Parton Distribution Functions

α_s, m_q, *the gluon density and all that* $\ldots$

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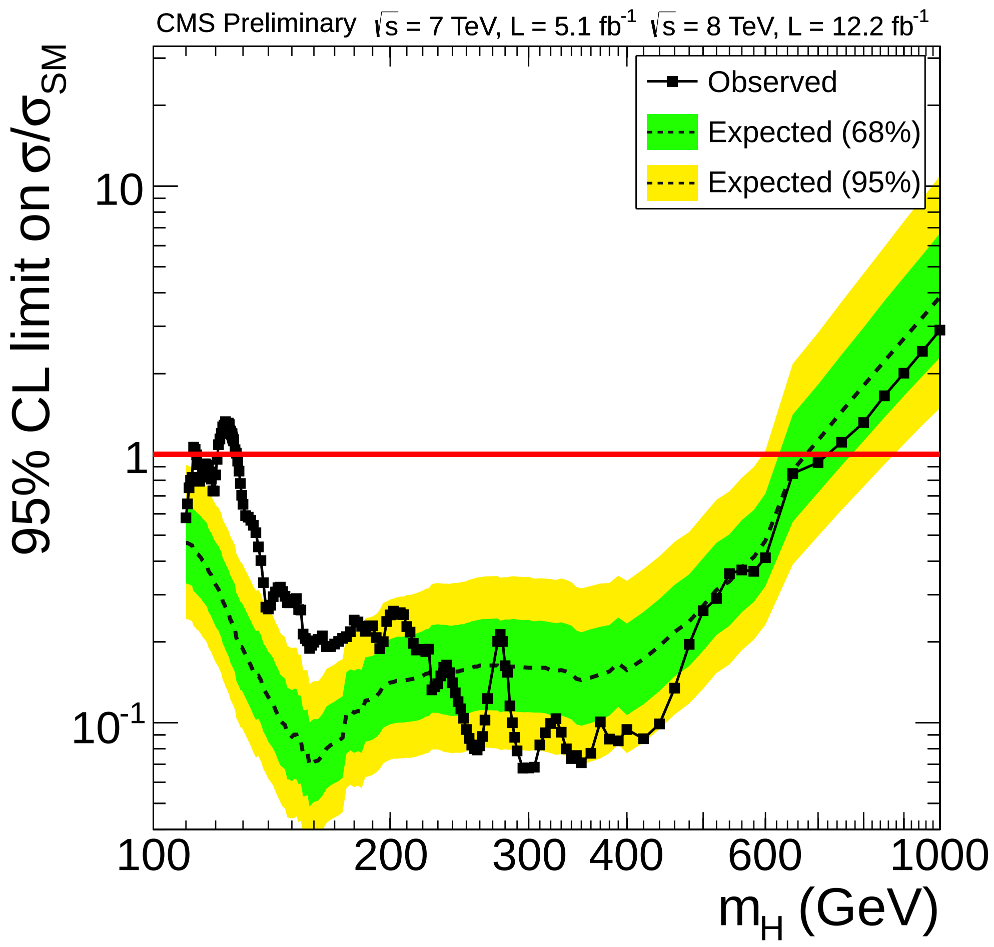
Plan

- Cross sections in perturbative QCD
- Non-perturbative input parameters
  - parton distributions
  - strong coupling $\alpha_s(M_Z)$
- LHC measurements
  - $W^\pm$- and Z-boson production
  - top-quark mass
- Implications for electroweak vacuum

Example from LHC Higgs measurements

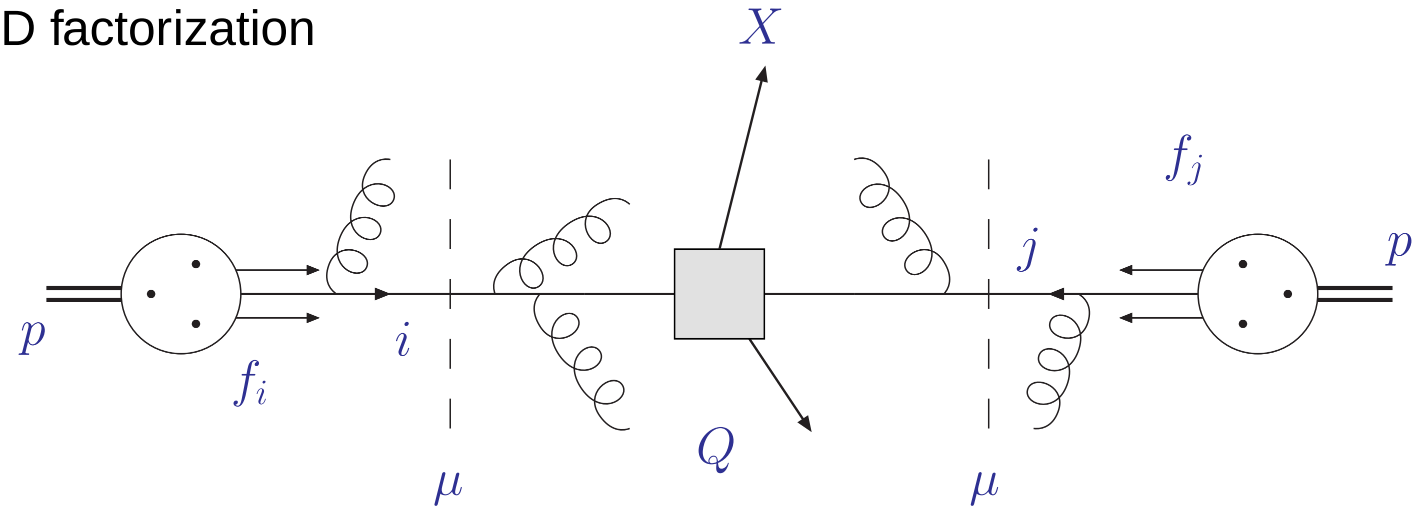
CMS coll. Dec 2012



- Signal strength of all analyzed decay modes
  - normalization to Standard Model expectation
  - accuracy of $\sigma_{\rm SM}$ crucial

QCD factorization

- QCD factorization

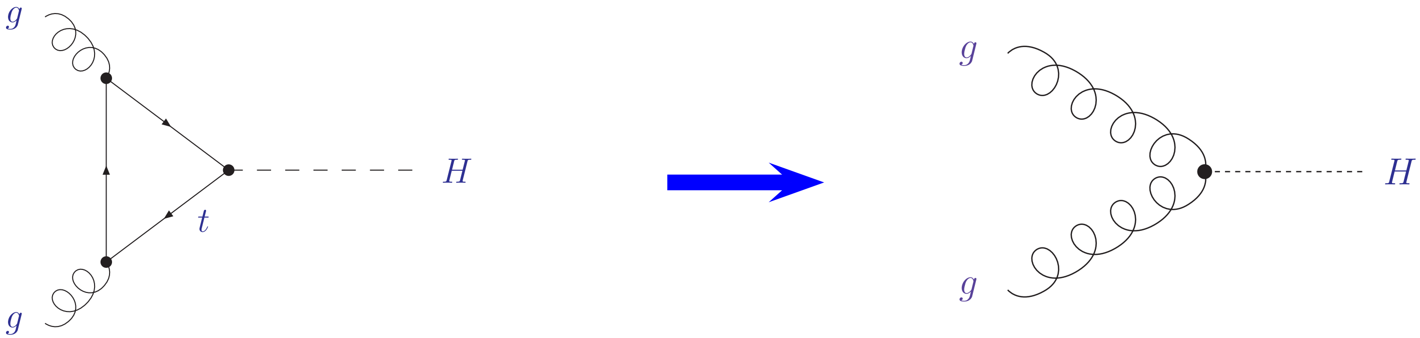


$$\sigma_{pp\to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij\to X}\left(\alpha_s(\mu^2), Q^2, \mu^2, m_X^2\right)$$

- Hard parton cross section $\hat{\sigma}_{ij\to X}$ calculable in perturbation theory
  - known to NLO, NNLO, . . . ($\mathcal{O}(\text{few}\%)$ theory uncertainty)
- Non-perturbative parameters: parton distribution functions f_i, strong coupling α_s, particle masses m_X
  - known from global fits to exp. data, lattice computations, . . .

Higgs production in gg-fusion

Effective theory



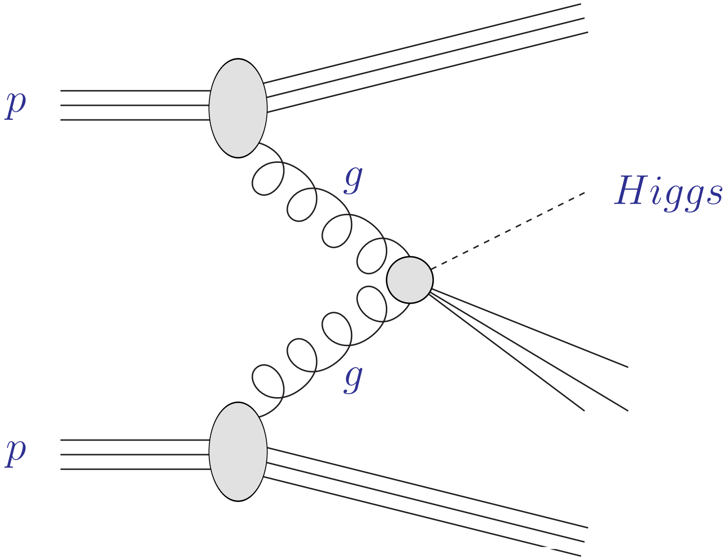
- Integration of top-quark loop (finite result)
  - decay width $H \to gg$ ($m_q = 0$ for light quarks, m_t heavy)

$$\Gamma_{H\to gg} = \frac{G_\mu \, m_H^3}{64\,\sqrt{2}\,\pi^3}\,\alpha_s^2\ f\left(\frac{m_H^2}{4m_t^2}\right)$$

- Effective theory in limit $m_t \to \infty$; Lagrangian $\mathcal{L} = -\frac{1}{4}\,\frac{H}{v}\,C_H\,G^{\mu\nu\,a}\,G^a_{\mu\nu}$
  - operator $HG^{\mu\nu\,a}\,G^a_{\mu\nu}$ relates to stress-energy tensor
  - additional renormalization proportional to QCD β-function required

    Kluberg-Stern, Zuber ‘75; Collins, Duncan, Joglekar ‘77

QCD corrections to ggF



- Hadronic cross section $\sigma_{pp\to H}$ with $\tau = m_H^2/S$
  - renormalization/factorization (hard) scale $\mu = \mathcal{O}(m_H)$

