Aspects of Transplanckian Scattering

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Outline

Various aspects of transplanckian scattering and some approaches to study this and related problems of QG

- Motivation and what problems we want to solve
- Eikonal approximation and beyond
- Scattering phases and QM of black holes
- New formulation of old problems

Introduction

- EM, Strong, Weak interactions of nature are incorporated within the framework of the Standard Model
- SM is a fundamental and consistent QFT describing nature with high accuracy
- However, with all the successes of the SM, we still don't know how to unite GR with QM.
- Moreover, the problem is not just UV but also IR!

Introduction

In an attempt to reconcile GR with QM one faces multiple difficulties. Three of these problems are:

- **P1.** GR is not renormalizable
- **P2.** Unitarity violation (information paradox) or the classical objects of GR, BH do not allow synthesis with QM
- **P3.** problem in defining gauge invariant observables in GR

P2 and P3 suggest that we might want to abandon the notion of locality in order to find consistent theory of QG

I want to understand the problem of unitarity throught the study of transplanckian collisions which produce BH states

Introduction

Taking into account the problems with locality and renormalizability, along with the lack of knowledge of exact QG, we will attempt to understand this problem by employing S-matrix description of gravity

If $|\alpha\rangle_{\rm in}$ and $|\beta\rangle_{\rm out}$ are asymptotic states in Minkowski space-time, then the scattering matrix is:

$$S_{\alpha\beta} = \langle\beta|\alpha\rangle$$
$$S = 1 + i\mathcal{T}$$

Main properties of the S-matrix:

- Unitarity $\quad S\,S^\dagger = 1$
- Analyticity
- Crossing $\quad \mathcal{T}(s^*, t^*) = \mathcal{T}^*(s, t)$ (hermitean analyticity)

Introduction

- We apply S-matrix approach to study $2 \to 2$ scattering of particles with the c.o.m. energy $\sqrt{s} \gg M_{Pl}$ and will vary momentum transfer $\sqrt{-t}$ or angle θ
- In general $(p_1, p_2) \to (p_3, p_4)$ scattering can be described by analytic function in (s, t), where $s = (p_1 + p_2)^2$, $t = (p_2 - p_3)^2$
- For example, at tree-level: $e^+e^- \to e^+e^-$ and $\pi^+\pi^- \to \pi^+\pi^-$ EM scattering (in case $s \gg m_i^2$ and $-t/s \ll 1$) gives:

$$\mathcal{T} \sim -\alpha\, \frac{s}{t} \approx \frac{\alpha}{\sin^2(\theta/2)}$$

- However, $\phi\phi \to \phi\phi$ gravitational scattering gives:

$$\mathcal{T} \sim -G_N\, \frac{s^2}{t} \approx \frac{G_N s}{\sin^2(\theta/2)}$$

for fixed θ we have $|\mathcal{T}|^2 \to \infty$, therefore, problems with unitarity

Introduction

- In gravity unitarity problem is different from one in Fermi theory and $W_L W_L \to W_L W_L$ scattering that lead to the theoretical discovery of Higgs
- For gravity, unitarity violations occur not only for short scales but also at long distances, not linked to UV-behavior of gravity
- Another difference of GR from the rest of the SM interactions is that GR generates an s-dependent scale - Schwarzschild radius

Parameters of Transplanckian Scattering

The relevant length scales involved in two particle scattering with c.o.m. energy $\sqrt{s}$ are:

- i) $\ell_P \equiv \sqrt{\hbar G_N}$, quantum Planck's length scale
- ii) $R = 2G_N\sqrt{s}$, classical characteristic Schwarzschild radius

$$f_D(r) = 1 - \frac{\kappa_D M}{M_D^{D-2} r^{D-3}}$$

- iii) b - impact parameter of scattering (used often instead of t)
- iv) $\ell_s = \sqrt{\hbar\alpha'}$, string length scale

We are interested in the regime: $\ell_s \ll \min\{\ell_p, R, b\}$.

