Symmetry energy in the neutron star equation of state and astrophysical observations

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comp star

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The Modern Physics of Compact Stars and Relativistic Gravity

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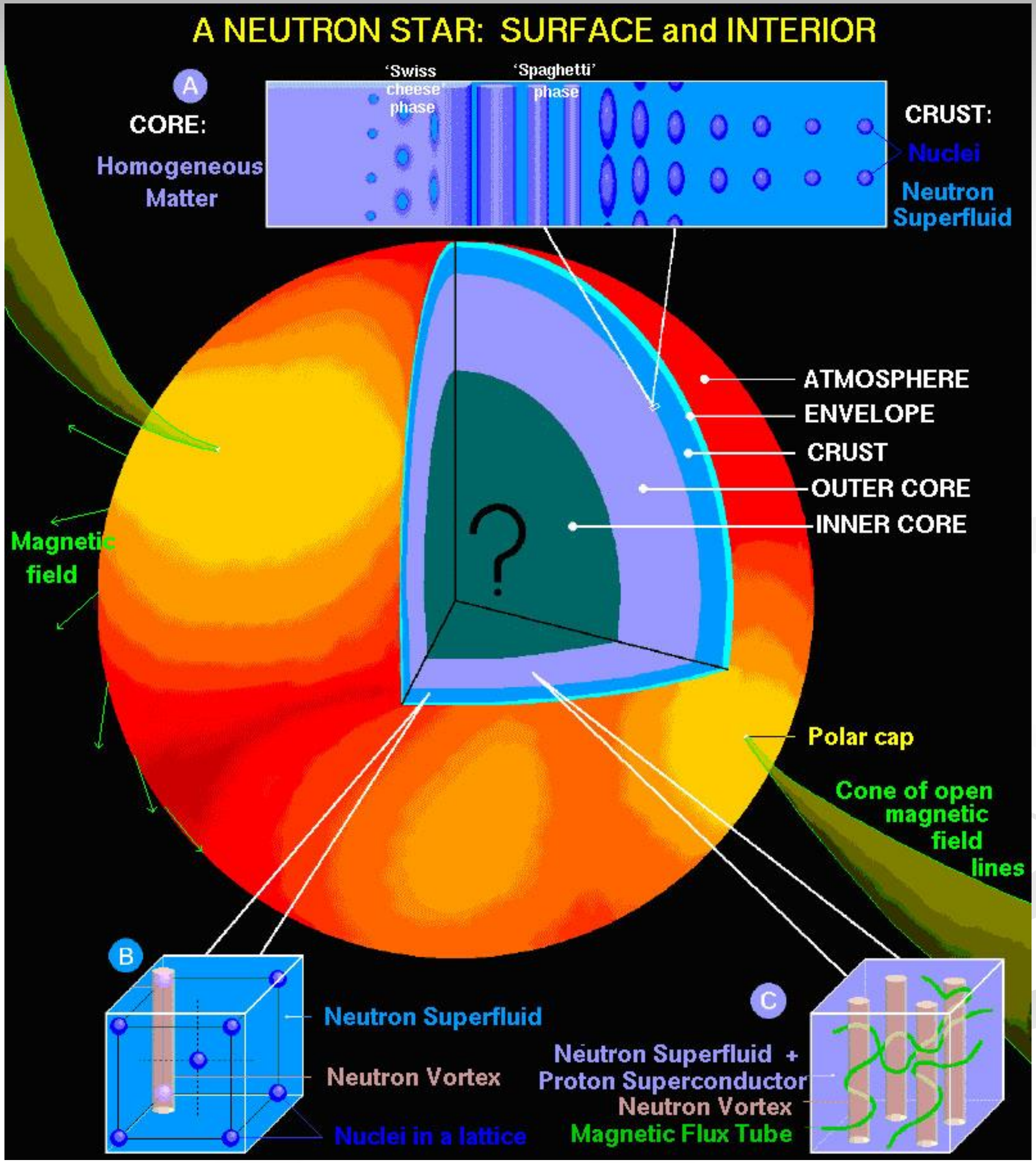
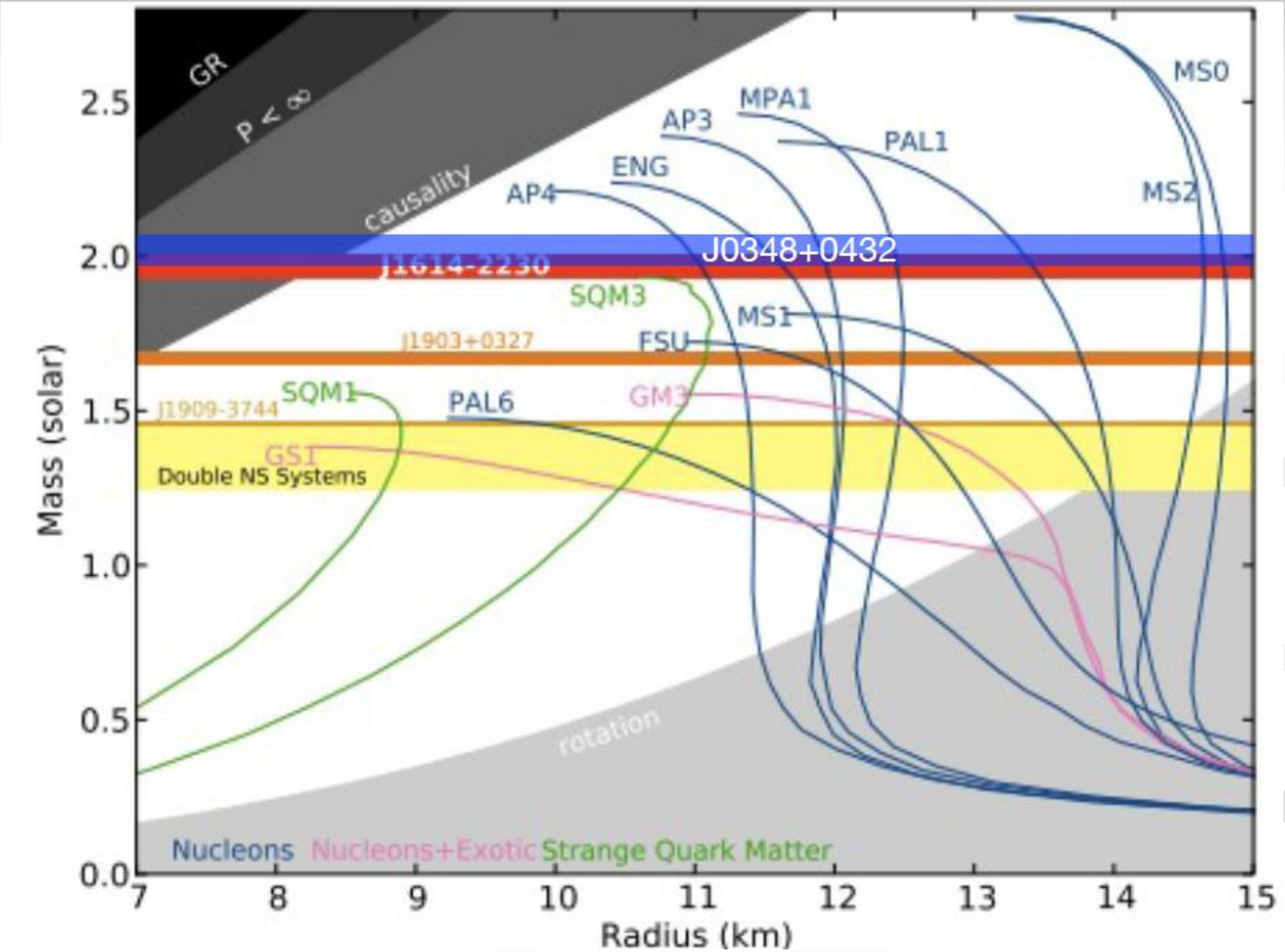
Outline

- Introduction to neutron stars
- Astronomical observations of neutron star related phenomena
- The symmetry energy from laboratory experiments
- Applications of the symmetry energy to neutron star phenomenology
- Maximum universal symmetry energy conjecture

