Soft Non-Global Structure at Two Loops in Soft-Collinear Effective Theory

Robert M. Schabinger

Randall Kelley, RMS, Matthew D. Schwartz, and Hua Xing Zhu (Phys. Rev. **D86** (2012) 054017)
and Andreas von Manteuffel, RMS, and Hua Xing Zhu (to appear)

BOOST 2013, August 12-16

Outline

Thrust in Soft-Collinear Effective Theory

Thrust,

$$T = \max_{\mathbf{x}} \left\{ \frac{\sum_i |\mathbf{p}_i \cdot \mathbf{x}|}{\sum_i |\mathbf{p}_i|} \right\}$$

is a well-studied e^+e^- event shape variable that requires resummation in the end-point region, $1 - T = \tau \to 0$.

(see *e.g.* Schwartz Phys. Rev. **D77** (2008) 014026; Becher and Schwartz JHEP **07** (2008) 034)

- The framework of soft-collinear effective theory is a convenient one in which to discuss factorization and resummation.
- In the context of thrust in the end-point region, the hard scale is simply Q and one defines the scaling behavior of a soft or collinear momentum by

$$p_{\eta_{\text{collinear}}} \approx Q\,(\tau, 1, \sqrt{\tau}) \quad p_{\text{soft}} \approx Q\,(\tau, \tau, \tau)$$
$$p = (p^+, p^-, p_\perp) \qquad p^2 = p^+ p^- - p_\perp^2$$

Factorization For Thrust-Like Observables (τ or τ_ω)

Catani et. al. Nucl. Phys. **B407**, 3 (1993); Schwartz arXiv:0709.2709;

Ellis et. al. **JHEP** 1011:101, 2010; Kelley et. al. arXiv:1102.0561

$$\frac{1}{\sigma_{tree}}\frac{d\sigma}{d\tau_{\rm alg}} = H(Q,\mu)\int dk_L dk_R dM_L^2 dM_R^2 J_{\mathbf{n}}(M_L^2 - Q\ k_L,\mu)\times$$
$$\times J_{\bar{\mathbf{n}}}(M_R^2 - Q\ k_R,\mu) S_{\rm alg}(k_L,k_R,\omega,r,\mu)\delta\left(\tau_{\rm alg} - \frac{M_L^2+M_R^2}{Q^2}\right)+\cdots$$

- H is a "hard function" which captures the effects associated with the short-distance hard scattering process.
- $J_{\mathbf{n}}$ and $J_{\bar{\mathbf{n}}}$ are "jet functions" which capture the effects associated with the radiation of collinear gluons off of the partons which emerge from the hard scattering.
- $S_{\rm alg}$ is a "soft function" which captures the effects associated with the radiation of soft gluons off of the partons which emerge from the hard scattering and the exchange of soft partons between them. It depends on the algorithm used to determine how the radiated soft partons are clustered into jets.

The Hemisphere Jet Algorithm

One defines two hemispheres (left and right) by dropping a plane perpendicular to the thrust axis at the collision point and defining

$$P^{\mu}_{L(R)} = \text{four} - \text{vector sum of all radiation in left(right) hemisphere.}$$

In soft-collinear effective theory, the $-(+)$ component of the $\eta(\bar{\eta})$ collinear momenta (of order Q) is fixed after the hard modes are integrated out but the $+(-)$ component is determined by the interaction of collinear fields with a soft background. It is therefore trivial to determine the parameters $M^2_{L(R)}$ and $k_{L(R)}$:

$$M_L^2 = P_L^2 \qquad M_R^2 = P_R^2 \qquad k_L = \eta \cdot P_L \qquad k_R = \bar{\eta} \cdot P_R$$

Clearly, this also fixes the sum of the $+(-)$ components of the soft momenta being radiated into the left(right) hemisphere to be $k_L(k_R)$.