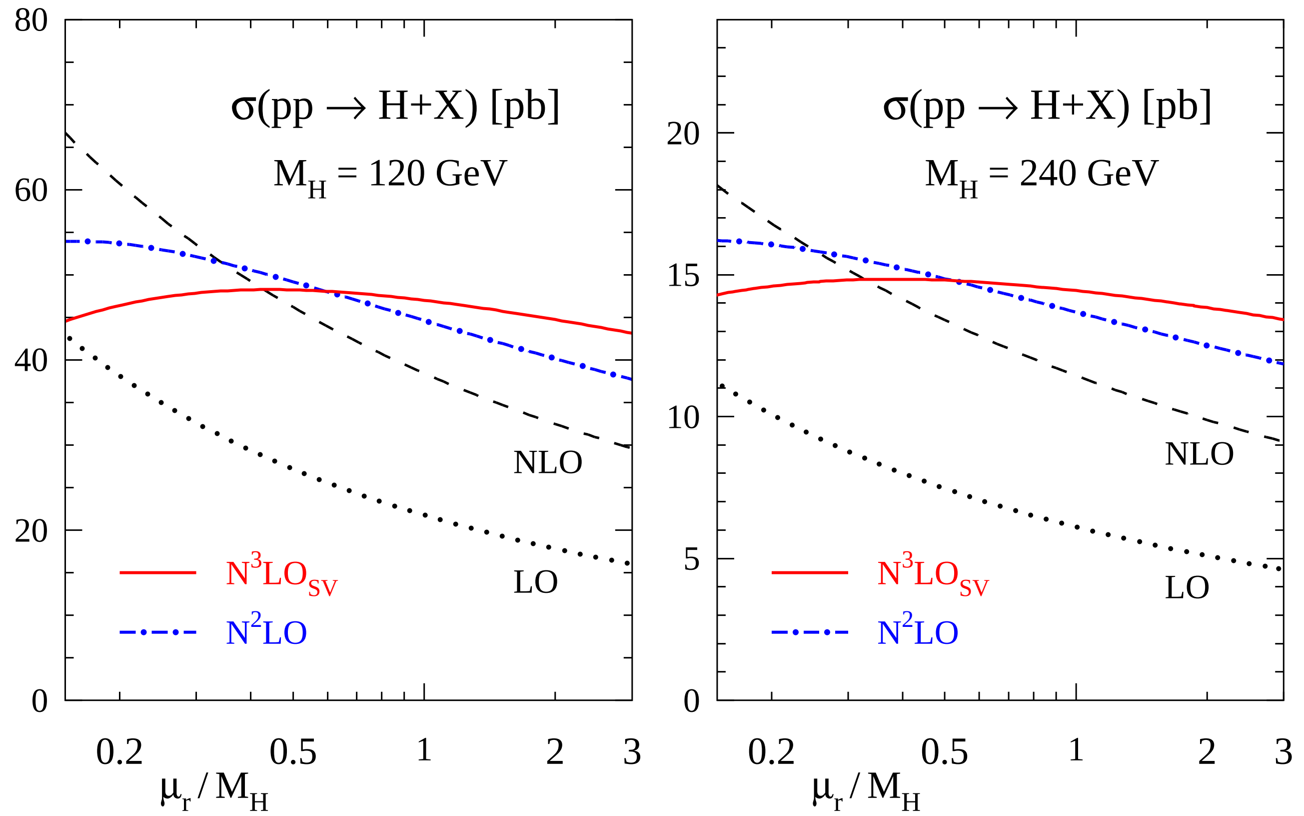
$$\sigma_{pp\to H} = \sum_{ij} \int_\tau^1 \frac{dx_1}{x_1} \int_{x_1}^1 \frac{dx_2}{x_2} f_i\left(\frac{x_1}{x_2}, \mu^2\right) f_j\left(x_2, \mu^2\right) \hat{\sigma}_{ij\to H}\left(\frac{\tau}{x_1}, \frac{\mu^2}{m_H^2}, \alpha_s(\mu^2)\right)$$

- Partonic cross section $\hat{\sigma}_{ij\to H}$

$$\hat{\sigma}_{ij\to H} \quad = \quad \underbrace{\alpha_s^2 \Big[\hat{\sigma}^{(0)}_{ij\to H} + \alpha_s\, \hat{\sigma}^{(1)}_{ij\to H}}_{} \; + \; \alpha_s^2\, \hat{\sigma}^{(2)}_{ij\to H} \; + \; \dots \Big]$$

NLO: standard approximation (large uncertainties)

Perturbation theory at work



- Apparent convergence of perturbative expansion
  - NNLO corrections still large

    Harlander, Kilgore ‘02; Anastasiou, Melnikov ‘02; Ravindran, Smith, van Neerven ‘03
  - improvement through complete soft N^3LO corrections S.M., Vogt ‘05 or NNLL resummtion Catani, de Florian, Grazzini, Nason ‘03, Ahrens et al. ‘10
- Perturbative stability under renormalization scale variation

Non-perturbative parameters

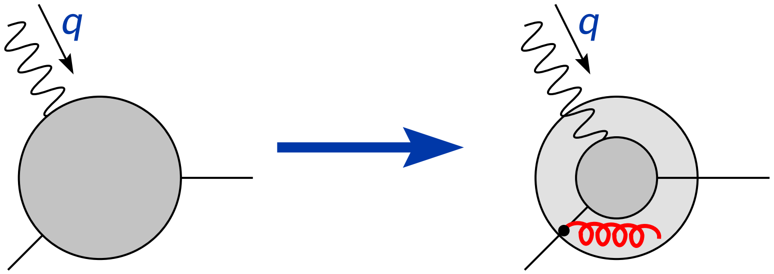
Input for collider phenomenology

- Non-perturbative parameters are universal
- Determination from comparision to experimental data
    - masses of heavy quarks m_c, m_b, m_t
    - parton distribution functions $f_i(x, \mu^2)$
    - strong coupling constant $\alpha_s(M_Z)$

Interplay with perturbation theory

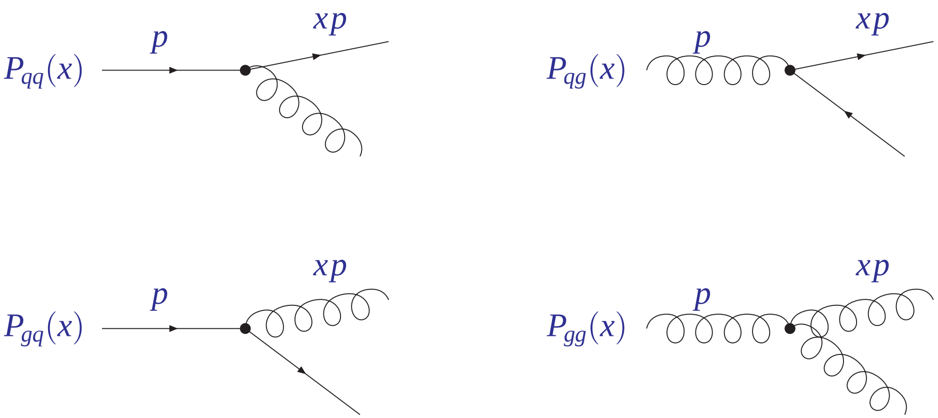
- Accuracy of determination driven by precision of theory predictions
- Non-perturbative parameters sensitive to
    - radiative corrections at higher orders
    - renormalization and factorization scales μ_R, μ_F
    - chosen scheme (e.g. ($\overline{MS}$ scheme)
    - ...

Parton evolution

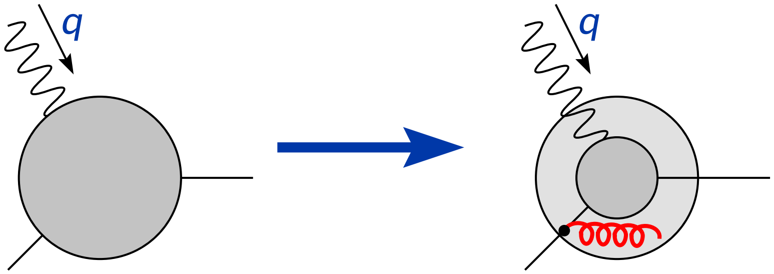


- Proton in resolution $1/Q$ $\longrightarrow$ sensitive to lower momentum partons

- Feynman diagrams in leading order



Parton evolution



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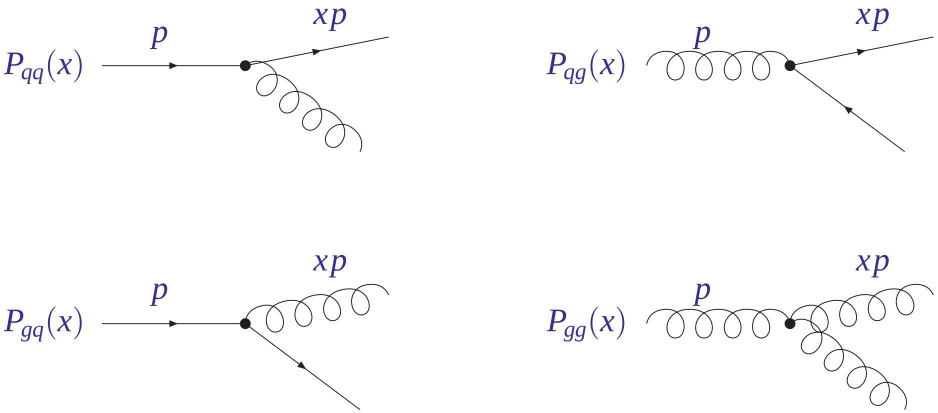


- Proton in resolution $1/Q \longrightarrow$ sensitive to lower momentum partons

- Evolution equations for parton distributions f_i
  - predictions from fits to reference processes (universality)

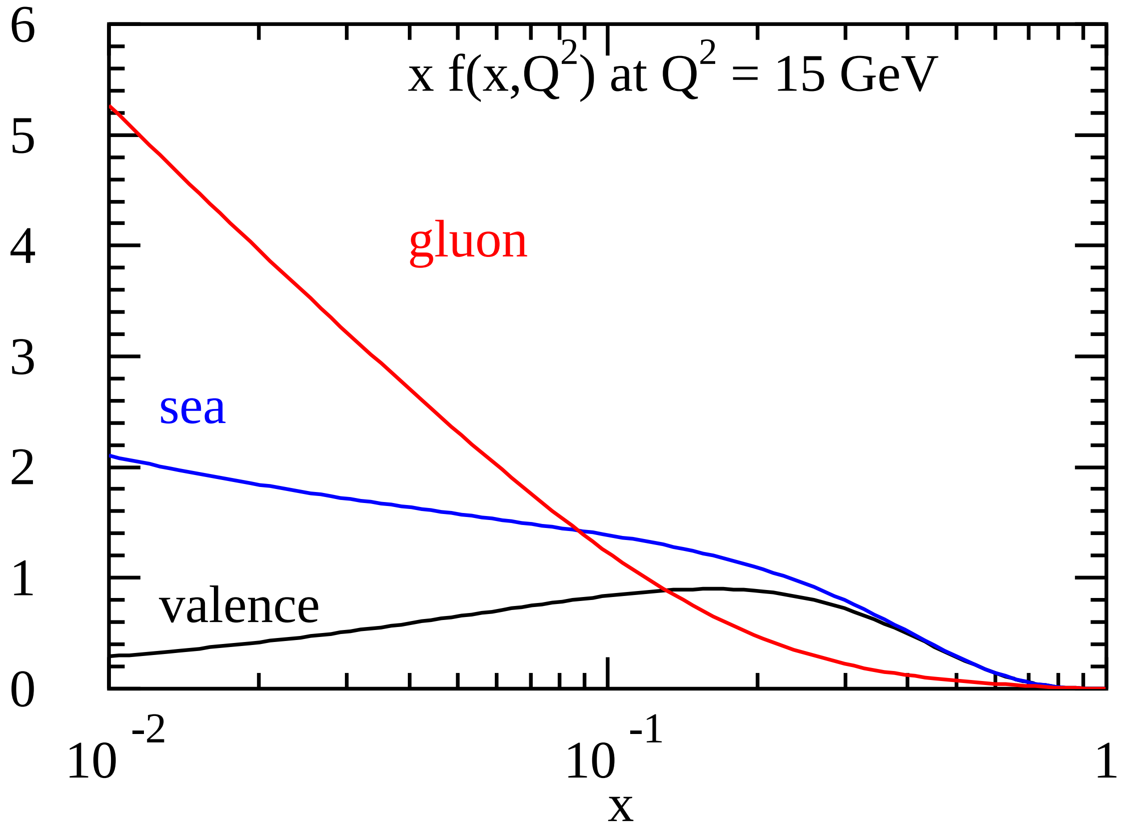
$$\frac{d}{d\ln\mu^2} f_i(x,\mu^2) = \sum_k \left[P_{ik}(\alpha_s(\mu^2)) \otimes f_k(\mu^2)\right](x)$$

- Splitting functions P

$$P = \underbrace{\alpha_s P^{(0)} + \alpha_s^2 P^{(1)}}_{\text{NLO: standard approximation (large uncertainties)}} + \alpha_s^3 P^{(2)} + \ldots$$

Parton distributions in proton

- Valence $q - \bar{q}$ (additive quantum numbers) sea (part with $q + \bar{q}$)



- Parameterization (bulk of data from deep-inelastic scattering)
  - structure function $F_2 \longrightarrow$ quark distribution
  - scale evolution (perturbative QCD) $\longrightarrow$ gluon distribution