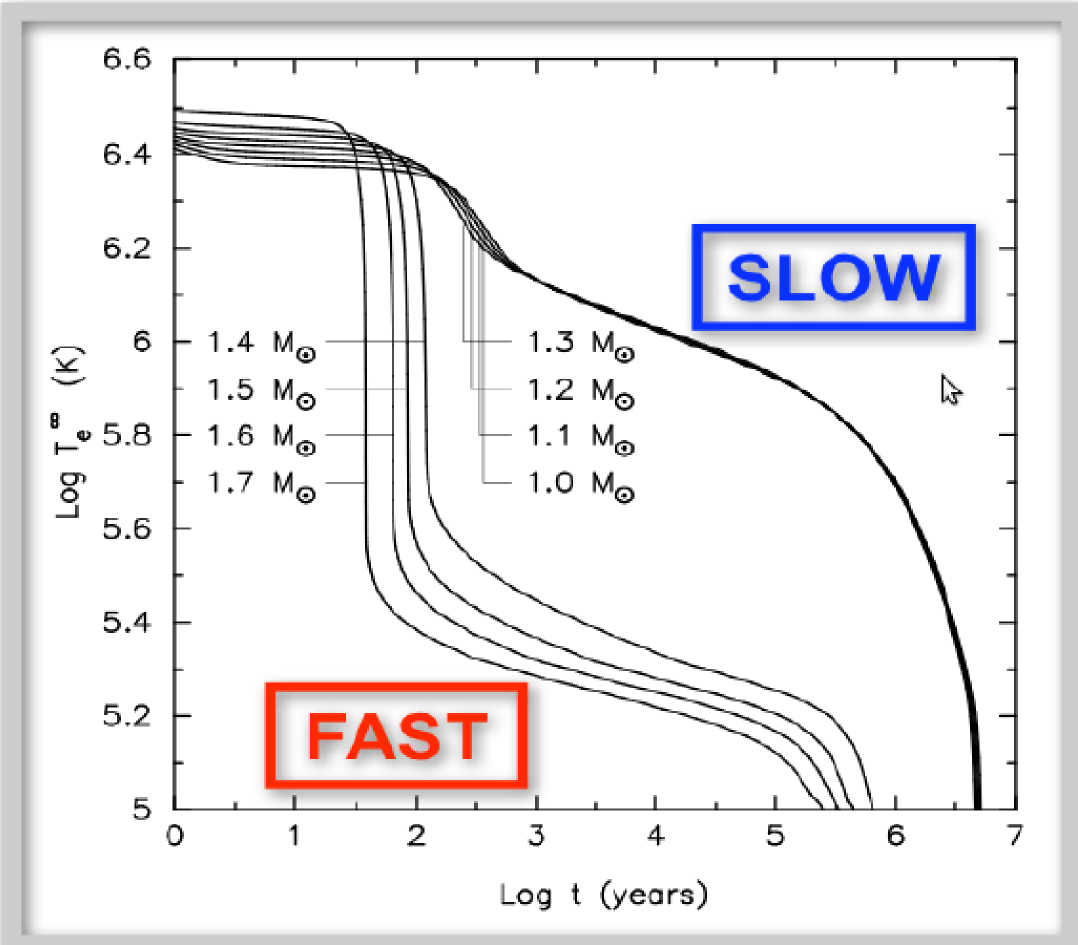
Neutron Star Composition



Mass vs. Radius Relation

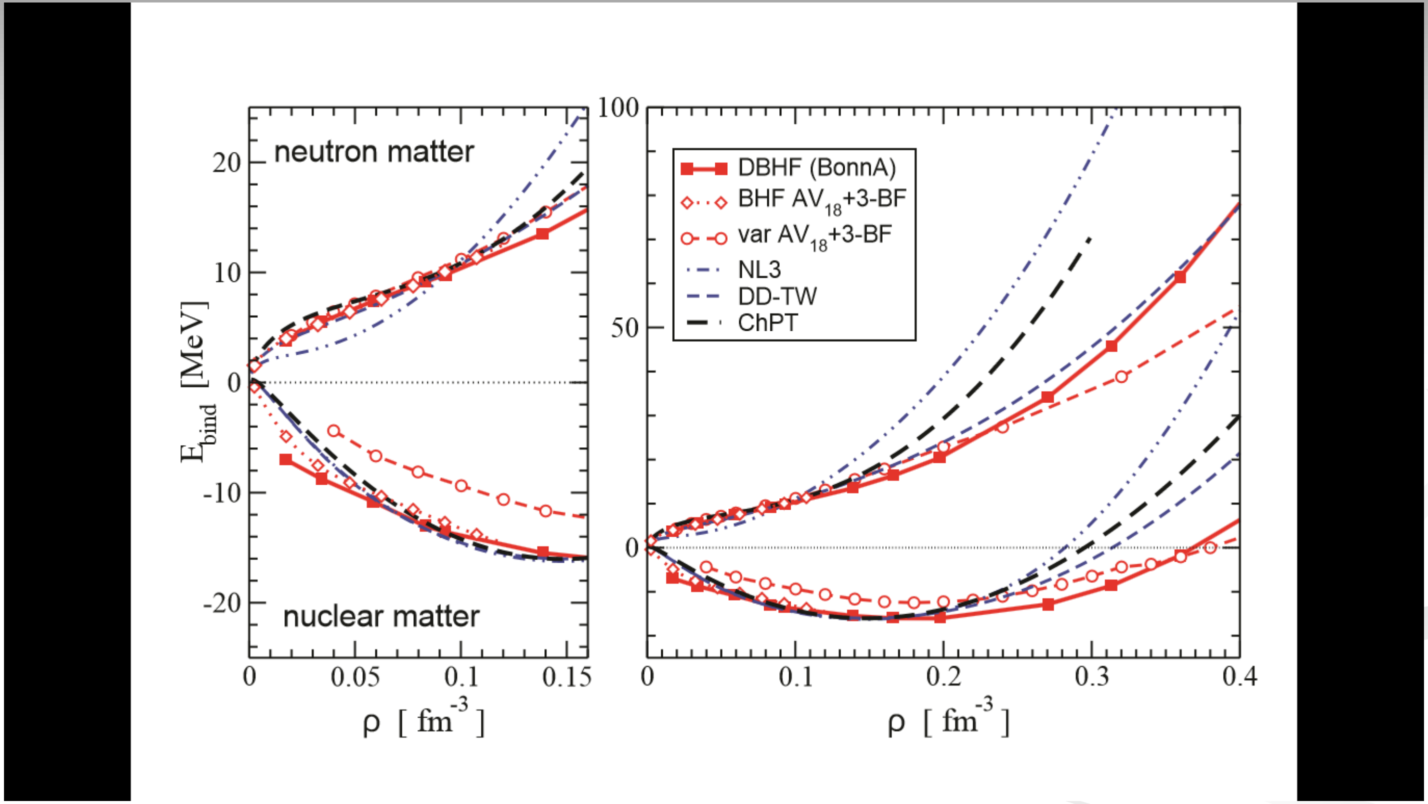


Cooling



6.6
6.4
6.2
6
5.8
5.6
5.4
5.2
5
Log T_e^{∞} (K)
0
1
2
3
4
5
6
7
Log t (years)
1.4 $M_\odot$
1.5 $M_\odot$
1.6 $M_\odot$
1.7 $M_\odot$
1.3 $M_\odot$
1.2 $M_\odot$
1.1 $M_\odot$
1.0 $M_\odot$
SLOW
FAST

The nuclear symmetry energy

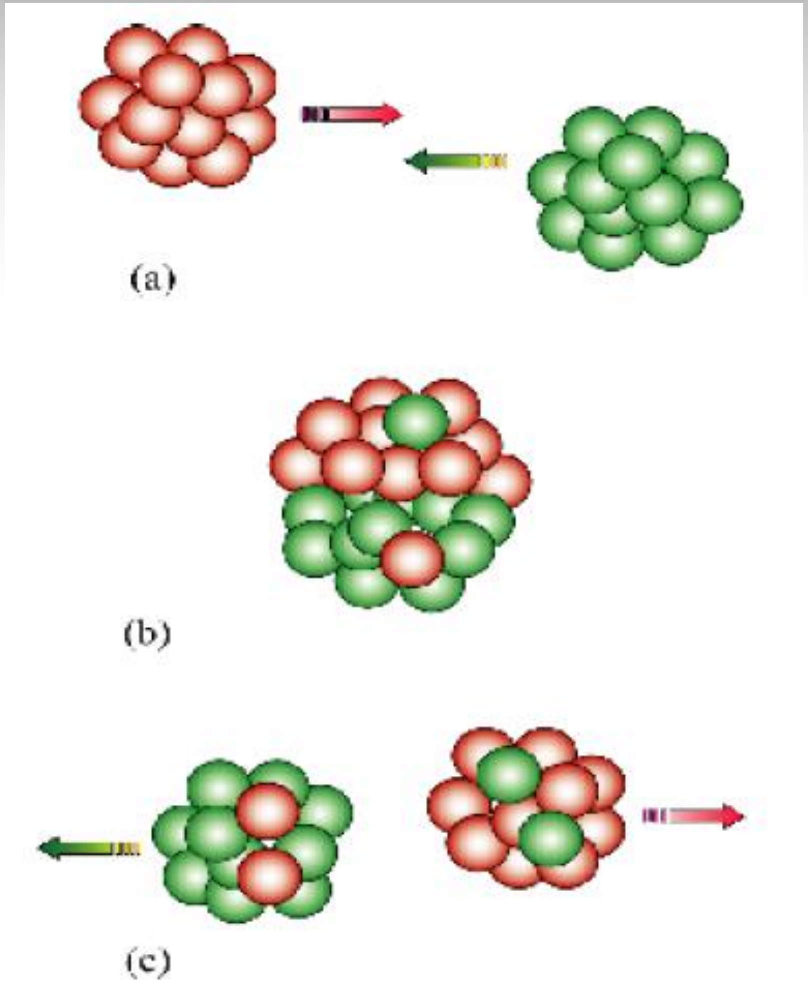


is the difference between symmetric nuclear matter and pure neutron matter in the parabolic approximation:

$$E(n,x) = E(n, x = 1/2) + E_s(n) * \alpha^2(x) + E_q(n) * \alpha^4(x) + \mathrm{O}(\alpha^6(x))$$

with $\alpha = 1-2x$

E_s measurements in the laboratory

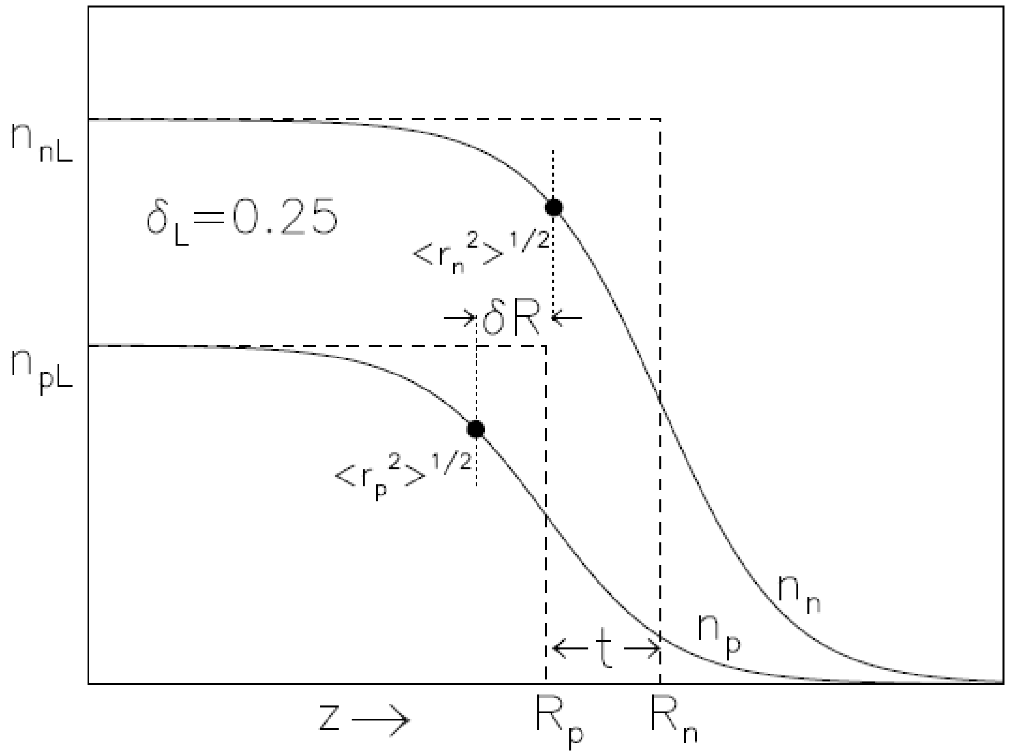


- Nuclear masses from the liquid droplet model
- Neutron skin thickness
- Isospin diffusion
- Giant dipole resonances