The Thrust Cone Jet Algorithm

- A soft parton in the left hemisphere is in the **n** jet if the quantity $\tan^2\left(\frac{\theta}{2}\right)$ is less than r, where θ is the direction of the soft parton as measured from the direction of the thrust axis.
- A soft parton in the right hemisphere is in the $\bar{\mathbf{n}}$ jet if the quantity $\tan^2\left(\frac{\theta}{2}\right)$ is less than r, where θ is again the direction of the soft parton as measured from the direction of the thrust axis.
- The algorithm is sensitive to the radiation in neither jet and for the algorithm to be well-defined an explicit veto procedure must be used to reject events with a significant amount of energy deposited out of all jets.

$$S_{TC}(k_L, k_R, \omega, r, \mu) = \int_0^\omega d\lambda\, S(k_L, k_R, \lambda, r, \mu)$$

Here ω is called the *veto scale* and $0 < r \leq 1$ is called the *jet radius*.

Operator Definition of the Hemisphere Soft Function

$$S_{hemi}(k_L, k_R, \mu) =$$
$$\frac{1}{N_c} \sum_{X_s} \delta\Big(k_L - \bar{\eta} \cdot P^L_{X_s}\Big)\delta\Big(k_R - \eta \cdot P^R_{X_s}\Big)\langle 0|Y_\eta Y_{\bar{\eta}}|X_s\rangle\langle X_s|Y^\dagger_{\bar{\eta}} Y^\dagger_\eta|0\rangle$$

- $P_s^{L(R)}$ is the total soft momentum of final state X_s entering the left(right) hemisphere.
- The $Y's$ are Fourier transformed soft Wilson lines encapsulating the interaction of the "frozen" collinear quark and anti-quark with the soft gluon background.
- At $\mathcal{O}(\alpha_s^2)$, there are two soft partons emitted which can either travel into the same hemisphere or into opposite hemispheres.
- In earlier work, Hornig et. al. (JHEP **08** (2011) 054) and our group (Phys. Rev. **D84** (2011) 045022) calculated $S_{hemi}(k_L, k_R, \mu)$ to $\mathcal{O}(\alpha_s^2)$.

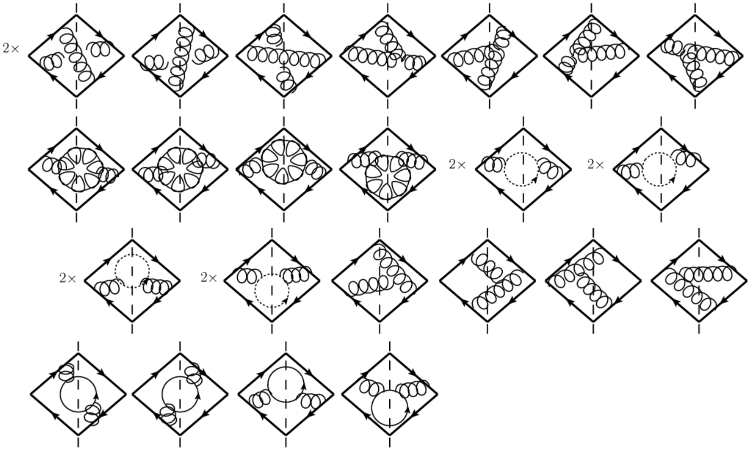
Operator Definition of the Thrust Cone Soft Function

$$S(k_L, k_R, \lambda, r, \mu) =$$
$$\frac{1}{N_c} \sum_{X_s} \delta\Big(k_L - \bar{\eta} \cdot P^L_{X_s}\Big)\delta\Big(k_R - \eta \cdot P^R_{X_s}\Big)\delta\Big(\lambda - E_{X_s}\Big)\langle 0|Y_\eta Y_{\bar{\eta}}|X_s\rangle\langle X_s|Y^\dagger_{\bar{\eta}} Y^\dagger_\eta|0\rangle$$

