Equilibrium properties and oscillation frequencies of charged spheres in GR

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Outline

1 Motivation

2 Derivation of pulsation equation

3 Results on strange and hybrid stars

Why care about charge in compact stars?

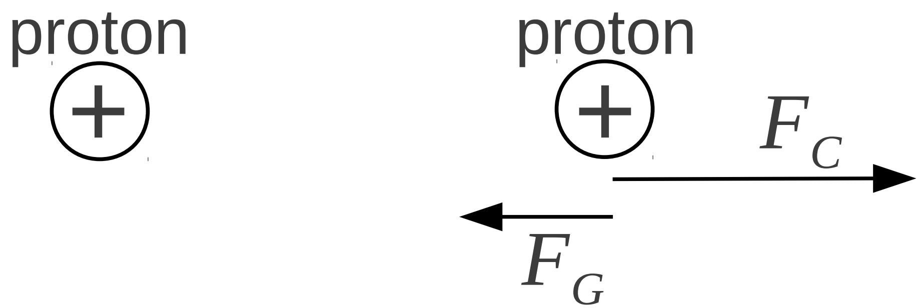
- charged balls might be natural candidates to form extremal black holes

- even if the global charge is zero, there can be separation of charges inside

- charge allows for exotic compact objects like hollow spheres = thick shells with its own M-R diagram (family parameterized by total charge and inner radius)

- charge might prevent gravitational collapse and support supermassive stars

How much charge allowed?

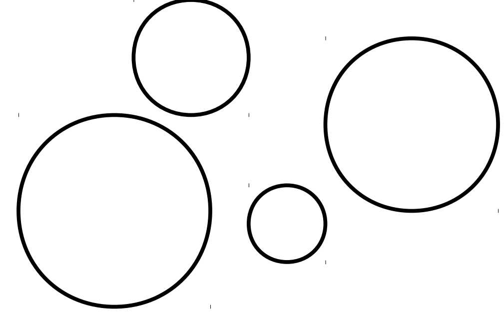


$$\frac{F_C}{F_G} \approx 10^{36}$$

Number of baryons in one neutron star: $N_B \approx 3 \cdot 10^{57}$

Number of net unit charges allowed to build "reasonable" charged compact stars: $N_c < 10^{-18} N_B$

Spheres in GR



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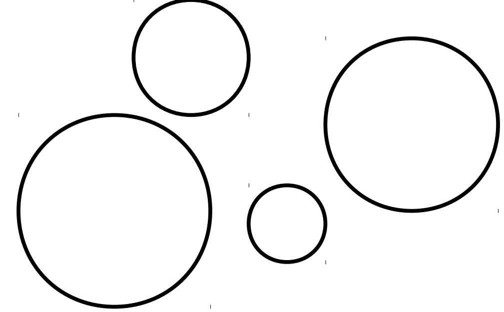

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prescription to find pulsation equation

- time-dependent Schwarzschild metric

- equations: $G_{\mu\nu} = 8\pi T_{\mu\nu} \quad T^{\mu\nu}{}_{;\nu} = 0 \quad (nu^{\mu})_{;\mu} = 0$

$$\partial_{\mu}\left[\sqrt{-g}F^{\nu\mu}\right] = 4\pi\sqrt{-g}\,j^{\nu}$$

- decompose variables: $A(r,t) = A_0(r) + \delta A(r,t)$

- linearize time-dependent equations

- subtract equilibrium equations from time-dependent equations and get perturbations: $\delta A(r,t)$

- substitute perturbations in $T^{\mu r}{}_{;\mu} = 0$ and get pulsation equation

