Soft Gluon Effects in Four Parton Hard-Scattering Processes

Alessandro Torre

Universität Zürich

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with Stefano Catani and Massimiliano Grazzini

Outline

1. Introduction: Sudakov Logs & Threshold Resummation

2. One-particle inclusive cross section:
   a general NLO calculation of the large logarithmic terms

3. Soft-gluon resummation at fixed rapidity

4. Summary

All-order Threshold Resummation

- Perturbative QCD predictions are at the heart of our understanding of hard-scattering processes at high-energy colliders
- At the phase space boundaries, the imbalance between real emission (strongly inhibited) and virtual corrections leads to enhanced logarithmic terms
- The large logarithms spoil the perturbative expansion and must be resummed in order to get reliable physical predictions [Sterman '87]

[Catani, Trentadue '89]

[Kidonakis, Laenen, Oderda, Sterman '98]

[Bonciani, Catani, Mangano, Nason '98]

One-particle-inclusive cross-section

$h_1 h_2 \to h_3 X$ $\qquad\qquad$ $a_1 a_2 \to a_3 a_4$

$$\frac{d\sigma_{h_1 h_2 h_3}}{[dP_3]}(P_i) = \sum_{a_i} f_{h_1/a_1} \otimes f_{h_2/a_2} \otimes d_{h_3/a_3} \otimes \frac{d\hat{\sigma}_{a_1 a_2 a_3}}{[dp_3]}(p_i)$$

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- $x = (s + t + u)/s \to 0$
  - Constant terms: $\delta(x) \to 1$
  - Large Logs: $\left(\frac{\ln^\ell x}{x}\right)_+ \to (\ln N)^{\ell+1}$

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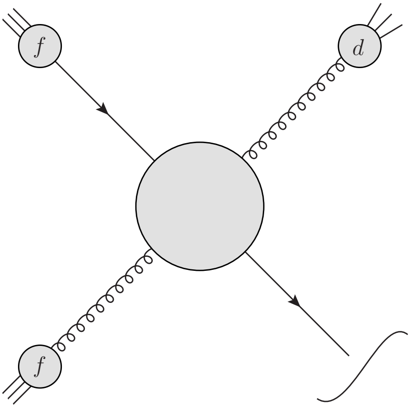
  - Large Logs: $\left(\frac{\ln^\ell x}{x}\right)_+ \to (\ln N)^{\ell+1}$
- While regular terms are suppressed: $\mathcal{O}(1/N)$

- Resummed Multiparton Cross Section [Bonciani, Catani, Mangano, Nason '03]

$$d\hat{\sigma}_{a_1 a_2 a_3, N} =$$



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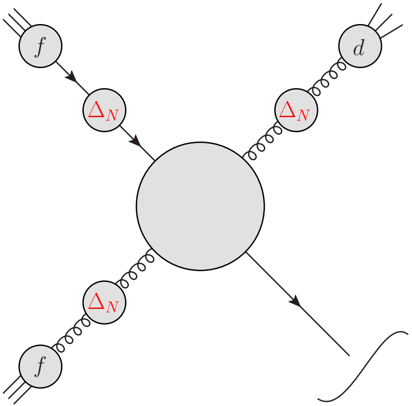

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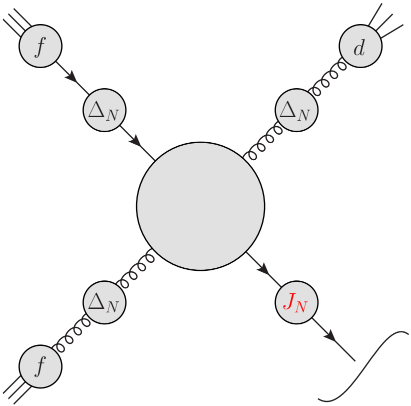

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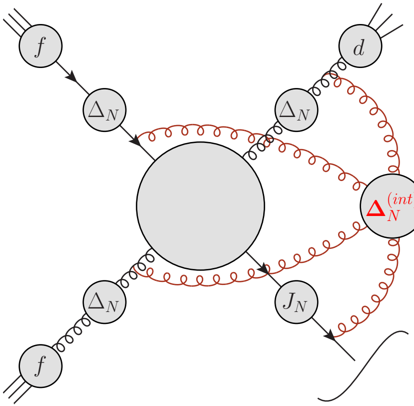

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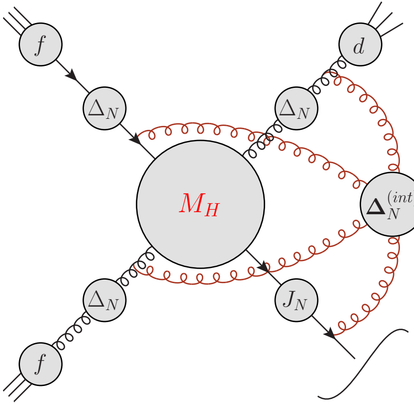

