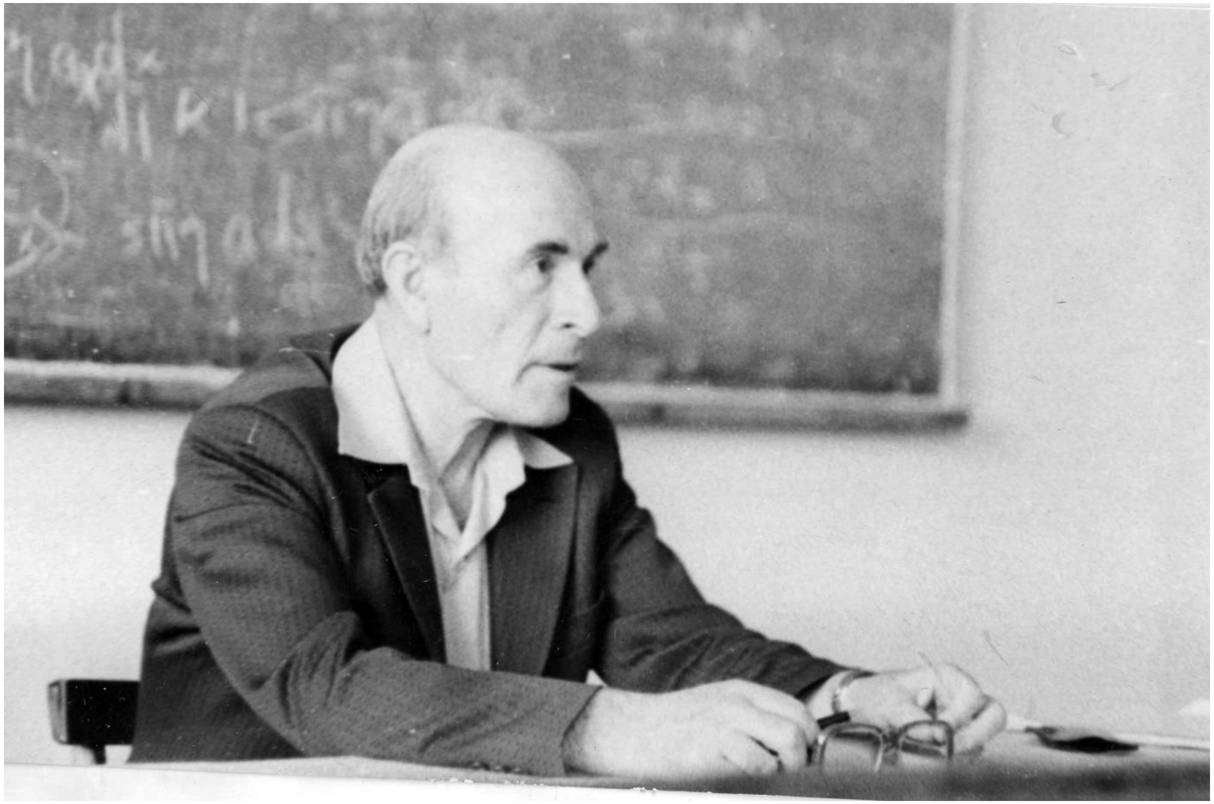
Dedicated to the bright memory of Prof. G.S. Sahakyan



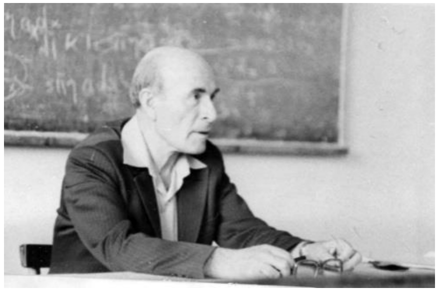
An Almost Einsteinian Theory of Gravitation

