



BRANE WORLD SCENARIOS:QUASAR LUMINOCITY

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PLAN

- Why brane world scenarios are important
- The field equations in 5D and 4D
- The idea and purpose (expansion of metric)
- CFP solutions
- Quasars and their luminosities



THE TALK IS BASED ON

- **Casadio–Fabbri–Mazzacurati black strings and braneworld-induced quasars luminosity corrections**

Roldão da Rocha, A Piloyan, A M Kuerten and C H Coimbra-Araújo

Roldão da Rocha *et al* 2013 *Class. Quantum Grav.* **30**
045014 [doi:10.1088/0264-9381/30/4/045014](https://doi.org/10.1088/0264-9381/30/4/045014)

- <http://arxiv.org/abs/1301.4483>



- More quantitatively:

$$F_{\text{EM}} = \frac{q_1 q_2}{r^2}$$

$$F_{\text{gravity}} = G_N \frac{m_1 m_2}{r^2}$$

Even for the heaviest elementary particles (e.g., W bosons, top quarks)

$$F_{\text{gravity}} \sim 10^{-32} F_{\text{EM}}$$

- The Hierarchy Problem: Why is gravity so weak?
Maybe it isn't...



EXTRA DIMENSIONS

- Suppose photons are confined to $D=4$, but gravity propagates in n extra dims of size L .

$$r \ll L, F_{\text{gravity}} \sim 1/r^{2+n} \quad r \gg L, F_{\text{gravity}} \sim 1/r^2$$



PROJECTED 4D EQUATIONS

$$^{(5)}G_{AB} = -\Lambda_5 \, ^{(5)}g_{AB} + \kappa_5^2 \, ^{(5)}T_{AB}$$

$$R_{ABCD} = ^{(5)}R_{EFGH} g_A^E g_B^F g_C^G g_D^H + 2K_{A[C} K_{D]B},$$

$$K_{\mu\nu} = -\frac{1}{2}\kappa_5^2 \left(T_{\mu\nu} + \frac{1}{3}(\lambda - T) g_{\mu\nu} \right)$$

$$G_{\mu\nu} = -\Lambda g_{\mu\nu} + \kappa^2 T_{\mu\nu} + 6\frac{\kappa^2}{\lambda} \mathcal{S}_{\mu\nu} - \mathcal{E}_{\mu\nu} + 4\frac{\kappa^2}{\lambda} \mathcal{F}_{\mu\nu}$$



JUNCTION CONDITION

- Lets define

$$T_{\mu\nu}^{\text{brane}} = T_{\mu\nu} - \lambda g_{\mu\nu}$$

$$g_{\mu\nu}^+ - g_{\mu\nu}^- = 0,$$

$$K_{\mu\nu}^+ - K_{\mu\nu}^- = -\kappa_5^2 \left[T_{\mu\nu}^{\text{brane}} - \frac{1}{3} T^{\text{brane}} g_{\mu\nu} \right]$$

- Junction cond.



$$K_{\mu\nu} = -\frac{1}{2} \kappa_5^2 \left[T_{\mu\nu} + \frac{1}{3} (\lambda - T) g_{\mu\nu} \right]$$



EACH MEMBER IN THE EQUATION

- High energy correction

$$\frac{|\kappa^2 \mathcal{S}_{\mu\nu} / \lambda|}{|\kappa^2 T_{\mu\nu}|} \sim \frac{|T_{\mu\nu}|}{\lambda} \sim \frac{\rho}{\lambda}.$$

$$\mathcal{S}_{\mu\nu} = \frac{1}{12} T T_{\mu\nu} - \frac{1}{4} T_{\mu\alpha} T^\alpha{}_\nu + \frac{1}{24} g_{\mu\nu} [3 T_{\alpha\beta} T^{\alpha\beta} - T^2]$$

- Vanishing if there is no brane-bulk energy exchange

$$\mathcal{F}_{\mu\nu} = {}^{(5)}T_{AB} g_\mu{}^A g_\nu{}^B + \left[{}^{(5)}T_{AB} n^A n^B - \frac{1}{4} {}^{(5)}T \right] g_{\mu\nu}$$

