Dense matter equation of state in strong magnetic field

Monika Sinha

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

Institute for Theoretical Physics
Goethe University Frankfurt am Main

Collaborators

Banibrata Mukhpadhyay (IISc, Bangalore)
Armen Sedrakian (Goethe University, Frankfurt)
Xu-Guang Huang (Indiana University, Bloomington)

Motivation

- Soft Gamma Repeaters (SGRs) and Anomalous X-ray Pulsars (AXPs) :
  - Very different from ordinary X-ray bursters and pulsars.
- $L_{peak} \sim 10^{41}$ ergs/s, $L_x \sim 10^{35}$ ergs/s.
- Association with supernova remnants, large luminosity, strong pulsation, :
  - These objects are neutron stars (NS).
- Increasing number of common properties:
  - Close relationship between SGRs and AXPs.
- No correlation between energy and time interval since the previous burst:
  - Trigger of the bursts is not accretion.
- AXPs: Softer spectrum, secular spin down rate:
  - Neither accretion powered, nor rotation powered.

Motivation

- Current model: Magnetar – Neutron stars with strong surface magnetic field $\sim 10^{14}$ - 10^{15} G.
- The field in the interior of the NS may have higher value.
- The density of matter in the NS interior is $\sim 10^{14}$ gm/cm^3.
- In this connection keeping in mind the existence of magnetars, we are interested in studying the properties of highly dense matter in the presence of strong magnetic field.

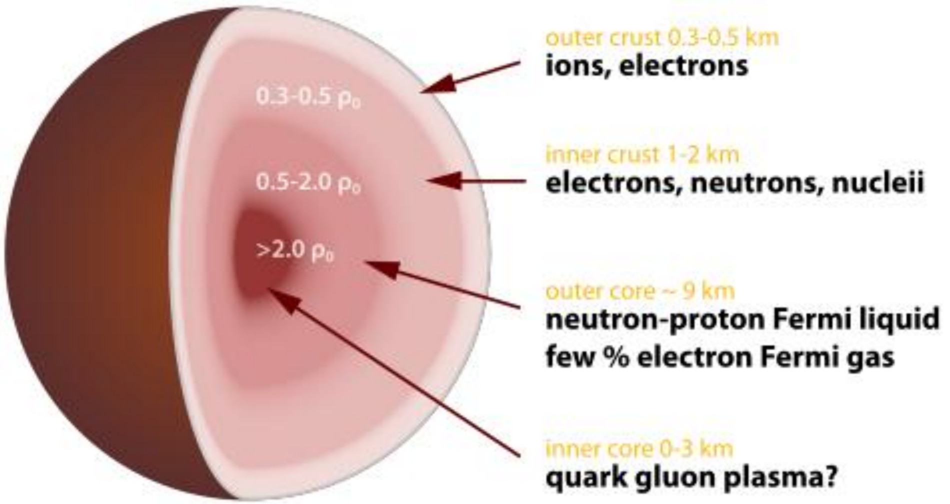
Structure of Neutron star

- Outer crust: ions + electrons
  a few hundred meters

- Inner crust: electrons + neutrons + neutron rich nuclei
  about one kilometer

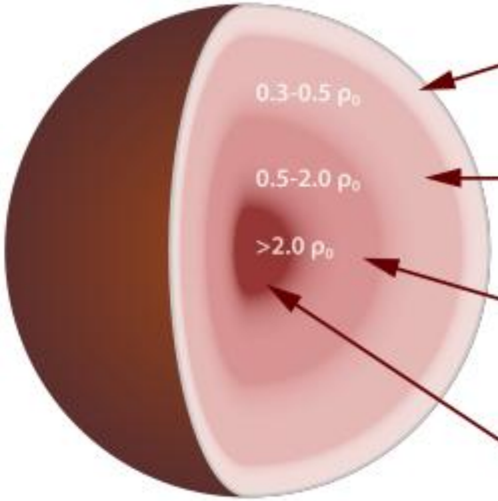
- Outer core: **neutrons** + protons + electrons + muons

- Inner core: ? number of possibilities



Structure of Neutron star

- ❖ Nuclear matter
- ❖ Hyperon matter
- ❖ Pion condensate
- ❖ Kaon condendate
- ❖ Quark matter



Neutrons in the core could also be in superfluid state.

Effect of magnetic field on matter

Energy momentum tensor of the system:

$$T^{\mu\nu} = T_m^{\mu\nu} + T_f^{\mu\nu}$$

with

$$T_m^{\mu\nu} = \varepsilon_m u^\mu u^\nu - P(g^{\mu\nu} - u^\mu u^\nu) + \frac{1}{2}(M^{\mu\lambda} F_\lambda^\nu + M^{\nu\lambda} F_\lambda^\mu)$$

$$T_f^{\mu\nu} = -\frac{1}{4\pi} F^{\mu\lambda} F_\lambda^\nu + \frac{1}{16\pi} g^{\mu\nu} F^{\rho\sigma} F_{\rho\sigma}$$

$\varepsilon_m \longrightarrow$ Matter energy density

$P \longrightarrow$ Thermodynamic pressure

$M^{\mu\lambda} \longrightarrow$ Magnetization tensor

Effect of magnetic field on matter

- We assume the direction of magnetic field as the z-direction of the system.

$$\vec{\mathcal{B}} = \mathcal{B}\hat{z} \qquad \vec{E} = 0$$

- With the above choice of magnetic field and in the rest frame of matter

$$T_m^{\mu\nu} = \begin{bmatrix} \varepsilon_m & 0 & 0 & 0 \\ 0 & P - M\mathcal{B} & 0 & 0 \\ 0 & 0 & P - M\mathcal{B} & 0 \\ 0 & 0 & 0 & P \end{bmatrix},$$

$$\mathcal{B}^\mu \mathcal{B}_\mu = -\mathcal{B}^2$$

$$T_f^{\mu\nu} = \begin{bmatrix} \frac{\mathcal{B}^2}{8\pi} & 0 & 0 & 0 \\ 0 & \frac{\mathcal{B}^2}{8\pi} & 0 & 0 \\ 0 & 0 & \frac{\mathcal{B}^2}{8\pi} & 0 \\ 0 & 0 & 0 & -\frac{\mathcal{B}^2}{8\pi} \end{bmatrix}.$$

$\mathcal{B} \longrightarrow$ Magnetic field

$M \longrightarrow$ Magnetization per unit volume

$$\varepsilon = \varepsilon_m + \frac{\mathcal{B}^2}{8\pi} \qquad P_\perp = P - M\mathcal{B} + \frac{\mathcal{B}^2}{8\pi} \qquad P_\parallel = P - \frac{\mathcal{B}^2}{8\pi}$$

Particle in magnetic field

- In magnetic field the motion of a charged particle is Landau quantized in the plane perpendicular to the direction of magnetic field.