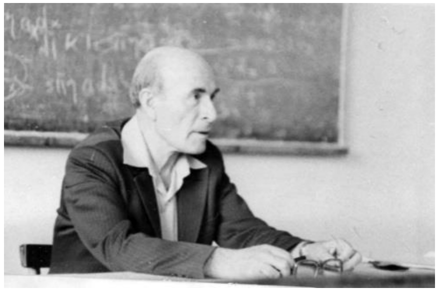
Levon Grigoryan

Institute of Applied Problems in Physics, Yerevan

Yerevan State University

Table of contents





1. Introduction

1.1. GR after 100 years since its creation

1

Some important conclusions of GR

(e.g., the black holes, gravitational waves)

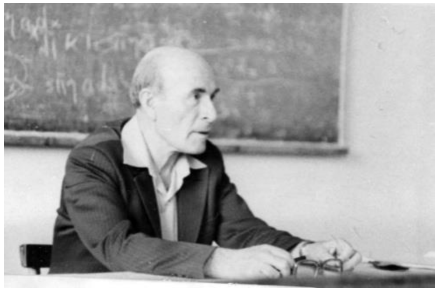
have not been directly confirmed by observational data.

2

There still exist some unresolved problems in GR

(e.g., the lack in the theory of the covariant energy-momentum tensor of the gravitational field).

GR after 100 years since its creation



3

Some important phenomena have been discovered

(e.g., accelerated expansion of Universe and

the associated problem of “dark energy”),

that had not found any consistent and non contradictory explanation in the frameworks of GR.

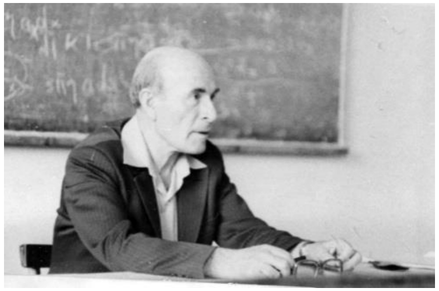
It is the desire to bypass the above difficulties

that justifies the efforts for development of more accurate,

improved

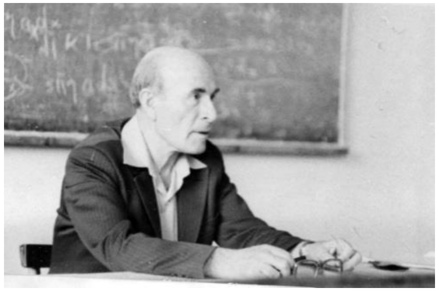
or, possibly, the modified GR.

1.2. The Alternative Theories of Gravitation



1
Scalar-tensor theories of gravitation, in which the gravitation «constant» is a function of space-time coordinates (e.g., the Jordan, Branse and Dikke theories, the theory of Sahakyan with co-authors).

2
Bimetric theories of gravitation (e.g., the Rosen's theory), in which along with the principal metric there is another («background») metric.



3

The theories in which the contributions of the

summands that are nonlinear in Riemann tensor,

its convolutions and covariant derivatives

are taken into account.

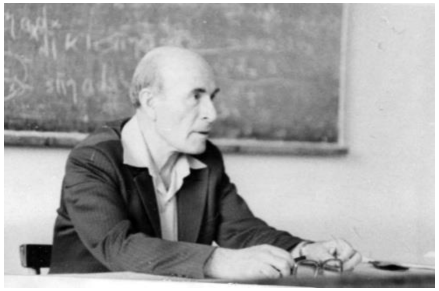
4.

Theories of gravitation, in which

the space-time is multi-dimensional.

5

Theories of gravitation with torsion.

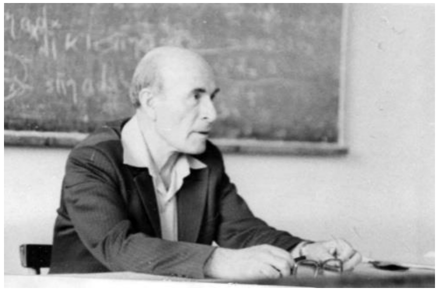


1.3. The aim of the report

In my report I would like to offer you a modified version of GR, in which

the General Relativity is supplemented with two postulates.

