}_s = 2\vec{\Omega} + \mathrm{rot}\, \vec{u}_s , \quad \vec{e}_k = \vec{\omega}_s / \omega_s$$



Equations

Linearized system of equations

$$-2\Omega\beta \vec{e}_\Omega \times [\vec{e}_\Omega \times \vec{u}_s] + 2\Omega(1+\beta')\,[\vec{e}_\Omega \times \vec{u}_s] + \nabla h = -[\dot{\vec{\Omega}} \times \vec{r}]$$

$$\mathrm{div}\, \rho_n^{(0)} \vec{u}_s = 0$$

$$\left[\rho_n^{(0)} \vec{e}_r \cdot \vec{u}_s\right]_{r=R_s} = 0$$

Here, $h = \mu_n^{(1)} + \Phi^{(1)}$.

Model 2: Strong coupling

Velocity field

$$\vec{u}_s = [\vec{\varpi} \times \vec{r}] - \frac{\dot{\Omega}_{||}}{2\Omega}\frac{\gamma - \beta\psi}{\gamma^2 + \beta^2}\tilde{r} + \vec{e}_z \frac{1}{\rho_s}\frac{\dot{\Omega}_{||}}{2\Omega}\int\limits_0^z \frac{1}{\tilde{r}}\frac{\partial}{\partial \tilde{r}}\left(\tilde{r}^2 \rho_s \frac{\gamma - \beta\psi}{\gamma^2 + \beta^2}\right) dz',$$

$$\vec{\varpi} = -\frac{\dot{\vec{\Omega}}_{||}}{2\Omega}\frac{\beta + \gamma\psi}{\gamma^2 + \beta^2} - \frac{\beta}{\gamma^2 + \beta^2}\frac{\dot{\vec{\Omega}}_{\perp}}{\Omega} + \frac{\gamma}{\gamma^2 + \beta^2}\left[\vec{e}_z \times \frac{\dot{\vec{\Omega}}_{\perp}}{\Omega}\right],$$

$$\psi(\tilde{r}) = \left[\int_0^{z_b} \frac{\rho_s \gamma}{\gamma^2 + \beta^2} dz'\right]\left[\int\limits_0^{z_b} \frac{\rho_s \beta}{\gamma^2 + \beta^2} dz'\right]^{-1}, \quad z_b = \sqrt{r_s^2 - \tilde{r}^2}.$$

Interaction torque

$$\vec{N} = -S_1 I_{core} \vec{e}_z (\vec{e}_\Omega \cdot \dot{\vec{\Omega}}) - S_2 I_{core} \vec{e}_\Omega \times [\vec{e}_\Omega \times \dot{\vec{\Omega}}] + S_3 I_{core} [\vec{e}_\Omega \times \dot{\vec{\Omega}}]$$

$$\vec{u}_{s||} \Rightarrow S_1 = 1, \qquad \vec{u}_{s\perp} \Rightarrow$$

$$S_2 = -\frac{I_c}{I_{core}} + \frac{8\pi}{3 I_{core}}\int_0^{R_s} \frac{\beta^2 - (1 - \beta')\beta'}{(1 - \beta')^2 + \beta^2}\rho_s r^4 dr, \quad S_3 = \frac{8\pi}{3 I_{core}}\int_0^{R_s} \frac{\beta}{(1 - \beta')^2 + \beta^2}\rho_s r^4 dr$$

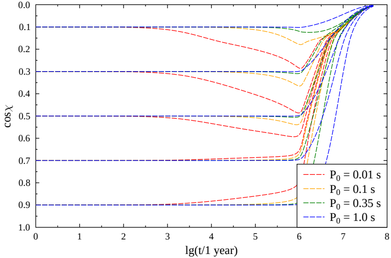
Mutual friction coefficients from M. A. Alpar, S. A. Langer, J. A. Sauls, *ApJ* **282**, 533 (1984)

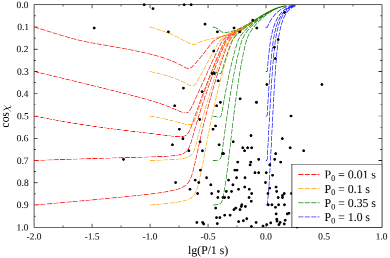
Inclination angle evolution

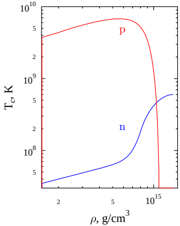
$$\dot{\chi} = -\frac{1}{\Omega}\frac{(I_{crust} - S_2 I_{core})K_m - S_3 I_{core} K_\perp}{(I_{crust} - S_2 I_{core})^2 + S_3^2 I_{core}^2}\sin\chi$$

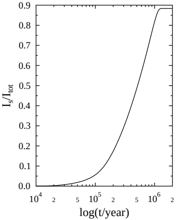
- $K_\perp \sim P$ – grows with P;
- I_s – grows with time.

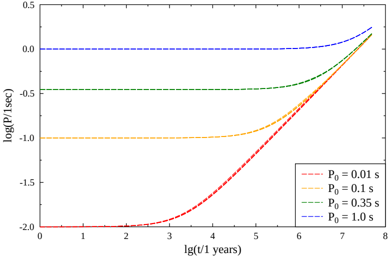
Trajectories







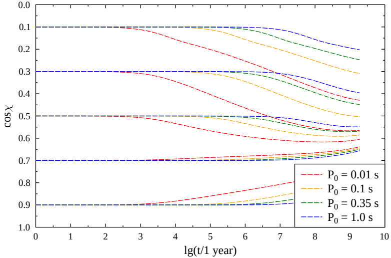


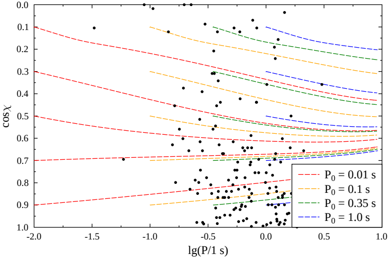


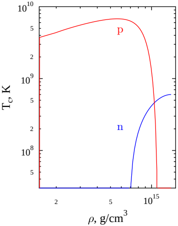
Observational data: J. M. Rankin, *ApJS* **85**, 145 (1993);

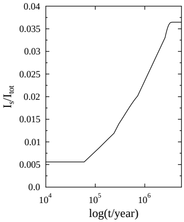
Cooling code: O. Y. Gnedin, D. G. Yakovlev, A. Y. Potekhin, *MNRAS* **324**, 725 (2001)

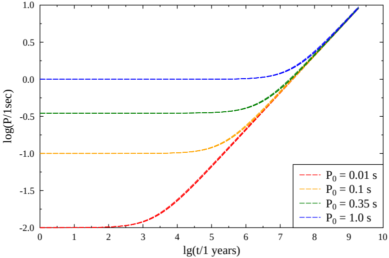
Trajectories











Observational data: J. M. Rankin, *ApJS* **85**, 145 (1993);

Cooling code: O. Y. Gnedin, D. G. Yakovlev, A. Y. Potekhin, *MNRAS* **324**, 725 (2001)

Conclusions

1. Inclination angle evolution significantly depends on the interior structure of the neutron stars.
2. The effect depends on the amount of the non-rigid rotating matter and the mechanism of its interaction with the rest of the star.
3. Since rapid inclination angle evolution seems to contradict the observation data, the results probably may be used as an additional test for the neutron star core matter theories.

Thank you for attention!