- The kinetic energy of any charged particle is

$$E_n = \sqrt{p_z^2 + m^2 + 2ne|Q|B}.$$

- The momentum in the plane perpendicular direction of the field is quantized:

$$\mathbf{p} = (p_z, \mathbf{p}_\perp) \qquad |\mathbf{p}_\perp| = 2ne|Q|B$$

- The phase space volume modifies as:

$$\int d^3p \;\longrightarrow\; e|Q|B\sum_n \int_{-p(n)}^{p(n)} dp_z \int_0^{2\phi} d\phi \qquad p(n) \;=\; \sqrt{p_F^2 - 2ne|Q|B}.$$

Hyperonic matter

Our Model

- In our study of effect of strong magnetic field on the highly dense matter we consider hypernuclear matter.

Non-linear Boguta-Bodmer-Walecka model

$$
\begin{aligned}
\mathcal{L}_B = & \sum_B \bar{\psi}_B \left(i\gamma_\mu D^\mu - m_B + g_{\sigma B}\sigma - g_{\omega B}\gamma_\mu \omega^\mu - g_{\rho B}\gamma_\mu \boldsymbol{\tau}_B \cdot \boldsymbol{\rho}^\mu \right) \psi_B \\
& + \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - U(\sigma) \\
& - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4}\boldsymbol{\rho}_{\mu\nu} \cdot \boldsymbol{\rho}^{\mu\nu} + \frac{1}{2} m_\rho^2 \boldsymbol{\rho}_\mu \cdot \boldsymbol{\rho}^\mu \\
& + \sum_{l=e,\mu} \bar{\psi}_l \left(i\gamma_\mu D^\mu - m_l \right) \psi_l - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} .
\end{aligned}
$$

The kinetic energy density for charged particle is given by

$$\varepsilon_k = e|Q|B\sum_n (2-\delta_{n_1,0})\int_{-p(n)}^{p(n)} dp_z \int_0^{2\phi} d\phi\, E_n$$

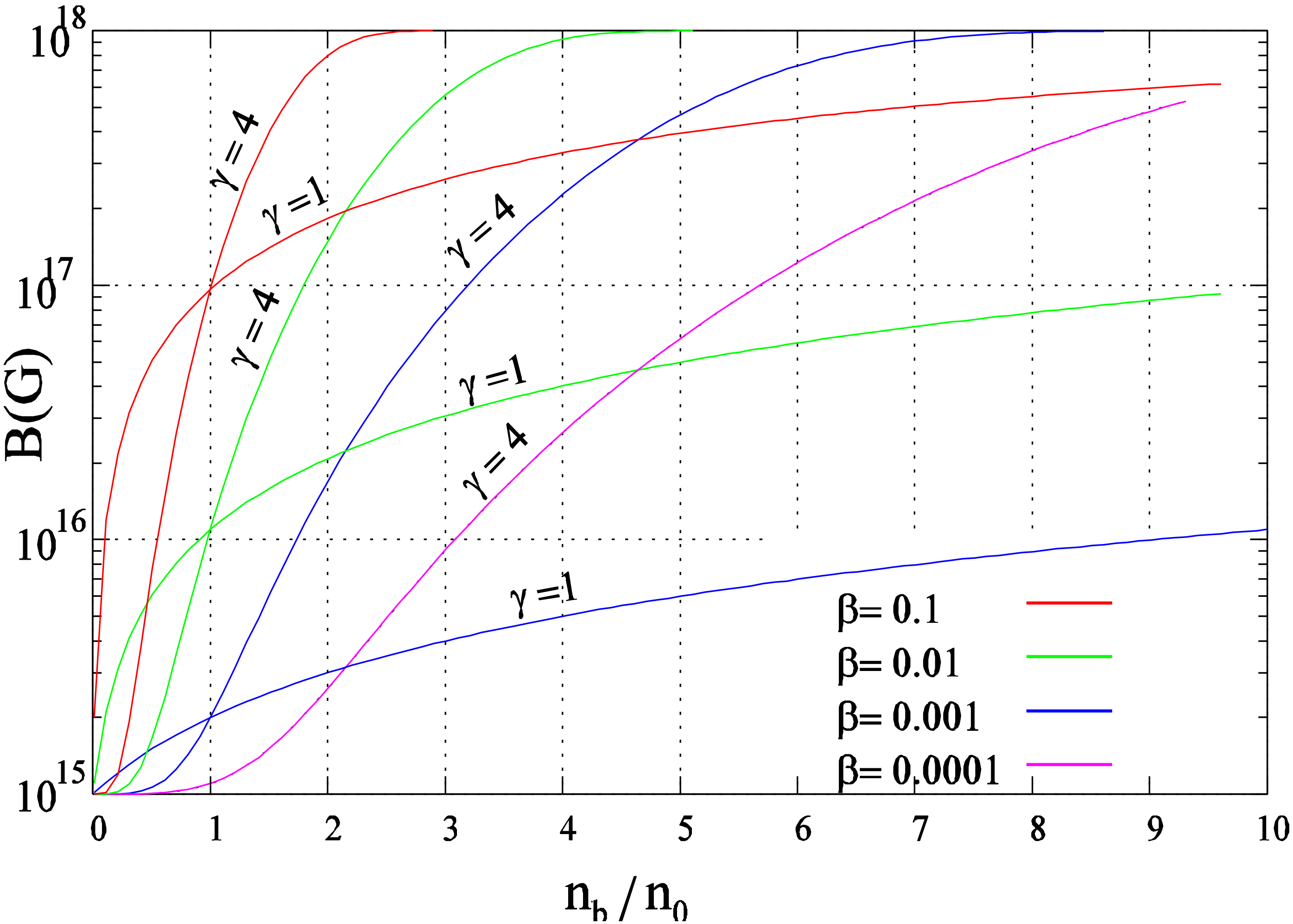
$$\varepsilon_k = \int_0^{p_F} d^3p E_p$$

$$E_p = \sqrt{p^2+m^2}$$

$$\begin{aligned}
\varepsilon_m &= \frac{1}{8\pi^2}\sum_B \left(2p_F^{(B)}\mu_B^{*^3} - p_F^{(B)} m_B^{*^2}\mu_B^* - m_B^{*^4}\ \ln\frac{p_F^{(B)}+\mu_B^*}{m_B^*}\right) \\
&+ \frac{e|Q|B}{(2\pi)^2}\sum_{B'}\sum_{n=0}^{n_{max}}(2-\delta_{n,0})\left[p_{B'}(n)\mu_{B'}^* \ + \ (m_{B'}^{*^2}+2ne|Q|B)\ln\left(\frac{p_{B'}(n)+\mu_{B'}^*}{\sqrt{m_{B'}^{*^2}+2ne|Q|B}}\right)\right] \\
&+ \frac{e|Q|B}{(2\pi)^2}\sum_{l=e,\mu}\sum_{n=0}^{n_{max}}(2-\delta_{n,0})\left[p_l(n)\mu_l \ + \ (m_l^2+2ne|Q|B)\ln\left(\frac{p_l(n)+\mu_l}{\sqrt{m_l^2+2ne|Q|B}}\right)\right] \\
&+ \frac{1}{2}m_\sigma^2\sigma^2 + U(\sigma) + \frac{1}{2}m_\omega^2\omega^{0^2} + \frac{1}{2}m_\rho^2\rho_3^{0^2},
\end{aligned}$$