- Projection Weyl tensor, KK

$$\nabla^\mu \mathcal{E}_{\mu\nu} = \frac{6\kappa^2}{\lambda} \nabla^\mu \mathcal{S}_{\mu\nu}$$

$$\mathcal{E}_{\mu\nu} = {}^{(5)}C_{ACBD} n^C n^D g_\mu{}^A g_\nu{}^B$$



CORRECTIONS TO THE KK

- Note

$$V(r) = \frac{GM}{c^2 r} \left(1 + \frac{2\ell^2}{3r^2} + \dots \right)$$

- and

$$K_{\mu\nu} = \frac{1}{2} \mathcal{L}_{\mathbf{n}} g_{\mu\nu} \qquad \Lambda_4 = \frac{\kappa_5^2}{2} \left(\frac{1}{6} \kappa_5^2 \lambda^2 + \Lambda \right)$$

$$G_5 = G \ell_{\text{Planck}} \qquad \ell_{\text{Planck}} = \sqrt{G\hbar/c^3}$$

$$\kappa_4^2 = \frac{1}{6} \lambda \kappa_5^4$$



THE MAIN IDEA OF THE WORK

- Suppressing of Hawking radiation
- Taylor expansion of the metric along the extra dimension by using extrinsic curvature

$$\mathcal{L}_n K_{\mu\nu} = K_{\mu\alpha} K^\alpha{}_\nu - \mathcal{E}_{\mu\nu} - \frac{1}{6} \Lambda_5 g_{\mu\nu}$$

- The calculation of the Black Hole horizon

$$\sqrt{g_{\theta\theta}(x,0)} = R,$$

$$g_{rr}^{-1} = 0$$

- The analysis of the luminosity corrections

$$L(\ell) = \eta(\ell) \dot{M} c^2$$

$$\eta = \frac{GM}{6c^2 R_S}$$



TEYLOR EXPANSION OF VACUM ON THE BRANE (I AM SORRY☺)

$$\begin{aligned}
 g_{\mu\nu}(x,y) = & g_{\mu\nu} - \frac{1}{3}\kappa_5^2\lambda g_{\mu\nu}|y| + \left[\frac{1}{6} \left(\frac{1}{6}\kappa_5^4\lambda^2 - \Lambda_5 \right) g_{\mu\nu} - \mathcal{E}_{\mu\nu} \right] y^2 \\
 & - \frac{1}{6} \left(\left(\frac{193}{36}\lambda^3\kappa_5^6 + \frac{5}{3}\Lambda_5\kappa_5^2\lambda \right) g_{\mu\nu} + \kappa_5^2 R_{\mu\nu} \right) \frac{|y|^3}{3!} + \\
 & + \left[\frac{1}{6}\Lambda_5 \left(\left(R - \frac{1}{3}\Lambda_5 - \frac{1}{18}\lambda^2\kappa_5^4 \right) + \frac{7}{324}\lambda^4\kappa_5^8 \right) g_{\mu\nu} + \left(R - \Lambda_5 + \frac{19}{36}\lambda^2\kappa_5^4 \right) \mathcal{E}_{\mu\nu} \right. \\
 & \left. + \frac{1}{6} \left(\frac{37}{36}\lambda^2\kappa_5^4 - \Lambda_5 \right) R_{\mu\nu} + \mathcal{E}^{\alpha\beta} R_{\mu\alpha\nu\beta} \right] \frac{y^4}{4!} + \dots
 \end{aligned}$$



WHAT ARE WE DEPARTING FROM

$$g_{\mu\nu}dx^\mu dx^\nu = -N(r)dt^2 + A(r)dr^2 + r^2d\Omega^2$$

- Schwarzschild?

$$H(r) = 1 - \frac{2GM}{c^2 r} + \psi(r) \quad \psi(r) \approx -\frac{4GM\ell^2}{3c^2 r^3}$$

- 4D solutions with corrected $1/r^3$ term

1. Cassadio-Fabbri-Mazzacurati I?

2. Cassadio-Fabbri-Mazzacurati II?



METRIC ON BRANE

- Cassadio-Fabbri-Mazzacurati I ($\beta = 5/4$)

$$N(r) = 1 - \frac{2GM}{c^2 r}$$

$$A(r) = \frac{1 - \frac{3GM}{2c^2 r}}{\left(1 - \frac{2GM}{c^2 r}\right) \left(1 - \frac{GM}{2c^2 r} (4\beta - 1)\right)}$$

- Cassadio-Fabbri-Mazzacurati II ($\beta = 3/2$)