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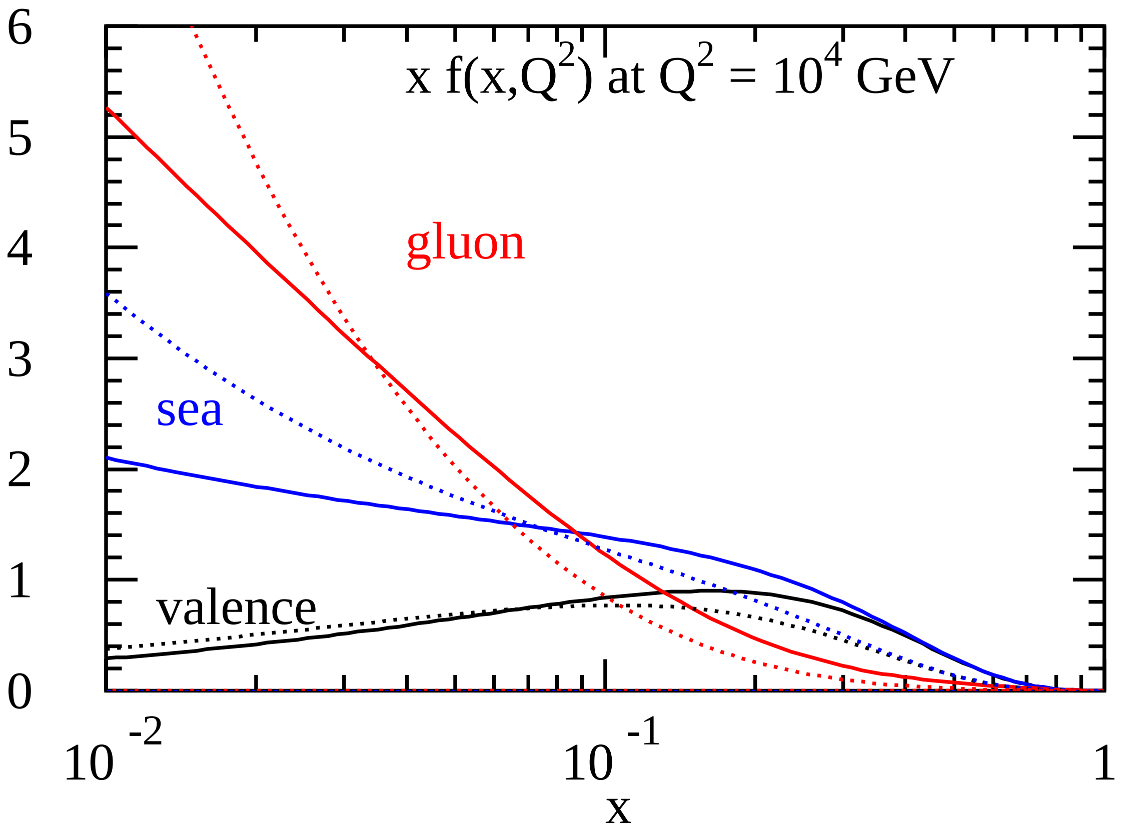

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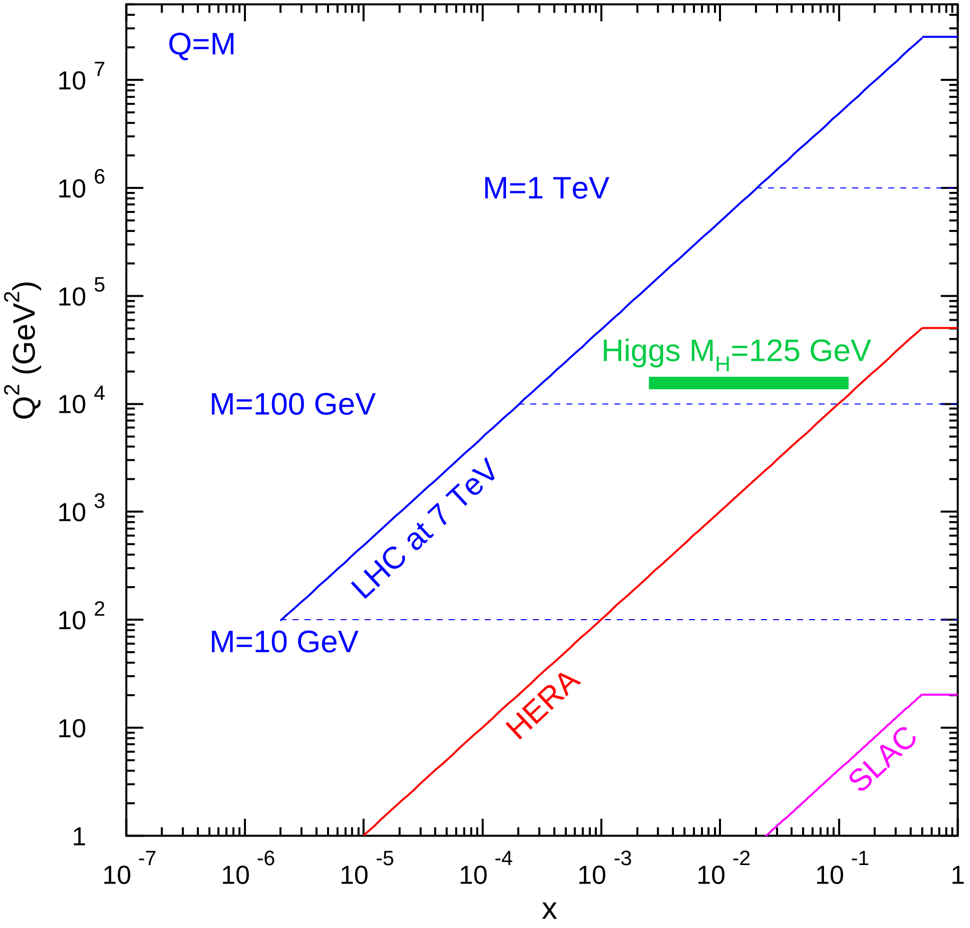
Parton luminosity at LHC



- LHC run at $\sqrt{s}$ = 7/8 TeV
  - parton kinematics well covered by HERA and fixed target experiments
- Parton kinematics at effective $\langle x \rangle = M/\sqrt{S}$
  - 100 GeV physics: small-x, sea partons
  - TeV scales: large-x

Parton distribution fits

Example

- ABM PDF set Alekhin, Blümlein, S.M. ‘12

Theory considerations

- Consistent theory description for consistent data sets
- Determination of PDFs and strong coupling constant α_s to NNLO QCD
- Consistent scheme for treatment of heavy quarks
    - fixed-flavor number scheme for $n_f = 3, 4, 5$
    - $\overline{\mathrm{MS}}$-scheme for quark masses and α_s
- Full account of error correlations

Data considered in the fit

- Analysis of world data for deep-inelastic scattering and fixed-target data for Drell-Yan process
    - inclusive DIS data HERA, BCDMS, NMC, SLAC
    - Drell-Yan data (fixed target) E-605, E-866
    - neutrino-nucleon DIS data (di-muon production) CCFR/NuTeV

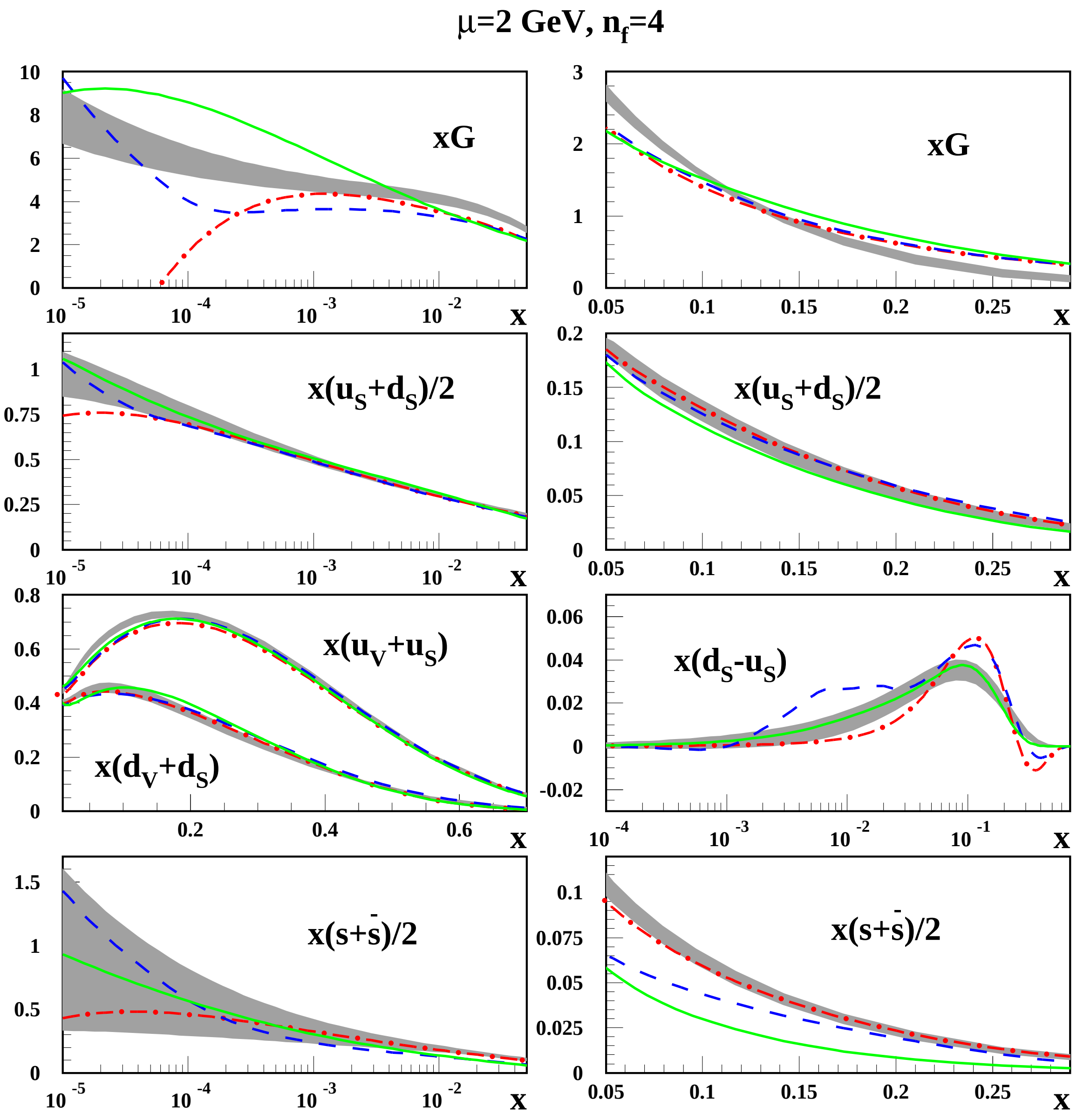
PDF ansatz

- ABM PDFs parameterized at scale $Q_0 = 3\mathrm{GeV}$ in scheme with $n_f = 3$
  Alekhin, Blümlein, S.M. ‘12
    - ansatz for valence-/sea-quarks, gluon with polynomial $P(x)$
    - strange quark is taken in charge-symmetric form
    - 24 parameters in polynomials $P(x)$
    - 4 additional fit parameters: $\alpha_s^{(n_f=3)}(\mu = 3$ GeV), m_c, m_b and deuteron correction

$$
\begin{aligned}
x q_v(x, Q_0^2) &= \frac{2\delta_{qu} + \delta_{qd}}{N_q^v} x^{a_q} (1-x)^{b_q} x^{P_{qv}(x)} \\
x u_s(x, Q_0^2) = x \bar{u}_s(x, Q_0^2) &= A_{us} x^{a_{us}} (1-x)^{b_{us}} x^{a_{us} P_{us}(x)} \\
x\Delta(x, Q_0^2) = x d_s(x, Q_0^2) - x u_s(x, Q_0^2) &= A_\Delta x^{a_\Delta} (1-x)^{b_\Delta} x^{P_\Delta(x)} \\
x s(x, Q_0^2) = x \bar{s}(x, Q_0^2) &= A_s x^{a_s} (1-x)^{b_s} \, , \\
x g(x, Q_0^2) &= A_g x^{a_g} (1-x)^{b_g} x^{a_g P_g(x)}
\end{aligned}
$$