2. Tensor and Monometric Theory of Gravitation (TMTG)



2.1.The principal provisions of the proposed theory

2.1a. As in GR

1) In TMTG g_ik describes the gravitational field as well as also the effects connected with the choice of non-inertial reference system.

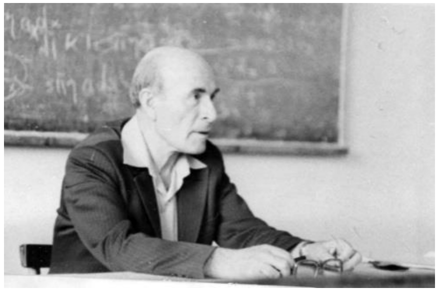
Thus, only a part of g_ik describes the gravitational field. If we denote this part of metric tensor as fi_ik, then

$$g^{ik} = g_*^{ik} + \varphi^{ik}$$

Here g*_ik *is not related to the gravitational field* (g*_ik is determined, e.g., by initial conditions of the problem under consideration and the choice of the reference frame).

$$g^{ik} = g_*^{ik} + \varphi^{ik}$$

As in GR



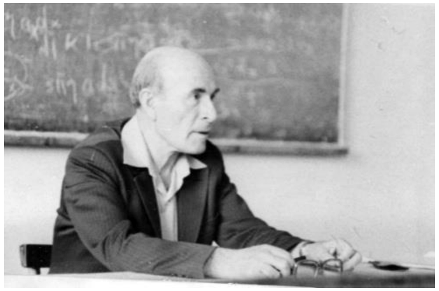
2) In TMTG the influence of gravitation on the matter and on non-gravitational fields is described by metrical tensor g_ik.

2.1в. Postulates of TMTG

In TMTG

(A) φ^{ik} is **a covariant symmetrical second rank tensor.**

$\Rightarrow g_*^{ik}$ is also a covariant tensor that does not change at variation of gravitational field action with respect to φ^{ik}

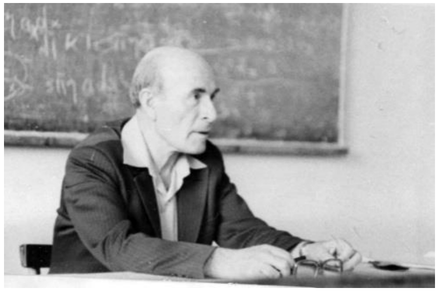


(B) the effect of gravitational field on the matter, non-gravitational fields and on itself is produced in the same way (via metric tensor g^{ik}).

For this reason the covariant density of gravitational field Lagrangian

$$L_g = L_g(g^{\alpha\beta}, \varphi^{\mu\nu}, \varphi^{\mu\nu}{}_{;\lambda})$$

(C) The influence of all the “remaining part of Universe” on the gravitating system under consideration is taken into account.



2.1c. Total Action of the System

$$S = S_g + S_M^* = c^{-1}\int \left(L_g + L_M^*\right)\sqrt{-g}\,d\Omega$$

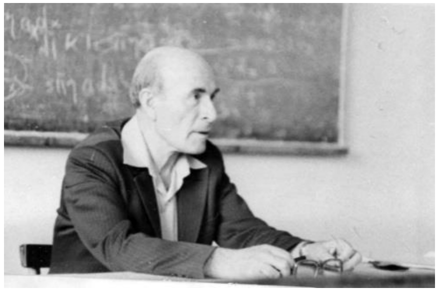
where L*_M is the covariant density of the Lagrangian of the matter and non-gravitation field.