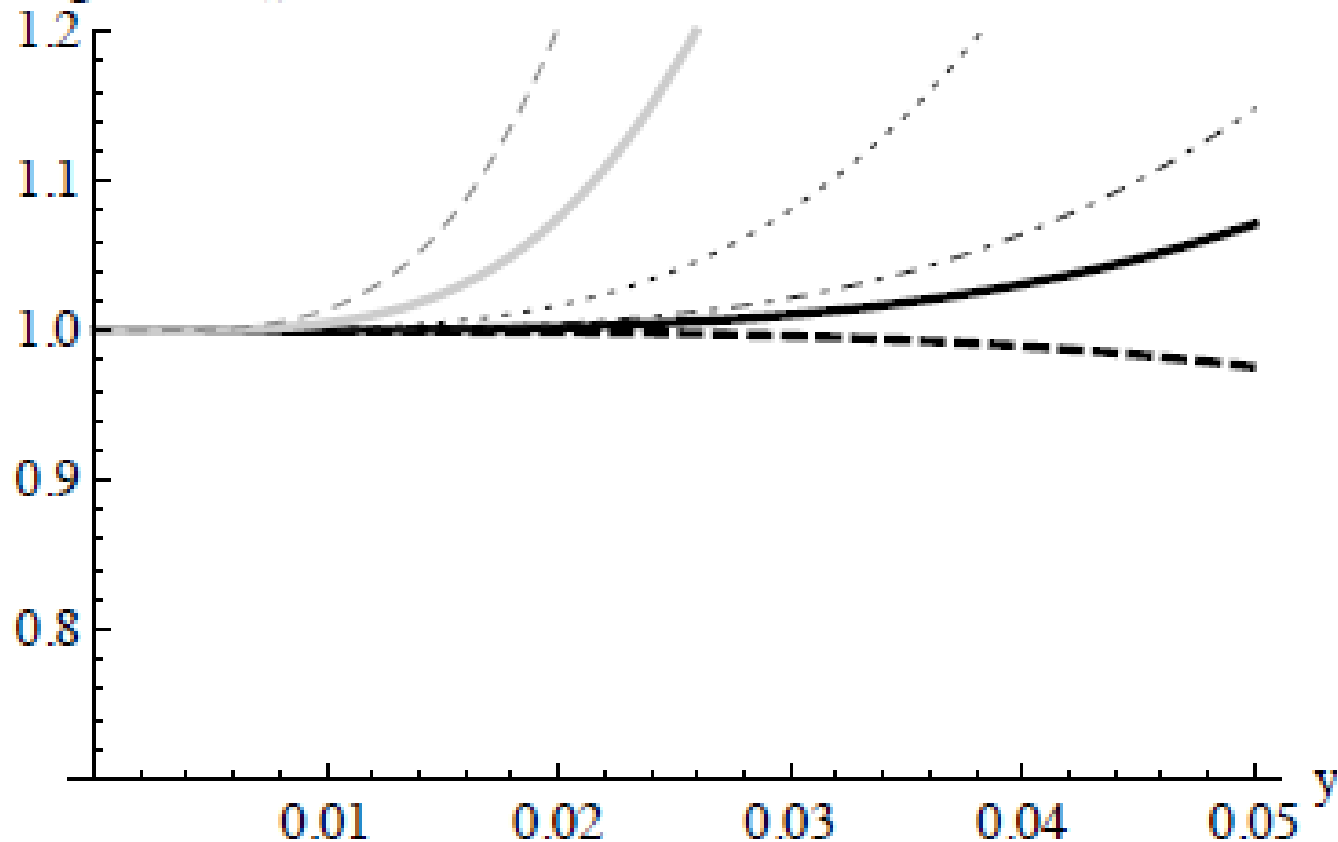
$$N(r) = 1 - \frac{2GM}{c^2 r} + \frac{2G^2 M^2}{c^4 r^2} (\beta - 1)$$

$$A(r) = \frac{1 - 3GM/2c^2 r}{\left(1 - \frac{2GM}{c^2 r}\right) \left(1 - \frac{GM}{2c^2 r} (4\beta - 1)\right)}$$