$$P = \sum_B \mu_B n_B + \sum_l \mu_l n_l - \varepsilon_m.$$

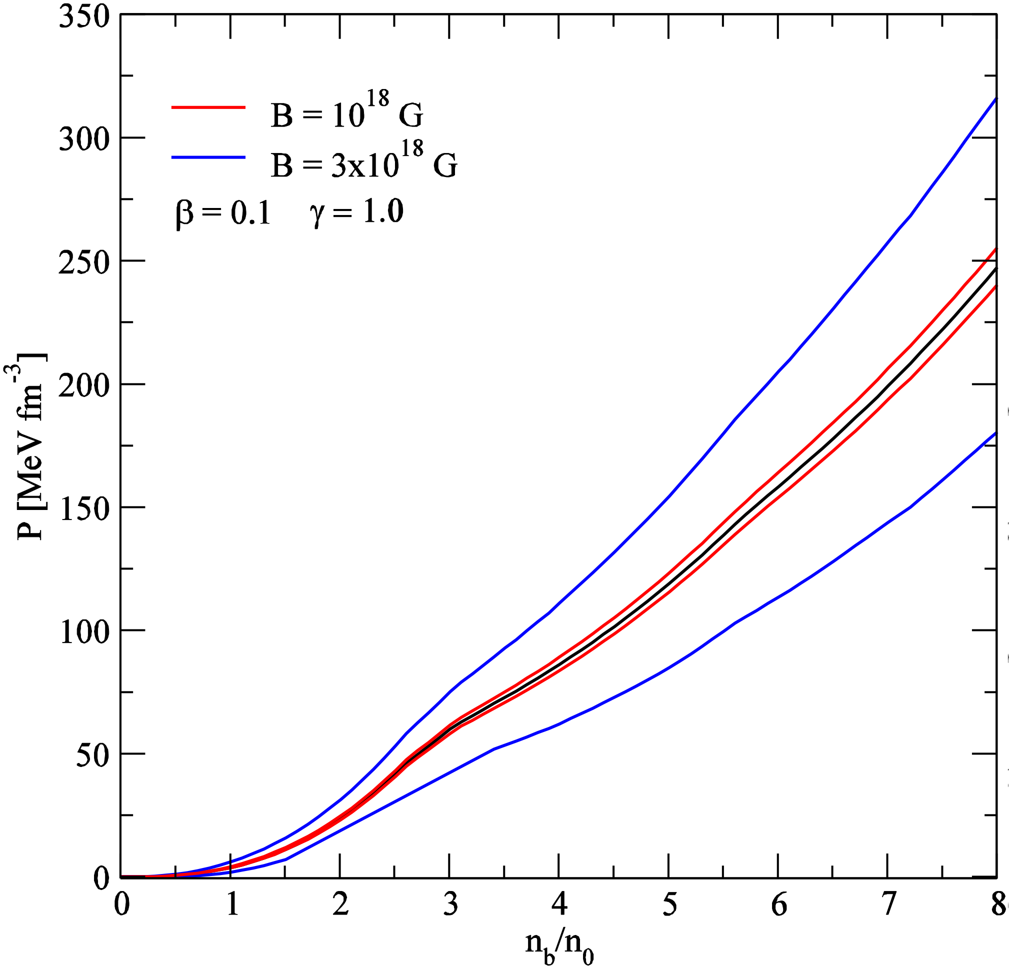
$$\mathcal{B}(n_b/n_0) = \mathcal{B}_s + \mathcal{B}_c \left\{ 1 - e^{-\beta \left(\frac{n_b}{n_0} \right)^{\gamma}} \right\}$$