2.2. Equations of the Gravitational Field

The equations of gravitational field are obtained from the principle of least total action: $S = S_g + S_M^*$

Subjected to variation is the gravitational field, i.e., the quantities φ^{ik}

It is important that $\delta g^{ik} = \delta\varphi^{ik}$ in virtue of $g^{ik} = g_*^{ik} + \varphi^{ik}$



From the condition

$$\delta S = \delta S_g + \delta S_M^* = 0$$

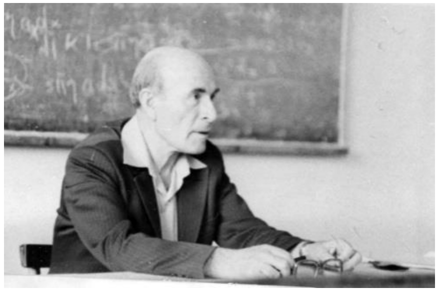
One obtains the following main equations of TMTG

$$G_{(ik)} = T_{(ik)}^{M*} + T_{(ik)}^{g*}$$

for the gravitational field φ^{ik}

T_{ik}^{M*} is the covariant energy-momentum tensor of the matter and of non-gravitational field: It is determined by the following well-known expression:

$$\delta \int L_M^* \sqrt{-g}\, d\Omega = 0.5 \int T_{(ik)}^{M*} \delta g^{ik} \sqrt{-g}\, d\Omega$$



$T^{g*}_{(ik)}$ Is determined by the expression:

$$\delta \int L_g(g^{\alpha\beta}, \varphi^{\mu\nu}, \varphi^{\mu\nu}{}_{;\lambda}) \sqrt{-g}\, d\Omega \equiv 0.5 \int T^{g*}_{(ik)} \delta g^{ik} \sqrt{-g}\, d\Omega$$

which is similar to that for T^{M*}_{ik}

$$G_{(ik)} = T^{M*}_{(ik)} + T^{g*}_{(ik)}$$

In this sense

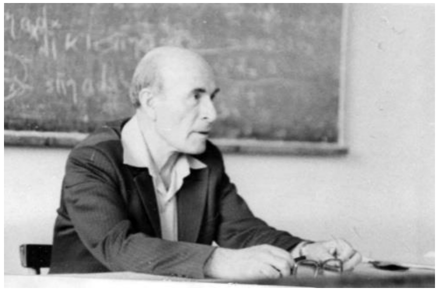
$T^{g*}_{(ik)}$ is a covariant tensor of the energy-momentum of the gravitational field.

Tensor G_ik is determined by a bit another expression

$$\delta \int L_g(g^{\alpha\beta}, \varphi^{\mu\nu}, \varphi^{\mu\nu}{}_{;\lambda}) \sqrt{-g}\, d\Omega \equiv -0.5 \int G_{(ik)} \delta \varphi^{ik} \sqrt{-g}\, d\Omega$$

$g^{\alpha\beta}$ is not subjected to variation.

2.3. Covariant conservation Laws



The action is a scalar and hence its variation must be zero: $\delta S_g = 0$ (1)

in case of transformation of coordinates $x'^i = x^i + \zeta^i(x^\mu)$

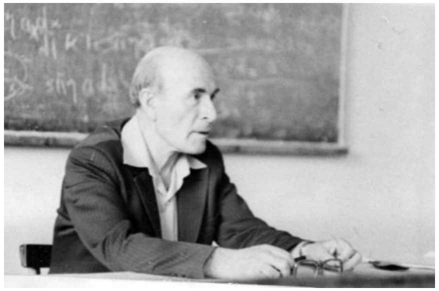
It follows from (1) in view of the arbitrariness of Ksi(x) the following differential covariant conservation law:

$$T^{g_*}_{(im);n} g^{mn} = \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^m} \sqrt{-g} G_{(in)} \varphi^{mn} + \frac{1}{2} G_{(mn)} \frac{\partial \varphi^{mn}}{\partial x^i} \quad (2)$$