Derivation of pulsation equation 1

$$ds^2 = -e^{2\Phi}dt^2 + e^{2\Lambda}dr^2 + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2$$

$$G_0{}^0 = -e^{-2\Lambda}\left[2r^{-1}\Lambda' - \left(1 - e^{2\Lambda}\right)r^{-2}\right]$$

$$G_1{}^1 = e^{-2\Lambda}\left[2r^{-1}\Phi' + r^{-2}\right] - r^{-2}$$

$$G_2{}^2 = e^{-2\Lambda}\left[\Phi'' - \Phi'\Lambda' + \Phi'^2 + r^{-1}\left(\Phi' - \Lambda'\right)\right]$$

$$+ e^{-2\Phi}\left[\dot{\Phi}\dot{\Lambda} - \ddot{\Lambda} - \dot{\Lambda}^2\right]$$

$$G_0{}^1 = 2r^{-1}e^{-2\Lambda}\dot{\Lambda}$$

$$T_\mu{}^\nu = (\rho + P)\, u_\mu u^\nu + P g_\mu{}^\nu + \frac{1}{4\pi}\left[F_{\mu\alpha}F^{\alpha\nu} - \frac{1}{4}g_\mu{}^\nu F^{\beta\gamma}F_{\beta\gamma}\right]$$

Derivation of pulsation equation 2

$$\delta\Lambda = -\left(\Phi_0' + \Lambda_0'\right)\xi$$

$$\delta\rho = -\xi\rho_0' - (\rho_0 + P_0)\,\frac{e^{\Phi_0}}{r^2}\left(r^2 e^{-\Phi_0}\xi\right)'$$

$$\delta\Phi' = 4\pi r e^{2\Lambda_0}\delta P + 2\Phi_0'\delta\Lambda + r^{-1}\delta\Lambda - \frac{Q_0\delta Q e^{2\Lambda_0}}{r^3}$$

$$\delta P = \frac{dP_0}{d\rho_0}\delta\rho = -\xi P_0' - \frac{\gamma P_0 e^{\Phi_0}}{r^2}\left(r^2 e^{-\Phi_0}\xi\right)'$$

Energy-momentum conservation:

$$e^{2\Lambda_0 - 2\Phi_0}\left(\rho_0 + P_0\right)\dot{v} + \delta P' + \frac{Q_0 Q_0'\xi'}{4\pi r^4} + \frac{Q_0 Q_0''\xi}{4\pi r^4}$$
$$+\ \frac{{Q_0'}^2\xi}{4\pi r^4} + \Phi_0'\left(\delta\rho + \delta P\right) + \left(\rho_0 + P_0\right)\delta\Phi' = 0$$

The pulsation equation

Chandrasekhar 1964

$$\sigma^2 e^{\lambda_0-\nu_0}(p_0+\epsilon_0)\,\xi = \frac{4}{r}\frac{dp_0}{dr}\xi - e^{-(\lambda_0+2\nu_0)/2}\frac{d}{dr}\left[e^{(\lambda_0+3\nu_0)/2}\frac{\gamma p_0}{r^2}\frac{d}{dr}\left(r^2 e^{-\nu_0/2}\xi\right)\right]$$

$$+\frac{8\pi G}{c^4}e^{\lambda_0}p_0(p_0+\epsilon_0)\,\xi - \frac{1}{p_0+\epsilon_0}\left(\frac{dp_0}{dr}\right)^2\xi\,.$$

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$$\omega^2 e^{2\Lambda_0-2\Phi_0}(\rho_0+P_0)\,\xi = -e^{-\Lambda_0-2\Phi_0}\left[e^{\Lambda_0+3\Phi_0}\frac{\gamma P_0}{r^2}\left(r^2 e^{-\Phi_0}\xi\right)'\right]'$$

$$-(\rho_0+P_0)\,{\Phi_0'}^2\xi + 4r^{-1}\xi P_0{}' + 8\pi(\rho_0+P_0)\,\xi e^{2\Lambda_0}P_0$$

