Electromagnetic two-point functions and Casimir densities for a conducting plate in de Sitter spacetime

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Motivation

- De Sitter (dS) spacetime is among the most interesting backgrounds
  - Maximally symmetric solution of Einstein's equations with a positive cosmological constant
  - Numerous physical problems are exactly solvable on this background
- In accordance with the inflationary cosmology scenario, in the early stages of the cosmological expansion our Universe passed through a phase in which the geometry is well approximated by dS spacetime
- During an inflationary epoch, quantum fluctuations in the inflaton field introduce inhomogeneities which play a central role in the generation of cosmic structures from inflation

Motivation

- Recent observations of high redshift supernovae, galaxy clusters and cosmic microwave background have indicated that at the present epoch the Universe is accelerating and can be approximated by a world with a positive cosmological constant

- If the Universe were to accelerate indefinitely, the Standard Cosmology would lead to an asymptotic dS Universe

- It is therefore important to investigate physical effects in dS spacetime for understanding both the early Universe and its future

Quantum vacuum

- The interaction of a fluctuating quantum field with the background gravitational field leads to vacuum polarization

- The boundary conditions imposed on the field give rise to another type of vacuum polarization. They can arise because of the presence of boundaries having different physical natures:
  - macroscopic bodies in QED
  - extended topological defects
  - horizons and branes in higher-dimensional models

- Boundary conditions modify the zero-point modes of a quantized field and, as a result, forces arise acting on constraining boundaries

➡ The Casimir effect

The particular features of the forces depend on the nature of a quantum field, the type of spacetime manifold, the boundary geometry, and the specific boundary conditions imposed on the field

Background geometry

- We consider the change in the properties of the electromagnetic vacuum induced by the presence of a perfectly conducting plate in the background of (D + 1)-dimensional dS spacetime.

- dS spacetime is described by the line element:

$$ds^2 = dt^2 - e^{2t/\alpha} \sum_{l=1}^{D} (dz^l)^2,$$

the parameter α is related to the positive cosmological constant Λ by the formula

$$\alpha^2 = D(D-1)/(2\Lambda)$$

conformal time τ: defined as $\tau = -\alpha e^{-t/\alpha}, \ -\infty < \tau < 0$

In terms of this coordinate the metric tensor takes a conformally flat form:

$$g_{ik} = (\alpha/\tau)^2 \mathrm{diag}(1, -1, ..., -1)$$

Electromagnetic modes and two-point functions

- All properties of the quantum vacuum are encoded in two-point functions

- Mode-sum formula for boundary-free dS spacetime

$$\langle A_l(x)A_m(x')\rangle_0 = \sum_{\sigma=1}^{D-1}\int d\mathbf{k}\, A_{(\sigma\mathbf{k})l}(x)A^*_{(\sigma\mathbf{k})m}(x')$$

$\langle\cdots\rangle_0$ → Vacuum expectation value (VEV)

↑ Complete set of normalized mode functions for the vector potential

We assume that the field is prepared in the Bunch-Davies vacuum state

- Complete set of mode functions for Bunch-Davies vacuum state

$$A_{(\sigma\mathbf{k})l}(x) = C\epsilon_{(\sigma)l}\eta^{D/2-1}H^{(1)}_{D/2-1}(k\eta)e^{i\mathbf{k}\cdot\mathbf{z}},\ \ l=1,\ldots,D$$

$\sigma = 1,\ldots,D-1$ correspond to different polarizations, and $\eta = |\tau|$.

For $D=3$, by taking into account that $H^{(1)}_{1/2}(z) = -i\sqrt{2/(\pi x)}e^{iz}$, we see that the mode functions coincide with the corresponding functions in Minkowski spacetime

This is a consequence of the conformal invariance of the electromagnetic field in $D=3$.

Two-point functions

- The two-point function for the vector potential is presented in the form

$$\langle A_l(x)A_m(x')\rangle_0 = \frac{\delta_{lm}\alpha^{3-D}\Gamma(D-1)}{(4\pi)^{(D-1)/2}\Gamma((D+1)/2)}F\left(D-1,1;\frac{D+1}{2};z\right) - \frac{\alpha^{3-D}(\eta\eta')^{D/2-1}}{4(2\pi)^{D-2}}\int d\mathbf{k}\,\frac{k_l k_m}{k^2}H^{(2)}_{D/2-1}(k\eta)H^{(1)}_{D/2-1}(k\eta')e^{i\mathbf{k}\cdot\Delta\mathbf{z}}$$

The second term does not contribute to the two-point functions for the electromagnetic field tensor and it will not be needed in the further consideration

