MAGNETOSONIC WAVES IN THE CRUST OF A NEUTRON STAR

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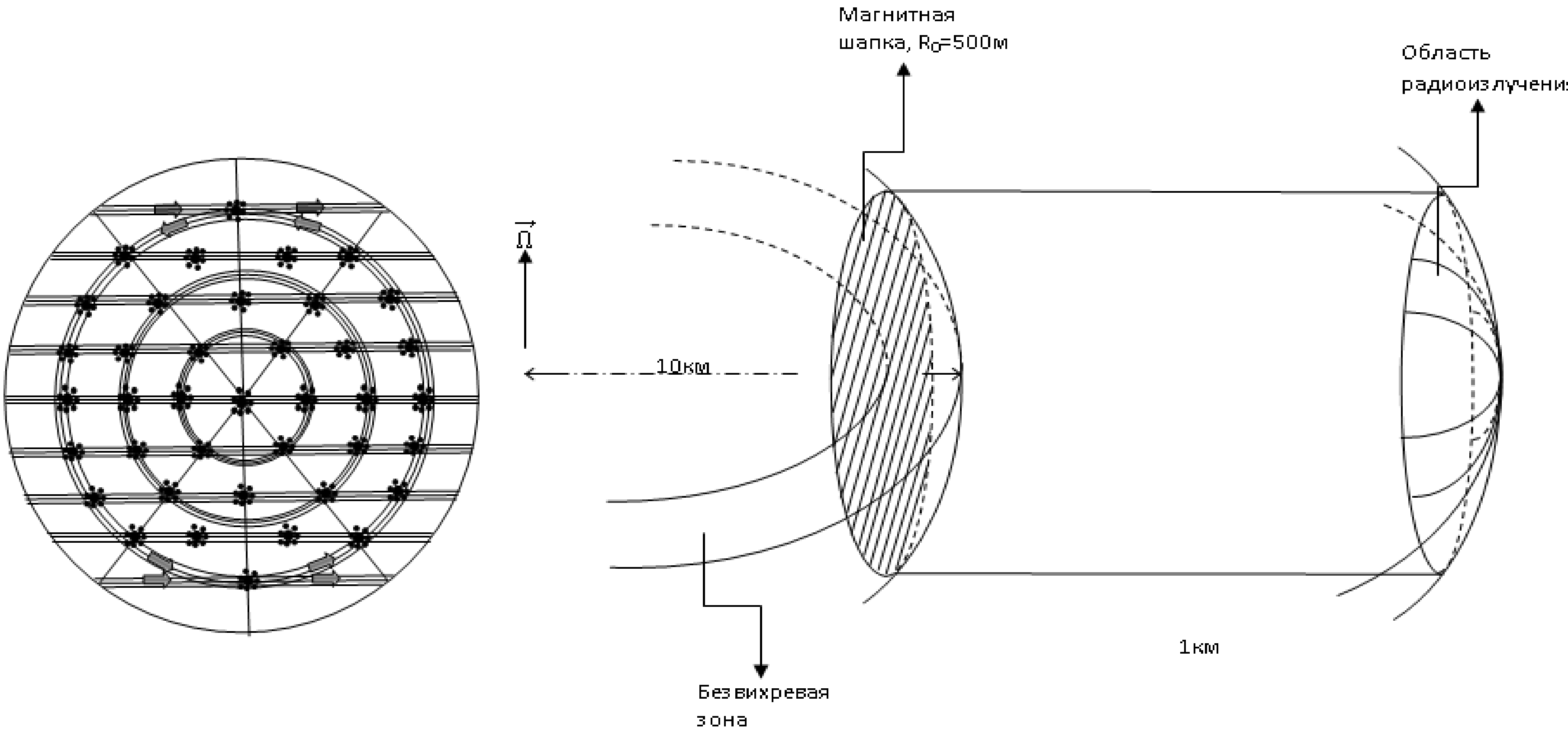
The equations of magnetohydrodynamics are used to show that the energy released at the inner surface of the crust of a neutron star generates magnetosonic wave beams that propagate to the star's surface. These equations can be linearized under the conditions of matter in the crust of neutron stars and for frequencies $10^7 \leq \omega \leq 10^{11}$Hz. *The solution has a form of the standing wave beam with a constant radius. The outer base of this beam on the star's surface is a source of radio emission. Electrical currents are excited in this source and it becomes an antenna that emits radio waves into the circumstellar space. The intensity of the radio emission decreases at higher frequencies, so that the spectrum of pulsars is bounded above* ($\omega \leq 10^{11}$Hz).