One can write it in an explicitly covariant form:

$$[\eta (T^{M_*}_{(im)} + T^{g_*}_{(im)})]_{|n} \cdot g_*^{mn} = 0 \quad (3)$$

$$[\eta(T_{(im)}^{M_*} + T_{(im)}^{g_*})]_{|n} \cdot g_*^{mn} = 0$$



The vertical bar implies the operation of covariant differentiation with respect to the metrical tensor in the particular case, when in the space-time «the gravitational field is mentally switched off»:

$$\varphi^{ik} = 0$$

In that case

$$g^{ik} = g_*^{ik} + \varphi^{ik} = g_*^{ik}$$

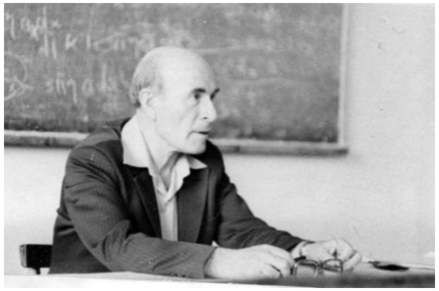
$$g^{ik} = g_*^{ik}$$

is so called

a “degenerate or naked, auxiliary” metric tensor.

Covariant components g_{*ik} are determined by usual way: $g_{*in} g_*^{nk} = \delta_i^k$

$$\eta = \sqrt{g / g_*}$$



2.5. Influence of the Universe

The total Lagrangian of matter and non-gravitational fields is

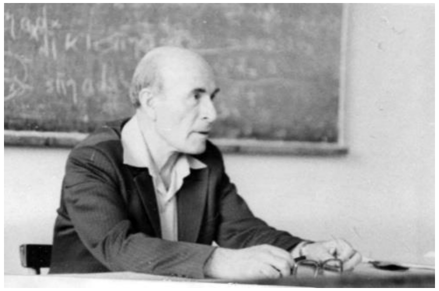
$$L_M^* = L_M(g^{\alpha\beta}; q, \partial q / x^\lambda) + L_{UN}(g^{\alpha\beta}; p, \partial p / \partial x^\lambda)$$

L_M describes the contribution of gravitating system under consideration.

L_UN describes the contribution of the «remaining part of Universe» (L_UN is a certain function of some quantities p describing the state of this part of Universe).

The «solutions of corresponding equations of motion»

$$\frac{\partial}{\partial x^\lambda}\frac{\partial\sqrt{-g}L_{UN}}{\partial p_{,\lambda}} - \frac{\partial\sqrt{-g}L_{UN}}{\partial p} = 0$$



The «solutions of corresponding equations of motion»

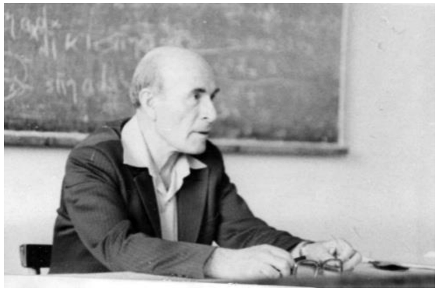
$$\frac{\partial}{\partial x^{\lambda}}\frac{\partial\sqrt{-g}L_{UN}}{\partial p_{,\lambda}}-\frac{\partial\sqrt{-g}L_{UN}}{\partial p}=0$$

depend on space-time coordinates and metric tensor $g^{\alpha\beta}$

$$p=p(g^{\alpha\beta},x^{\lambda})\quad\Rightarrow$$

$$L_{UN}[g^{\alpha\beta},x^{\lambda}]$$

$$L_M^* = L_M(g^{\alpha\beta}; q, \partial q / x^\lambda) + L_{UN}[g^{\alpha\beta}, x^\lambda]$$