***"P-Rex" experiment**
(Pb radius experiment - C.J. Horowitz)

***Uses parity violating electron scattering to measure the neutron radius in Pb208 at Hall A in Jeferson Lab**

Measurements in Laboratory Experiments



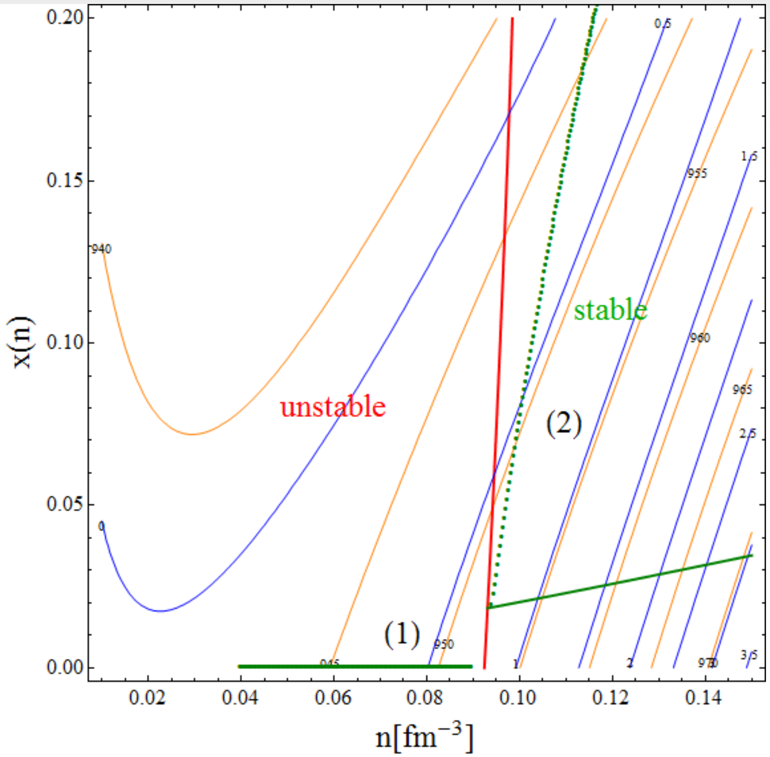
Measured values

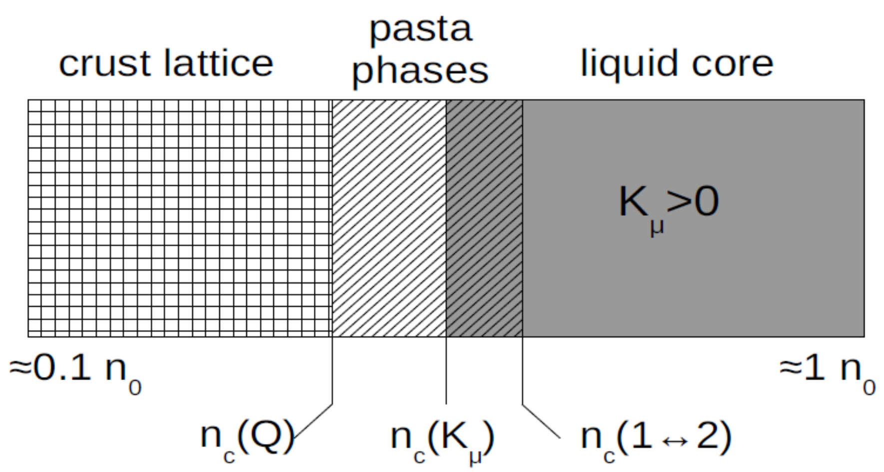
$$E_s(n_0) = 30 \pm 2\,\text{MeV}$$

$$L = 3n_0 \left.\frac{dE_s}{dn}\right|_{n_0} = 88 \pm 25\,\text{MeV}$$

E_s is highly undetermined both above and below n_0

Crust-Core Transition



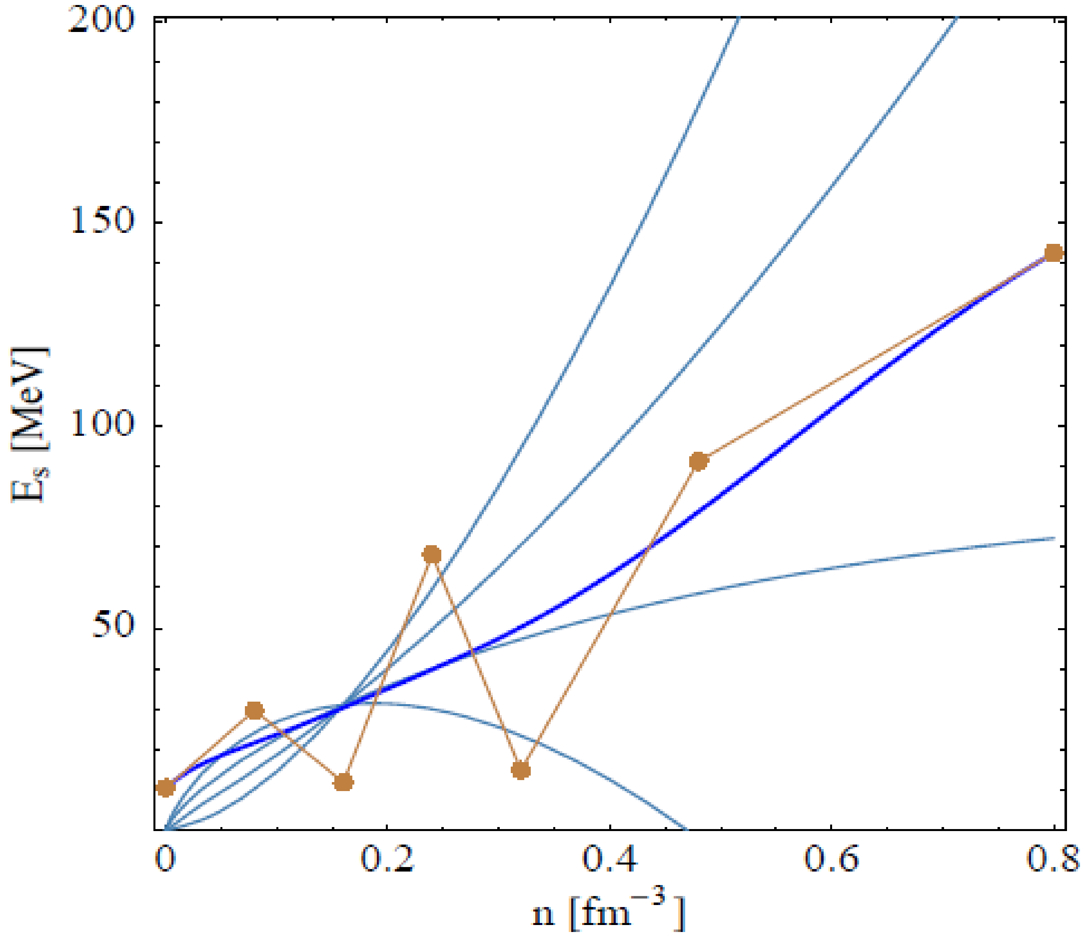


SLy4 Ioffe EoS used to model the NS crust

Kubis, Alvarez-Castillo: arXiv:1205.6368

Kubis, Porebska, Alvarez-Castillo : arXiv:0910.5066

Bézier Curves

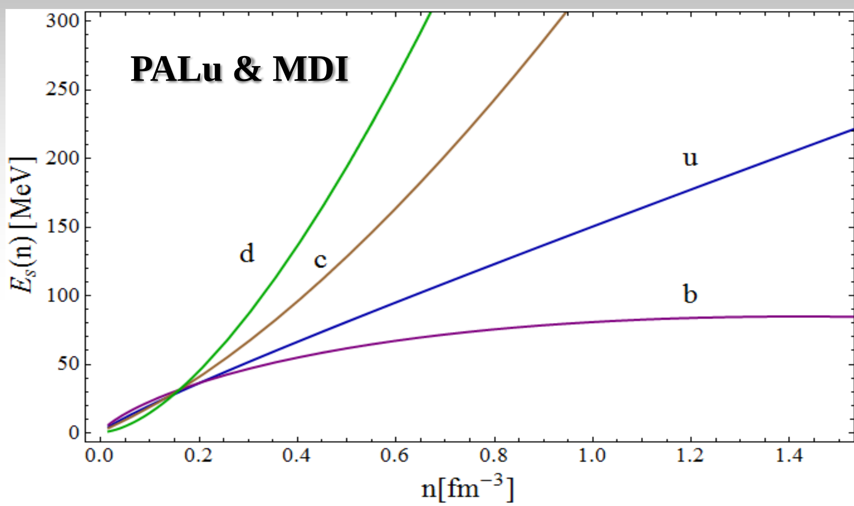


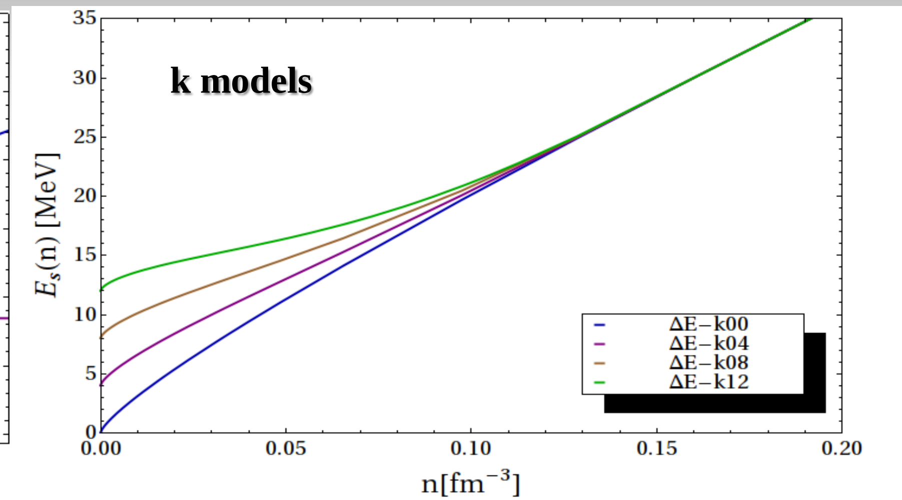
Features:

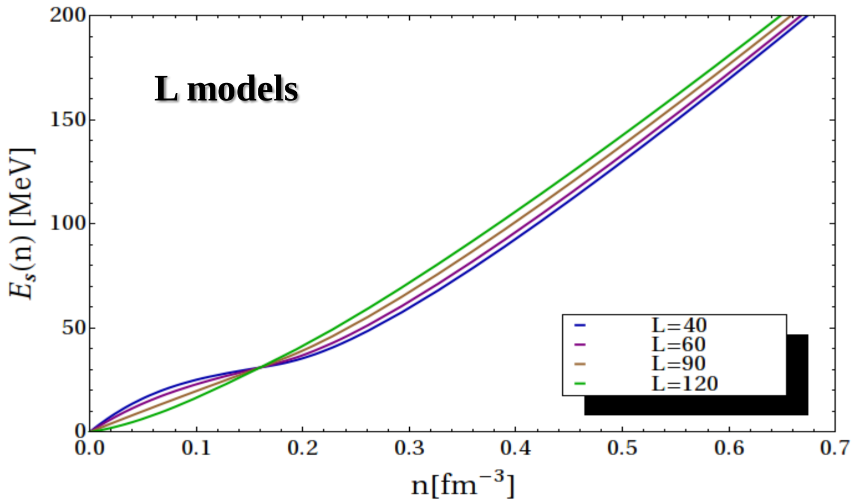
- **Full control of shapes and analicity**
- **Only few parameters**
- **Easy to adjust to experimental data**