By using two-point function for the vector potential, one finds the expressions for the two-point functions of the electromagnetic field tensor

$$\langle F_{pl}(x)F_{qm}(x')\rangle_0$$

Geometry with a conducting plate

- We consider the change in the properties of the electromagnetic vacuum induced by the presence of a perfectly conducting plate placed at $z^D = 0$

- We consider the region $z^D > 0$. Boundary condition $n^{\nu_1} {}^*F_{\nu_1 \cdots \nu_{D-1}} = 0$,
$^*F_{\nu_1 \cdots \nu_{D-1}}$ dual to $F_{\mu\nu}$, n^μ is the normal to the plate

- Mode-functions for the vector potential are given by the expressions

$$
\begin{aligned}
A_{(\sigma \mathbf{k})l}(x) &= iC_b \epsilon_{(\sigma)l} \eta^{D/2-1} H^{(2)}_{D/2-1}(k\tau) \sin(k_D z^D) e^{i\mathbf{k}_\parallel \cdot \mathbf{z}_\parallel}, \\
A_{(\sigma \mathbf{k})D}(x) &= C_b \epsilon_{(\sigma)D} \eta^{D/2-1} H^{(2)}_{D/2-1}(k\tau) \cos(k_D z^D) e^{i\mathbf{k}_\parallel \cdot \mathbf{z}_\parallel},
\end{aligned}
$$

where $l = 1, \ldots, D-1$, $\mathbf{k}_{||} = (k_1, \ldots, k_{D-1})$, $\mathbf{z}_{||} = (z^1, \ldots, z^{D-1})$ and $k = \sqrt{k_D^2 + \mathbf{k}_{||}^2}$.

- Two-point functions are presented in the decomposed form

$$
\langle A_l(x) A_m(x') \rangle = \langle A_l(x) A_m(x') \rangle_0 + \langle A_l(x) A_m(x') \rangle_\mathrm{b},
$$

Part induced by the plate

Plate induced parts in two-point functions

$$\langle A_l(x)A_m(x')\rangle_{\mathrm{b}} = -\langle A_l(x)A_m(x'_-)\rangle_0,$$
$$\langle A_l(x)A_D(x')\rangle_{\mathrm{b}} = \langle A_l(x)A_D(x'_-)\rangle_0,$$

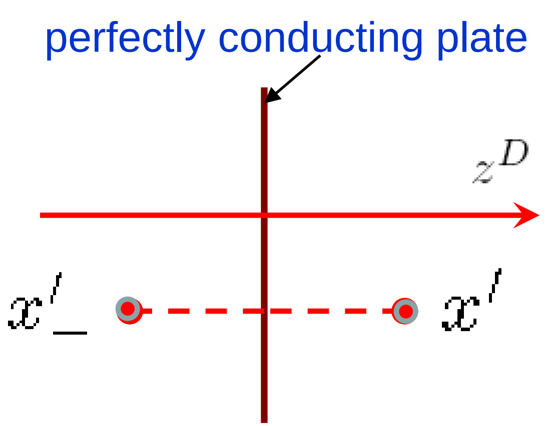
$l = 1, \ldots, D,\ m = 1, \ldots, D-1,$, x'_- is the image of x' with respect to the plate:

$$x'_- = (\tau', z^{1\prime}, \ldots, z^{D-1\prime}, -z^{D\prime}).$$

Plate induced part in the two-point function of the electromagnetic field tensor

$$\langle F_{pl}(x)F_{qm}(x')\rangle_{\mathrm{b}} = -\langle F_{pl}(x)F_{qm}(x'_-)\rangle_0,$$
$$\langle F_{pl}(x)F_{Dm}(x')\rangle_{\mathrm{b}} = \langle F_{pl}(x)F_{Dm}(x'_-)\rangle_0,$$

with $p, l = 0, 1, \ldots, D,$ and $q, m = 0, 1, \ldots, D-1.$



Electric field squared

- VEV of the electric field squared

$$\langle E^2\rangle = -g^{00}g^{lm}\lim_{x'\to x}\langle F_{0l}(x)F_{0m}(x')\rangle$$

- VEV is decomposed as:

$$\langle E^2\rangle = \langle E^2\rangle_0 + \langle E^2\rangle_{\rm b}.$$

- Plate induced part

$$\langle E^2\rangle_{\rm b} = \frac{D-1}{2B_D\alpha^{D+1}}\left[2(1-y)\partial_y - D + 2\right]G_D(y) \qquad y = 1-(z^D/\eta)^2$$

with the notations $B_D = (4\pi)^{(D-1)/2}\Gamma((D+3)/2)$, $G_D(z) = 2\Gamma(D-1)F\left(D-1,3;\frac{D+3}{2};z\right)$

The plate-induced part depends on z^D and $\eta = |\tau|$. in the combination z^D/η

For *D = 3* $\Rightarrow$ $\langle E^2\rangle_{\rm b} = 3(\alpha z^D/\eta)^{-4}/(4\pi)$

One can obtain this result from the corresponding result in Minkowski spacetime by the standard conformal transformation.