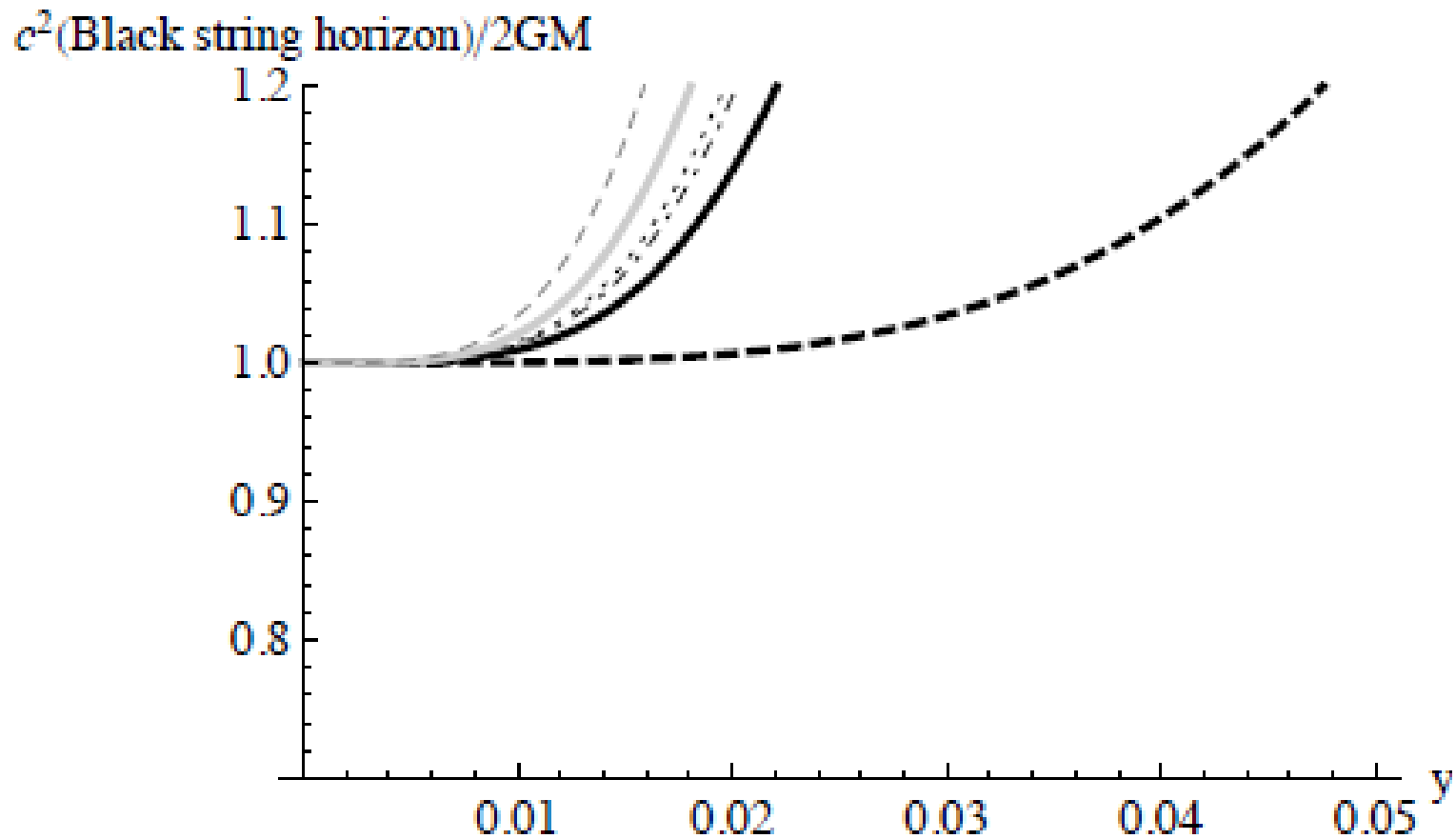
CASSADIO-FABBRI-MAZZACURATI I (HORIZON CORRECTIONS)

$c^2(\text{Black string horizon})/2GM$



CASSADIO-FABBRI-MAZZACURATI II

(HORIZON CORRECTIONS)



LUMINOSITY CORRECTIONS

- As mentioned above:

$$\Delta L = \frac{GM}{6c^2} (R_{\text{brane}}^{-1} - R_S^{-1}) \dot{M} c^2 = \frac{1}{12} \left(\frac{R_S}{R_{\text{brane}}} - 1 \right) \dot{M} c^2$$