Asymmetric release of energy at the boundary of the superfluid core of a neutron star

- Sedrakian D. M.; Sedrakian, A. D. Sov. Phys. - JETP, Vol. 73, No. 2, p. 193 – 197, 1991
- Sedrakian, D. M.; Hayrapetyan, M. V. Astrophysics, Volume 55, Issue 3, pp.377-386, 2012

Vortex structure of the superfluid core of a neutron star is consederd, taking into account the residual (RP), generated poloidal (GP) and toroidal (TP) magnetic fields. The motion of the vortex structure to the boundary of the core and the crust during the spin-down of a star leads to the formation of two local "magnetic caps", where the magnetic field increases with time and reaches the value of second critical field. It is shown that the intensity of energy output in the "magnetic caps" due to the collapse of the magnetic field is of the order of the pulsars' radioluminosity.

Asymmetric release of energy at the boundary of the superfluid core of a neutron star



$$I = 10^{29} \bullet \left| \frac{\dot{\Omega}}{\Omega} \right|_{-15} R_6^3$$

MHD Equations

- $\frac{\partial\rho}{\partial t} + \nabla\left(\rho\vec{V}\right) = 0,$
- $\rho\frac{d\vec{V}}{dt} - \frac{1}{4\pi}\left(\vec{H}\nabla\right)\vec{H} = -\nabla\left(p + \frac{H^2}{8\pi}\right) + \eta\nabla^2\vec{V} + \left(\xi + \frac{\eta}{3}\right)\nabla\left(\nabla\vec{V}\right),$
- $\frac{d\vec{H}}{dt} - \left(\vec{H}\nabla\right)\vec{V} + \vec{H}\left(\nabla\vec{V}\right) = \nu_m\Delta\vec{H},$
- $\rho\frac{Tds}{dt} = -(\nabla\vec{q}) - \pi_{\alpha\beta}\frac{\partial V_\alpha}{\partial x_\beta} + \frac{j^2}{\sigma},$
- $\rho T\frac{ds}{dt} = \frac{1}{\gamma-1}\left(\frac{dp}{dt} - c_s^2\frac{d\rho}{dt}\right).$
- $\vec{H} = \vec{H}_0 + \vec{h},\ \rho = \rho_0 + \rho',\ p = p_0 + p'$

A wave beam in the crust of a neutron star

- we take the unknown function to be the velocity of matter, $V_x \equiv u$, then

$$u(x, y, z, t) = u_1(\tau_1, \tau_1', y, z) + u_2(\tau_2, \tau_2', y, z)$$

$$\tau_{1,2} = \int_{\pm x}^{l} \frac{dx}{c_n(x)} - t = \tau_{1,2}' - t$$

$$c_n^2 = c_s^2 + c_A^2, \qquad c_A^2 = \frac{H_0^2}{4\pi\rho}$$

A wave beam in the crust of a neutron star

- The density of matter in the crust of a neutron star is highly nonuniform, i.e., $\rho(x)$ varies over $x\in[0, l]$, but the magnetosonic wave speed $c_n(x)$ in the crust of a neutron star is almost constant . For the eikonals we can use the expressions