$$+(\rho_0+P_0)\,r^{-4}\xi e^{2\Lambda_0}Q_0{}^2$$ ← CHARGE TERM

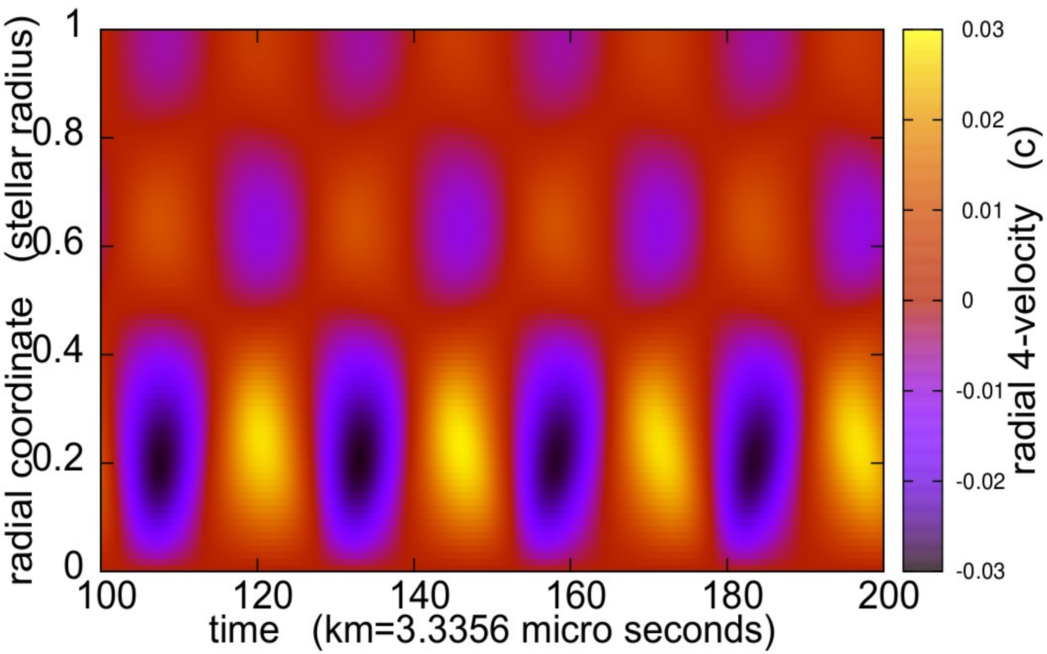
Radial oscillations

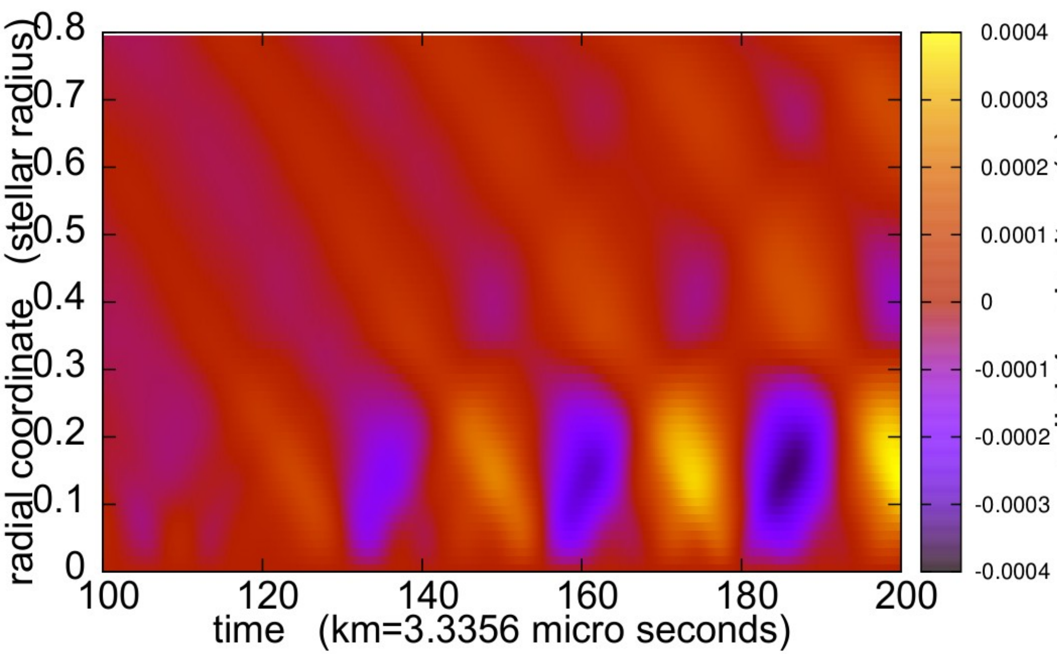
- spherical symmetry preserved

- type of oscillation described by number of nodes