Parameters of Transplanckian Scattering

To establish a perturbative expansion, it is convenient to work with:

- **a.** $\left(\frac{\ell_P}{b}\right)^2 = \frac{\hbar G_N}{b^2} \equiv \alpha(b)$
- **b.** $\left(\frac{R}{\ell_P}\right)^2 = \frac{4G_N s}{\hbar} \equiv N(s)$
- **c.** $\left(\frac{R}{b}\right)^2 = \frac{\hbar G_N}{b^2} \cdot \frac{4G_N s}{\hbar} = \alpha N$

While α and N are quantum, αN is a classical parameter

Cases:

- For $b \gg R$ or $\alpha N \ll 1$, we have a small angle scattering
- For $b \sim R$ or $\alpha \sim 1/N$, we have a large angle scattering
- For $b \leq R$ or $\alpha N \geq 1$, collision produces a trapped surface, therefore a black hole (Eardley, Giddings '02)

Eikonal Regime

For $b \gg R, \ell_P$ - single graviton exchange dominates the scattering amplitude

- $A_0(s,t) \approx -8\pi G_N s^2/t$ (the leading Born amplitude)

When b becomes smaller, but still much larger than R, the scattering enters the *eikonal regime*, in which case (Amati et al. '90)

$$iA_{eik}(s,t) = 2s \int d^2x_\perp \, e^{-i\mathbf{q}_\perp \mathbf{x}_\perp} \left(e^{i\chi(x_\perp, s)} - 1 \right) ,$$

$$\chi(x_\perp, s) = \frac{1}{2s} \int \frac{d^2k_\perp}{(2\pi)^2} \, e^{i\mathbf{k}_\perp \mathbf{x}_\perp} \, A_0(s, -k_\perp^2) ,$$

where $|\mathbf{x}_\perp| = b$, and $\chi \sim N(s) \ln(b\lambda_{IR})$ is an eikonal phase with λ_{IR} being an IR cutoff

Soft graviton divergences can be eliminated in case $D \geq 5$ for which $\chi \sim G_D s / b^{D-4}$, see, e.g. (Giddings '07).

Eikonal Amplitude

- Eikonal amplitude starts to dominate the Born amplitude, when $\chi \geq 1$
- Elastic amplitude A_0 grows too fast with energy $\rightarrow$ breaks unitarity
- To maintain unitarity the full sum over ladder and cross-ladder diagrams is required
- The crossover between Born and eikonal approximatinos occur when $b \sim (G_D s)^{1/(D-4)}$ for $D > 4$

Eikonal Amplitude

- Decreasing b, so that $b > R$, we will enter a regime, where intermediate graviton exchange diagrams among different legs of the ladder will start to contribute
- Simplest H-diagram $\sim G_N^3 s^2 \sim G_N s (R/b)^2 \sim \alpha N^2$
- These H-diagrams will be described by $(R/b)^2 = \alpha N$ classical expansion (Amati et al '90). In this case, the S matrix can be written as:

$$\ln S(\alpha, N) = iNF(\alpha N, \ln N, \ln \alpha) \left[1 + \mathcal{O}(\hbar)\right] \ ,$$

  where F is some function of R/b, $\ln s$ and $\ln b$ respectively
- Entering the region where $\alpha N \sim 1$, we have to add all H-type of diagrams, which in certain respect is a classical problem

What is the Meaning of Imaginary N?

- The scattering amplitude of one particle, seen as a test body, in the background metric of the other particle, an Aichelburg-Sexl metric was computed by 't Hooft (same can be obtained by eikonal approximation)
- When $\theta^2 \sim -t/s \ll 1$, the important contribution comes from $b \approx R/\theta \gg R$. In this case, the small-angle amplitude can be computed by just extending the previous result up to $b = 0$. As a result:

$$iA_{eik}(s,t) = \frac{2i\pi sN}{-t} \frac{\Gamma(1 - iN/4)}{\Gamma(1 + iN/4)} \left(\frac{4\lambda_{IR}^2}{-t} \right)^{-iN/4} ,$$

  where the first fraction is just the Born amplitude. (We also assumed $m \ll \sqrt{s}$, where m is the mass of the scatteried particle.)
- The complex N-poles of this amplitude are analogous to those in the Coulomb scattering, due to the $1/r$ potential.
- Therefore, 't Hooft poles originate from the singularity at $b = 0$, that we attempted to ignore, without consistently treating the $b < R$ region

The Black Hole Ansatz in the Large N limit

- The partial-wave expansion of the scattering amplitude (in D dimensions) can be written as ;

$$A(s,t) = \psi_h s^{2-D/2} \sum_{\ell=0}^{\infty} (\ell + h) C_\ell^h (1 + 2t/s) f_\ell(s) \ ,$$

$$f_\ell(s) = \frac{1}{2i} \left(e^{2i\delta_\ell(s) - 2\beta_\ell(s)} - 1 \right) \ ,$$

where $h = (D-3)/2$, $\psi_h \equiv 2^{4h+3} \pi^h \Gamma(h)$

- δ_ℓ is the phase shift, and β_ℓ is an absorption parameter.
- $\ell = Eb/2$ and $L \equiv ER_S/2 = (G_D E^{D-2})^{1/(D-3)}$ also $\ell \leq L$ when BH is expected to be formed