- Ansatz provides sufficient flexibility; no additional terms required to improve the quality of fit

Parton distributions for the LHC



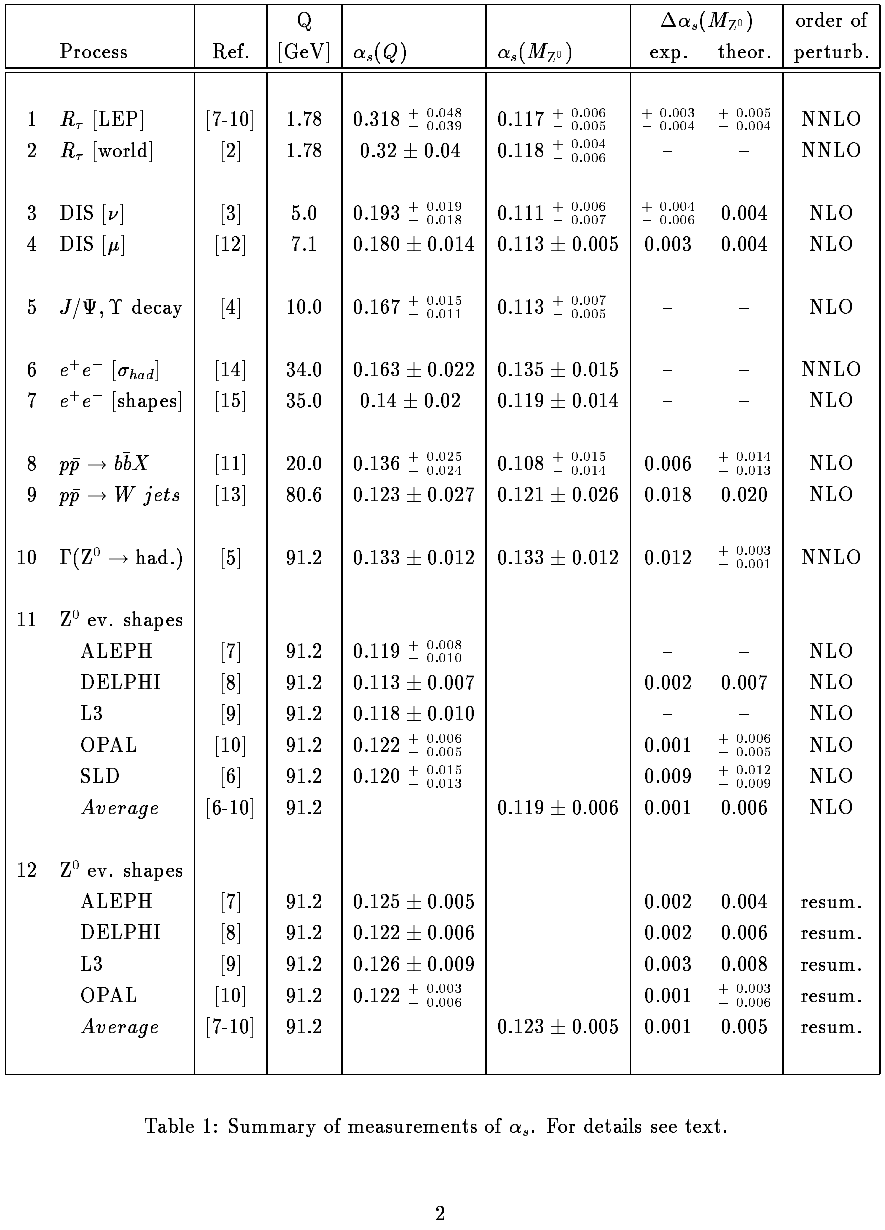
- 1σ band for ABM11 PDFs (NNLO, 4-flavors) at $\mu = 2\,\mathrm{GeV}$
  Alekhin, Blümlein, S.M.‘12
- comparison with:
  JR09 (solid lines),
  MSTW (dashed dots) and
  NN21 (dashes)
- Some interesting observations to be made . . .

Strong coupling constant

Essential facts

- $\alpha_s(M_Z)$ from e^+e^- data high
- $\alpha_s(M_Z)$ from DIS data low
- World average 1992
  $\alpha_s(M_Z) = 0.117 \pm 0.004$

Bethke, Catani CERN TH-6484/92



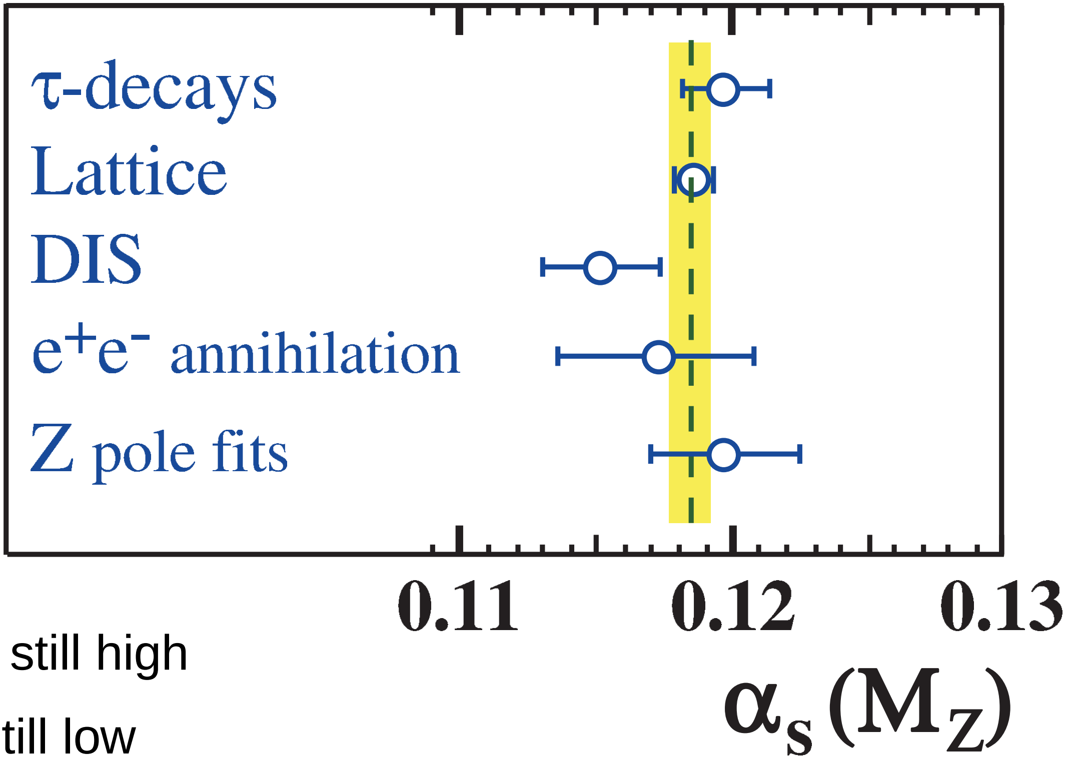
	Process	Ref.	Q [GeV]	$\alpha_s(Q)$	$\alpha_s(M_{Z^0})$	$\Delta\alpha_s(M_{Z^0})$ exp.	theor.	order of perturb.
1	R_τ [LEP]	[7-10]	1.78	$0.318^{+0.048}_{-0.039}$	$0.117^{+0.006}_{-0.005}$	$^{+0.003}_{-0.004}$	$^{+0.005}_{-0.004}$	NNLO
2	R_τ [world]	[2]	1.78	0.32 ± 0.04	$0.118^{+0.004}_{-0.006}$	–	–	NNLO
3	DIS [ν]	[3]	5.0	$0.193^{+0.019}_{-0.018}$	$0.111^{+0.006}_{-0.007}$	$^{+0.004}_{-0.006}$	0.004	NLO
4	DIS [μ]	[12]	7.1	0.180 ± 0.014	0.113 ± 0.005	0.003	0.004	NLO
5	$J/\Psi, \Upsilon$ decay	[4]	10.0	$0.167^{+0.015}_{-0.011}$	$0.113^{+0.007}_{-0.005}$	–	–	NLO
6	e^+e^- [σ_{had}]	[14]	34.0	0.163 ± 0.022	0.135 ± 0.015	–	–	NNLO
7	e^+e^- [shapes]	[15]	35.0	0.14 ± 0.02	0.119 ± 0.014	–	–	NLO
8	$p\bar{p} \to b\bar{b}X$	[11]	20.0	$0.136^{+0.025}_{-0.024}$	$0.108^{+0.015}_{-0.014}$	0.006	$^{+0.014}_{-0.013}$	NLO
9	$p\bar{p} \to W\ jets$	[13]	80.6	0.123 ± 0.027	0.121 ± 0.026	0.018	0.020	NLO
10	$\Gamma(Z^0 \to \text{had.})$	[5]	91.2	0.133 ± 0.012	0.133 ± 0.012	0.012	$^{+0.003}_{-0.001}$	NNLO
11	Z^0 ev. shapes							
	ALEPH	[7]	91.2	$0.119^{+0.008}_{-0.010}$		–	–	NLO
	DELPHI	[8]	91.2	0.113 ± 0.007		0.002	0.007	NLO
	L3	[9]	91.2	0.118 ± 0.010		–	–	NLO
	OPAL	[10]	91.2	$0.122^{+0.006}_{-0.005}$		0.001	$^{+0.006}_{-0.005}$	NLO
	SLD	[6]	91.2	$0.120^{+0.015}_{-0.013}$		0.009	$^{+0.012}_{-0.009}$	NLO
	Average	[6-10]	91.2		0.119 ± 0.006	0.001	0.006	NLO
12	Z^0 ev. shapes							
	ALEPH	[7]	91.2	0.125 ± 0.005		0.002	0.004	resum.
	DELPHI	[8]	91.2	0.122 ± 0.006		0.002	0.006	resum.
	L3	[9]	91.2	0.126 ± 0.009		0.003	0.008	resum.
	OPAL	[10]	91.2	$0.122^{+0.003}_{-0.006}$		0.001	$^{+0.003}_{-0.006}$	resum.
	Average	[7-10]	91.2		0.123 ± 0.005	0.001	0.005	resum.

Table 1: Summary of measurements of α_s. For details see text.