- Sturm-Liouville equation as in Newtonian gravity

- discrete set of frequencies given by boundary conditions: $\xi(r=0)=0, \Delta P(r=R)=0$





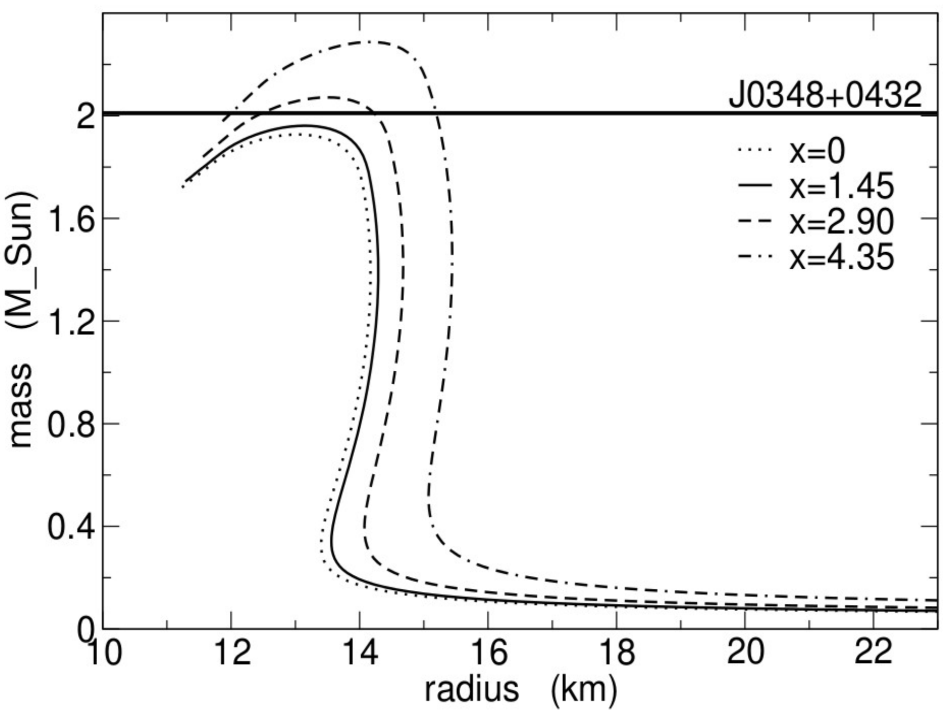
Results for hybrid stars – Gibbs constr.