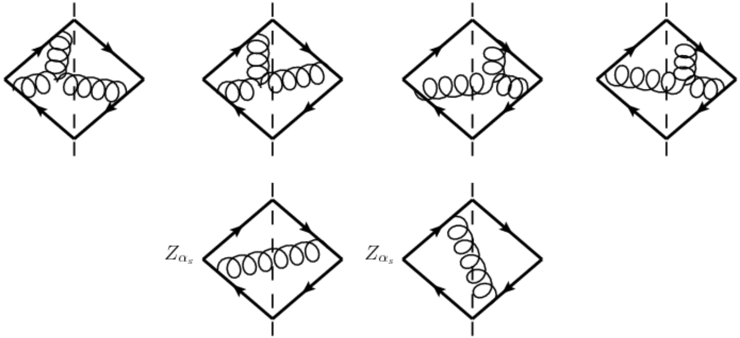
At $\mathcal{O}\left(\alpha_s^2\right)$, the phase-space of the two soft partons naturally splits up into four different contributions:

- Both soft partons clustered into the same jet.
- One soft parton clustered into the $\mathbf{n}$ jet and one soft parton clustered into the $\bar{\mathbf{n}}$ jet.
- One soft parton clustered into a jet and the other out of all jets.
- Both soft partons out of all jets.

$\mathcal{O}(\alpha_s^2)$ Double-Cut Soft Feynman Diagrams



$\mathcal{O}(\alpha_s^2)$ Single-Cut Soft Feynman Diagrams



$$Z_{\alpha_s} = 1 - \left(\frac{11}{3} C_A - \frac{4}{3} T_F n_f \right) \frac{\alpha_s}{4\pi\epsilon} + \mathcal{O}(\alpha_s^2)$$

Integrated Distributions

- It is difficult to work with soft functions directly because they contain singular distributions.
- It is technically easier to work with fully integrated quantities:

$$\Sigma(X,Y,\mu) = \int_0^X \mathrm{d}k_L \int_0^Y \mathrm{d}k_R \; S_{hemi}(k_L,k_R,\mu)$$

$$K_{TC}(\tau_\omega,\omega,r,\mu) = \int_0^{\tau_\omega} \mathrm{d}\tau'_\omega \int \mathrm{d}k_L \mathrm{d}k_R S_{TC}(k_L,k_R,\omega,r,\mu)\delta\left(\tau'_\omega - \frac{k_L+k_R}{Q}\right)$$

The Integrated Two-Loop Hemisphere Soft Function

$$\begin{aligned}\Sigma^{(2)}(X,Y,\mu) &= \int_0^X dk_L \int_0^Y dk_R\; S^{(2)}_{hemi}(k_L,k_R,\mu)\\ &= \Sigma^{(2)}_\mu\left(\frac{X}{\mu},\frac{Y}{\mu}\right)+\Sigma^{(2)}_f\left(\frac{X}{Y}\right)\end{aligned}$$

$$\Sigma^{(2)}_\mu\left(\frac{X}{\mu},\frac{Y}{\mu}\right) = \Bigg[\frac{88}{9}\ln^3\left(\frac{X}{\mu}\right)+\frac{4\pi^2}{3}\ln^2\left(\frac{X}{\mu}\right)-\frac{268}{9}\ln^2\left(\frac{X}{\mu}\right)-\frac{11\pi^2}{9}\ln\left(\frac{XY}{\mu^2}\right) + \frac{404}{27}\ln\left(\frac{XY}{\mu^2}\right)-14\zeta_3\ln\left(\frac{XY}{\mu^2}\right)+X\leftrightarrow Y\Bigg]C_F C_A + \Bigg[-\frac{32}{9}\ln^3\left(\frac{X}{\mu}\right) + \frac{80}{9}\ln^2\left(\frac{X}{\mu}\right)+\frac{4\pi^2}{9}\ln\left(\frac{XY}{\mu^2}\right)-\frac{112}{27}\ln\left(\frac{XY}{\mu^2}\right)+X\leftrightarrow Y\Bigg]C_F T_F n_f$$