$$\tau'_{1,2} = \frac{l \mp x}{c_n} \quad \tau_{1,2} = \tau'_{1,2} - t$$

- equations for the unknowns $u_{1,2}$

$$\frac{\partial^2 u_{1,2}}{\partial \tau'_{1,2} \partial \tau_{1,2}} - \frac{1}{2} \hat{L} u_{1,2} - \frac{\partial u_{1,2}}{\partial \tau_{1,2}} \frac{d ln \Phi_{1,2}}{d \tau'_{1,2}} =$$

$$= -\frac{1}{c_n} \frac{\partial}{\partial \tau_{1,2}} \left(\Gamma u_{1,2} \frac{\partial u_{1,2}}{\partial \tau_{1,2}} + D \frac{\partial^2 u_{1,2}}{\partial \tau_{1,2}^2} \right)$$

A wave beam in the crust of a neutron star

$$\Gamma = \frac{\gamma+1}{2}\frac{c_s^2}{c_n^2} + \frac{3}{2}\frac{c_A^2}{c_n^2}$$

$$D = -\frac{1}{2c_n}\left\{\frac{1}{\rho}\left(\xi + \frac{4}{3}\eta\right) + \frac{c_A^2}{c_n^2}\upsilon_m + \frac{(\gamma-1)^2\kappa T}{\rho c_n^2}\right\}$$

$$\upsilon_m = \frac{c^2}{4\pi\sigma_0}$$

A wave beam in the crust of a neutron star

$$\hat{L} = -c_n^2\left[\left(1-\frac{c_A^2 c_s^2}{c_n^4}\right)\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right]$$

$$c_A^2 c_s^2 / c_n^4 << 1 \Longrightarrow \hat{L} = -c_n^2\left(\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right) = -c_n^2\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}\right)$$

The perturbations in the unknown physical quantities are related to V_x *by the following equations*

$$V_y = V_z = 0, \quad h_x = h_z = 0, \quad h_y = \mp\frac{H_0}{c_n}V_x\,, \quad \rho' = \mp\frac{\rho_0}{c_n}V_x\,, \quad p' = c_s^2\rho'$$

A wave beam in the crust of a neutron star

- The functions $\Phi_{1,2}$ in the evolution equation are related to the inhomogeneity of the medium and represent the one-dimensional ray solution in the direction perpendicular to the wave front. This solution is determined from the law of energy conservation, according to which the flux of energy of a one-dimensional wave is constant on planes perpendicular to the propagation direction of the wave.

$$\rho V_x^2 c_n = const = \rho(0) V_x^2(0) c_n(0),$$

$$\Phi_{1,2}^2 = \left(\frac{V_x(\tau'_{1,2})}{V_x(0)}\right)^2 = \frac{\rho(0) c_n(0)}{\rho(\tau'_{1,2}) c_n(\tau'_{1,2})} \approx \frac{\rho(0)}{\rho(\tau'_{1,2})}\,.$$

A wave beam in the crust of a neutron star

- $$\frac{\partial^2 u}{\partial\tau'\partial\tau}+\frac{c_n^2}{2}\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right)-\frac{\partial u}{\partial\tau}\frac{dln\Phi}{d\tau'}=-\frac{1}{c_n}\frac{\partial}{\partial\tau}\left(\Gamma u\frac{\partial u}{\partial\tau}+D\frac{\partial^2 u}{\partial\tau^2}\right)$$
$$u=\frac{1}{2}\left(\mathrm{v}_1 e^{i\omega\tau}+\mathrm{v}_2 e^{2i\omega\tau}\right)+\mathrm{c.\,c.}$$

- $$i\omega\frac{\partial \mathrm{v}_1}{\partial\tau'}+\frac{c_n^2}{2}\left(\frac{\partial^2 \mathrm{v}_1}{\partial r^2}+\frac{1}{r}\frac{\partial \mathrm{v}_1}{\partial r}\right)-i\omega\mathrm{v}_1\frac{dln\Phi}{d\tau'}=\frac{\Gamma\omega^2}{2c_n}\mathrm{v}_1^*\mathrm{v}_2+i\omega\mathrm{v}_1\frac{\omega^2 D}{c_n}$$