$\mathcal{B}_c$ = 10^{18} G

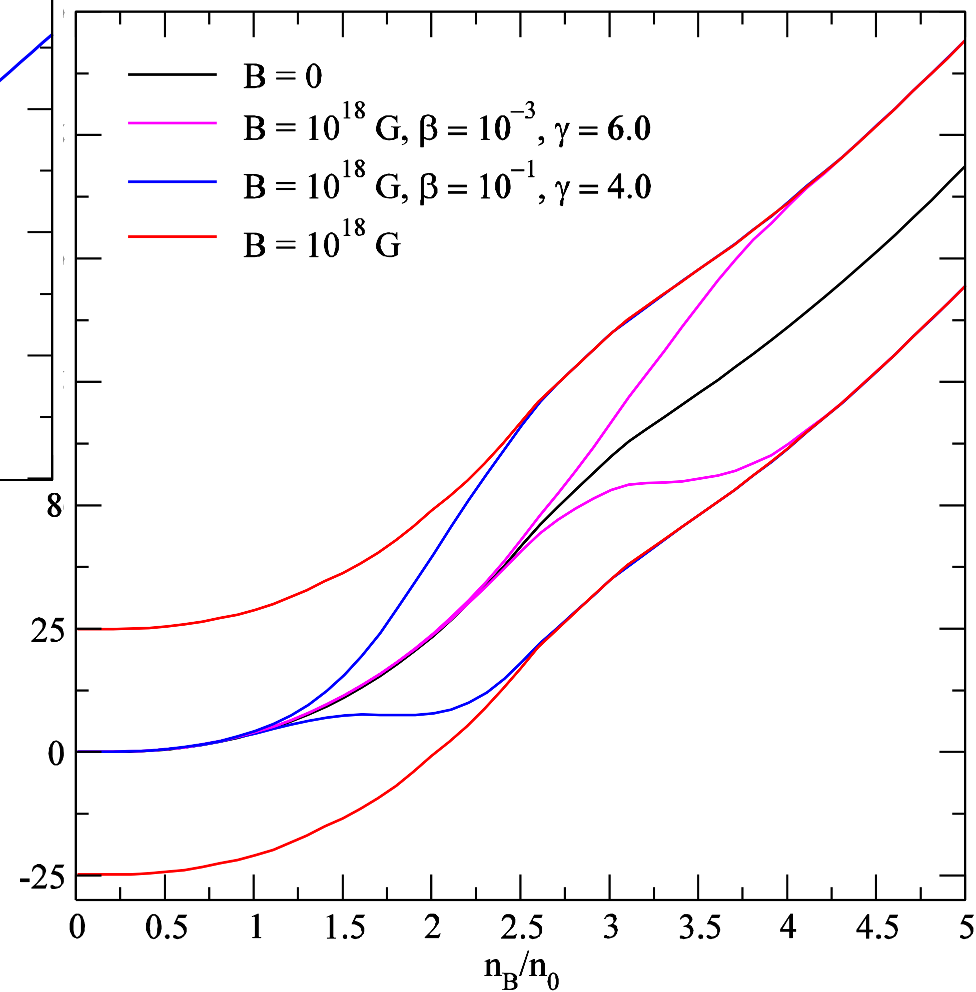
$\mathcal{B}_s$ = 10^{15} G

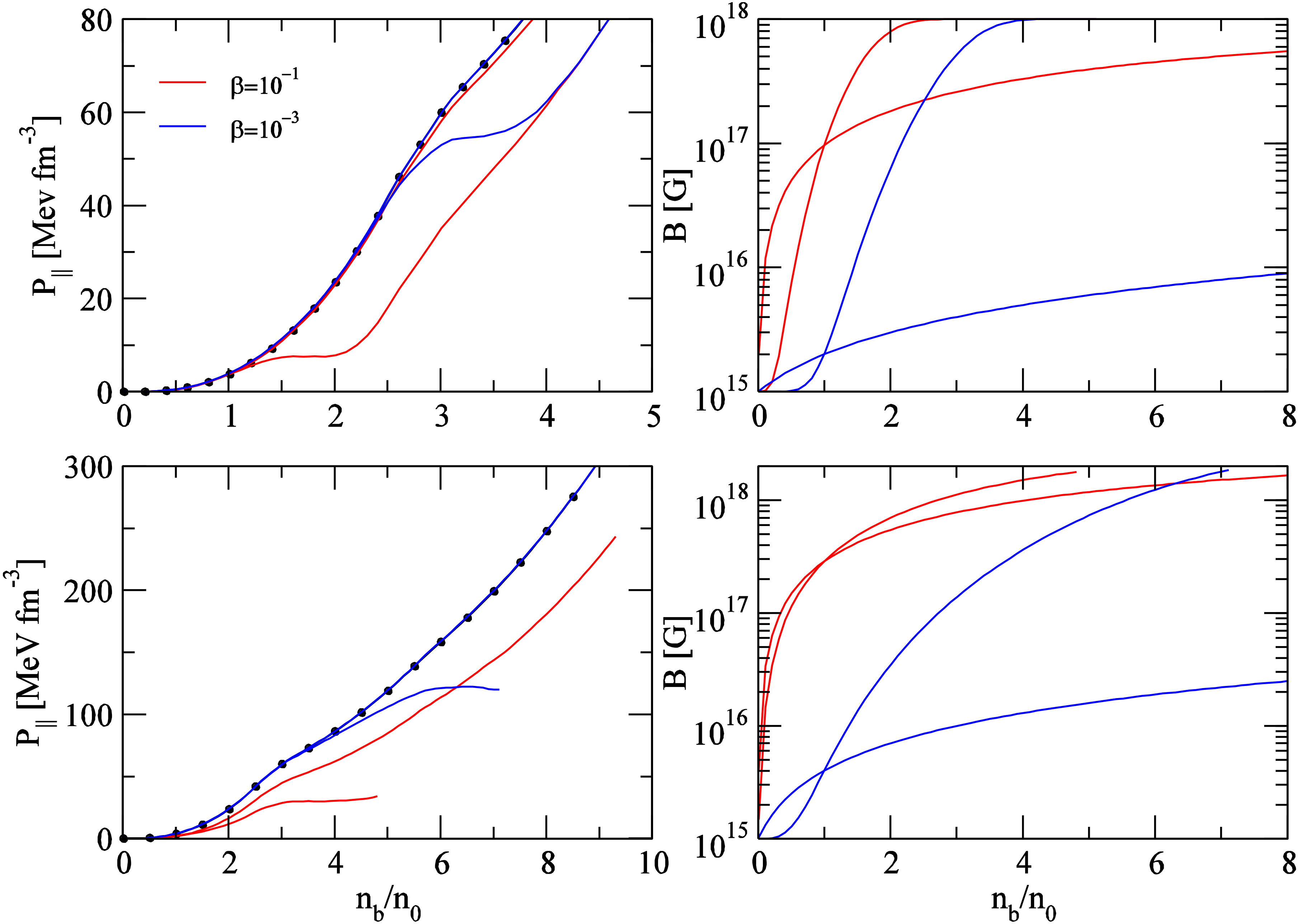
Bandyopadhyay, Chakrabarty and Pal, PRL **79**, 2167 (1998)



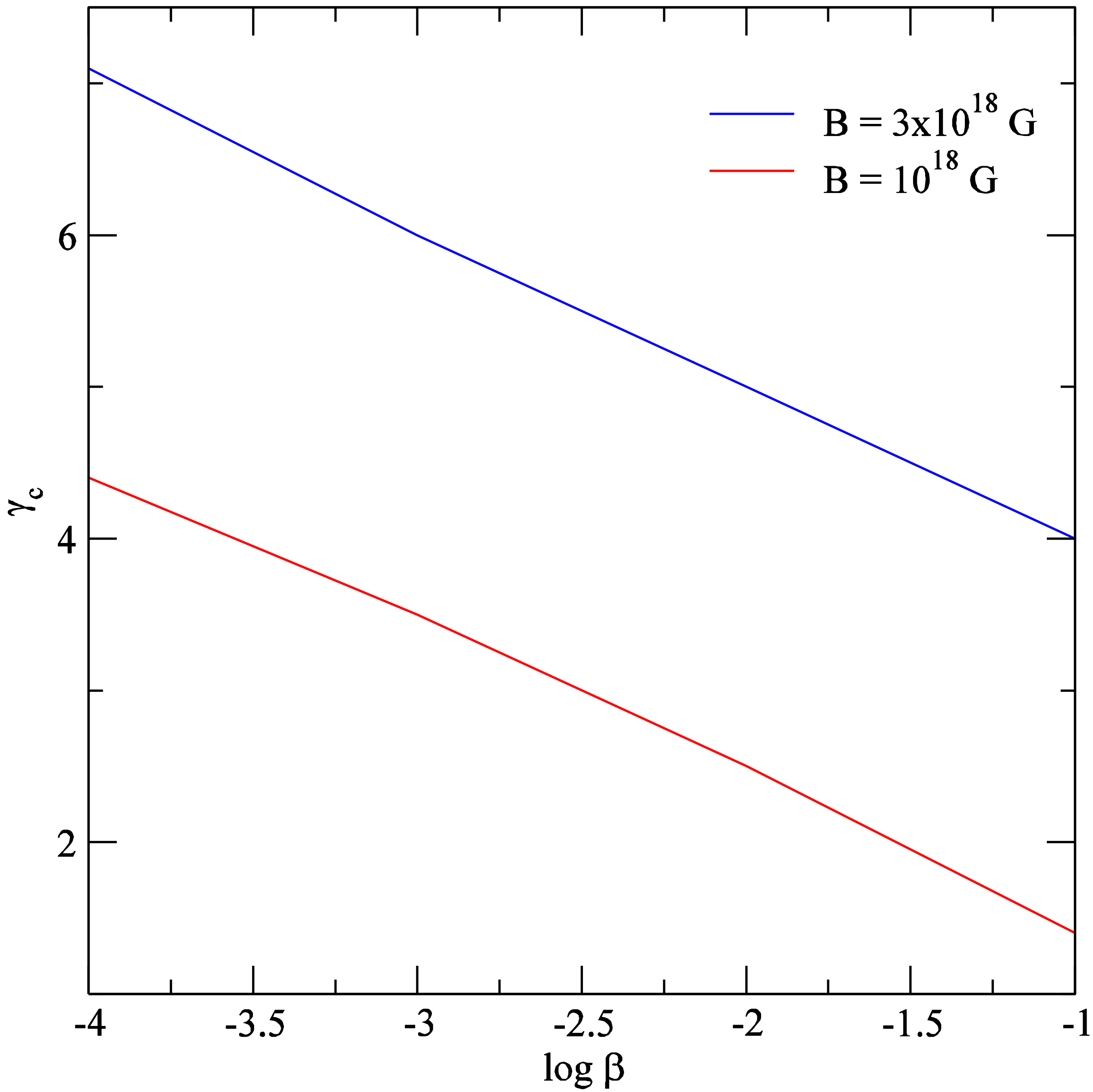
$$P_\perp = P - M\mathcal{B} + \frac{\mathcal{B}^2}{8\pi}$$