$$\Sigma_f^{(2)}\left(\frac{X}{Y}\right) = \left[-88\mathrm{Li}_3\left(-\frac{X}{Y}\right) - 16\mathrm{Li}_4\left(\frac{1}{\frac{X}{Y}+1}\right) - 16\mathrm{Li}_4\left(\frac{\frac{X}{Y}}{\frac{X}{Y}+1}\right) + 16\times\right.$$
$$\times\mathrm{Li}_3\left(-\frac{X}{Y}\right)\ln\left(\frac{X}{Y}+1\right) + \frac{88\mathrm{Li}_2\left(-\frac{X}{Y}\right)\ln\left(\frac{X}{Y}\right)}{3} - 8\mathrm{Li}_3\left(-\frac{X}{Y}\right)\ln\left(\frac{X}{Y}\right) - 16\zeta_3\times$$
$$\times\ln\left(\frac{X}{Y}+1\right) + 8\zeta_3\ln\left(\frac{X}{Y}\right) - \frac{4}{3}\ln^4\left(\frac{X}{Y}+1\right) + \frac{8}{3}\ln\left(\frac{X}{Y}\right)\ln^3\left(\frac{X}{Y}+1\right)$$
$$+\frac{4\pi^2}{3}\ln^2\left(\frac{X}{Y}+1\right) - \frac{4\pi^2}{3}\ln^2\left(\frac{X}{Y}\right) - \frac{4\left(3\left(\frac{X}{Y}-1\right)+11\pi^2\left(\frac{X}{Y}+1\right)\right)\ln\left(\frac{X}{Y}\right)}{9\left(\frac{X}{Y}+1\right)}$$
$$\left.-\frac{154\zeta_3}{9} + \frac{4\pi^4}{3} - \frac{335\pi^2}{54} - \frac{2032}{81}\right]C_F C_A + \left[32\mathrm{Li}_3\left(-\frac{X}{Y}\right) - \frac{32}{3}\mathrm{Li}_2\left(-\frac{X}{Y}\right)\times\right.$$
$$\left.\times\ln\left(\frac{X}{Y}\right) + \frac{8\left(\frac{X}{Y}-1\right)\ln\left(\frac{X}{Y}\right)}{3\left(\frac{X}{Y}+1\right)} + \frac{16\pi^2}{9}\ln\left(\frac{X}{Y}\right) + \frac{56\zeta_3}{9} + \frac{74\pi^2}{27} - \frac{136}{81}\right]C_F n_f T_F$$

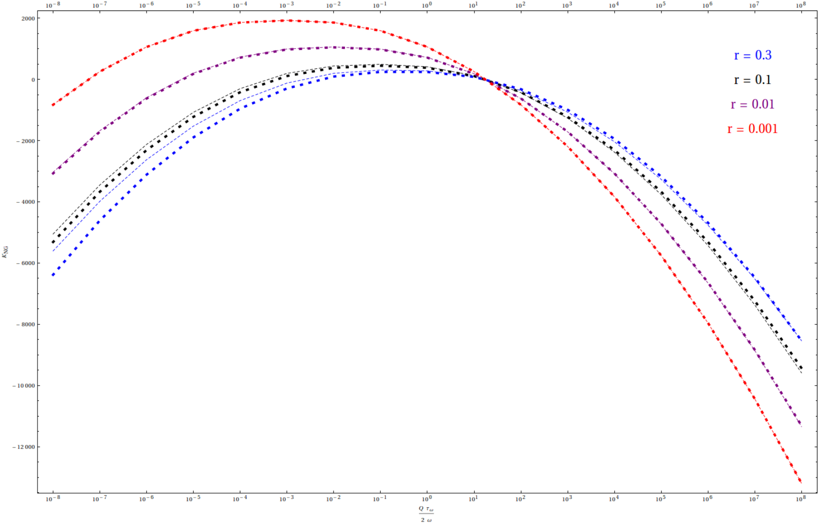
The Small r Non-Global Contribution To The Integrated Jet Thrust Distribution

Remarkably, taking the small r limit before integrating captures not only the extreme small r asymptotics but also the dominant power-corrections to them!