The Black Hole Ansatz in the Large N limit

- As one can check, partial-wave amplitudes satisfy unitarity condition: $\mathrm{Im}\, f_\ell \geq |f_\ell|^2$ for each ℓ.
- In theory with mass gap the expansion converges in a certain region (Lehman-ellipse) and one can derive Froissart-Martin bound $\sigma_{tot} < c(\ln s)^{D-2}$ (if there is a mass gap)
- Since gravity is massless, we have problems!
- For long distances (large b) or Born approximation $\mathcal{A} \sim 1/\theta^2$, therefore, integral in

$$f_\ell(s) = s^{D/2-2} \int_0^\pi d\theta \;\sin^{D-3}\theta \; C_\ell^h(\cos\theta) A(s,t)$$

converges only when $D > 4$

The Black Hole Ansatz in the Large N limit

- In the eikonal regime, and when $D = 4$, one has: $\delta_\ell^{eik}(s) \sim \frac{4G_N s}{\hbar} \sim N(s)$ and $\beta_\ell^{eik}(s) = 0$
- The contribution from the graviton bremsstrahlung of soft gravitons can be estimated to give the following correction to the absorptive part: $\beta_\ell^{soft} \sim \alpha N^2(s)$ (same order as H-diagram)
- According to Giddings and Srednicki, if $S_{BH}(s, b) \sim N(s)$ is the black hole entropy, and $R \leq b$ or $\ell \leq N$, then

$$\delta_\ell^{GS}(s) \sim \pi e^{S_{BH}} \;, \quad \beta_\ell^{GS} \sim S_{BH}$$

- As a result, the elastic and absorptive cross sections can be shown to be approximately equal, and:

$$\sigma_{el} \approx \sigma_{abs} \sim \sum_{\ell=0}^{N} (\ell + h) \sim N^2$$

Number of BH Microstates

Is the number of BH state resonances really $\sim e^{S_{BH}}$?

- Consider multiparticle collision with the c.o.m. energy E that results to the production of black hole (BH) states
- Assume that the initial state is $|E, a\rangle_{in}$, where a run through the number of states that can be produced from the collapse of matter of energy E

$$\rightarrow a \text{ takes } \exp\{E^{\frac{(D-2)(D-1)}{D(D-3)}}\} \text{ integer values}$$

- Since not all of the energy goes into the production of BH, the produced state will be a mixture of BH state and radiation:

$$|E, a\rangle_{in} = \sum_{M,I,i} \mathcal{A}(E, M)_{aIi} |M, I\rangle |E - M, i\rangle_{rad} \ ,$$

where $|M, I\rangle$ and $|E - M, i\rangle_{rad}$ are orthonormal states in the Hilbert spaces of BH, $\mathcal{H}_{BH}$, and radiation, $\mathcal{H}_R$, correspondingly.

Number of BH Microstates

The dimensions of $\mathcal{H}_{BH}$ and $\mathcal{H}_R$ are:

$$\dim\mathcal{H}_{BH} = \exp\{M^{\frac{D-2}{D-3}}\}\ , \quad \dim\mathcal{H}_R = \exp\{E^{\frac{D-2}{D-3}}\}\ .$$

Moreover, as mentioned above,

$$\dim\{|M\rangle_{ai}\} = \exp\{E^{\frac{(D-2)(D-1)}{D(D-3)}}\} \ll \dim\mathcal{H}_{BH}\ ,$$

therefore, not all BH states can be accessed by the collision, since $E/2 < M < E$

This suggests that the correct phase shift should be different and depend not on S_{BH} but on S_{bh}

Summary

Different Regimes of Scattering when $b \gg \ell_s$

- Coulomb (or Born) regime: $b \geq (G_D s)^{1/(D-4)}$
- Eikonal regime: $(G_D s)^{1/(D-4)} \geq b \geq R_S \sim (G_D \sqrt{s})^{1/(D-3)}$
- Strong dynamics: $b \leq R_S$

Questions:

- What is the number of BH states accessed by the collision?
- Can this knowledge help us to resolve the Information Paradox?