$$P_\parallel = P - \frac{\mathcal{B}^2}{8\pi}$$

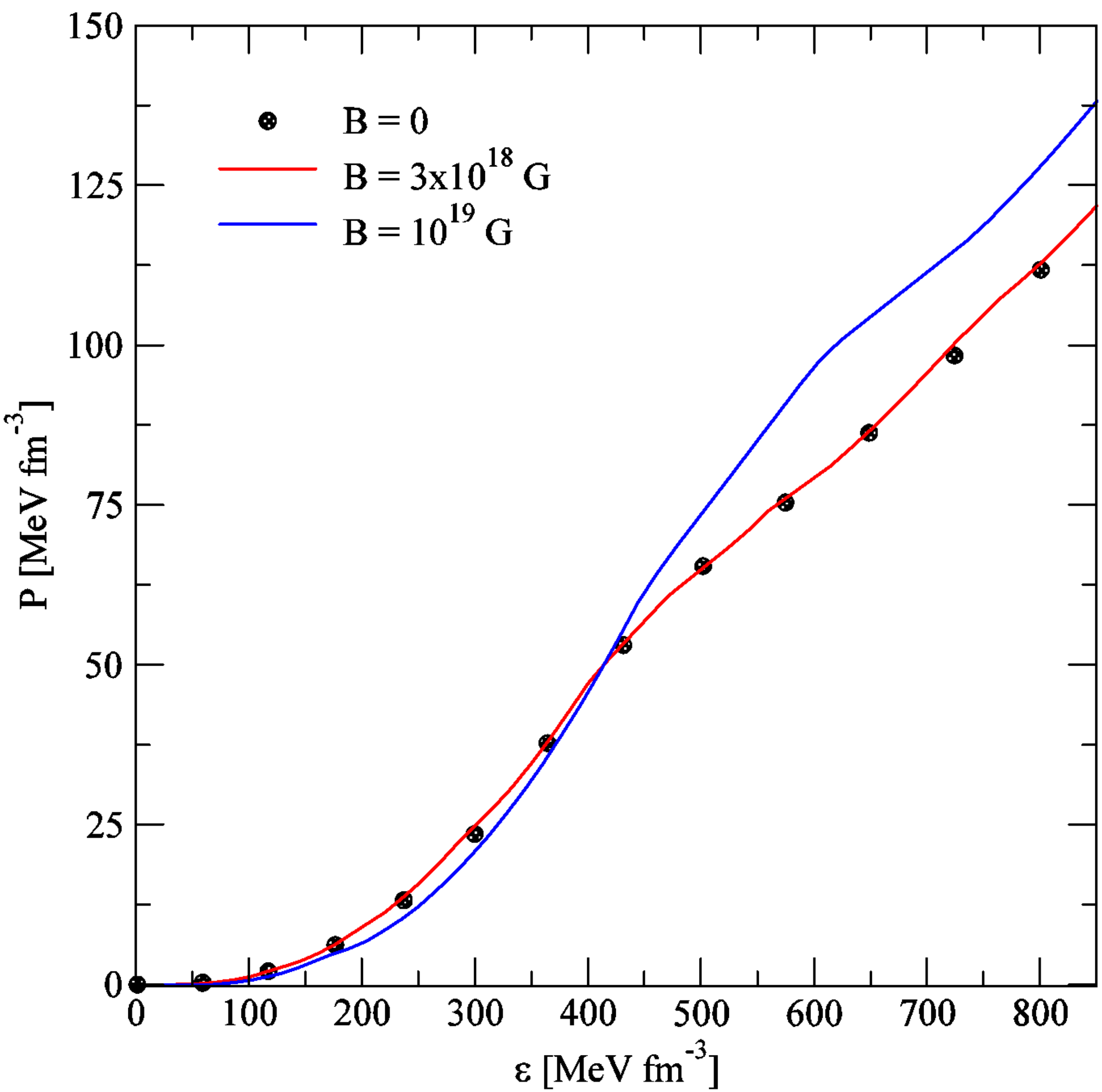




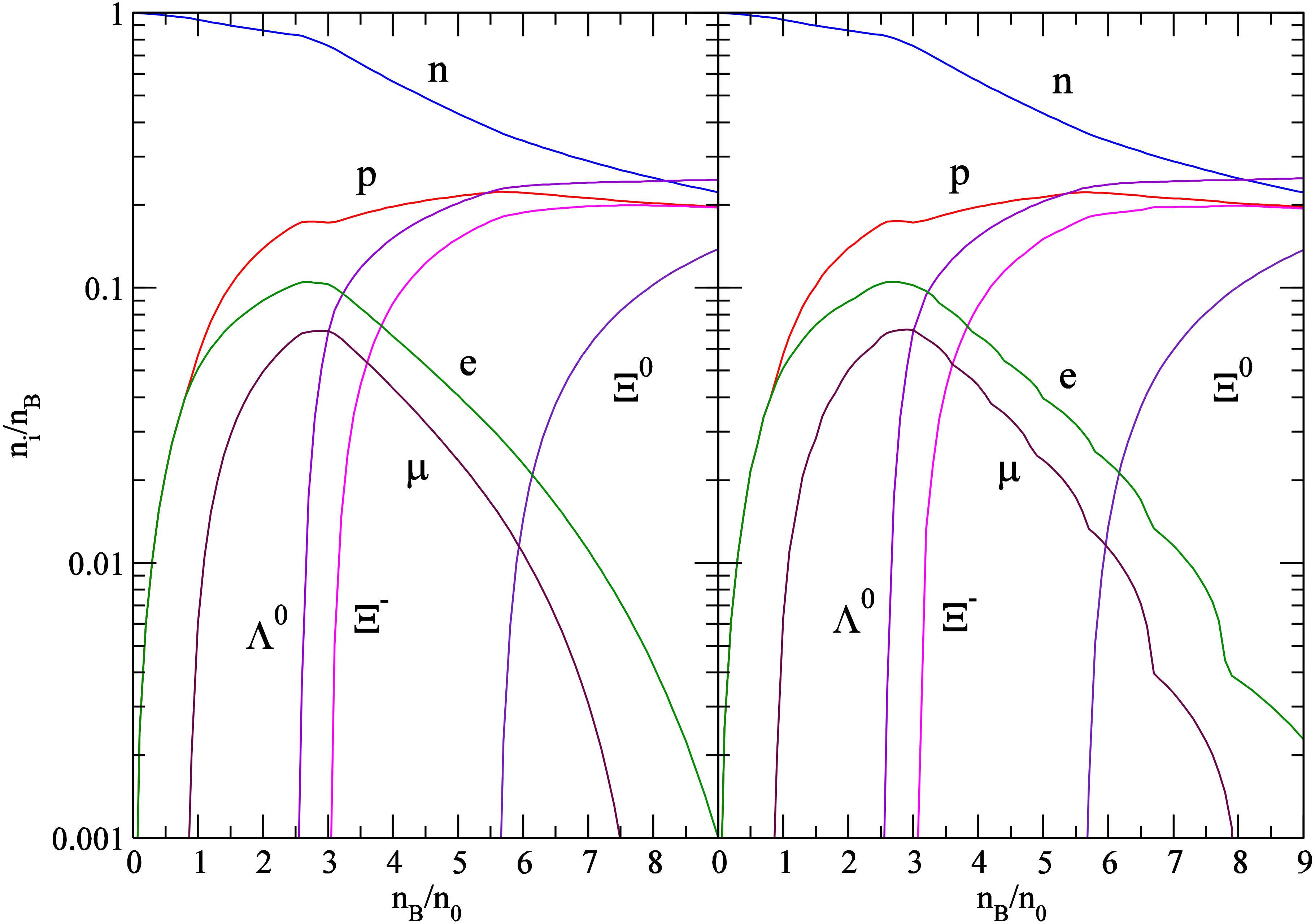
β=10^−1
β=10^−3
P_|| [Mev fm^-3]
B [G]
P_|| [MeV fm^-3]
B [G]
n_b/n_0
n_b/n_0



B = 3x10^18 G
B = 10^18 G
6
4
2
γc
-4
-3.5
-3
-2.5
-2
-1.5
-1
log β



B = 0
B = 3x10^18 G
B = 10^19 G
P [MeV fm^-3]
ε [MeV fm^-3]
0
25
50
75
100
125
150
0
100
200
300
400
500
600
700
800



1
0.1
0.01
0.001
n_i/n_B
0 1 2 3 4 5 6 7 8 0 1 2 3 4 5 6 7 8 9
n_B/n_0
n_B/n_0
n
p
e
μ
Λ^0
Ξ^-
Ξ^0
n
p
e
μ
Λ^0
Ξ^-
Ξ^0

Strange quark matter

Our Model

Quark masses are density dependent, restoring chiral symmetry at high density

$$m_i = M_i + M_q \operatorname{sech}\left(\nu \frac{n_b}{n_0}\right)$$

- Quarks interact among themselves via two-body Richardson potential

$$V(q^2) = -\frac{4}{9}\,\frac{\pi}{\ln[1 + (q^2 + m_g^2)/\Lambda^2]}\,\frac{1}{(q^2 + m_g^2)}$$