Energy-momentum tensor

- Another important characteristic of the vacuum state is the VEV of the energy-momentum tensor. In addition to describing the local structure of the vacuum state, it acts as the source of gravity in the quasi classical Einstein's equations and plays an important role in modelling self-consistent dynamics involving the gravitational field.

- VEV of the energy-momentum tensor is decomposed into the boundary-free and plate-induced parts:

$$\langle T_\mu^\nu \rangle = \langle T_\mu^\nu \rangle_0 + \langle T_\mu^\nu \rangle_{\rm b}.$$

- For points outside the plate the renormalization is required for the boundary-free part only. Because of the maximal symmetry of the background geometry, the latter is proportional to the metric tensor:

$$\langle T_\mu^\nu \rangle_0 = {\rm const} \cdot \delta_\mu^\nu.$$

Plate-induced energy-momentum tensor

- For the VEVs of the diagonal components one finds (no summation over $l=1,\ldots,D-1$)

$$\langle T_0^0\rangle_{\rm b} = \frac{\alpha^{-D-1}}{A_D}\{[2(1-y)\partial_y - D+2]G_D(y) - [2(1-y)\partial_y + D-4]F_D(y)\},$$

$$\langle T_D^D\rangle_{\rm b} = \frac{\alpha^{-D-1}}{A_D}[2(1-y)\partial_y - D][F_D(y) - G_D(y)],$$

$$\langle T_l^l\rangle_{\rm b} = -\frac{\alpha^{-D-1}}{A_D}\frac{D-3}{D-1}\left\{\left[2(1-y)\partial_y - (D-4)\frac{D-1}{D-3}\right]G_D(y) + \left[2(1-y)\partial_y + (D-4)\frac{D-1}{D-3} - 4\right]F_D(y)\right\},$$

where $A_D = (4\pi)^{(D+1)/2}(D+1)\Gamma\left(\frac{D-1}{2}\right)$

- Off-diagonal component

$$\langle T_0^D\rangle_{\rm b} = \frac{\alpha^{-D-1}}{A_D}\frac{4z^D}{\eta}[(1-y)\partial_y - 2]F_D(y)$$

corresponds to the energy flux along the direction normal to the plate

Asymptotics

- VEV of the energy-momentum tensor depends on the coordinates z^D and η in the combination z^D/η. This is a consequence of the maximal symmetry of the background spacetime and of the Bunch-Davies vacuum state. For $D=3$ we can see that $\langle T_\mu^\nu \rangle_{\rm b} = 0$.

- Let us consider the behavior of the VEV of the energy-momentum tensor at small and large distances from the plate.
  - At small distances: $z^D/\eta \ll 1$ (compared with the de Sitter curvature scale)

$$\langle T_l^l \rangle_{\rm b} \approx -\frac{\eta}{z^D} \langle T_0^D \rangle_{\rm b} \approx \frac{D-1}{(z^D/\eta)^2} \langle T_D^D \rangle_{\rm b} \approx -\frac{(D-3)\,(D-1)\Gamma((D+1)/2)}{2(4\pi)^{(D+1)/2}\alpha^{D+1}(z^D/\eta)^{D+1}}$$

  - At large distances from the plate, for $D>4$, $D=4$, $|y| \gg 1$

$$\langle T_0^0 \rangle_b \approx \frac{D}{D-4} \langle T_l^l \rangle_b \approx -\frac{2^{D-4} D(D-1)\Gamma(D/2-1)}{(4\pi)^{D/2+1}\alpha^{D+1}(z^D/\eta)^4},$$

$$\langle T_0^D \rangle_b \approx \frac{2^{D-2}(D-1)\Gamma(D/2-1)}{(4\pi)^{D/2+1}\alpha^{D+1}(z^D/\eta)^5}.$$