- where

$$R_{\text{brane}} = \sqrt{g_{\theta\theta}(r = R_S, y)}$$

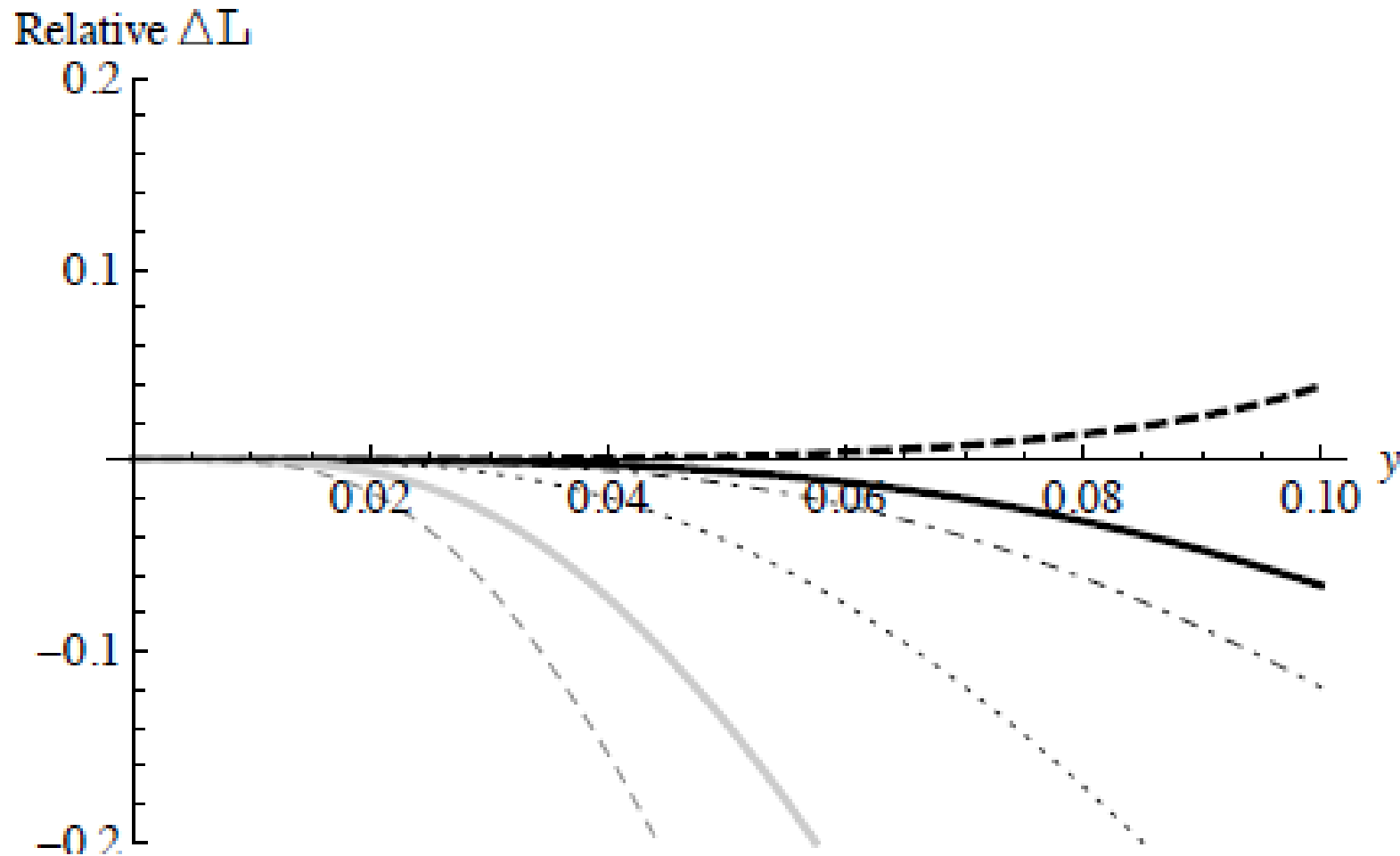
- For quasars $\dot{M} \approx 2.1 \times 10^{19} \text{kg s}^{-1}$

$$L = L_{\text{Edd}} \sim 1.2 \times 10^{45} \left(\frac{M}{10^7 M_{\odot}} \right) \text{erg s}^{-1}$$