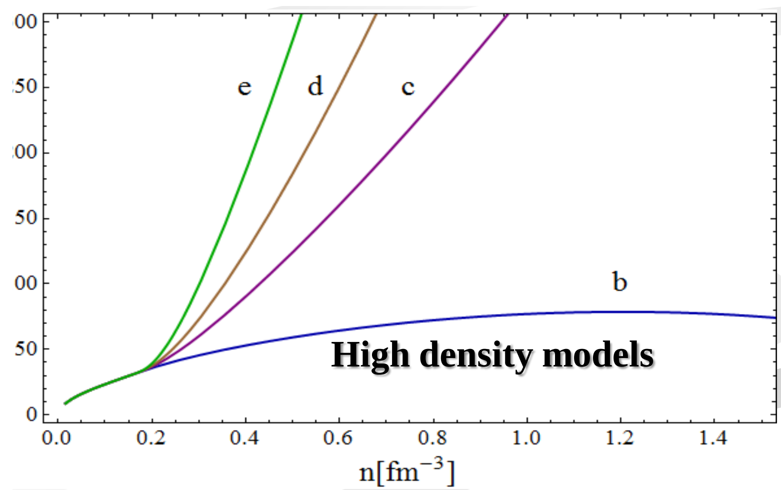
***Kubis, Alvarez-Castillo 2012**

Implemented models of E_s for neutron stars









Neutron star modeling

- Nuclear interaction: $E(n,x) = E(n,x=1/2) + E_s(n) * \alpha^2(x)$,

  E_s described by a Bézier curve

  E(n,x=1/2) taken from PAL
- Beta equilibrium: $\mu_n - \mu_p = \mu_e = \mu_\mu$
- 2 phase construction under Gibbs conditions

$$p^I = p^{II} \qquad \mu_n^I = \mu_n^{II} \qquad \mu_e^I = \mu_e^{II}$$

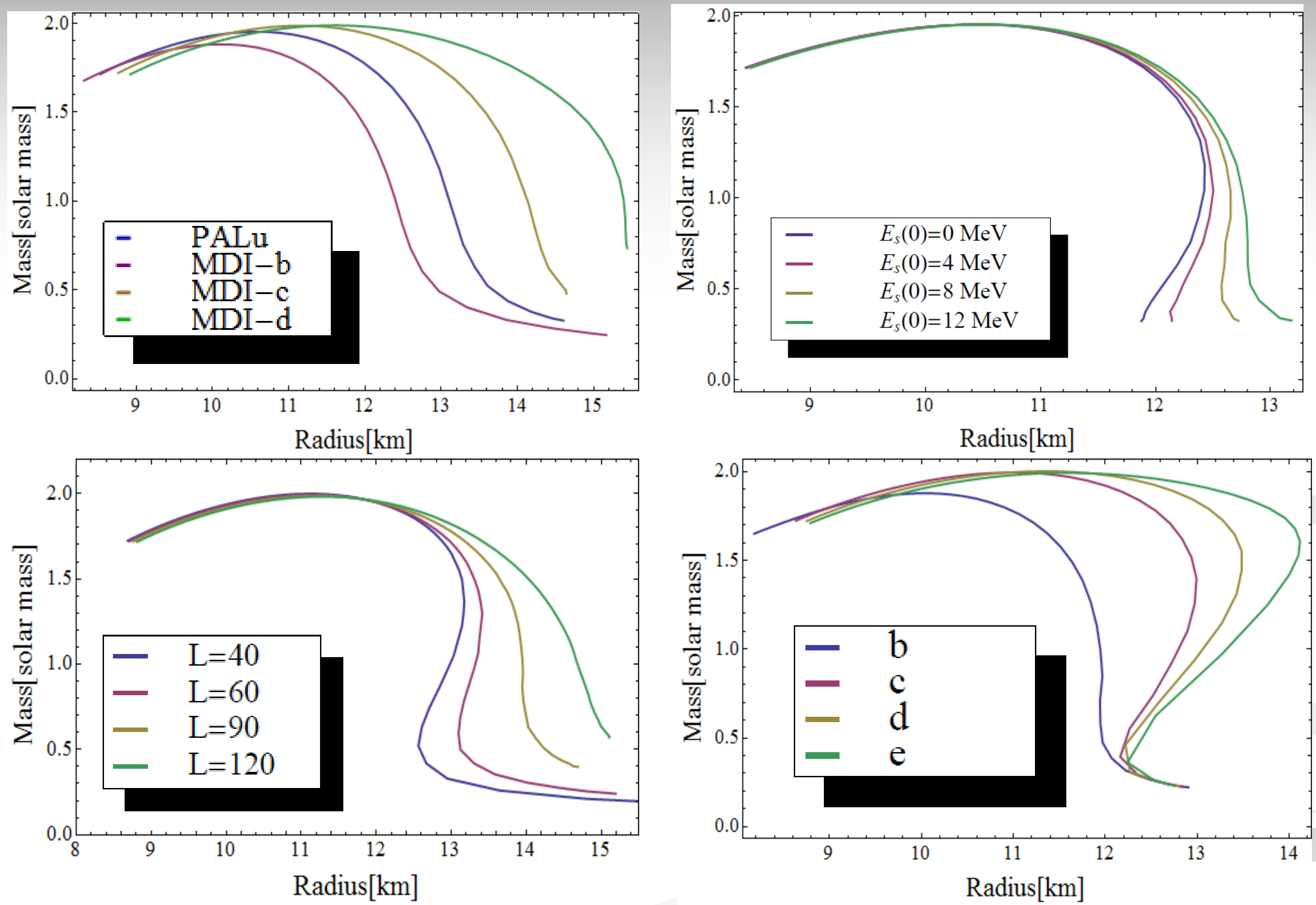
- TOV equations + Equation of State

$$\frac{dp}{dr} = -\frac{(\rho + p/c^2)G(m + 4\pi r^3 p/c^2)}{r^2(1-2Gm/rc^2)}$$

with the EoS as input $p(\rho)$

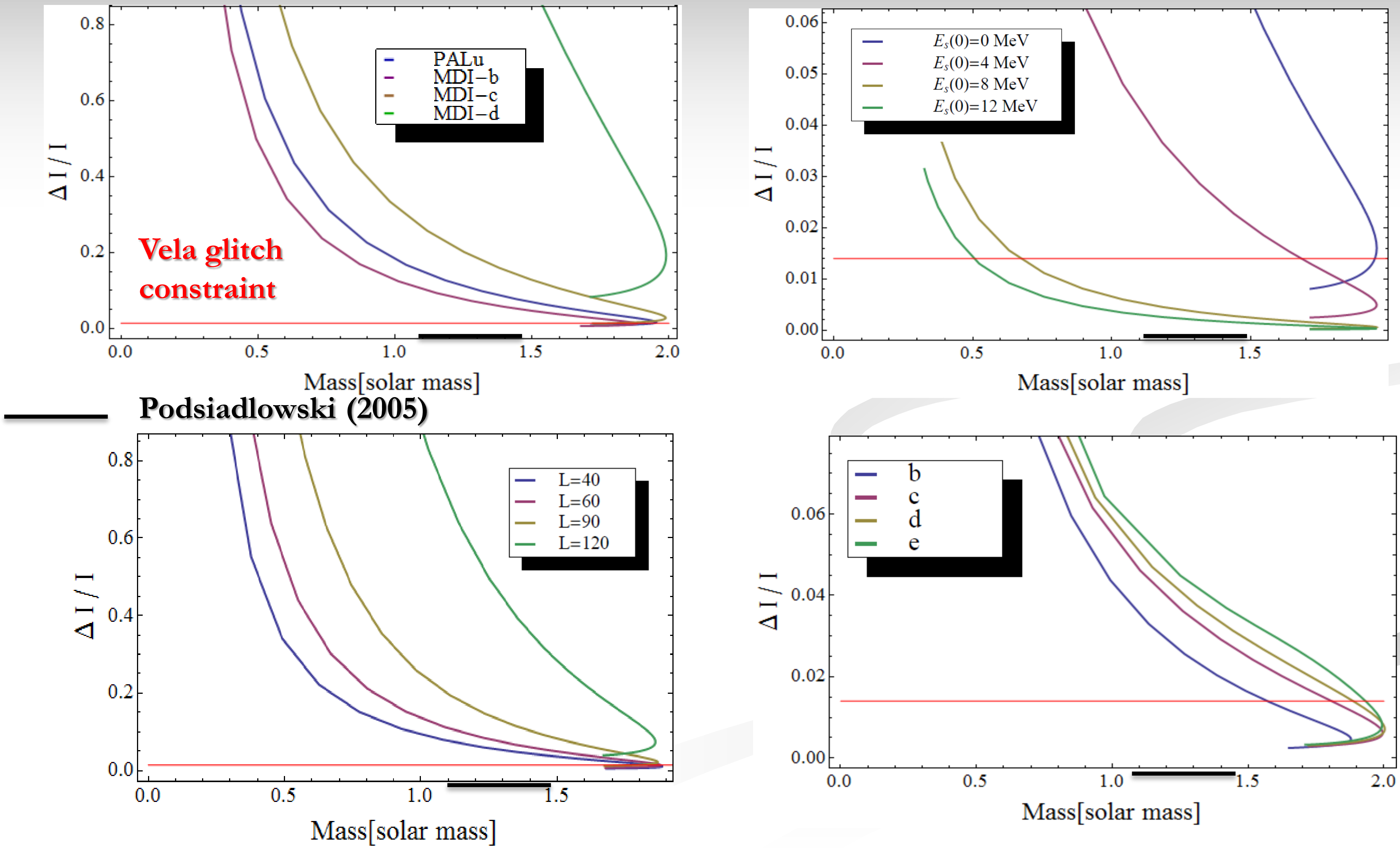
$$\frac{dm}{dr} = 4\pi r^2 \rho$$

Mass vs Radius Relations

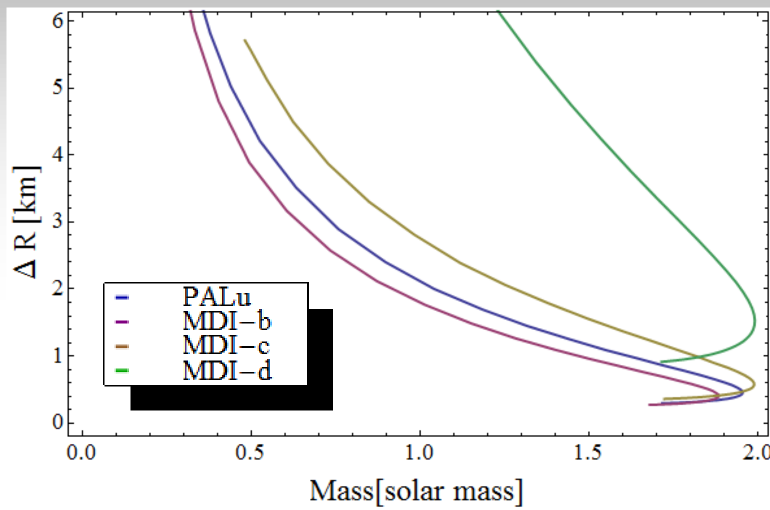


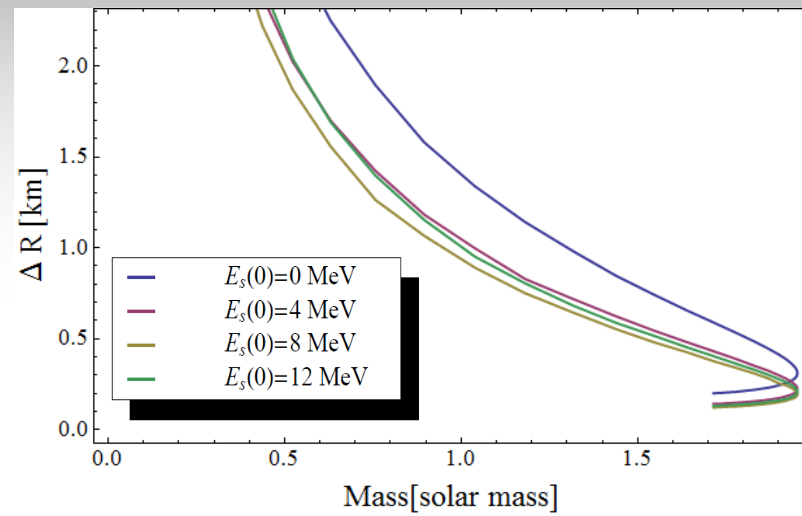
$E(n, x=1/2)$ given by the PAL parameterization