- $$2i\omega\frac{\partial \mathrm{v}_2}{\partial\tau'}+\frac{c_n^2}{2}\left(\frac{\partial^2 \mathrm{v}_2}{\partial r^2}+\frac{1}{r}\frac{\partial \mathrm{v}_2}{\partial r}\right)-2i\omega\mathrm{v}_2\frac{dln\Phi}{d\tau'}=\frac{\Gamma\omega^2}{c_n}\mathrm{v}_1^2+\mathrm{v}_2\frac{8i\omega^3 D}{c_n}$$

A wave beam in the crust of a neutron star

- $\frac{\omega|v_1|}{\tau'} \gg \frac{\Gamma\omega^2}{c_n}|v_1 v_2|$ ⟹ $|v_2| \ll \frac{c_n}{\tau'\Gamma\omega}$
- $\frac{\omega|v_2|}{\tau'} \sim \frac{\Gamma\omega^2|v_1^2|}{c_n}$ ⟹ $\left|\frac{v_2}{v_1}\right| \sim \frac{\Gamma\omega\tau'}{c_n}|v_1|$

⟹ $\left|\frac{v_2}{v_1}\right| << \left|\frac{v_1}{v_2}\right|$

$\left|\frac{v_2}{v_1}\right| \sim \frac{\Gamma\omega l}{c_n^2}|v_1| << 1$

- $i\omega\frac{\partial v_1}{\partial\tau'} + \frac{c_n^2}{2}\left(\frac{\partial^2 v_1}{\partial r^2} + \frac{1}{r}\frac{\partial v_1}{\partial r}\right) - i\omega v_1\frac{d\ln\widetilde{\Phi}}{d\tau'} = 0$

$\widetilde{\Phi}(\tau') = \Phi(\tau')exp\left(-\omega^2\int_0^{\tau'}\mu(\tau)\,d\tau\right) \qquad \mu(\tau) = \frac{D}{c_n}$

- $v_1 = \widetilde{\Phi}U$ ⟹ $i\omega\frac{\partial U}{\partial\tau'} + \frac{c_n^2}{2}\left(\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r}\right) = 0$

$$u_{1,2} = \frac{\Phi(\tau'_{1,2})b_{1,2}}{2f(\tau'_{1,2})}exp\left\{-\frac{r^2}{2r_0^2f^2(\tau'_{1,2})} + i\left[\sigma(\tau'_{1,2}) + \frac{r^2}{2Q(\tau'_{1,2})}\right] + i\omega\tau_{1,2} - \omega^2\int_0^{\tau'_{1,2}}\mu\,d\tau\right\} + c.c$$

A wave beam in the crust of a neutron star

- $\rho' \big|_{x=0} = \left(-\frac{\rho_0}{c_n} u_1 + \frac{\rho_0}{c_n} u_2\right) \big|_{x=0} = 0$
- $u_1(0) = u_2(0), \tau_1' \big|_{x=0} = \tau_2' \big|_{x=0} = l/c_n$
  $b_1 = b_2 \equiv b$
- $|u| =$
  $\frac{2\Phi(l/c_n)b}{f(l/c_n)} exp\left\{-\frac{r^2}{2r_0^2 f^2(l/c_n)} - \omega^2 \int_0^l \frac{\mu}{c_n} dx\right\}$
- $h_y = \frac{H_0}{c_n}(u_2 - u_1)\big|_{x=0} = 0$
- $j_z = \frac{c}{4\pi}\frac{\partial h_y}{\partial x} = \frac{c}{4\pi}\frac{H_0}{c_n}\left(\frac{\partial u_2}{\partial x} - \frac{\partial u_1}{\partial x}\right) \big|_{x=0}$