$$m_g^2 = D^{-2} = \frac{2\alpha_0}{\pi} \sum_{i=u,d,s} k_{F_i} \sqrt{k_{F_i}^2 + m_i^2}$$

Dey et al., PLB **438**, 123 (1998)

- With this features, the energy density is composed of two parts:

$$\varepsilon_k = \frac{3}{\pi^2}\int_0^\infty dp\,\sqrt{p^2+m^2}\;f(\epsilon)\;p^2 \qquad \varepsilon_p = \frac{1}{2\pi^3}\int_0^\infty dp_i\int_0^\infty dp_j\int_{-1}^{1} d(\cos\theta)\;F(\mathbf{p}_i,\mathbf{p}_j)$$

- In presence of magnetic field the kinetic energy and potential energy density get modified as:

$$\varepsilon_k = e|Q|B\sum_n (2-\delta_{n,0})\int_{-p(n)}^{p(n)} dp_z \int_0^{2\phi} d\phi\, E_n$$

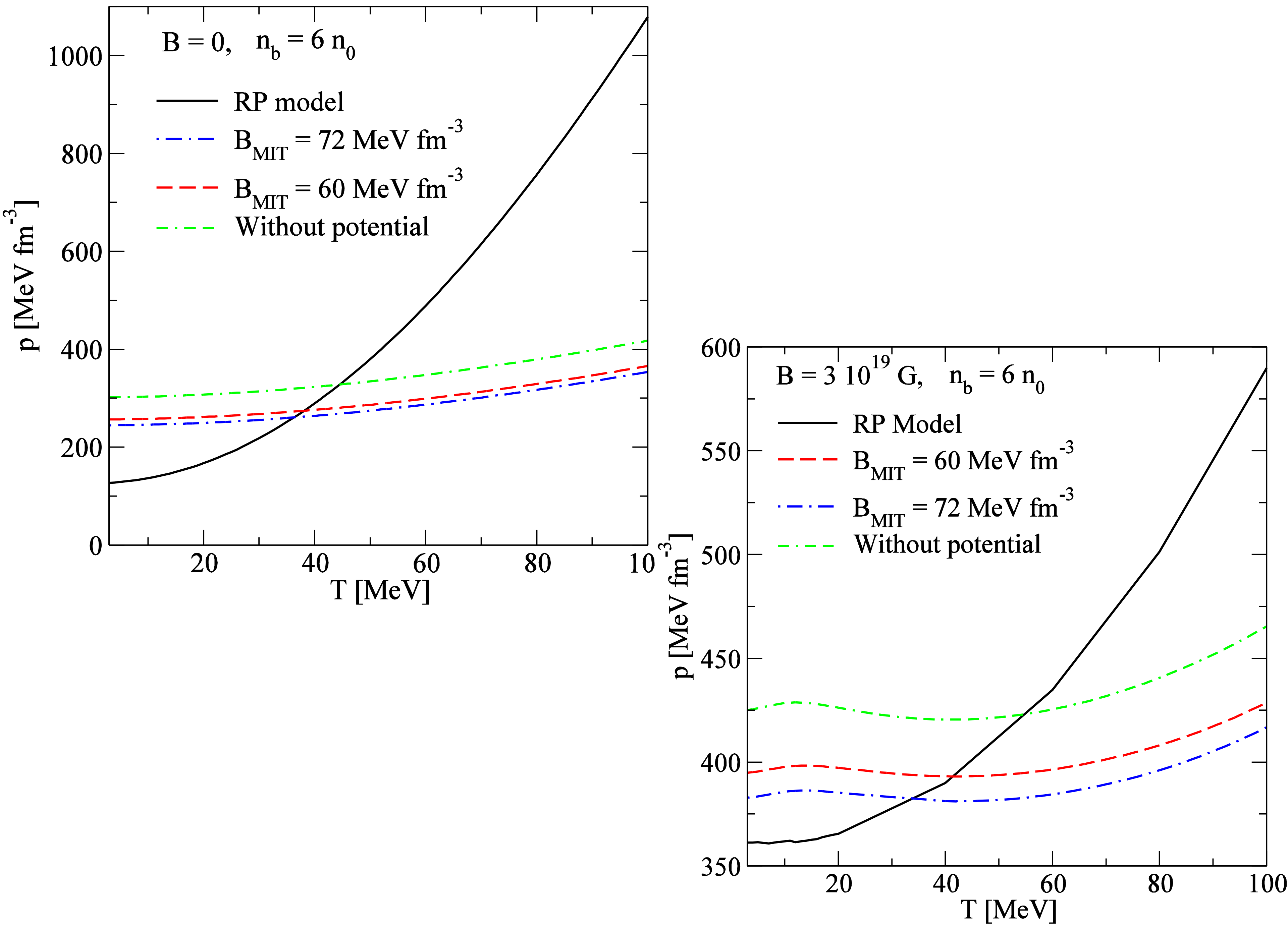
$$\begin{aligned}\varepsilon_{pot}^{ij} &= \frac{e^2|Q_i||Q_j|}{(2\pi)^5}B^2\sum_{n_i}\sum_{n_j}(2-\delta_{n_i,0})(2-\delta_{n_j,0}) \\ &\times\int_0^{2\pi}d\phi_i\int_0^{2\pi}d\phi_j\int_{-\infty}^{\infty}dk_z^i\int_{-\infty}^{\infty}dk_z^j f(\epsilon_i)f(\epsilon_j)F(\mathbf{p}_i,\mathbf{p}_j)\end{aligned}$$