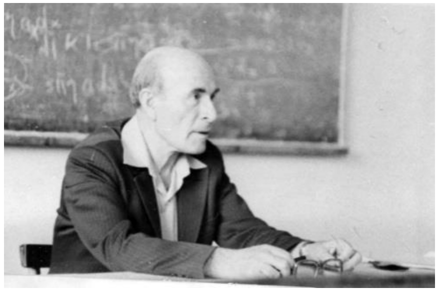
According to this equation the covariant energy-momentum tensor of the matter and of non-gravitational field splits into two summends:

$$T_{(ik)}^{M*} = T_{(ik)}^{M} + W_{(ik)}$$

$T_{(ik)}^{M}$ is the energy-momentum tensor of matter and non-gravitational fields for the gravitating system under consideration,

$W_{(ik)}$ is the tensor describing the contribution of the «remaining parts of Universe». It is determined by the equation

$$\delta \int L_{UN}[g^{\alpha\beta}, x^\lambda] \sqrt{-g}\, d\Omega = 0.5 \int W_{(ik)} \delta g^{ik} \sqrt{-g}\, d\Omega$$



Therefore the field equations of TMTG

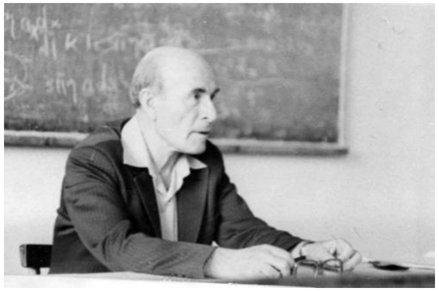
$$G_{(ik)} = T^{M_*}_{(ik)} + T^{g_*}_{(ik)}$$

one may rewrite in the following form

$$G_{(ik)} = T^{M}_{(ik)} + T^{g_*}_{(ik)} + W_{(ik)}$$

Were $W_{(ik)}$ is the tensor describing the contribution of the Universe.

3. The Simplest version of the Theory



3.1.

The simplest Lagrangian of the gravitational field

$$L_g = (a_1 \varphi^{im;r} \varphi^{kn;s} + a_2 \varphi^{im;r} \varphi^{ks;n}) g_{ik} g_{mn} g_{rs} +$$

$$a_3 \varphi_{,n} \varphi^{,n} + a_4 \varphi_{,n} \varphi^{nr}{}_{;r} +$$

$$L_g =$$

$$a_5 \varphi^{mr}{}_{;s} \varphi^{ns}{}_{;r} g_{mn} + a_6 \varphi^2 + a_7 \varphi^{im} \varphi^{kn} g_{ik} g_{mn}$$

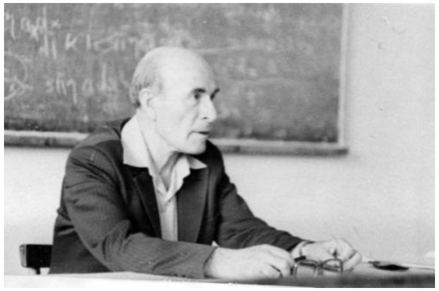
The simplest Lagrangian describing the contribution of the Universe

$$L_{UN} = b_1 + b_2 R$$

$$\varphi = g_{\varepsilon\beta} \varphi^{\alpha\beta}$$

Free parameters:

$$a_1 \div a_7 \qquad b_1, b_2$$



3.2. Partial Choice of the Values of Free Parameters of TMTG

If

$$a_1 = a_2 = a_3 = a_4 = a_5 = a_6 = a_7 = 0$$

$$b_1 = c^4 \Lambda / 8\pi G \qquad b_2 = -c^4 / 16\pi G \qquad (3)$$

Then

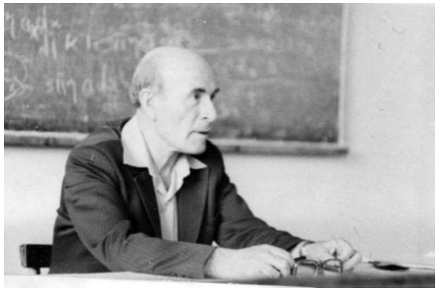
$$G_{(ik)} = T^{M}_{(ik)} + T^{g_*}_{(ik)} + W_{(ik)}$$

Is reduced to the Einstein equations with the cosmological term:

$$R_{ik} - 0.5 g_{ik} R + g_{ik} \Lambda = 8\pi G T^{M}_{(ik)} / c^4$$

In such case of the choice of the free parameters the problem of matching TMTG with observation data is reduced to the problem of matching GR with these data.