A wave beam in the crust of a neutron star

$$\frac{\partial u_{1,2}}{\partial x}\,\Big|_{x=0} = \mp \frac{i\omega b \Phi(l/c_n)}{2c_n f(l/c_n)}$$

$$exp\left\{-\frac{r^2}{2r_0^2 f^2(l/c_n)} + i\omega\left(\frac{l}{c_n} - t\right) - \omega^2 \int_0^l \frac{\mu}{c_n} dx + i\left(\sigma(l/c_n) + \frac{r^2}{2Q(l/c_n)}\right)\right\} + \text{c.c}$$

$$j_z(r,t) = \frac{cH_0}{4\pi}\frac{i\omega b \Phi(l/c_n)}{c_n^2 f(l/c_n)} \exp\left\{-\frac{r^2}{2r_0^2 f^2(l/c_n)} + i\omega\left(\frac{l}{c_n} - t\right) - \omega^2 \int_0^l \frac{\mu}{c_n} dx + i\left(\sigma(l/c_n) + \frac{r^2}{2Q(l/c_n)}\right)\right\} + \text{c. c.}$$

The coefficient before exponent is the main term of the exact solution

$$\frac{\partial u_{1,2}}{\partial x} = \frac{\partial u_{1,2}}{\partial \tau_{1,2}}\frac{\partial \tau_{1,2}}{\partial x} + \frac{\partial u_{1,2}}{\partial \tau'_{1,2}}\frac{\partial \tau'_{1,2}}{\partial x}$$
$$= \mp \frac{b}{2c_n f_{1,2}}\left\{i\omega \Phi_{1,2} + \Phi'_{1,2} - \frac{\Phi_{1,2} f'_{1,2}}{f_{1,2}} + \frac{\Phi_{1,2} f'_{1,2}}{r_0^2 f_{1,2}^3} r^2 + i\Phi_{1,2}\sigma'_{1,2} - \frac{i\Phi_{1,2} Q'_{1,2}}{2Q_{1,2}^2} r^2 - \Phi_{1,2}\nu\omega^2\right\} exp\left(-\frac{r^2}{2r_0^2 f_{1,2}^2} + i\left(\sigma_{1,2} + \frac{r^2}{2Q_{1,2}}\right) + i\omega\tau_{1,2} - \omega^2 \int_0^{\tau'} \mu\, d\tau\right) + c.c.$$

Estimating *b* from observations of the radioemission from pulsars

- $I_1 = \frac{2}{3c^3}\left|\ddot{\vec{d}}\right|^2$ $\qquad \dot{\vec{d}} = \sum_k e_k \vec{V}_k \approx \lambda^3 \sum_a e_a n_a \vec{V}_a = \vec{j}\lambda^3$
- $I_1 = \frac{2\lambda^6}{3c^3}\left|\dot{\vec{j}}\right|^2 \qquad N \sim \left(\frac{r_0}{\lambda}\right)^2 \quad \Longrightarrow \quad I = I_1 N = \frac{2}{3c^3}\left|\dot{\vec{j}}\right|^2 \lambda^4 r_0^2$

$$\left|\dot{\vec{j}}\right| = \frac{c}{4\pi}\frac{\omega^2 H_0}{c_n^2}\left|u(l/c_n)\right| \quad \Longrightarrow \quad I = \frac{2\pi^2 c}{3}\left(\frac{cH_0 r_0}{c_n^2}\right)^2 \left|u(l/c_n)\right|^2$$

$$\left|u(l/c_n)\right| \sim b\Phi(l/c_n) \sim 10^{-8} c_n$$

$$\omega \ll \frac{c_n^2}{\Gamma l|u|} \qquad \Gamma \sim 1 \qquad \omega \ll 10^{12}\,\mathrm{Hz} \qquad 10^7\,\mathrm{Hz} \le \omega \le 10^{11}\,\mathrm{Hz}$$

$$I \sim 10^{30}\,\mathrm{erg/s}$$