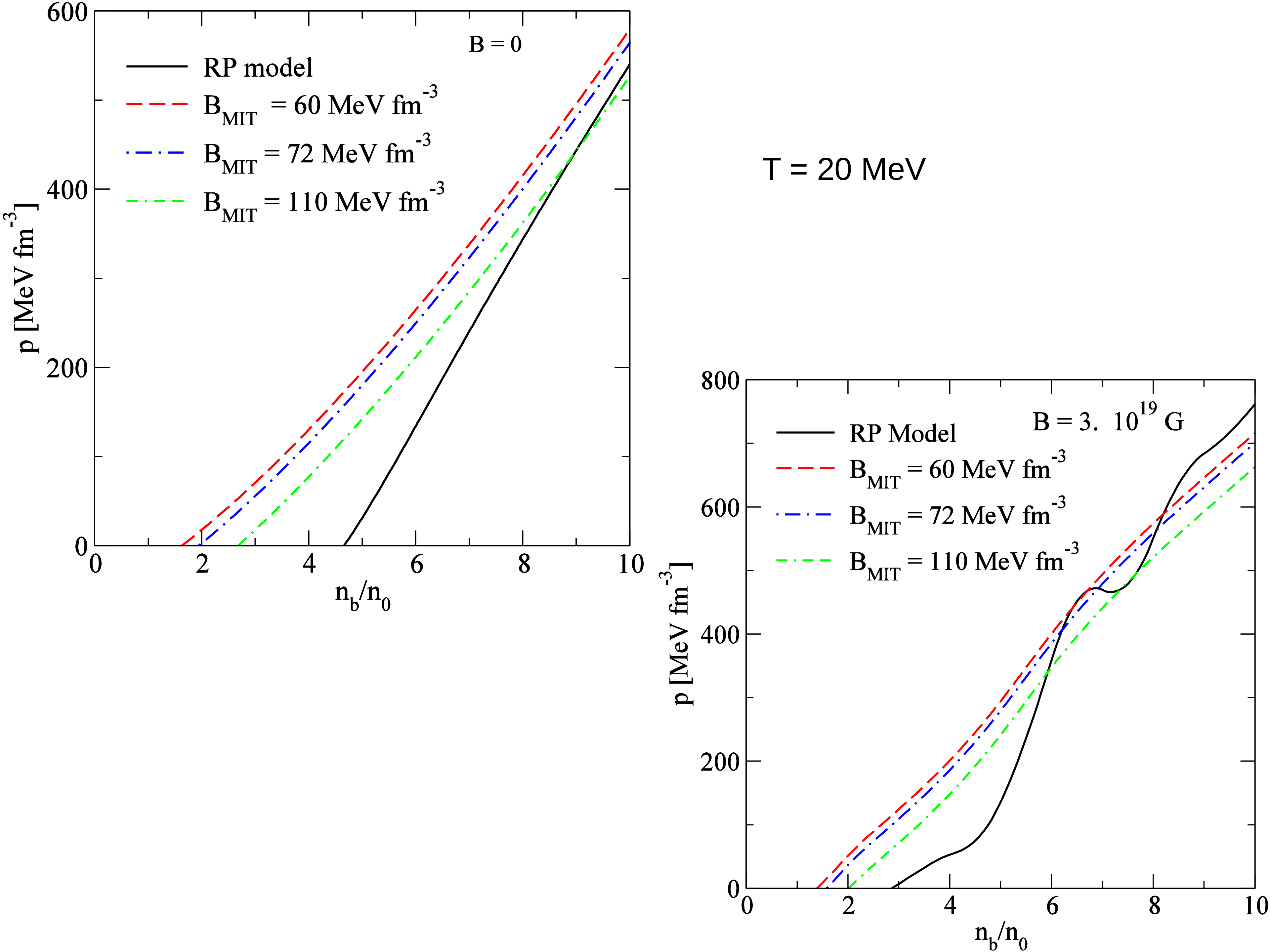
$$F(\mathbf{p}_i,\mathbf{p}_j) = V(q^2)\,\frac{(\epsilon_i+m_i)(\epsilon_j+m_j)}{4\epsilon_i\epsilon_j}\left\{1+\frac{p_i^2p_j^2}{(\epsilon_i+m_i)^2(\epsilon_j+m_j)^2}+\frac{2\,\mathbf{p}_i\cdot\mathbf{p}_j}{(\epsilon_i+m_i)(\epsilon_j+m_j)}\right\}$$

- Here we introduce the finite temperature effect.