3.4. Comparison with Observation Data



$$a_1 = a_2 = a_3 = a_4 = a_5 = a_6 = a_7 = 0$$

(3)

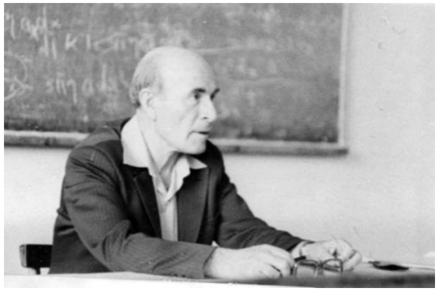
$$b_1 = c^4 \Lambda / 8\pi G \qquad b_2 = -c^4 / 16\pi G$$

When a_i and b_i insignificantly differ from (3)

one can consider TMTG as an «almost Einsteinian Theory of Gravitation».

The problem consists only in the following:

To what extent These insignificant differences from (3) are essential for astrophysical applications of TMTG?



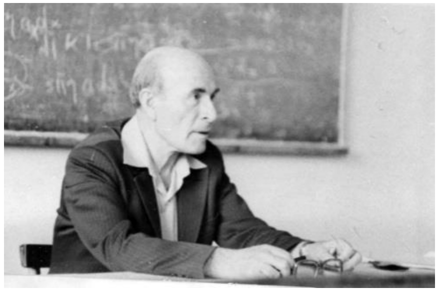
$$+a_6\varphi^2 + a_7\varphi^{im}\varphi^{kn}g_{ik}g_{mn}$$

The question is not idle since a_6 and a_7 determine the energy of gravitational field that is constant in time and in space.

The difference of a_6 and a_7 from zero may be crucial for models of Universe within the frameworks of TMTG.

In future it is supposed to determine the PPN parameters of TMTG to afford the determination of the limits of permissible values of a_6 and a_7.

3.4. Comparison with alternative theories of gravitation



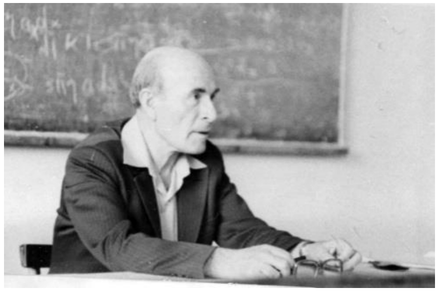
In bimetric theories of gravitation

1) the secondary («background») metric of space-time is introduced

2) the operation of covariant differentiation with respect to this background metric is based on the determination of gravitational field action.

In TMTG the «supplementary metrical tensor» is also introduced into consideration. However, in contrast to biometric theories, in the action of gravitational field the operation of covariant differentiation with respect to this «supplementary metrical tensor» is not generally used.

Comparison with alternative theories of gravitation

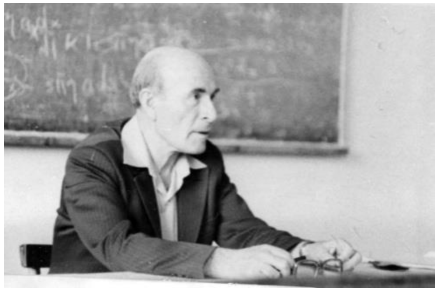


In TMTG it is not assumed that:

1) the gravitational constant is a variable quantity

2) the multidimensional space-time is not subject to consideration.