2

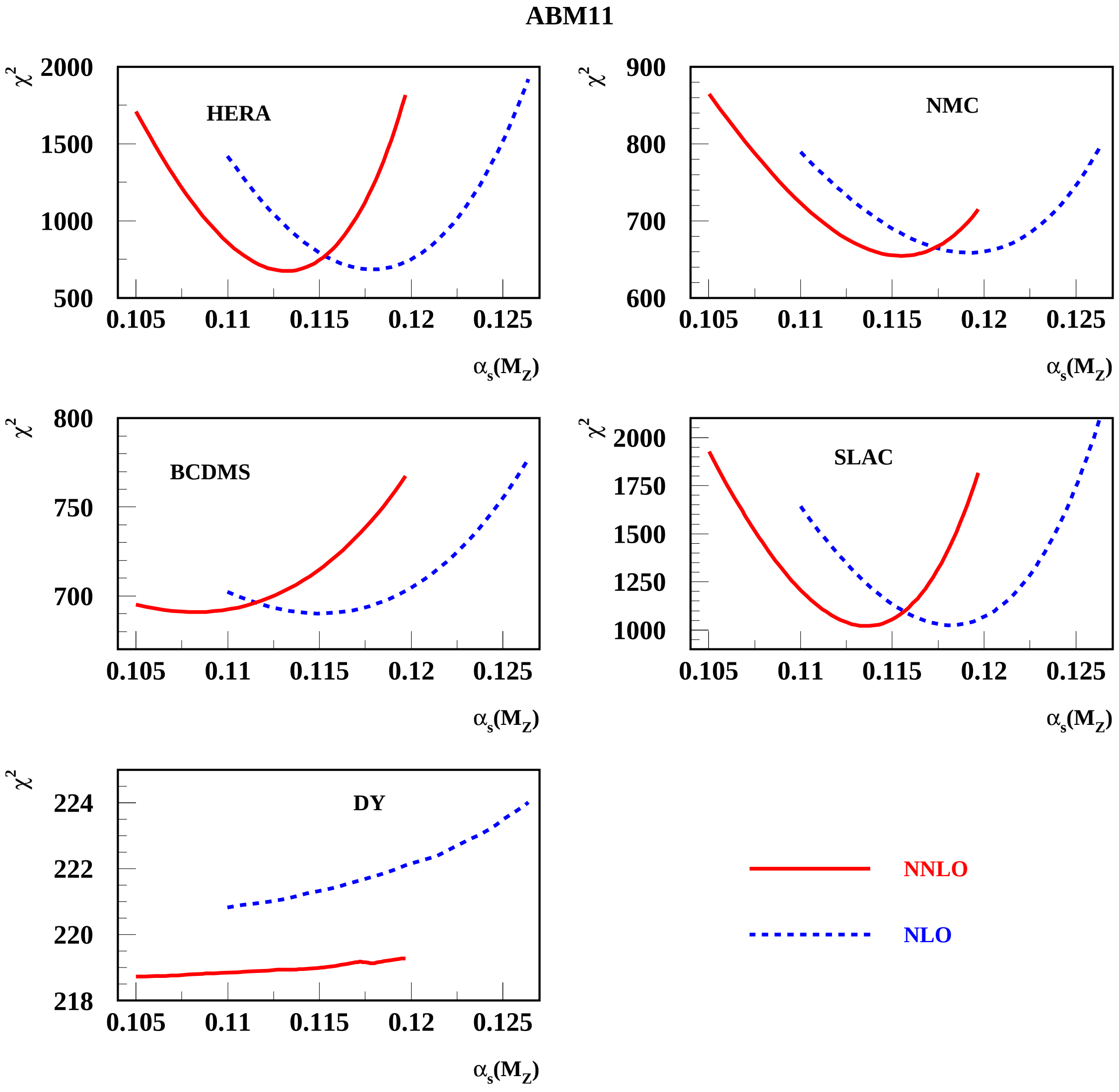
α_s 2012

Bethke in PDG 2012



- $\alpha_s(M_Z)$ from e^+e^- data still high
- $\alpha_s(M_Z)$ from DIS data still low
- World average for $\alpha_s(M_Z)$ based on arithmetic average of (pre-averaged) $\alpha_s(M_Z)$ values from different methods/processes

α_s *from DIS and PDFs*



- Profile of χ^2 for different data sets in ABM11 PDF fit Alekhin, Blümlein, S.M. '12

Comparison of α_s determinations

- Differences in α_s values:
  - result from different physics models and analysis procedures
    - target mass corrections (powers of nucleon mass M_N^2/Q^2)
    - higher twist $F_2^{\mathrm{ht}} = F_2 + ht^{(4)}(x)/Q^2 + ht^{(6)}(x)/Q^4 + \dots$
    - error correlations
- Effects for differences between ABM, MSTW and NN21 understood
  - variants of ABM with no higher twist etc. reproduce larger α_s values

	α_s at NNLO	target mass corr.	higher twist	error correl.
ABM11	0.1134 ± 0.0011	**yes**	**yes**	**yes**
NNPDF21	0.1166 ± 0.0008	**yes**	**no**	**yes**
MSTW	0.1171 ± 0.0014	**no**	**no**	**no**

Impact on Higgs production rates

- Rates for Higgs production at LHC for m_H = 125 GeV
- Cross section differences of $\mathcal{O}(10\%)$
  - diferences are statistically significant wrt. to PDF uncertainty

LHC at $\sqrt{s} = 7$ TeV	ABM11	MSTW
$\sigma(H)$ [pb]	$13.23\ ^{+1.35}_{-1.31}\ ^{+\mathbf{0.30}}_{-\mathbf{0.30}}$	$14.39\ ^{+1.54}_{-1.47}\ ^{+\mathbf{0.17}}_{-\mathbf{0.22}}$

LHC at $\sqrt{s} = 8$ TeV	ABM11	MSTW
$\sigma(H)$ [pb]	$16.99\ ^{+1.69}_{-1.63}\ ^{+\mathbf{0.37}}_{-\mathbf{0.37}}$	$18.36\ ^{+1.92}_{-1.82}\ ^{+\mathbf{0.21}}_{-\mathbf{0.28}}$

LHC measurements

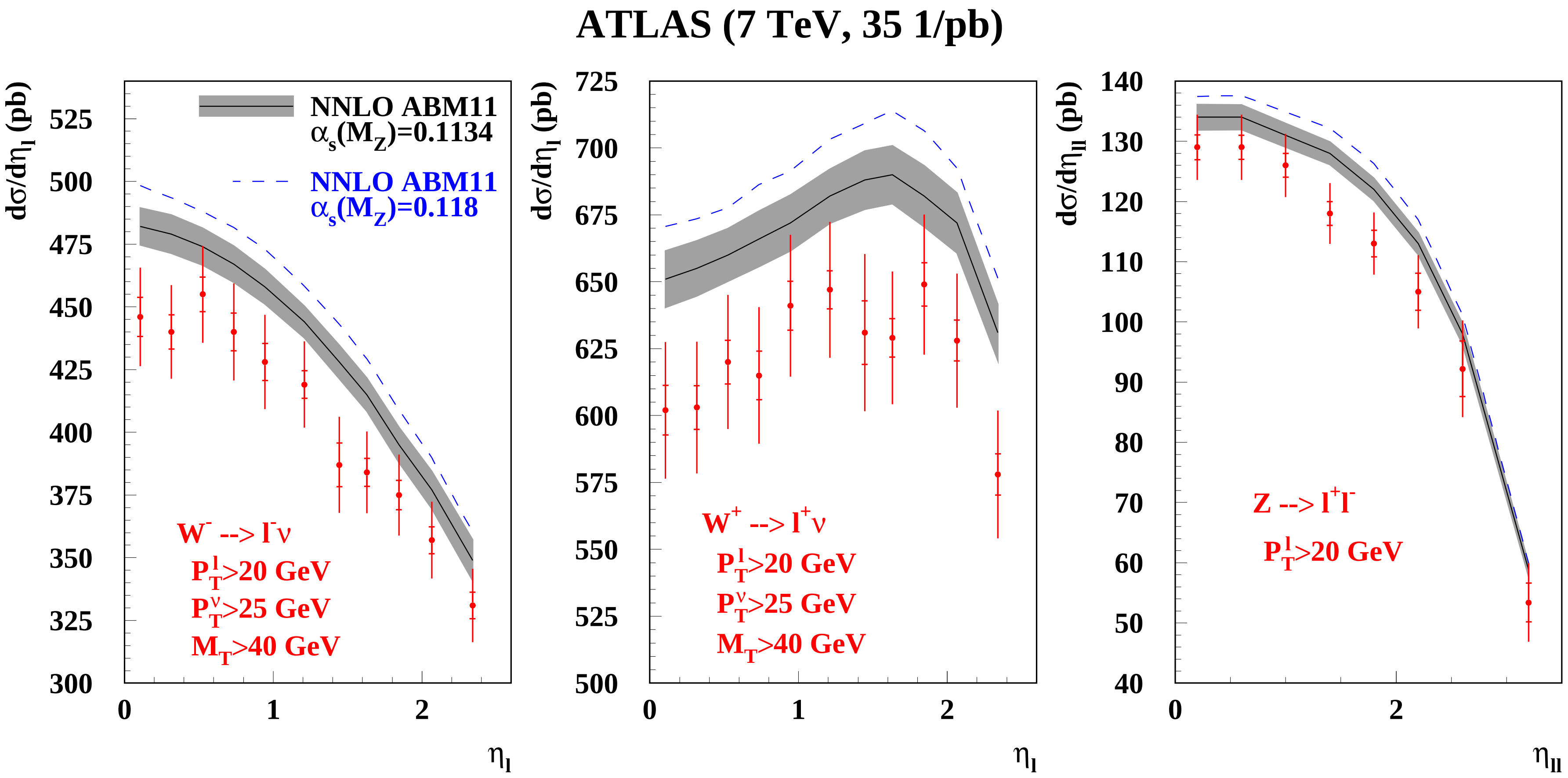
General remarks

- QCD corrections important
  - require theory predictions to NNLO accuracy
- PDF fits with 3-flavors for DIS, 5-flavors for jets (matching from 3 to 5-flavors)
  - QCD evolution over large range

Benchmark processes

- $W^{\pm}$- and Z-boson production
- top-quark hadro-production

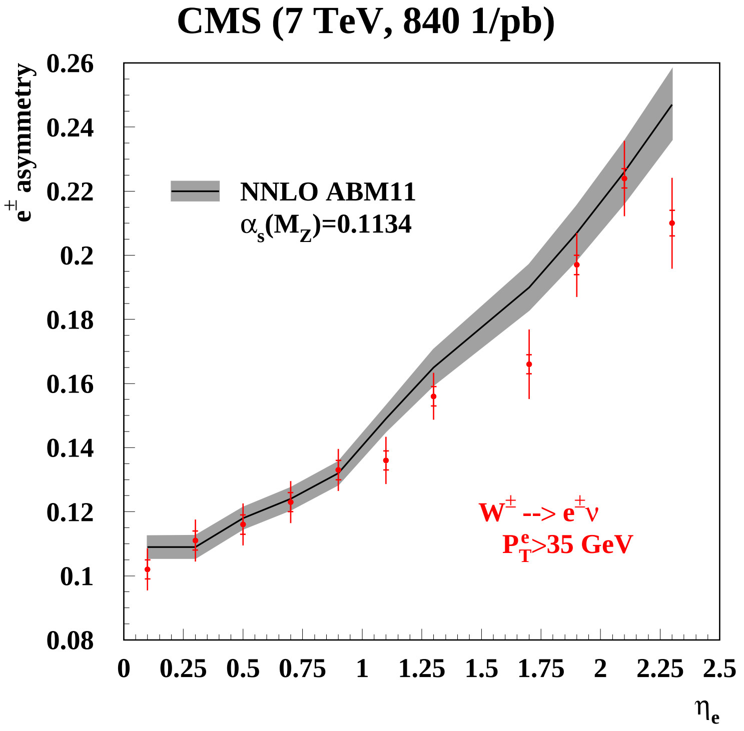
Drell-Yan codes at NNLO

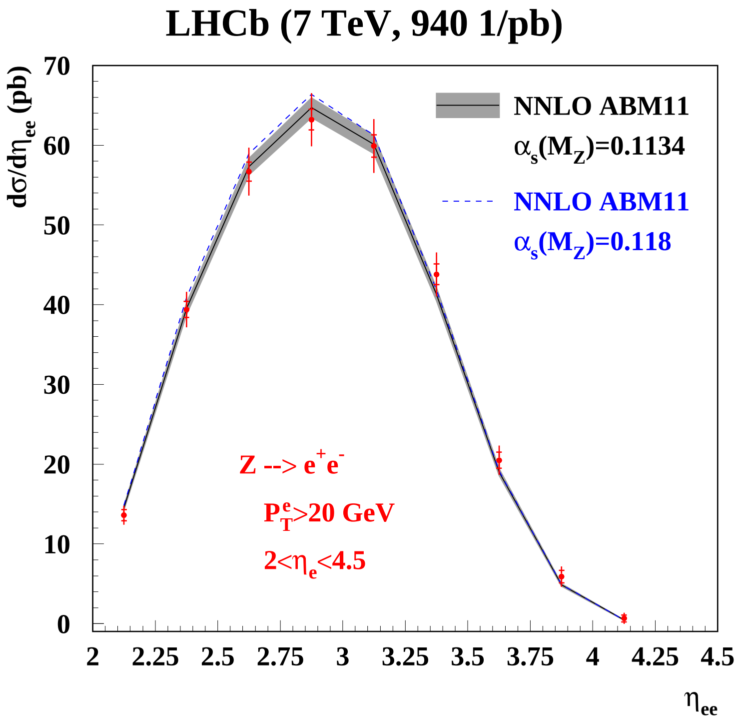


- DYNNLO 1.3 provides better numerical stability for the W-production in central region ($\sim$ 200h) Catani, Cieri, Ferrera, de Florian, Grazzini ‘09
- FEWZ 3.1 more convenient/stable for estimation of the PDF uncertainties ($\sim$ 2d x 24 processors) Li, Petriello ‘12
- Central values are computed with DYNNLO and the PDF errors are obtained with FEWZ