For D = 4 for the stresses one has

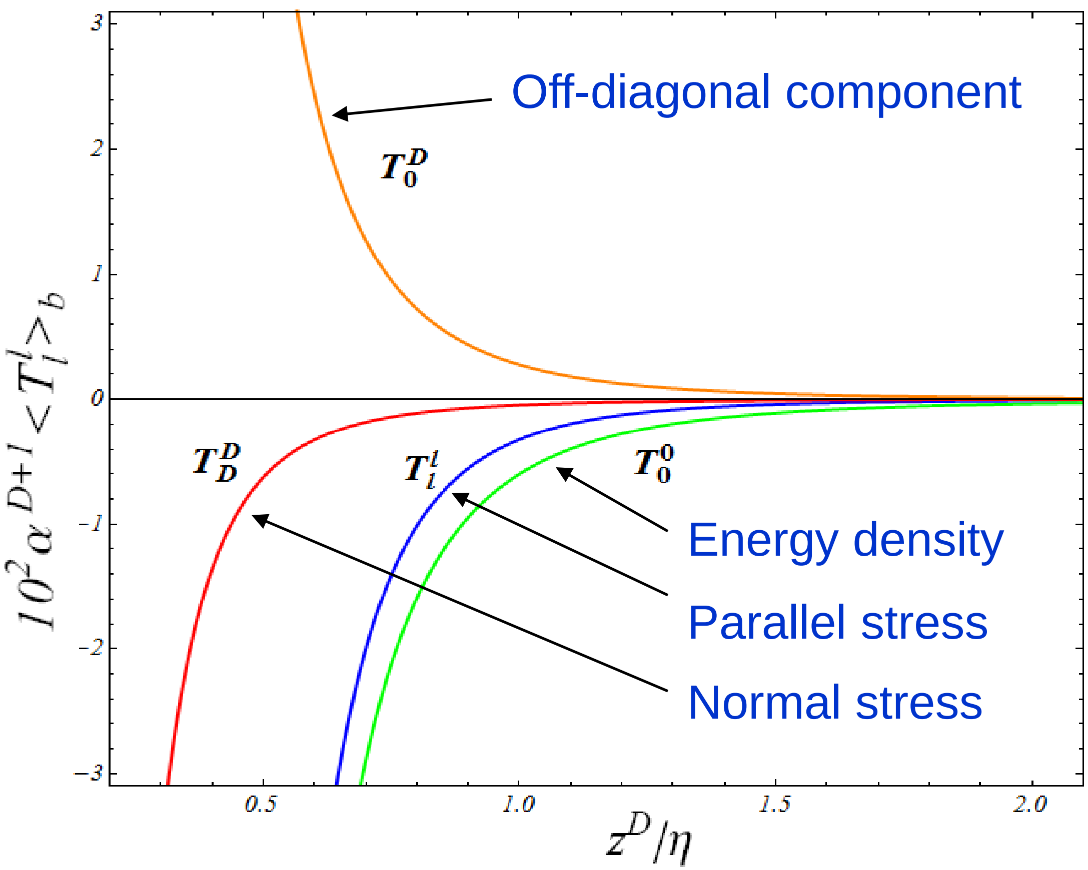
$$\langle T_l^l \rangle_{\rm b} = -\frac{3\alpha^{-5}\ln(z^D/\eta)}{16\pi^3 (z^D/\eta)^6}$$

At large distances the stresses are isotropic

Numerical results

Plate-induced parts in the components of the vacuum energy-momentum tensor as functions of z^D/η for $D = 4$

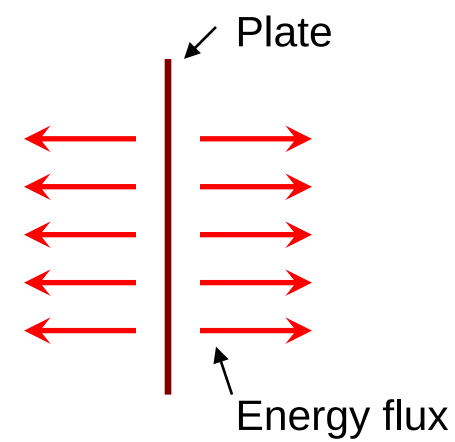
Proper distance from the plate measured in units of dS curvature radius



Diagonal components are negative, whereas the off-diagonal component corresponding to the energy flux is positive

This is the case for other values of D .

Energy flux is directed from the plate



Conclusions

- Two-point function for the electromagnetic field tensor is evaluated in (D + 1)-dimensional de Sitter spacetime in the presence of a conducting plate assuming that the field is prepared in Bunch-Davies vacuum state

- For $3 \leqslant D \leqslant 8$ the plate-induced part in the VEV of the electric field squared is positive everywhere, whereas for $D \geqslant 9$ it is positive near the plate and negative at large distances.

- Plate-induced part in the vacuum energy density is negative and the effective pressures are positive

- VEV of the energy-momentum tensor contains a nonzero off-diagonal component which corresponds to the energy flux along the direction normal to the plate

- Results obtained can be used for the investigation of the vacuum characteristics in the geometry of two parallel plates including the Casimir forces acting on the plates