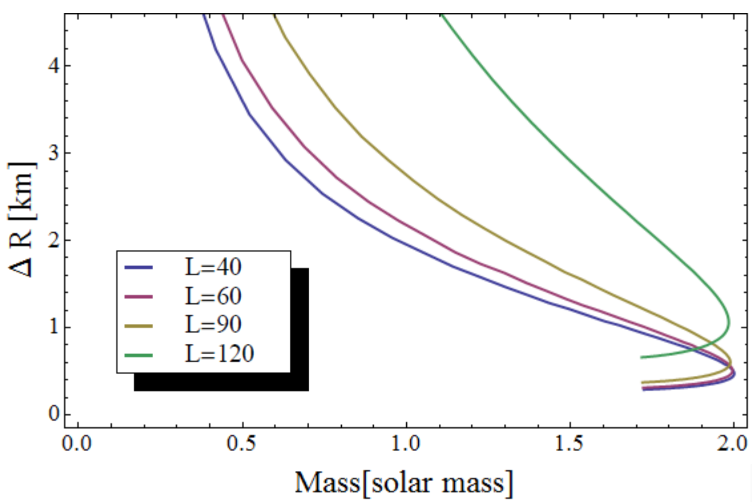
Crustal Fraction Moment of Inertia and Glitch Constraint

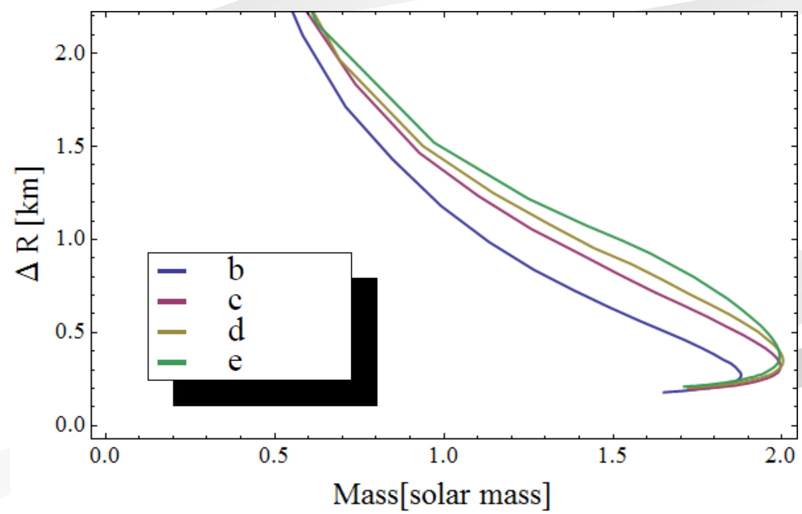


Crust Thickness

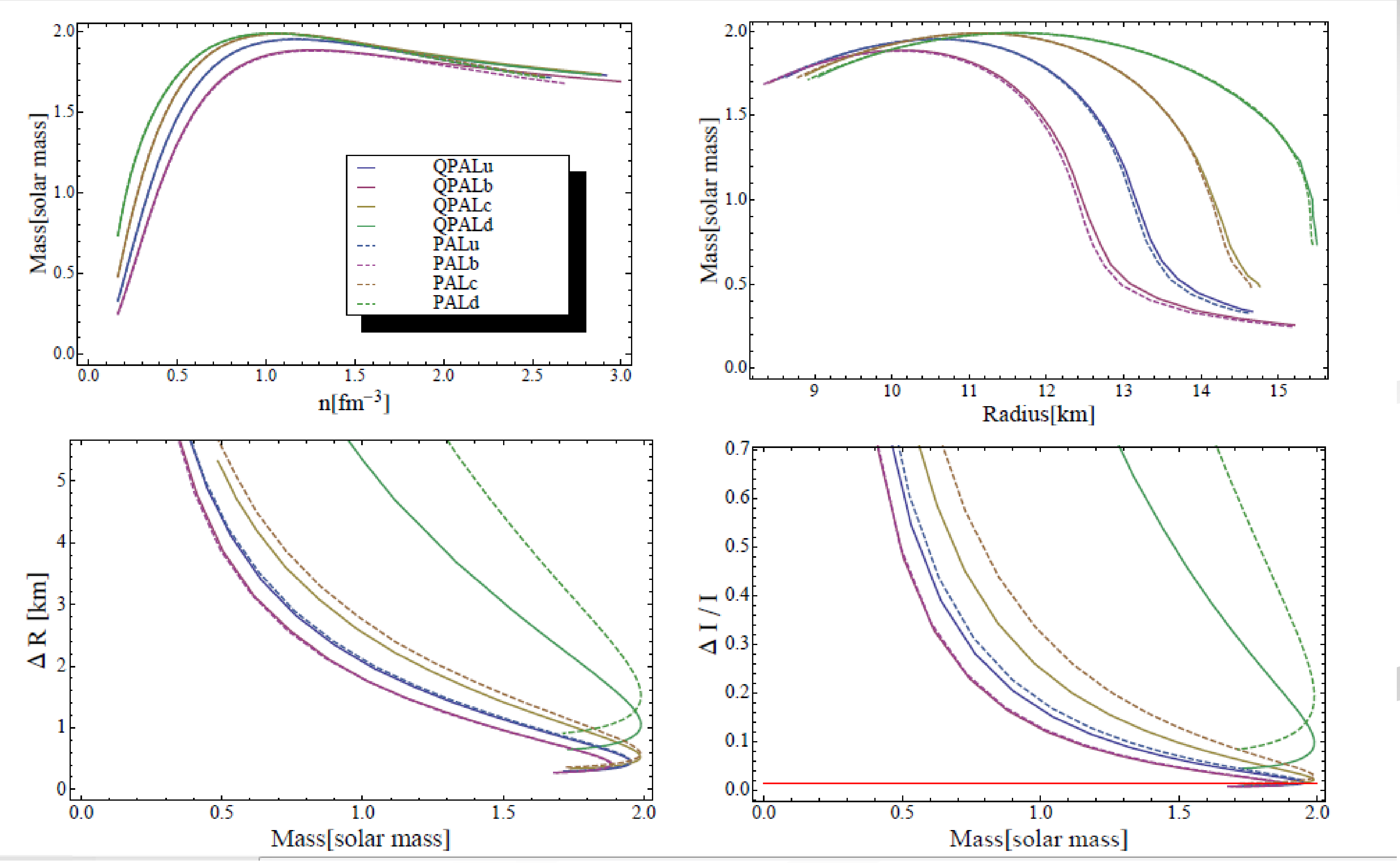








Quartic order effects



Universal Symmetry Energy Conjecture

Symmetry Energy Conjecture

Klaehn et. al., PHYSICAL REVIEW C **74**, 035802 (2006)

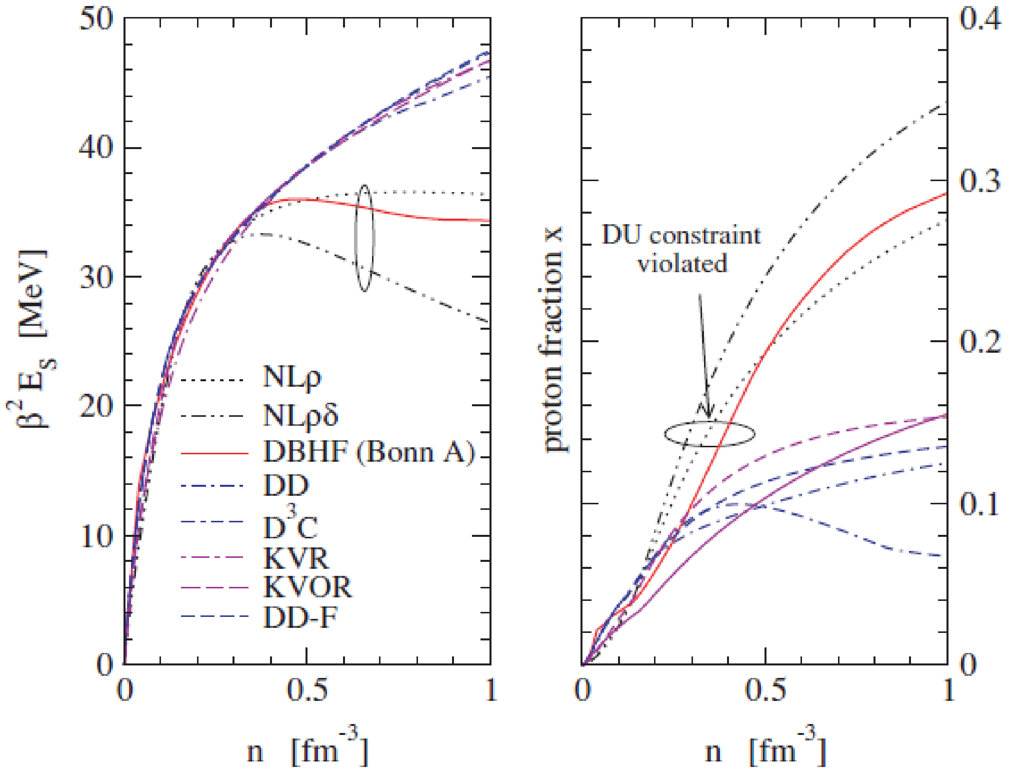


FIG. 7. (Color online) Density dependence of the asymmetry contribution to the energy per particle (left panel) and of the proton fraction (right panel) in NSM. Encircled curves correspond to EoSs that violate the DU-constraint.

Important considerations

- For EoS whose proton fraction stays low a maximum energy contribution to the asymmetry part is observed
- Direct Urca cooling requires the proton fraction to be above 1/9
- Astrophysical observations suggest that DUrca cooling doesn't take place for low mass neutron stars that are hot enough to be seen

Beta equilibrium condition from energy minimization

Consider the total energy of a system consisting of neutrons,protons and electrons, $\epsilon(n_n, n_p, n_e) = \epsilon(n_n) + \epsilon(n_p) + \epsilon(n_e)$ at fixed baryon density and under the constraint of charge neutrality $n_p = n_e$. Using the method of Lagrange multipliers one starts with a function like

$$F(n_n, n_p, n_e) = \epsilon(n_n, n_p, n_e) + \alpha(n - n_n - n_p) + \beta(n_e - n_p)$$

which has to vanish for arbitrary variations of the particle densities

$$\frac{\partial F}{\partial n_n} = 0, \qquad \frac{\partial F}{\partial n_p} = 0, \qquad \frac{\partial F}{\partial n_e} = 0.$$