Benchmarking of ABM11 PDFs

Comparision with LHC Drell-Yan data Alekhin, Bümlein, S.M. ‘13





- Good overall agreement with data of CMS ‘10 and LHCb ‘12, ‘13

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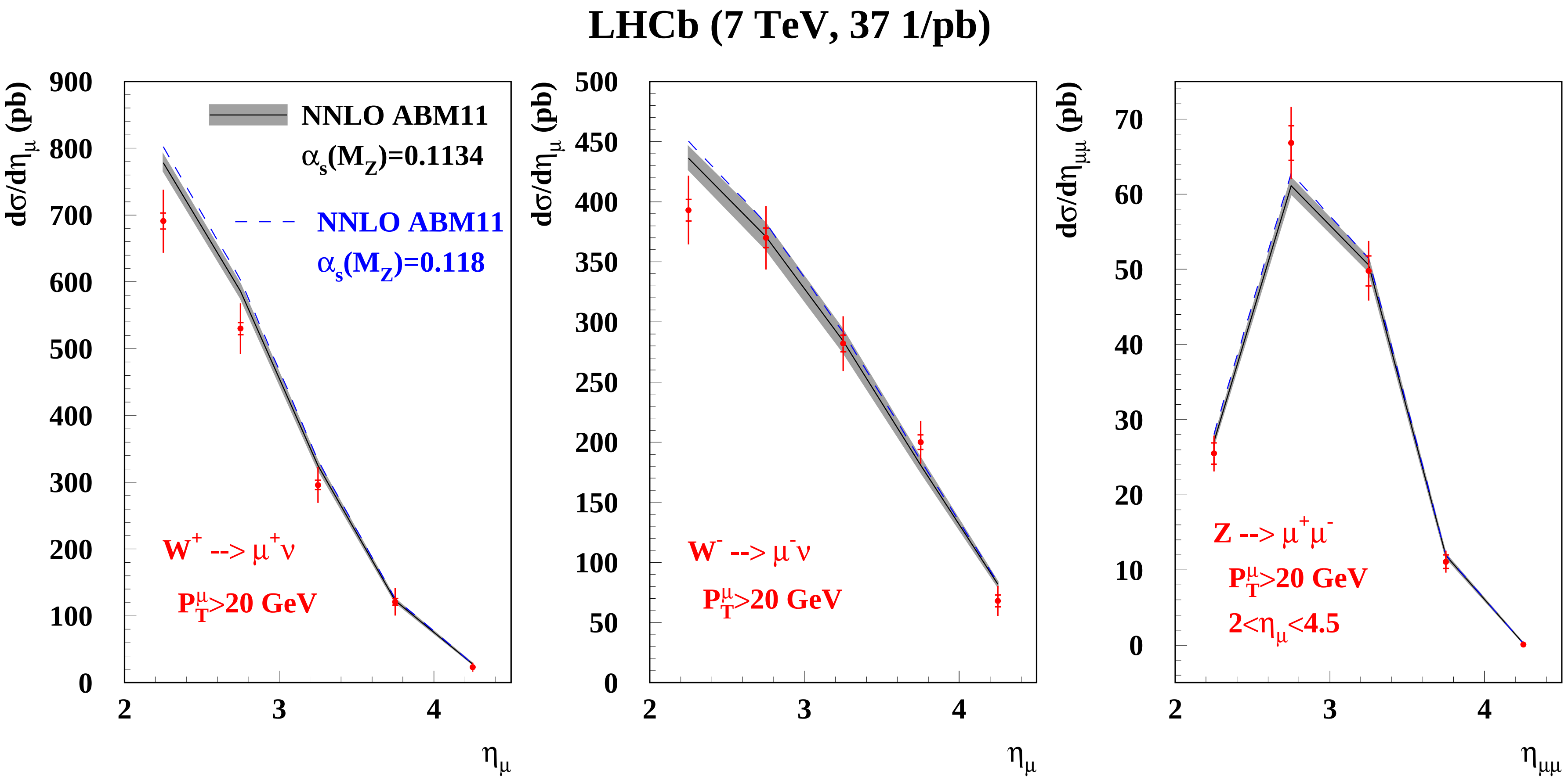

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Benchmarking of ABM11 PDFs

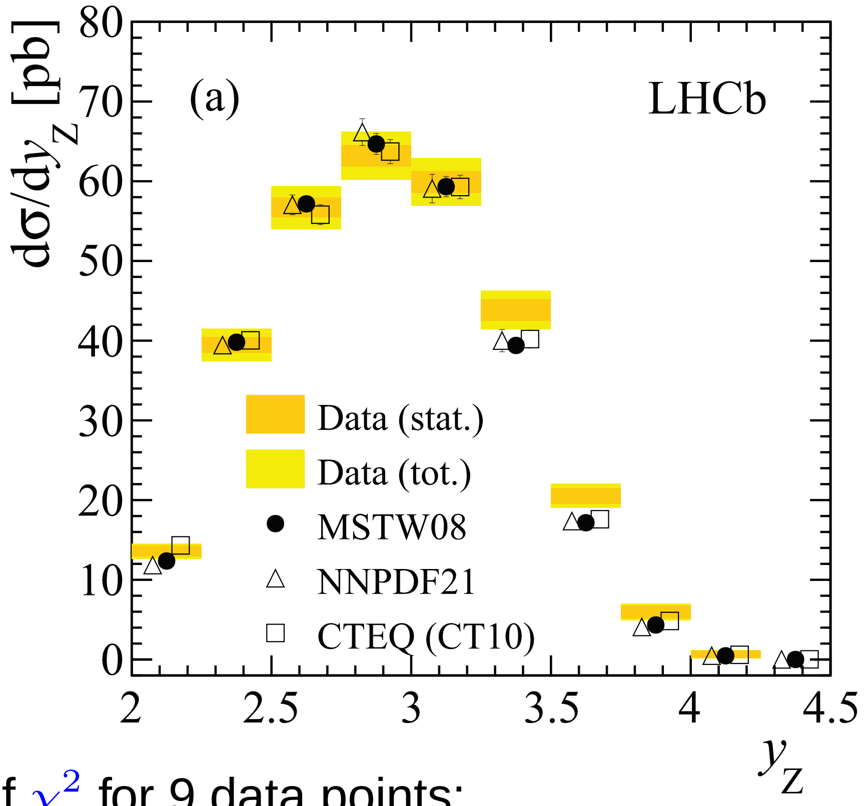
Comparision with LHC Drell-Yan data Alekhin, Bümlein, S.M. ‘13

Experiment	ATLAS ‘11	CMS ‘12	LHCb ‘12	LHCb ‘13
Final states	$W^+ \to l^+\nu$ $W^- \to l^-\nu$ $Z \to l^+l^-$	$W^+ \to e^+\nu$ $W^- \to e^-\nu$	$W^+ \to \mu^+\nu$ $W^- \to \mu^-\nu$	$Z \to e^+e^-$
Luminosity (1/pb)	35	840	37	940
NDP	30	11	10	9
χ^2	35.7(7.7)	10.6(4.7)	13.1(4.5)	11.3(4.2)

- value of χ^2 for Drell-Yan data at the LHC with NNLO ABM11 PDFs (+ one standard deviation of χ^2 equal to $\sqrt{2NDP}$)
- ABM11 benchmarking in arXiv:1211.5142 reports wrong χ^2 values for PDF comparison (NLO MCFM with K-factors, no PDF errors, shifted α_s)

Update of PDF benchmarking

Recent LHCb result LHCb '13



- Check value of χ^2 for 9 data points:
  MSTW08: 27.6
  NNPDF23: 24.6 (average)
  CT10: 9.8
  ABM11: 11.3

Drell Yan data in global fit

Current technology

- (N)NLO calculations are quite time-consuming; fast tools are employed (FASTNLO, Applegrid,.....)
  - corrections for certain basis of PDFs stored in grid
  - fitted PDFs are expanded over the basis
  - (N)NLO cross section in PDF fit calculated as combination of expansion coefficients with pre-prepared grids

Drell Yan data in global fit

Further optimization

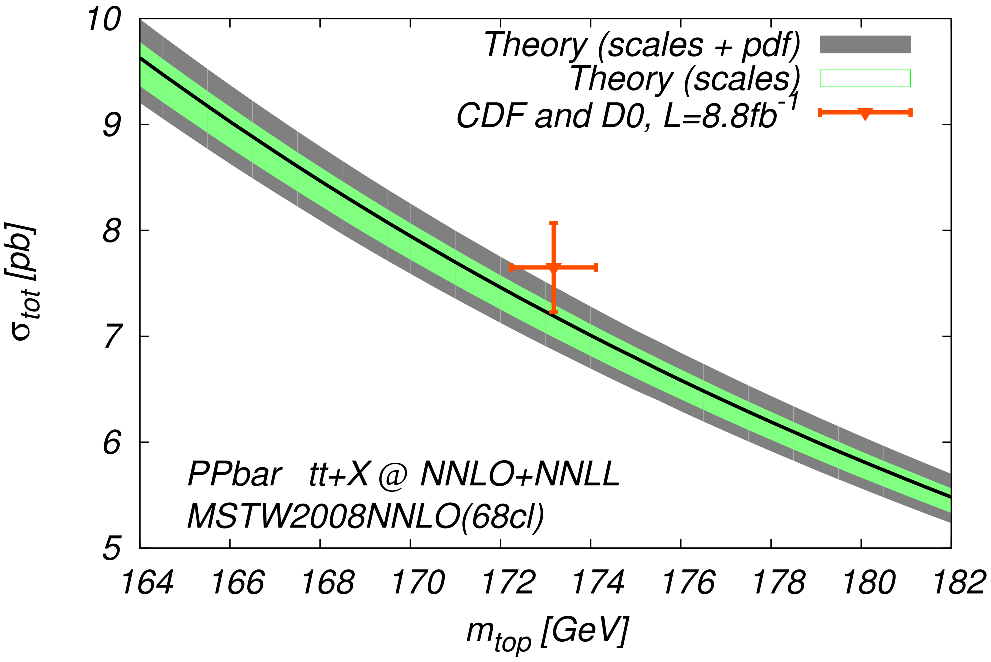
- General PDF basis not necessary (PDFs already constrained by data)
- Use PDF basis provided by eigenvalue PDF sets obtained in previous version of fit
  - $P_0 \pm \Delta P_0$ (vector of PDF parameters with errors from previous fit)
  - E: error matrix
  - P: current value of PDF parameters in fit
- Iterative cycle
  - store (N)NLO cross section for all PDF sets defined by eigenvectors of E
  - transform variation of fitted PDF parameters $(P - P_0)$ into this eigenvector basis
  - calculate (N)NLO cross section in PDF fit as combination of transformed $(P - P_0)$ with stored eigenvector values

Top-quark pair-production

Exact result at NNLO in QCD

Czakon, Fiedler, Mitov ‘13

- Illustration of mass dependence for Tevatron



- NNLO perturbative corrections (e.g. at LHC8)
  - K-factor (NLO → NNLO) of $\mathcal{O}(10\%)$
  - scale stability at NNLO of $\mathcal{O}(\pm 5\%)$

Heavy-quark masses in Standard Model

- Higgs boson gives mass to matter fields via Higgs-Yukawa coupling
  - large top quark mass m_t