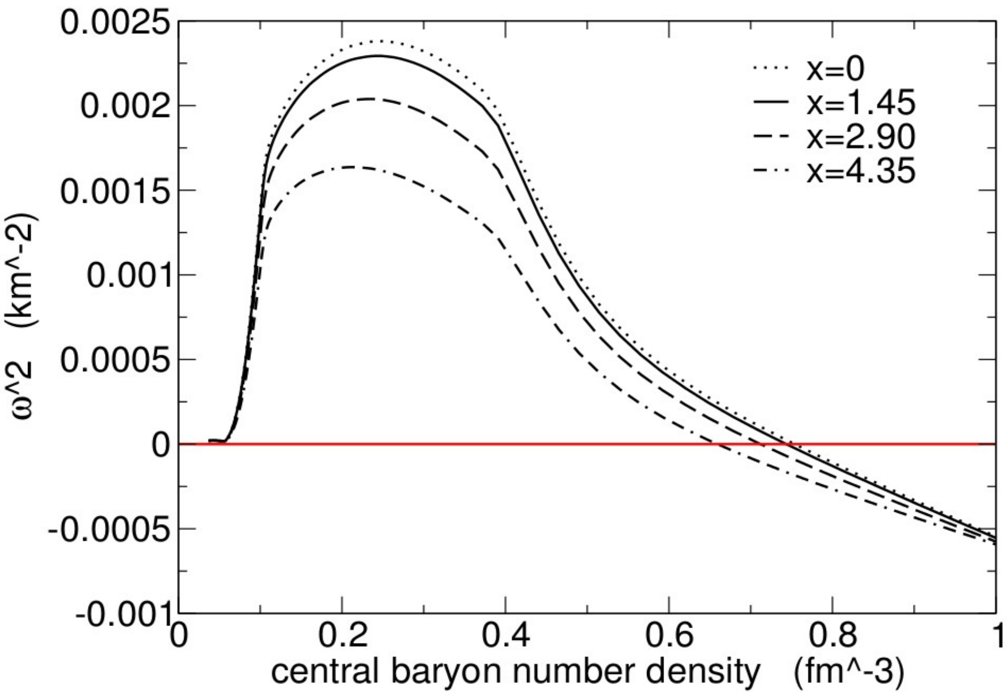
Hadronic phase: relativistic mean-field model, TM1 parameter set