These conditions yield:

$$\begin{aligned} \alpha &= \sqrt{k_n^2 + m_n^2} \equiv \mu_n, \\ \alpha + \beta &= \sqrt{k_p^2 + m_p^2} \equiv \mu_p, \\ -\beta &= \sqrt{k_e^2 + m_e^2} \equiv \mu_e, \end{aligned}$$

where $\partial\epsilon/\partial n_i = \sqrt{k_i^2 + m_i^2}$ for each i species has been used. From the above relations, by eliminating the Lagrange multipliers the beta equilibrium condition can be derived:

$$\mu_e + \mu_p = \mu_n.$$

Maximum bound for $\delta^2 E_s$

Energy per baryon in the parabolic approximation

$$E(n,x) = E_0(n) + \delta^2(x) E_s(n)$$

$$n = n_n + n_p \qquad \delta = 1 - 2x \qquad x = n_p/n$$

Beta equilibrium conditions and charge neutrality

$$n \rightarrow p + e \qquad \mu_n = \mu_p + \mu_e$$

$$\mu_e(n,\delta) = 4\delta E_S(n), \qquad xn = \frac{1}{3\pi^2}\mu_e^3.$$

$$E_S(n,x) = (an)^{1/3}\frac{x^{1/3}}{1-2x}$$

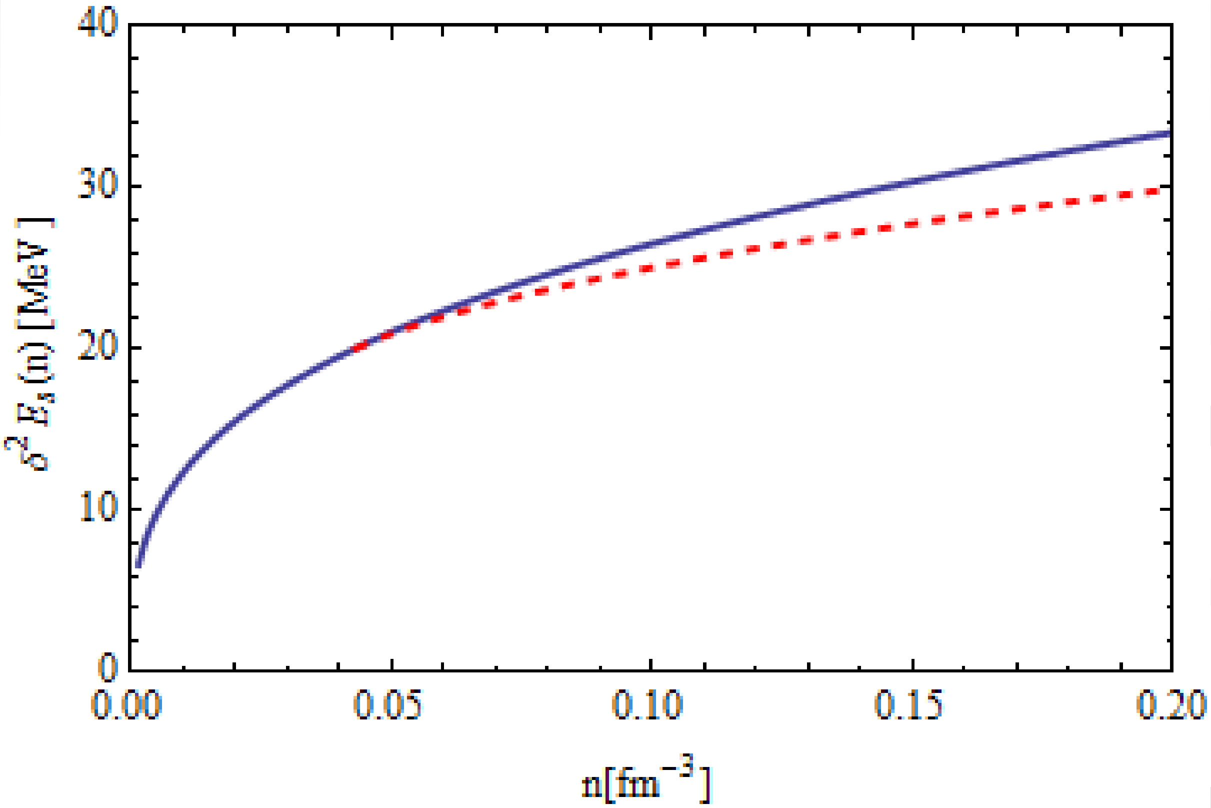
***Klaehn, Blaschke, Alvarez-Castillo in preparation**

$$\frac{\mathrm{d}}{\mathrm{d}x}(1-2x)^2 E_S(n,x)|_{x=x_c} = (an)^{1/3}\left\{\frac{1}{3}x^{-2/3} - \frac{8}{3}x^{1/3}\right\}\Big|_{x=x_c} = 0.$$

extremum

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}(1-2x)^2 E_S(n,x)|_{x=x_c} = (an)^{1/3}\left\{-\frac{2}{9}x^{-5/3} - \frac{8}{9}x^{-2/3}\right\}\Big|_{x=x_c} < 0.$$

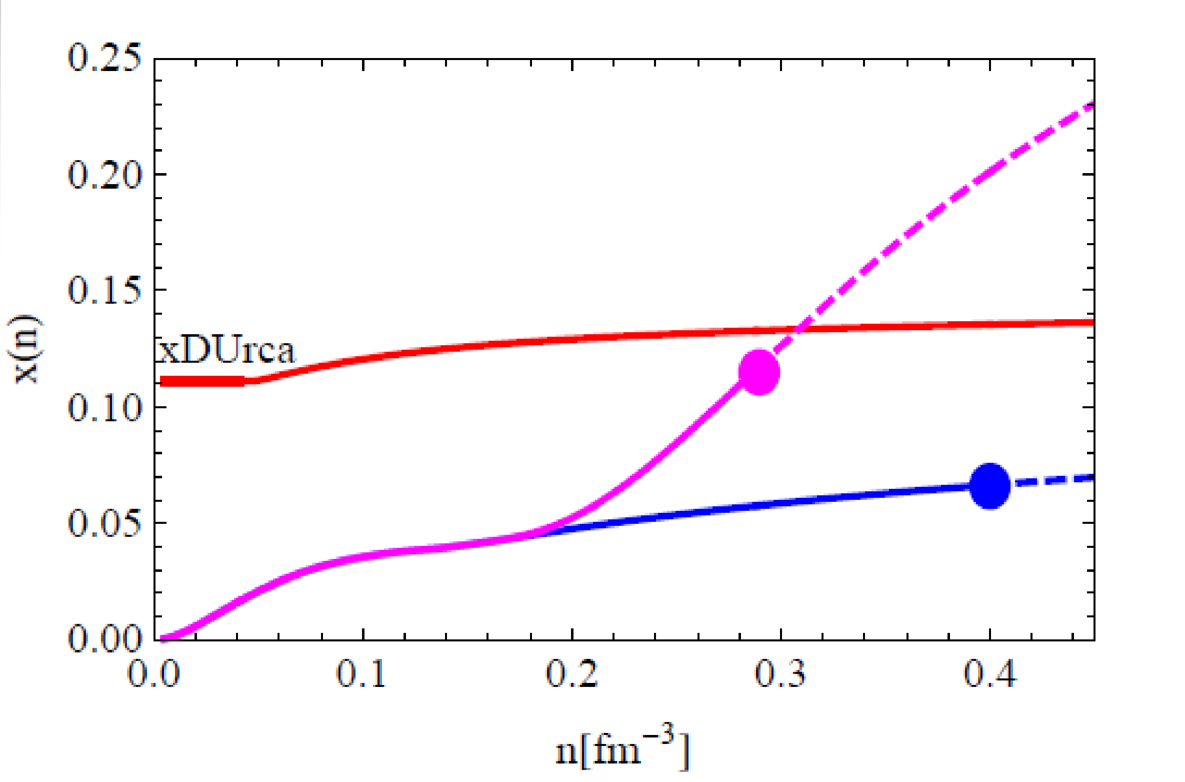
maximum



Only electrons (solid blue)
Electrons + Muons (dashed red)

For only electrons:
x=1/8

Direct Urca threshold



Cooling phenomenology of NS suggest Durca process should not occur for NS with typical masses, e.g., 1.3 <M/M$_{SUN}$ <1.5.
Two possibilities: 1. Maximum mass before the onset is reached (pink)
2. Bounded symmetry energy (blue)

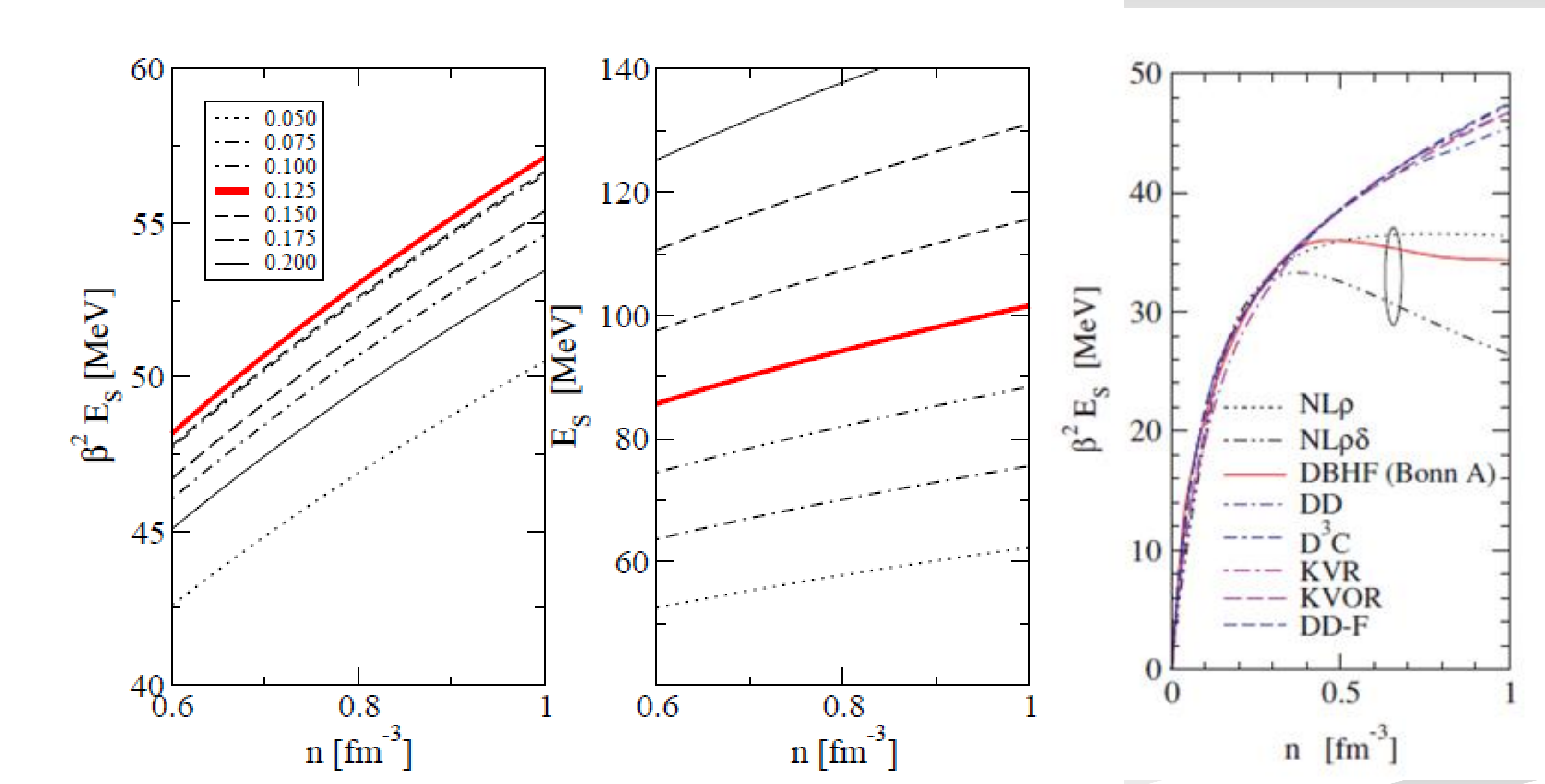
Neutron Star Cooling Processes

Process Name	Process	Emissivity Q_ν (erg cm^{-3} s^{-1})	Reference
Bremsstrahlung	$n+n \to n+n+\nu_e+\bar{\nu}_e$ $n+p \to n+p+\nu_e+\bar{\nu}_e$ $p+p \to n+p+\nu_e+\bar{\nu}_e$	$\simeq 10^{19} T_9^8$	Page, Geppert and Weber [92]
Modified Urca	$n+n \to n+p+e^-+\bar{\nu}_e$ $n+p+e^- \to n+n+\nu_e$	$\simeq 10^{20} T_9^8$	Friman and Maxwell [93]
Direct Urca	$n \to p+e^-+\bar{\nu}_e$ $p+e^- \to n+\nu_e$	$\simeq 10^{27} T_9^6$	Lattimer et al. [94]
Quark Urca	$d \to u+e^-+\bar{\nu}_e$ $u+e^- \to d+\nu_e$	$\simeq 10^{26} \alpha_c T_9^6$	Iwamoto [95]
Kaon Condensate	$n+K^- \to n+e^-+\bar{\nu}_e$ $n+e^- \to n+K^-+\nu_e$	$\simeq 10^{24} T_9^6$	Brown et al. [96]
Pion Condensate	$n+\pi^- \to n+e^-+\bar{\nu}_e$ $n+e^- \to n+\pi^-+\nu_e$	$\simeq 10^{26} T_9^6$	Maxwell et al. [97]

Direct Urca is the fastest cooling process.
Threshold for onset: $p_{F,n} < p_{F,p} + p_{F,e}$. For electrons only then $x_{DU}=1/9$.