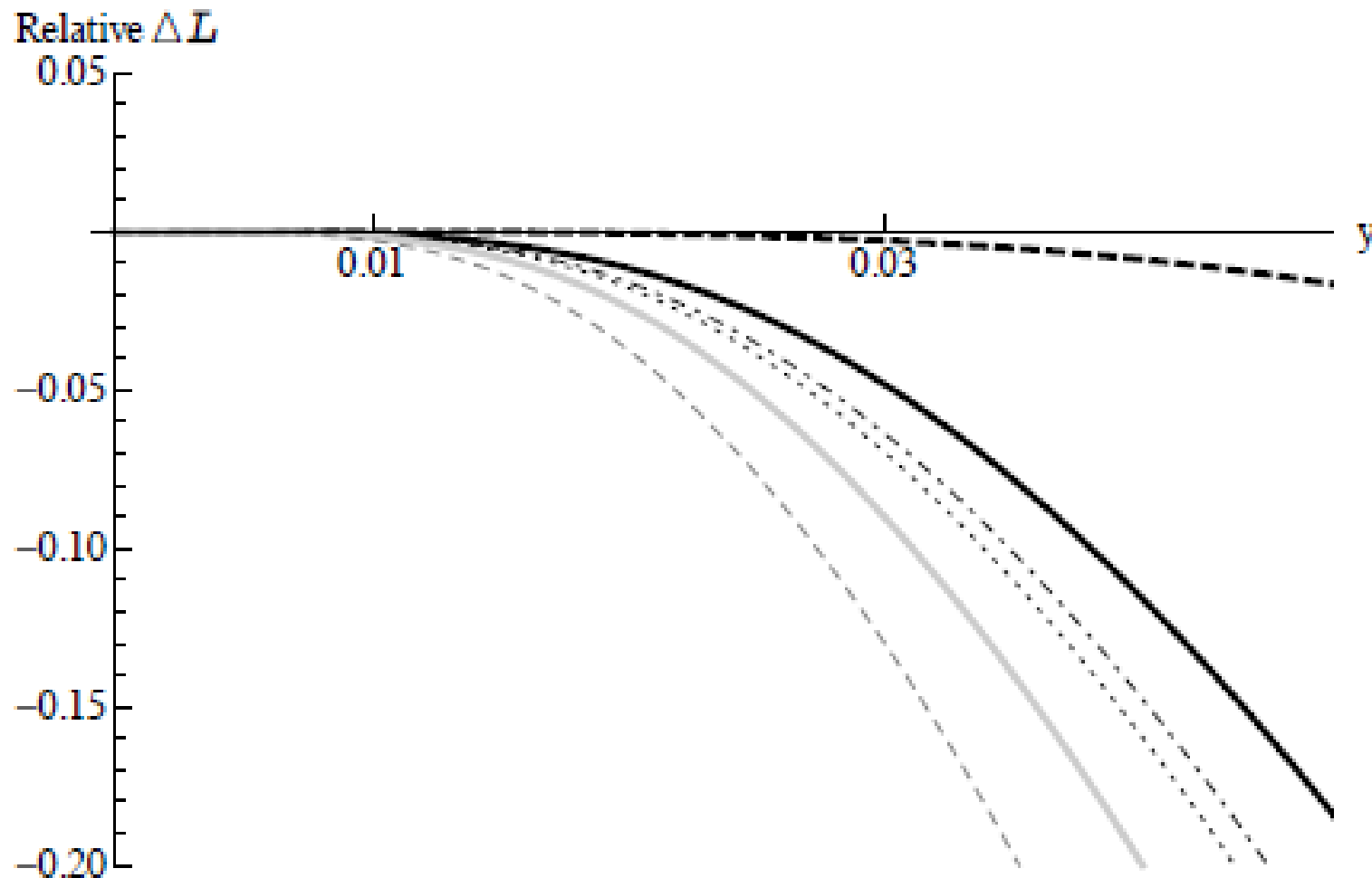
$$L \sim 10^{47} \text{erg s}^{-1}$$



CASSADIO-FABBRI-MAZZACURATI I (LUMINOSITY CORRECTIONS)



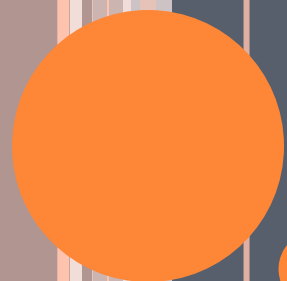
CASSADIO-FABBRI-MAZZACURATI II (LUMINOSITY CORRECTIONS)



CONCLUSION

- It is shown how the horizon vary due to the extra dimension
- Thus, Illustrated Luminosity variation only by extra dimensional effects





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