QCD

- Classical part of QCD Lagrangian

$$\mathcal{L} \quad = \quad -\frac{1}{4} F^a_{\mu\nu} F_b^{\mu\nu} + \sum_{\text{flavors}} \bar{q}_i \left(\mathrm{i} \not{D} - m_q \right)_{ij} q_j$$

  - field strength tensor $F^a_{\mu\nu}$ and matter fields $q_i, \bar{q}_j$
  - covariant derivative $D_{\mu,ij} = \partial_\mu \delta_{ij} + \mathrm{i} g_s \left(t_a \right)_{ij} A^a_\mu$

- Formal parameters of the theory (no observables)
  - strong coupling $\alpha_s = g_s^2/(4\pi)$
  - quark masses m_q

Challenge

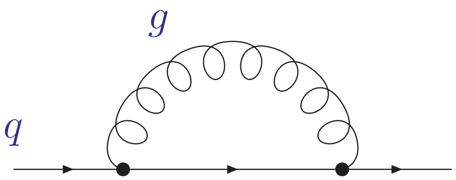
- Suitable observables for measurements of α_s, m_q, . . .
  - comparison of theory predictions and experimental data

Heavy-quark mass renormalization

Pole mass

- Based on (unphysical) concept of top-quark being a free parton

$$\not{p} - m_q - \Sigma(p, m_q)\Big|_{p^2=m_q^2}$$



  - heavy-quark self-energy $\Sigma(p, m_q)$ receives contributions from regions of all loop momenta – also from momenta of $\mathcal{O}(\Lambda_{QCD})$
- Definition of pole mass ambiguous up to corrections $\mathcal{O}(\Lambda_{QCD})$
  - bound from lattice QCD: $\Delta m_q \geq 0.7 \cdot \Lambda_{QCD} \simeq 200$ MeV

    Bauer, Bali, Pineda ’11

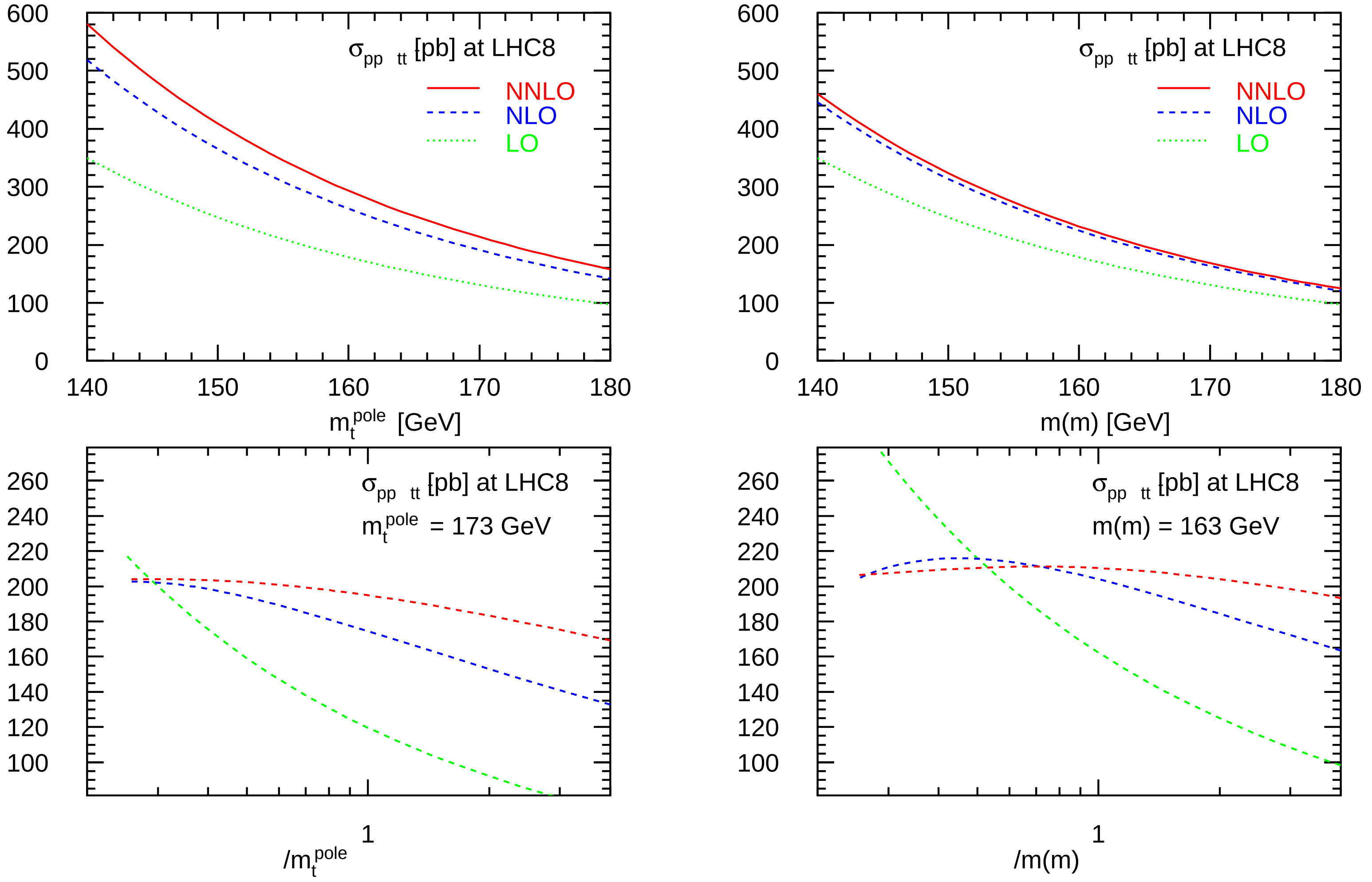
Running quark masses

- $\overline{MS}$ mass definition $m(\mu_R)$ realizes running mass (scale dependence)
  - short distance mass probes at scale of hard scattering

    $m_{\rm pole} = m_{\rm short\ distance} + \delta m$
  - conversion between $m_{\rm pole}$ and $\overline{MS}$ mass $m(\mu_R)$ perturbation theory

Total cross section with running mass

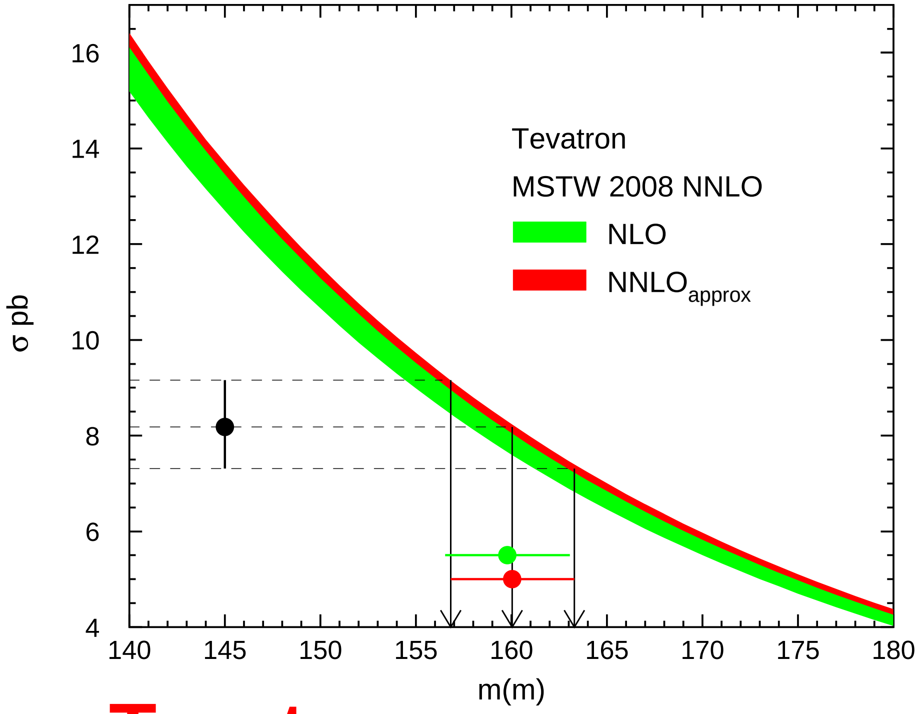
Comparison pole mass vs. $\overline{MS}$ mass



- good apparent convergence of perturbative expansion
- small theoretical uncertainity form scale variation

Top mass from total cross section

- Total top quark cross section as function of $\overline{MS}$ mass
  Langenfeld, S.M., Uwer ‘09



Tevatron

Tevatron

- Determine top quark mass from Tevatron cross section data
  - $\sigma_{\mathrm{t\bar{t}}} = 7.56^{+\,0.63}_{-\,0.56}$ pb D0 coll. arXiv:1105.5384
  - $\sigma_{\mathrm{t\bar{t}}} = 7.50\,^{+0.48}_{-0.48}$ pb CDF coll. CDF-note-9913
- Fit of m_t for individual PDFs
  - parton luminosity at Tevatron driven by $q\bar{q}$
  - $\overline{MS}$-scheme for $m_t^{\overline{\mathrm{MS}}}(m_t)$, then scheme transformation to pole mass m_t^{pole} at NNLO

	ABM11	JR09	MSTW08	NN21
$m_t^{\overline{\mathrm{MS}}}(m_t)$	162.0 $^{+2.3\,+0.7}_{-2.3\,-0.6}$	163.5 $^{+2.2\,+0.6}_{-2.2\,-0.2}$	163.2 $^{+2.2\,+0.7}_{-2.2\,-0.8}$	164.4 $^{+2.2\,+0.8}_{-2.2\,-0.2}$
m_t^{pole}	171.7 $^{+2.4\,+0.7}_{-2.4\,-0.6}$	173.3 $^{+2.3\,+0.7}_{-2.3\,-0.2}$	173.4 $^{+2.3\,+0.8}_{-2.3\,-0.8}$	174.9 $^{+2.3\,+0.8}_{-2.3\,-0.3}$
(m_t^{pole})	(169.9 $^{+2.4\,+1.2}_{-2.4\,-1.6}$)	(171.4 $^{+2.3\,+1.2}_{-2.3\,-1.1}$)	(171.3 $^{+2.3\,+1.4}_{-2.3\,-1.8}$)	(172.7 $^{+2.3\,+1.4}_{-2.3\,-1.2}$)

- Good consistency within errors for $m_t^{\mathrm{pole}} = 171.7 \dots 174.9$ at NNLO

The fine print

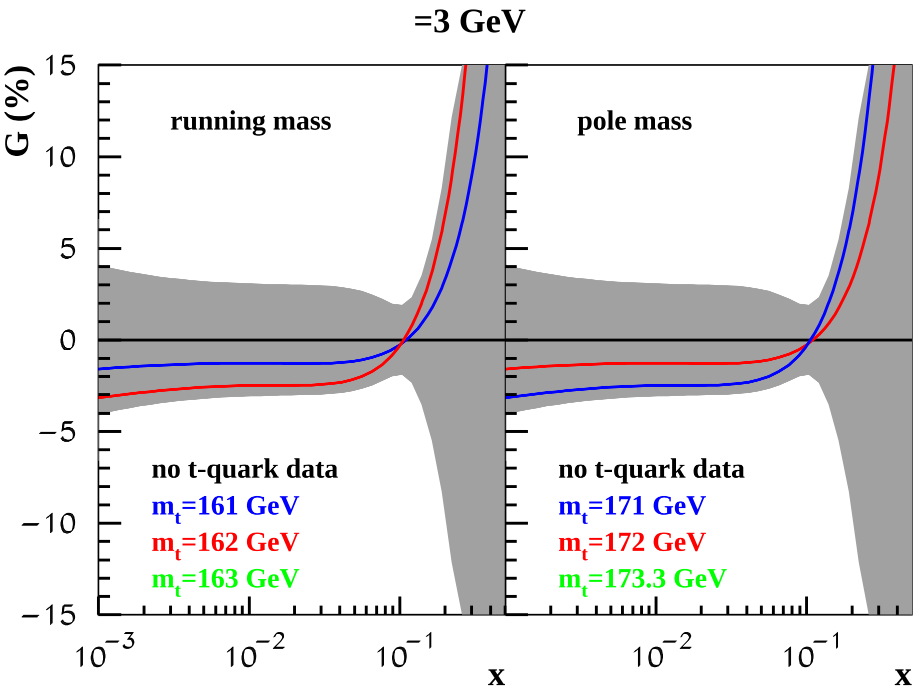
- Intrinsic limitation of sensitivity in total cross section