4. Conclusions

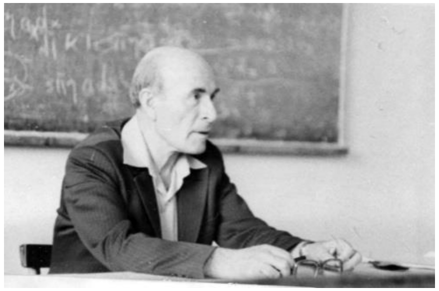


In this report a modified version of GR, named the Tensor Monometric Theory of Gravitation (TMTG), is proposed, in which the GR is supplemented with two postulates.

It is assumed that:

(A) the part of metrical tensor describing the gravitational field, is a covariant tensor.

(B) the gravitational field acts on the matter, on non-gravitational fields and on itself in a similar way (through the metric tensor g_ik).

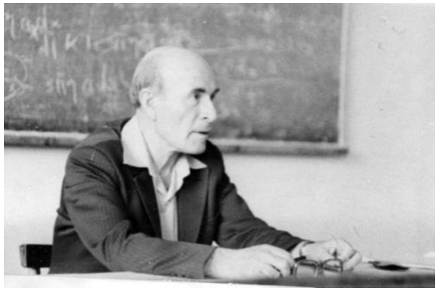


These postulates allow one to introduce

a covariant symmetrical energy-momentum tensor
for the gravitational field,
that is defined in the same way as that for the matter and non-gravitational fields.

In addition in TMTG

(C) The influence of the Universe
on the gravitating system under consideration
is taking into account.



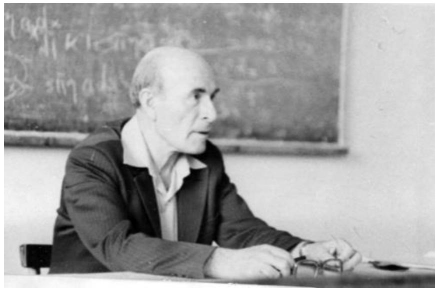
Further:

the equations describing the gravitational field in the framework of TMTG are derived and

the covariant differential law of conservation stemming from an invariance of gravitational field action with respect to transformation of four-dimensional coordinates of space-time is obtained.

This conservation law represents the conservation of the total energy-momentum tensor of the matter and of all fields

including the gravitational field.



$$+a_6\varphi^2 + a_7\varphi^{im}\varphi^{kn}g_{ik}g_{mn}$$

Two of nine free parameters of TMTG:

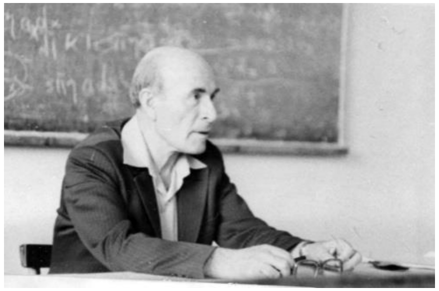
specify the energy of constant (in time and space) gravitational field, and, hence,

may be crucial for models of Universe in the framework of TMTG.

Thank you indeed for your attention

Levon Grigoryan

levonshg@mail.ru



$$\delta\int L_g(g^{\alpha\beta},\varphi^{\mu\nu},\varphi^{\mu\nu}{}_{;\lambda})\sqrt{-g}\,d\Omega \equiv -0.5\int G_{(ik)}\delta\varphi^{ik}\sqrt{-g}\,d\Omega$$

$g^{\alpha\beta}$ Is not subjected to variation

In GR

$$\delta\int L_g^{GR}(g^{\alpha\beta})\sqrt{-g}\,d\Omega \equiv 0.5\int G_{(ik)}^{GR}\delta\varphi^{ik}\sqrt{-g}\,d\Omega$$

$$L_g^{GR} = -\frac{c^4}{16\pi G}R$$

$$\delta\int L_g^{GR}(g^{\alpha\beta})\sqrt{-g}\,d\Omega = 0$$

$$G_{(ik)}^{GR} \equiv 0$$

In TMTG $G_{(ik)} \neq 0$

$$G_{(ik)} = T_{(ik)}^{M*} + T_{(ik)}^{g*}$$