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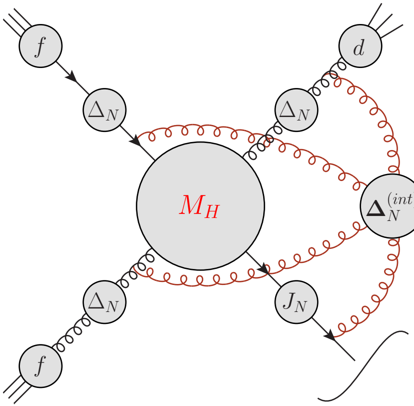

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NLO result near threshold

- $\mathcal{M}_{\mathrm{H}}$ perturbatively computable as a power series in α_{S}
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$$\sum_{pol\{g\}} |\mathcal{M}_{\rm LO+g}|^2 \simeq -4\pi\alpha_{\rm S} \sum_{i,j} \frac{2p_i p_j}{p_i q \; p_j q} \langle \mathcal{M}_{\rm LO} | \, \boldsymbol{T}_i \boldsymbol{T}_j \, | \mathcal{M}_{\rm LO} \rangle$$

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- Not enough! a_4 splitting hard-collinear too
  - Collinear matching to the full AP behaviour

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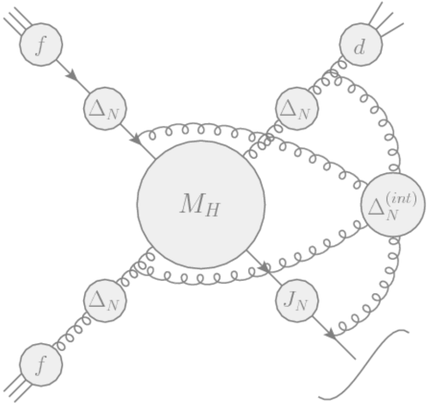

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- Our fixed-order result is consistent with known (color-specific) results
  - Photoproduction from qg and $q\overline{q}$ channels
    [Aurenche et al. '87] [Contogouris et al. '90] [Gordon and Vogelsang '93]
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$$|\mathcal{M}_{\mathrm{H}}\rangle$$

Full Color Structure

$$@\text{NLO:} \qquad |\mathcal{M}_{\rm H}\rangle = \left(1 + \frac{\alpha_{\rm S}}{\pi}\,\mathcal{H}_1 + \mathcal{O}(\alpha_{\rm S}^2)\right)|\mathcal{M}_{\rm LO}\rangle + \mathcal{O}(1/N)$$

$$2\,\mathcal{H}_1(r, p_T) = \boldsymbol{I}_{\rm real} + \boldsymbol{I}_{\rm virt} \qquad \left(r = e^{-2\eta}\right)$$

- $\boldsymbol{I}_{\rm real}$: (some) constant color correlations from real corrections

$$2\,\boldsymbol{I}_{\rm real} = \left[K_4 - C_4\left(\frac{1}{2}\ln^2 r + \frac{\pi^2}{3}\right) - \gamma_4 \ln\frac{(1+r)^2 p_T^2}{r\mu^2}\right] + \sum_{i\neq 4}\left[C_i\left(\frac{1}{2}\ln^2 r + \frac{\pi^2}{3}\right) - \gamma_i \ln\frac{\mu_i^2}{\mu^2}\right] - 4\ln\frac{r}{1+r}\ln\frac{1}{1+r}\,\boldsymbol{T}_1\boldsymbol{T}_2 + \ln^2\frac{1}{1+r}(\boldsymbol{T}_1\boldsymbol{T}_3 + \boldsymbol{T}_2\boldsymbol{T}_4) + \ln^2\frac{r}{1+r}(\boldsymbol{T}_1\boldsymbol{T}_4 + \boldsymbol{T}_2\boldsymbol{T}_3)$$

$$K_q = K_{\bar q} = \left(\frac{7}{2} - \frac{\pi^2}{6}\right)C_F, \qquad K_g = \left(\frac{67}{18} - \frac{\pi^2}{6}\right)C_A - \frac{10}{9}n_F T_R$$

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- $\boldsymbol{I}_{\rm real}$: (some) constant color correlations from real corrections
- $\boldsymbol{I}_{\rm virt}$: (all the) constant color correlations from virtual corrections

$$\mathrm{Re}\left|\mathcal{M}_{(1\mathrm{loop})}\right\rangle = \frac{\alpha_s}{2\pi}\left(\boldsymbol{I}_{\rm sing} + \boldsymbol{I}_{\rm virt}\right)|\mathcal{M}_{\rm LO}\rangle + \mathcal{O}(1/N)$$

$$\boldsymbol{I}_{\rm sing} = \frac{1}{2}\frac{1}{\Gamma(1-\epsilon)}\left[\sum_i\left(\frac{1}{\epsilon^2} + \frac{\gamma_i}{\boldsymbol{T}_i^2}\frac{1}{\epsilon}\right)\sum_{j\neq i}\boldsymbol{T}_i\boldsymbol{T}_j\left(\frac{4\pi\mu^2}{2p_ip_j}\right)^{\epsilon} - \sum_i\frac{\gamma_i}{\boldsymbol{T}_i^2}\sum_{j\neq i}\boldsymbol{T}_i\boldsymbol{T}_j\ln\frac{\mu^2}{2p_ip_j}\right]$$