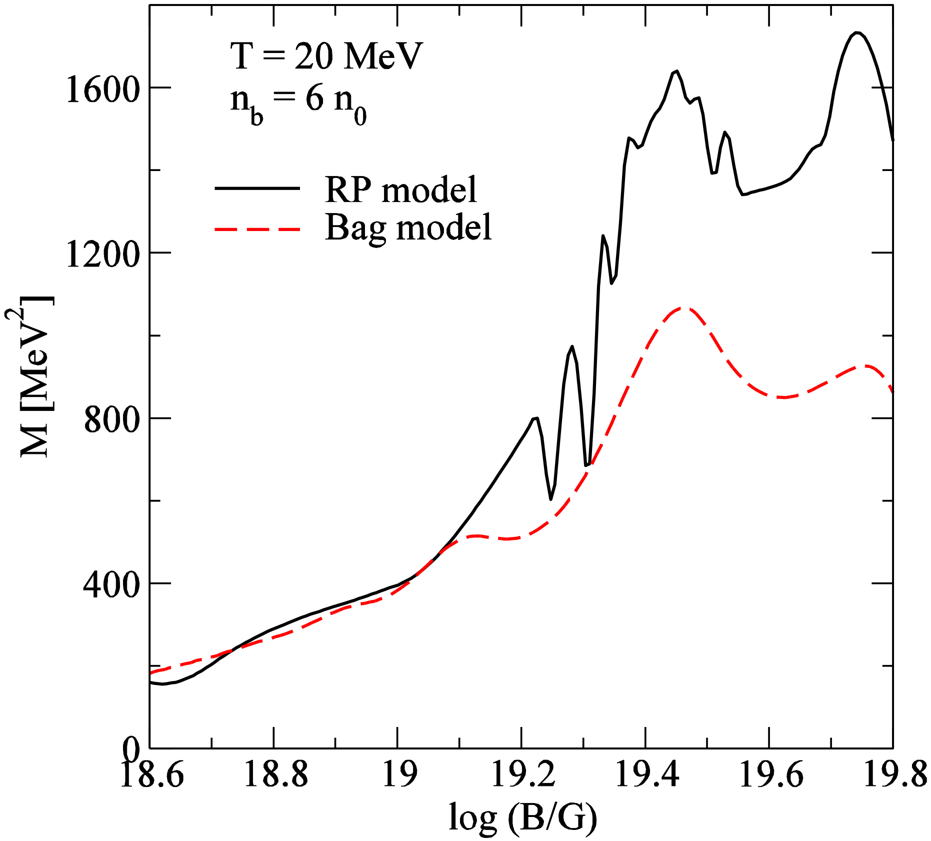
- The thermodynamic pressure is given by

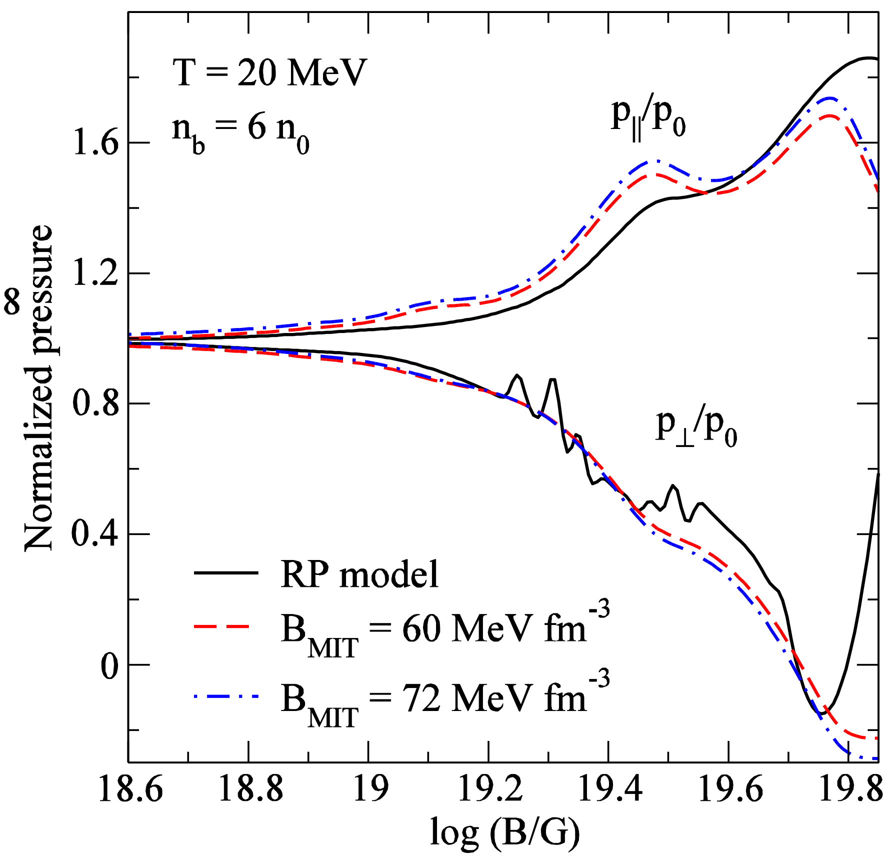
$$p = \sum_i \mu_i n_i + Ts - \varepsilon, \quad i = u, d, s$$


B = 0, n_b = 6 n_0
RP model
B_MIT = 72 MeV fm^-3
B_MIT = 60 MeV fm^-3
Without potential
p [MeV fm^-3]
T [MeV]
B = 3 10^19 G, n_b = 6 n_0
RP Model
B_MIT = 60 MeV fm^-3
B_MIT = 72 MeV fm^-3
Without potential
p [MeV fm^-3]
T [MeV]



B = 0
RP model
B_{MIT} = 60 MeV fm^{-3}
B_{MIT} = 72 MeV fm^{-3}
B_{MIT} = 110 MeV fm^{-3}
p [MeV fm^{-3}]
n_b/n_0
T = 20 MeV
B = 3. 10^{19} G
RP Model
B_{MIT} = 60 MeV fm^{-3}
B_{MIT} = 72 MeV fm^{-3}
B_{MIT} = 110 MeV fm^{-3}
p [MeV fm^{-3}]
n_b/n_0


T = 20 MeV
n_b = 6 n_0
RP model
Bag model
M [MeV^2]
1600
1200
800
400
0
18.6
18.8
19
19.2
19.4
19.6
19.8
log (B/G)


T = 20 MeV
n_b = 6 n_0
p_∥/p_0
p_⊥/p_0
RP model
B_MIT = 60 MeV fm^-3
B_MIT = 72 MeV fm^-3
Normalized pressure
1.6
1.2
0.8
0.4
0
18.6
18.8
19
19.2
19.4
19.6
19.8
log (B/G)

Summary

- Presence of magnetic field introduces the anisotropic pressure in the system.
- In the range of the field considered in our study the magnetization and anomalous magnetic moment effect is negligible. However, field pressure is significant.
- Consequently, the pressure in the perpendicular directions to the field is enhanced and in the parallel direction to the field is reduced compared to the field free case.
- With increase of field strength with density, when pressure ceases to increase with density due to the negative contribution of field pressure in the parallel direction of field, the matter becomes unstable.
- Our result gives one possible explanation of maximum possible magnetic field within NS.