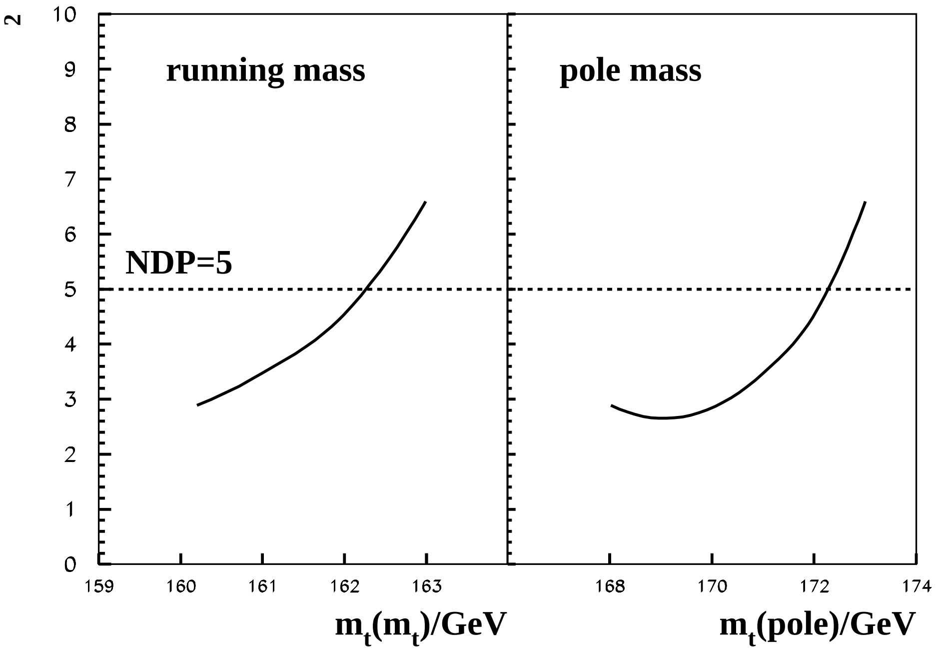
$$\left|\frac{\Delta\sigma_{t\bar{t}}}{\sigma_{t\bar{t}}}\right| \simeq 5 \times \left|\frac{\Delta m_t}{m_t}\right|$$

- Cross section at LHC has correlation of m_t, $\alpha_S(M_Z)$, gluon PDF
  $\sigma_{t\bar{t}} \sim \alpha_s^2 \, m_t^2 \, g(x) \otimes g(x)$
  - effective parton $\langle x \rangle \sim 2m_t/\sqrt{s} \sim 2.5 \dots 5 \cdot 10^{-2}$
  - fit with fixed values of m_t and $\alpha_S(M_Z)$ carries significant bias
    Czakon, Mangano, Mitov, Rojo ‘13

The fine print

- Fit with correlations
  - $g(x)$ and $\alpha_s(M_Z)$ already well constrained by global fit (no changes)
  - for fit with $\chi^2/NDP = 5/5$ obtain value of $m_t(m_t) = 162$ GeV Alekhin, Blümlein, S.M. [in progress]





Higgs potential

Renormalization group equation

- Quantum corrections to Higgs potential $V(\Phi) = \lambda \left|\Phi^\dagger \Phi - \frac{v}{2}\right|^2$
- Radiative corrections to Higgs self-coupling λ
  - electro-weak couplings g and g' of $SU(2)$ and $U(1)$
  - top-Yukawa coupling y_t

$$16\pi^2 \frac{d\lambda}{dQ} = 24\lambda^2 - \left(3g'^2 + 9g^2 - 12y_t^2\right)\lambda + \frac{3}{8}g'^4 + \frac{3}{4}g'^2 g^2 + \frac{9}{8}g^4 - 6y_t^4 + \dots$$

Higgs potential

Triviality

- Large mass implies large λ
  - renormalization group equation dominated by first term

$$16\pi^2 \frac{d\lambda}{dQ} \simeq 24\lambda^2 \qquad \longrightarrow \qquad \lambda(Q) = \frac{m_H^2}{2v^2 - \frac{3}{2\pi^2} m_H^2 \ln(Q/v)}$$

- $\lambda(Q)$ increases with Q
- Landau pole implies cut-off Λ
  - scale of new physics smaller than Λ to restore stability
  - upper bound on m_H for fixed Λ

$$\Lambda \leq v \exp\left(\frac{4\pi^2 v^2}{3m_H^2}\right)$$

- Triviality for $\Lambda \to \infty$
  - vanishing self-coupling $\lambda \to 0$ (no interaction)

Higgs potential

Vacuum stability

- Small mass
  - renormalization group equation dominated by y_t

$$16\pi^2 \frac{d\lambda}{dQ} \simeq -6y_t^4 \qquad \longrightarrow \qquad \lambda(Q) = \lambda_0 - \frac{\frac{3}{8\pi^2} y_0^4 \ln(Q/Q_0)}{1 - \frac{9}{16\pi^2} y_0^2 \ln(Q/Q_0)}$$

- $\lambda(Q)$ decreases with Q
- Higgs potential unbounded from below for $\lambda < 0$
- $\lambda = 0$ for $\lambda_0 \simeq \frac{3}{8\pi^2} y_0^4 \ln(Q/Q_0)$
- Vacuum stability

$$\Lambda \leq v \exp\left(\frac{4\pi^2 m_H^2}{3y_t^4 v^2}\right)$$

  - scale of new physics smaller than Λ to ensure vacuum stability
  - lower bound on m_H for fixed Λ

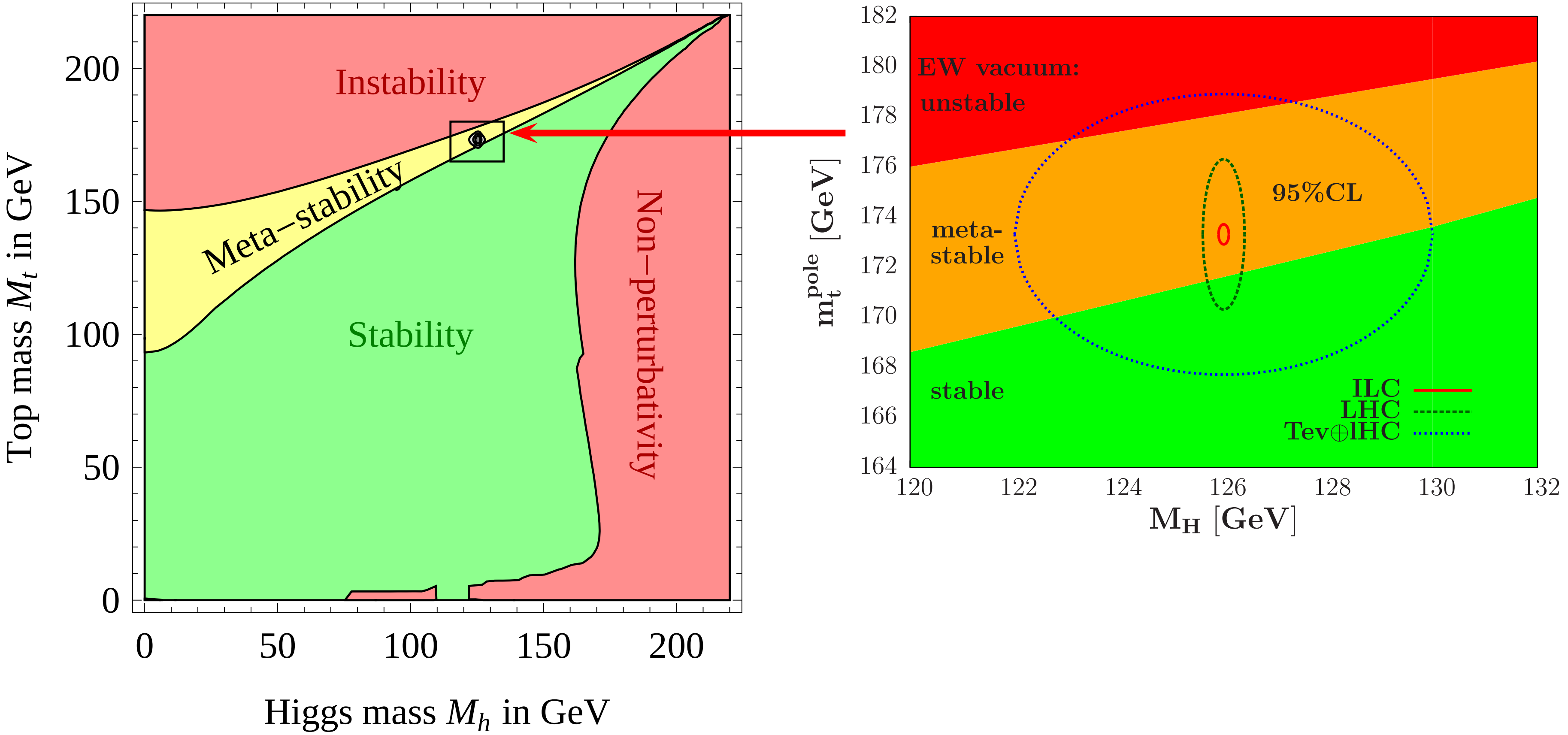
Implications on electroweak vacuum

- Relation between Higgs mass m_H and top quark mass m_t
  - condition of absolute stability of electroweak vacuum $\lambda(\mu) \geq 0$
  - extrapolation of Standard Model up to Planck scale M_P
  - $\lambda(M_P) \geq 0$ implies lower bound on Higgs mass m_H

$$m_H \;\geq\; 129.2 + 1.8 \times \left(\frac{m_t^{\text{pole}} - 173.2\ \text{GeV}}{0.9\ \text{GeV}} \right) - 0.5 \times \left(\frac{\alpha_s(M_Z) - 0.1184}{0.0007} \right) \pm 1.0\ \ \text{GeV}$$

  - recent NNLO analyses Bezrukov, Kalmykov, Kniehl, Shaposhnikov ‘12; Degrassi, Di Vita, Elias-Miro, Espinosa, Giudice et al. ‘12
  - uncertainity in results due to α_s and m_t (pole mass scheme)
- Top quark mass from Tevatron in well-defined scheme
  - $m_t^{\overline{\text{MS}}}(m_t) \;=\; 163.3 \pm 2.7\ \text{GeV}$ implies in pole mass scheme
    $m_t^{\text{pole}} \;=\; 173.3 \pm 2.8\ \text{GeV}$
  - good consistency of mass value between different PDF sets

Fate of the universe



Degrassi, Di Vita, Elias-Miro, Espinosa, Giudice et al. '12; Alekhin, Djouadi, S.M. '12; Masina '12

- Uncertainty in Higgs bound due to m_t from in $\overline{MS}$ scheme
  - bound relaxes $m_H \geq 129.4 \pm 5.6$ GeV
  - "fate of universe" still undecided

Summary

Physics at the Terascale

- Discovery of (SM like) Higgs boson opens new avenue for studies of Standard Model physics and beyond
- Precision determinations of non-perturbative parameters is essential
    - masses m_t, M_W, m_H, $\dots$
    - coupling constants $\alpha_s(M_Z)$
    - parton content of proton (PDFs)
- Precision measurements require careful definition of observable
    - top-quark mass m_t in well defined scheme
- Radiative corrections at higher orders in QCD and EW are mandatory
    - continuous benchmarking mandatory
    - theory improvements driven by experimental precision
- Lots of challenging tasks for young researchers