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- For any process one needs to:
  1. Determine $\boldsymbol{I}_{\rm virt}$ from the one-loop corrections
  2. Explicitely determine all the $\langle\mathcal{M}_{\rm LO}|\,\boldsymbol{T}_i\boldsymbol{T}_j\,|\mathcal{M}_{\rm LO}\rangle$ correlations

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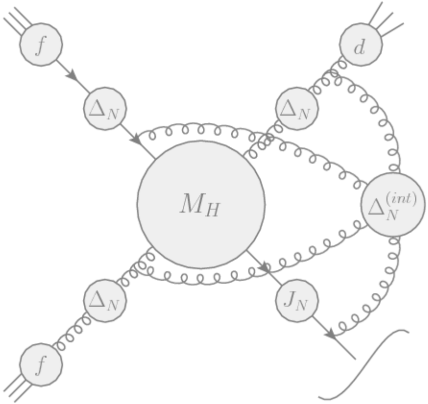

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- Real-Virtual $\mathcal{H}_1^* \boldsymbol{\Delta}_N^{(int)}$ interference → (L)

  & Virtual-Virtual $\mathcal{H}_1^* \mathcal{H}_1$ interference → (1)

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- We have considered the one-particle-inclusive cross section at high transverse energies in hadron collisions
- We have presented the general structure of the logarithmically enhanced terms at NLO
  - ✓ Agreement with previous specific results in the literature
- We have presented the all-order resummation formula of the logarithmically enhanced terms at fixed rapidity and extracted the colour structure of the hard coefficient at $\mathcal{O}(\alpha_{\rm S})$
  - → The same technique can be applied to other multiparton hard scattering processes

Thanks!

Backup (1)

$$\ln \Delta_{a,N}(Q^2;\mu^2) = \int_0^1 \frac{z^{N-1}-1}{1-z} \int_{\mu^2}^{(1-z)^2 Q^2} \frac{dq^2}{q^2} A_a(\alpha_{\rm S}(q^2))$$

$$\ln J_{a,N}(Q^2) = \int_0^1 \frac{z^{N-1}-1}{1-z} \Big[\int_{(1-z)^2 Q^2}^{(1-z)Q^2} \frac{dq^2}{q^2} A_a(\alpha_{\rm S}(q^2)) + \frac{1}{2} B_a \left(\alpha_{\rm S}\left((1-z)Q^2\right)\right) \Big]$$

- The coefficients A_a and B_a have perturbative expansion

$$A_a(\alpha_{\rm S}) = \sum_{k=1}^{\infty} \left(\frac{\alpha_{\rm S}}{\pi}\right)^n A_a^{(n)} \qquad B_a(\alpha_{\rm S}) = \sum_{k=1}^{\infty} \left(\frac{\alpha_{\rm S}}{\pi}\right)^n B_a^{(n)}$$

$$A_a^{(1)} = C_a \qquad A_a^{(2)} = \frac{1}{2} C_a \left[C_A \left(\frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{10}{9} n_F T_R \right]$$

$$B_a^{(1)} = -\gamma_a$$

Backup (2)

$$\boldsymbol{\Delta}_N^{(\mathrm{int})} = \bar{\boldsymbol{V}}_N \boldsymbol{V}_N$$

$$\boldsymbol{V}_N = P_z \; \exp\left\{ \sum_{i \neq j} \int_0^1 \frac{z^{N-1} - 1}{1 - z} \, \boldsymbol{\Gamma}\left(\alpha_{\mathrm{S}}\left((1-z)^2 p_T^2\right), r\right) \right\}$$

- P_z denotes z-ordering in the expansion of the exponential matrix
- The anomalous-dimension matrix $\boldsymbol{\Gamma}(\alpha_{\mathrm{S}}, r)$ has the perturbative expansion

$$\boldsymbol{\Gamma}(\alpha_{\mathrm{S}}, r) = \frac{\alpha_{\mathrm{S}}}{\pi} \boldsymbol{\Gamma}^{(1)}(r) + \mathcal{O}(\alpha_{\mathrm{S}}^2)$$

$$\boldsymbol{\Gamma}^{(1)}(r) = \frac{1}{4} \sum_{i \neq j} \boldsymbol{T}_i \cdot \boldsymbol{T}_j \ln \zeta_{ij}(r) + i\pi \left(\boldsymbol{T}_1 + \boldsymbol{T}_2\right)^2$$