Leptonic contribution to EoS

- Direct Urca constrain allows to keep the proton fraction bounded x<1/9.
- Therefore, leptonic contribution (x dependent) also bounded.



Understanding of the universal behavior of $\delta^2 E_s$, symmetry energy contribution to the NS for EoS which obey the DUrca constraint.

Numerical exploration

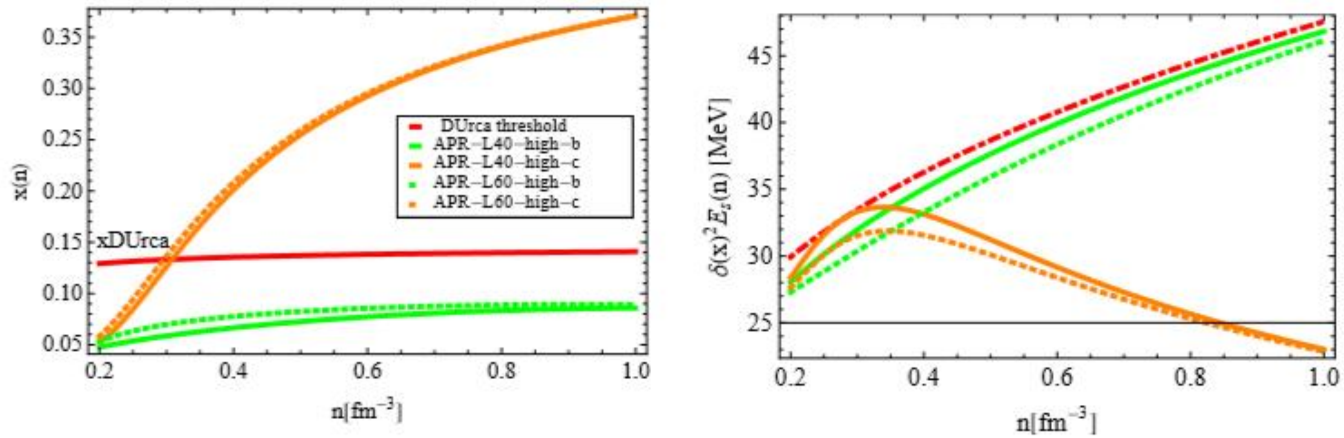


FIG. 4: APR-Bezier models that either violate or not the DUrca constraint. *Left*: proton fraction as a function of density and DUrca threshold. *Right*: symmetry energy contribution to the nuclear energy and universal symmetry energy line bound.

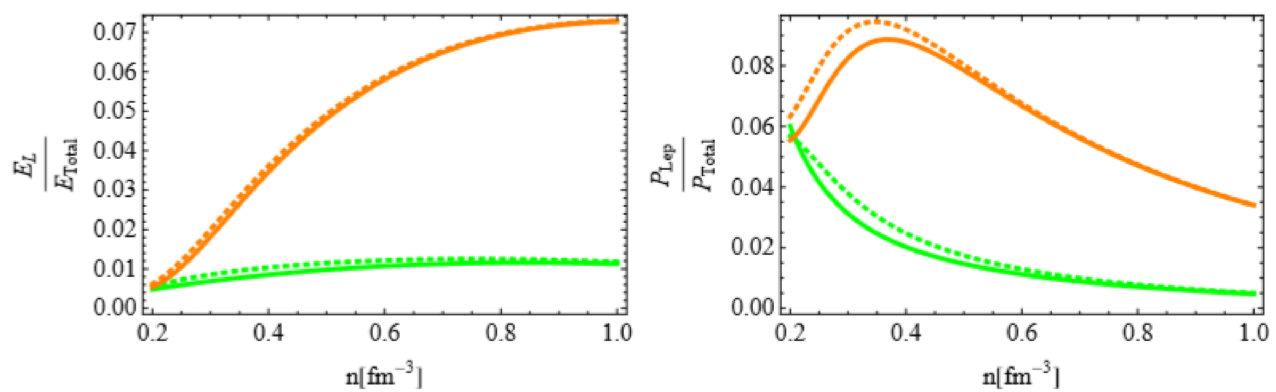


FIG. 5: APR-Bezier models displaying the leptonic contribution to the total energy density (left) and to the total presure (right).

Bayesian TOV Analysis

Nuclear interactions:

$\varepsilon_{trans} < \varepsilon < \varepsilon_1$

$$\varepsilon = n_B \left\{ m_B + B + \frac{K}{18}(u-1)^2 + \frac{K'}{162}(u-1)^3 + (1-2x)^2[S_k u^{2/3} + S_p u^{\gamma}] + \frac{3}{4}\hbar c x (3\pi^2 n_b x)^{1/3} \right\}$$

$\varepsilon_{trans} \approx \varepsilon_0/2$

Beta equilibrium and leptonic contribution:

$$\frac{\partial \varepsilon}{\partial x} = \hbar c (3\pi^2 n_B x)^{1/3} - 4[S_k u^{2/3} + S_p u^{\gamma}](1-2x) = 0$$

$$x = \frac{1}{4}[(\sqrt{d+1}+1)^{1/3} - (\sqrt{d+1}-1)^{1/3}]^3$$

$$d = \frac{\pi^2 n_B}{288} \left(\frac{\hbar c}{[S_k u^{2/3} + S_p u^{\gamma}]} \right)^3$$

Steiner, A. W., Lattimer, J. M., & Brown, E. F. 2010, ApJ, 722, 33

Parametrization of the EoS

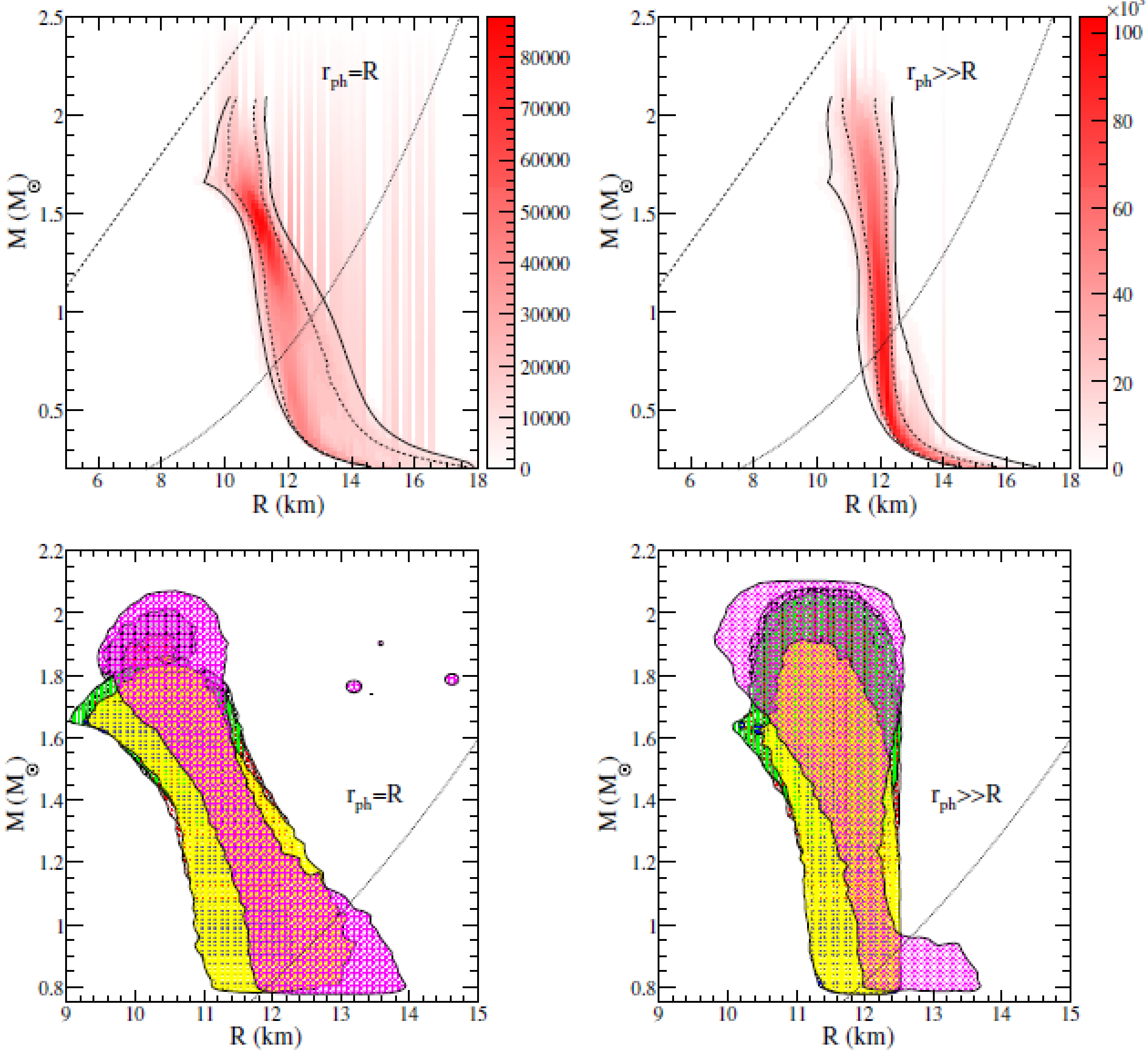
$$P = K_1 \varepsilon^{1+1/n_1}$$ $\varepsilon_1 < \varepsilon < \varepsilon_2$