Quark phase: MIT bag model, $m_s = 100\,MeV$, $a_4 = 0.8$, $B^{1/4} = 200\,MeV$

$$x = 10^{19}\frac{N_c}{N_b}$$

M. Alford, M. Braby, M. Paris, S. Reddy, ApJ, 629, 969, (2005)



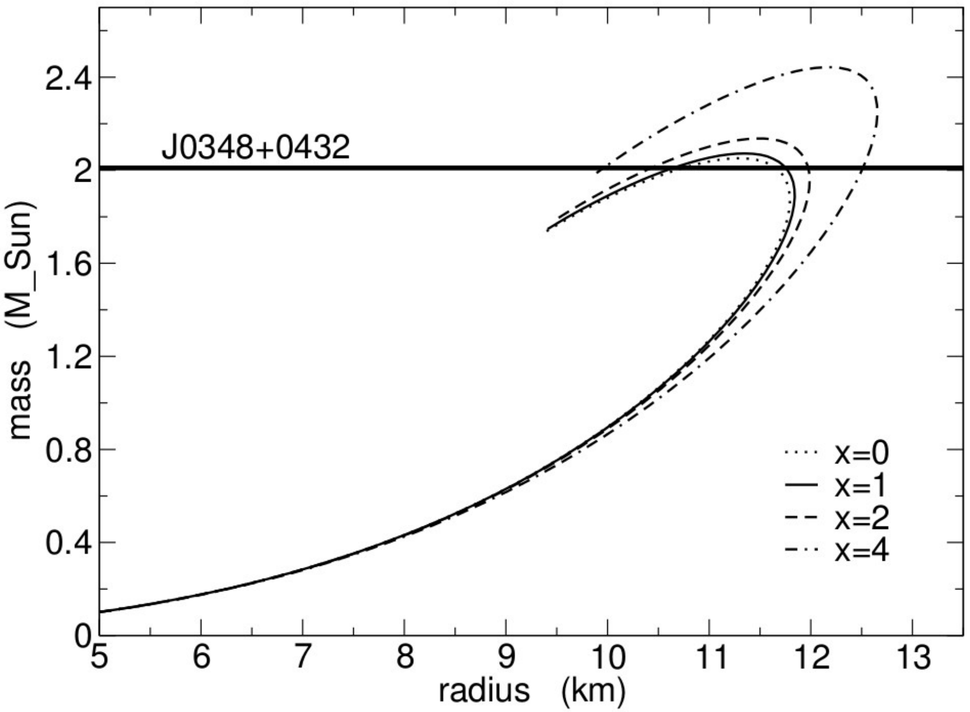


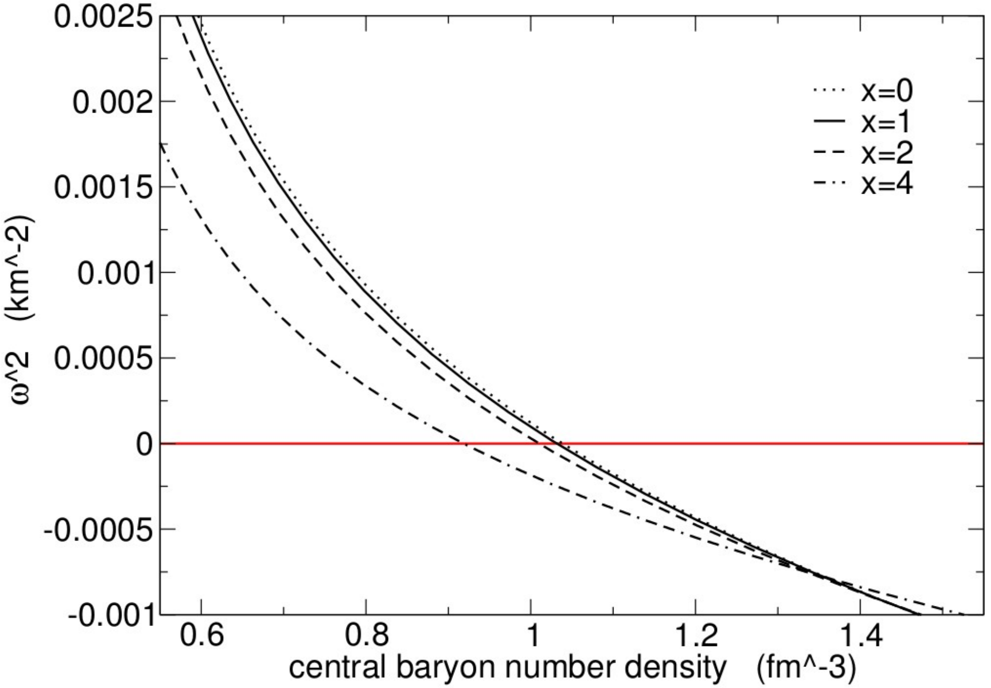
Results for pure strange stars

Quark phase: MIT bag model, $m_s = 100\,MeV$, $a_4 = 1.0$, $B^{1/4} = 140\,MeV$

$$x = 10^{19} \frac{N_c}{N_b}$$

M. Alford, M. Braby, M. Paris, S. Reddy, ApJ, 629, 969, (2005)





Results

- generalization of Chandrasekhar's equation to charge

- enhancement of masses of hybrid and strange stars due to Coulomb repulsion

- lower frequencies of radial eigenmodes at given central density

- A naïve application to stars with sharp density discontinuity contradicts with the *static stability criterion*.

- We constructed hollow spheres in hydrostatic equilibrium with positive pressure gradient.

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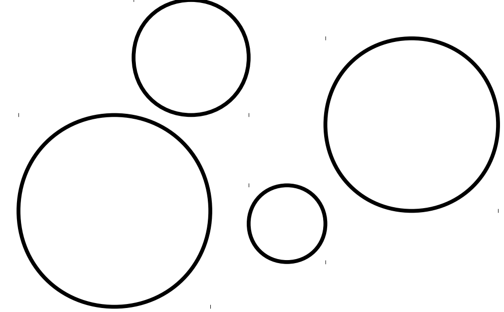

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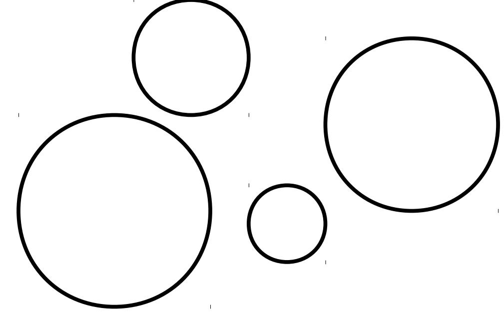

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