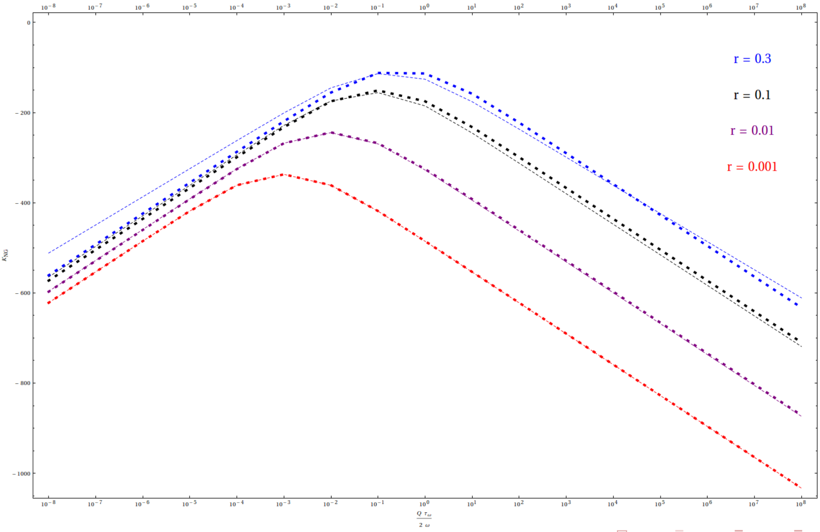
$$K_{TC}^{(2)}(\tau_\omega,\omega,\mu)\Big|_{\text{NG};\ r\to 0} = 2\,\Sigma_f^{(2)}\left(\frac{\tau_\omega Q}{2r\omega}\right) + C_A C_F\left(\frac{8}{3}\pi^2\ln^2(r)\right.$$
$$\left.+16\zeta_3\ln(r) - \frac{88}{9}\pi^2\ln(r) - \frac{4\ln(r)}{3} - \frac{16\pi^4}{5} + \frac{1012\zeta_3}{9} + \frac{871\pi^2}{27} + \frac{4064}{81}\right)$$
$$+C_F n_f T_F\left(\frac{32}{9}\pi^2\ln(r) + \frac{16\ln(r)}{3} - \frac{368\zeta_3}{9} - \frac{308\pi^2}{27} + \frac{272}{81}\right)$$

(see Kelley et. al. Phys. Rev. **D86** (2012) 054017 for partial results)

Robustness Of The Small r Approximation: $C_A C_F$



Robustness Of The Small r Approximation: $C_F n_f T_F$



Soft Non-Global Logarithms

We found that the logarithms that appear in the $\tau_\omega Q >> 2\omega R$ limit have the form (confirms partial results of Hornig et. al. JHEP **1201** (1012) 149)

$$C_F C_A \left[-\frac{8\pi^2}{3}\ln^2\left(\frac{\tau_\omega Q}{2R\omega}\right) + \left(-\frac{8}{3} + \frac{88\pi^2}{9} - 16\zeta_3 \right) \ln\left(\frac{\tau_\omega Q}{2R\omega}\right) \right]$$
$$+ C_F n_f T_F \left(\frac{16}{3} - \frac{32\pi^2}{9} \right) \ln\left(\frac{\tau_\omega Q}{2R\omega}\right) + \cdots$$

The resummation of the non-global logarithms that appear in this and other contexts have not yet been understood analytically.

(however, see Hatta and Ueda Nucl. Phys. **B874** (2013) 808)

Outlook

We have shown that the thrust cone algorithm, investigated here in the context of a jet mass observable, has some very nice theoretical properties.

- Is the general NGL resummation problem any easier for observables defined using a hemisphere jet algorithm? If so, our results would facilitate a resummation of the NGLs that appear in (integrated) jet mass distributions.
- Technical developments, which will appear in the paper upon which this talk is based, strongly indicate that the door to the calculation of the hemisphere NGLs at $\mathcal{O}\left(\alpha_s^3\right)$ is now wide open.
- Can we reproduce/improve upon large N_c analyses of NGLs?

(see *e.g.* Dasgupta and Salam Phys. Lett. **B512** (2001) 323; Banfi et. al. JHEP **08** (2010) 064)