$$P = K_2 \varepsilon^{1+1/n_2}$$ $\varepsilon > \varepsilon_2$

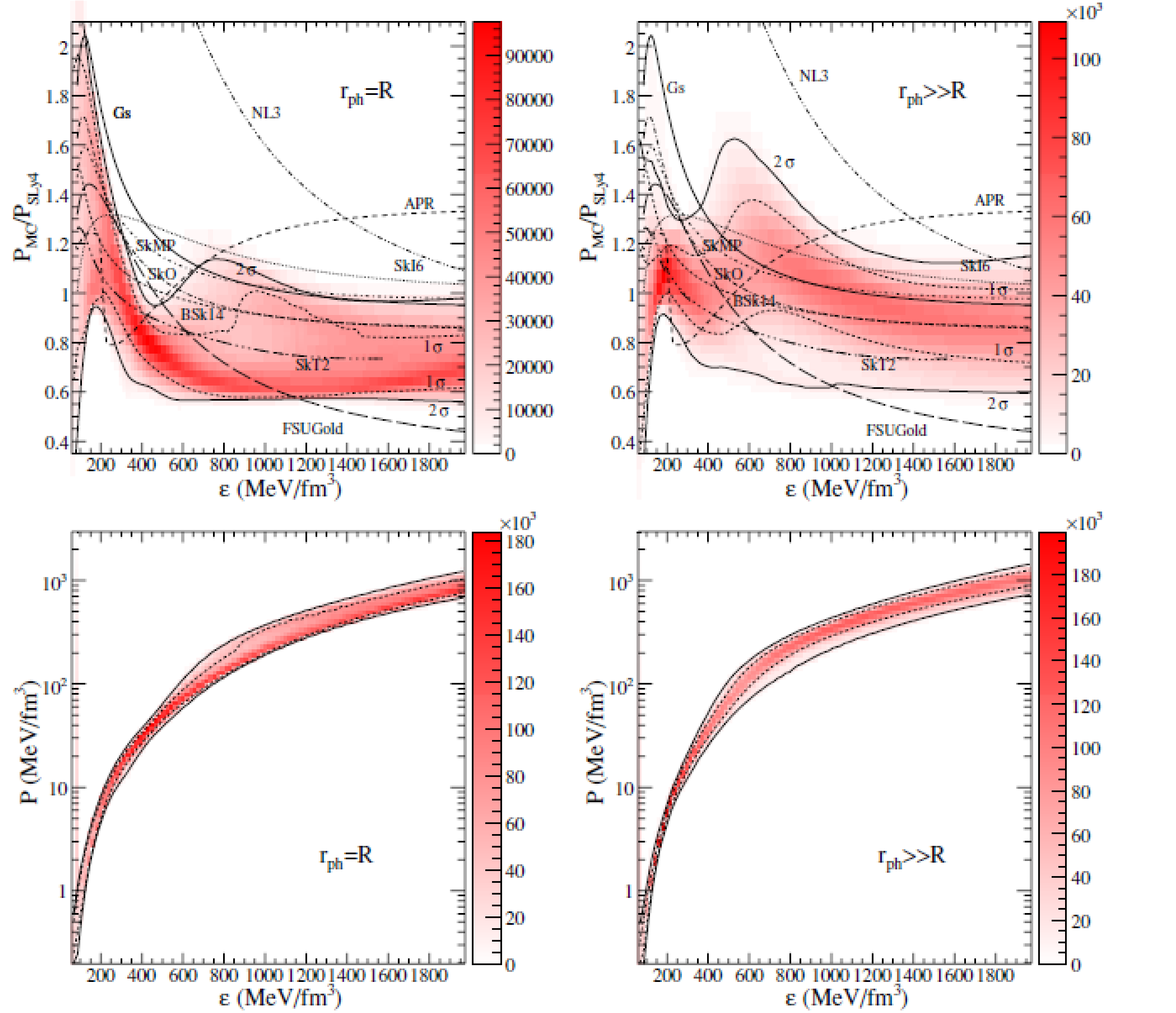
Criteria to follow (rejection rules):

1. the maximum mass is smaller than $1.66\,M_\odot$, which is 2σ below the mass of PSR J1903+0327, $1.74 \pm 0.04\,M_\odot$ (Champion et al. 2008);
2. the EOS becomes acausal below the central density of the maximum mass star;
3. the EOS is anywhere hydrodynamically unstable, i.e., has a pressure that decreases with increasing density; and
4. the maximum mass star has a maximum stable rotation rate less than 716 Hz, the spin frequency of the fastest known pulsar, Ter 5AD (Hessels et al. 2006). The spin frequency at which equatorial mass shedding commences is given to within a few percent by Haensel et al. (2009):

$$f_K \simeq 1.08 \left(\frac{M}{M_\odot}\right)^{1/2} \left(\frac{10\,\mathrm{km}}{R}\right)^{3/2} \mathrm{kHz}\,. \qquad (39)$$

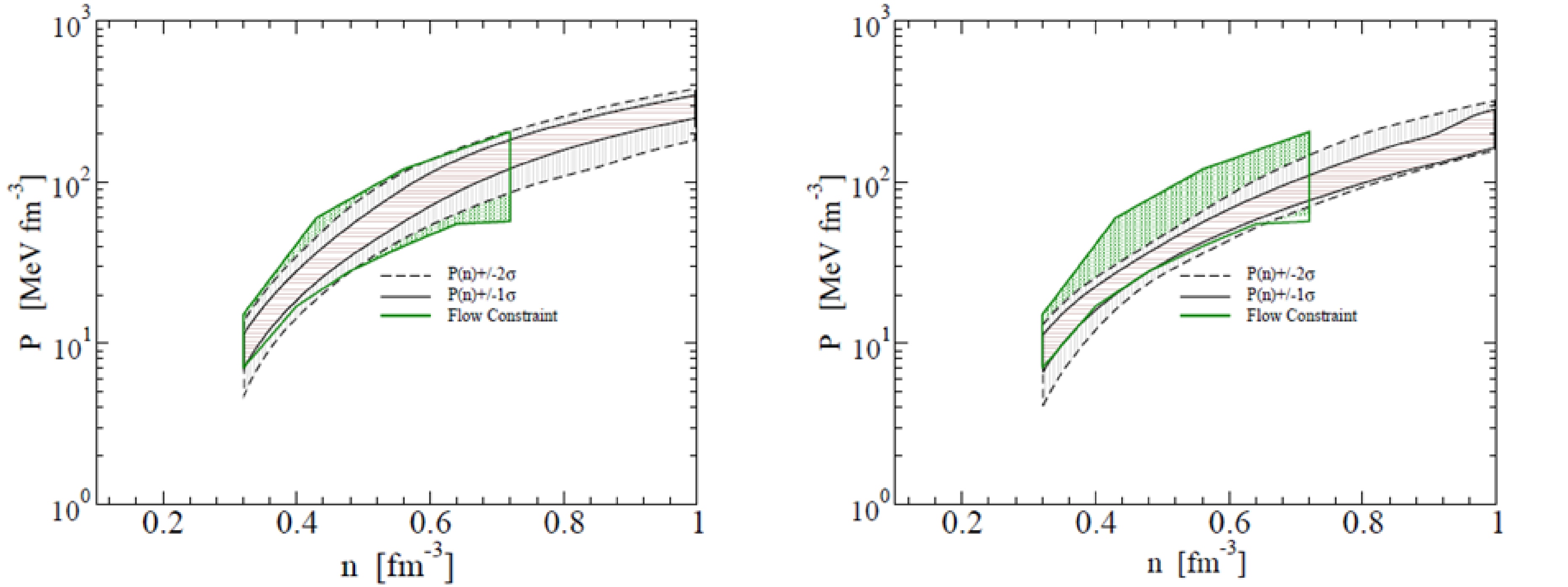


r_{ph}=R
r_{ph}>>R
r_{ph}=R
r_{ph}>>R
M ($M_\odot$)
R (km)



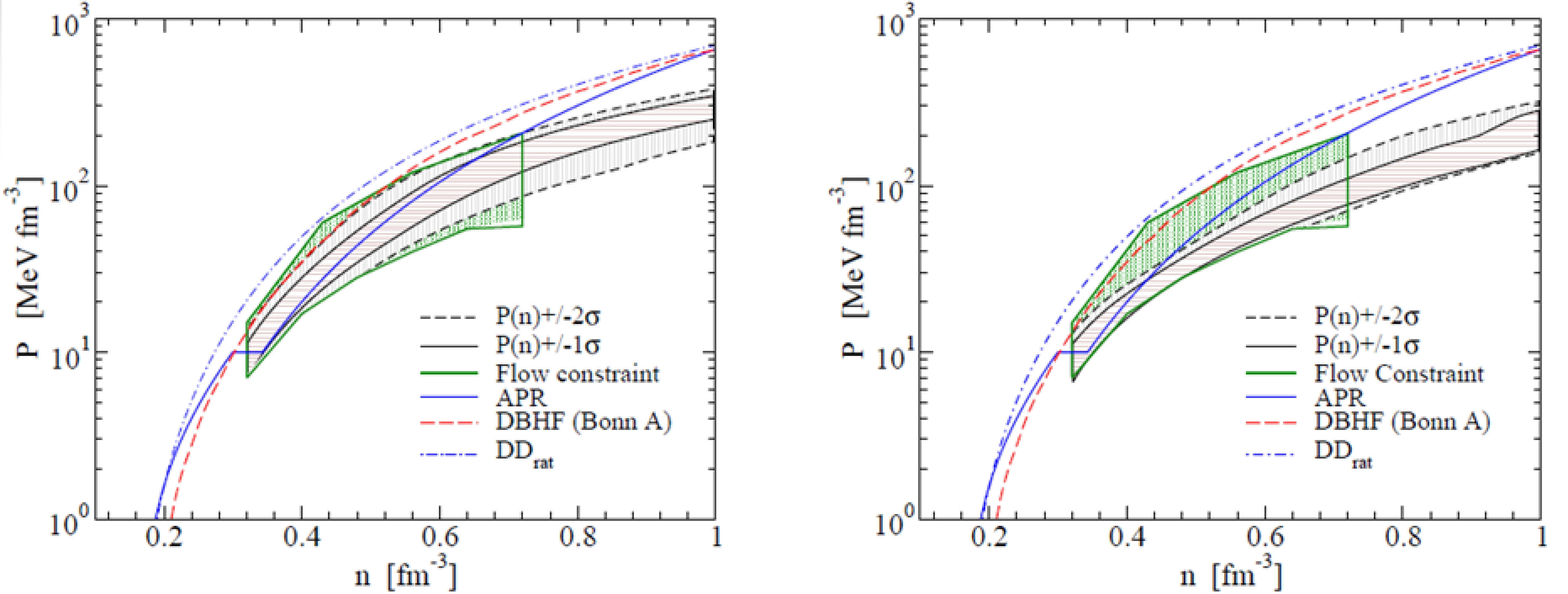
r_{ph}=R
r_{ph}>>R
P_{MC}/P_{SLy4}
ε (MeV/fm^3)
P (MeV/fm^3)
Gs
NL3
APR
SkMP
SkO
SkI6
BSk14
SkI2
FSUGold
1σ
2σ
×10^3

$E_0(n)$ (Preliminary Results)



EoS for Symmetric Nuclear Matter extracted from NS observations (Bayesian TOV inversion) and Universal Symmetry Energy compared to Flow Constraint from Heavy Ion Collisions.

$E_0(n)$ (Preliminary Results)



Flow Constraint and NS Constraint to SNM compared to three microscopic EoS.

Conclusions

- *k-models: Es at low density. A*ccording to these models, the mass of the Vela pulsar should be very low, with much less than 1 Solar Mass. A need for a better understanding on uniform matter and cluster formation.
- *Neutron star cooling can constrain the EoS*. Low mass NS should not cool by direct Urca process therefore some models can be ruled out.
- *Different determination of the critical density*. Finite size effects derived from Coulomb interactions lower the values of the thickness of neutron stars.
- *Effects of the quartic term in the energy expansion*. Neutron star crusts are the most affected. For the models with thick crust the effect is so large that cannot be neglected. This is where the parabolic approximation breaks down.

Conclusions

- There exists a Maximal Contribution from the Symmetry Energy of Nuclear Matter to the NS EoS for proton fractions in a not too narrow region around x=1/8, e.g., between 0.05 and 0.2
- This is close to the Direct Urca threshold (1/9 for electrons only)
- Violating the DUrca threshold consequently results in deviations from the Universal Symmetry Energy (USE)
- Applications to Compact Stars: Bayesian analysis from mass radius relation could result in predictions for the cold symmetric matter beyond the flow constraint.
- From laboratory measurements of symmetric nuclear matter one can predict the NS EoS using the USE

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