Energy Correlation Functions for Jet Substructure

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What qualities do we want in jet observables?

- Sensitive to N subjet structure: can distinguish QCD from boosted particle decays
- Simple: can be understood analytically
- Insensitive to soft recoil: preferred from the calculation point of view as well as for increased discrimination power

Energy Correlation Functions

$$\mathrm{ECF}(N,\beta) = \sum_{i_1<i_2<\ldots<i_N\in J} \left(\prod_{a=1}^{N} p_{T\,i_a}\right)\left(\prod_{b=1}^{N-1}\prod_{c=b+1}^{N} R_{i_b i_c}\right)^{\beta}$$

$$\mathrm{ECF}(0,\beta) = 1,$$

$$\mathrm{ECF}(1,\beta) = \sum_{i\in J} p_{T\,i},$$

$$\mathrm{ECF}(2,\beta) = \sum_{i<j\in J} p_{T\,i}\,p_{T\,j}\,(R_{ij})^{\beta},$$

Banfi, Salam, Zanderighi 2004
Jankowiak, AL 2011

$$\mathrm{ECF}(3,\beta) = \sum_{i<j<k\in J} p_{T\,i}\,p_{T\,j}\,p_{T\,k}\,(R_{ij}R_{ik}R_{jk})^{\beta}\,,$$

$$\mathrm{ECF}(4,\beta) = \sum_{i<j<k<\ell\in J} p_{T\,i}\,p_{T\,j}\,p_{T\,k}\,p_{T\,\ell}\,(R_{ij}R_{ik}R_{i\ell}R_{jk}R_{j\ell}R_{k\ell})^{\beta}$$

Energy Correlation Function Double Ratios

$$C_N^{(\beta)} = \frac{\mathrm{ECF}(N+1,\beta)\,\mathrm{ECF}(N-1,\beta)}{\mathrm{ECF}(N,\beta)^2}$$

Some nice properties:

- Dimensionless: reduced dependence on the energy scale of the jet
- Simple scaling under boosts: scales like $\gamma^{-\beta}$ so β can be used to emphasize different angular scales

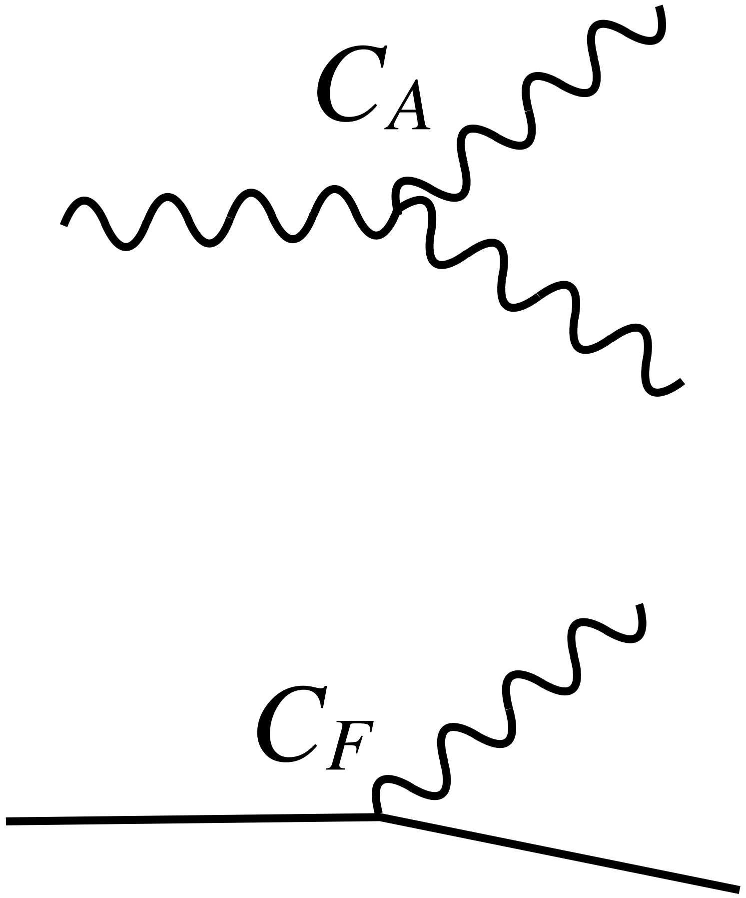
Why this combination of ECFs?

Three applications:

- Quarks vs. Gluons: Two-point correlation function
- Z/H/W vs. QCD: Three-point correlation function
- t vs. QCD: Four-point correlation function

Why use C₁ for quark vs. gluon discrimination?

$$C_1^{(\beta)} = \frac{\sum\limits_{i,j \in J} p_{Ti} p_{Tj} R_{ij}^{\beta}}{\left(\sum\limits_i p_{Ti}\right)^2}$$



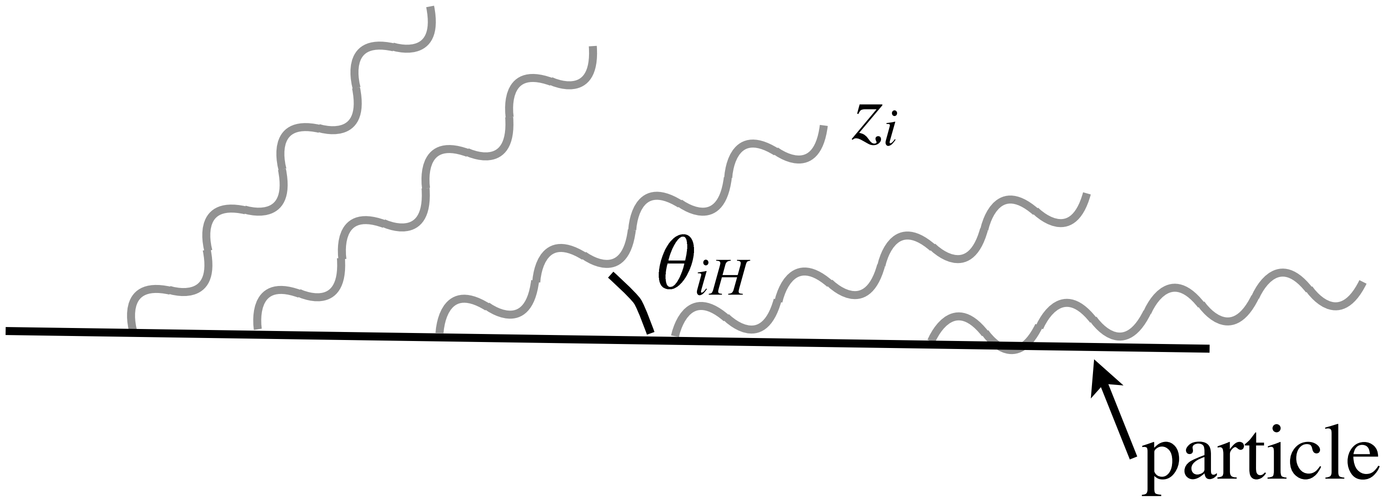
C_1 is sensitive to emissions about a single hard prong

For emissions about a single hard prong:

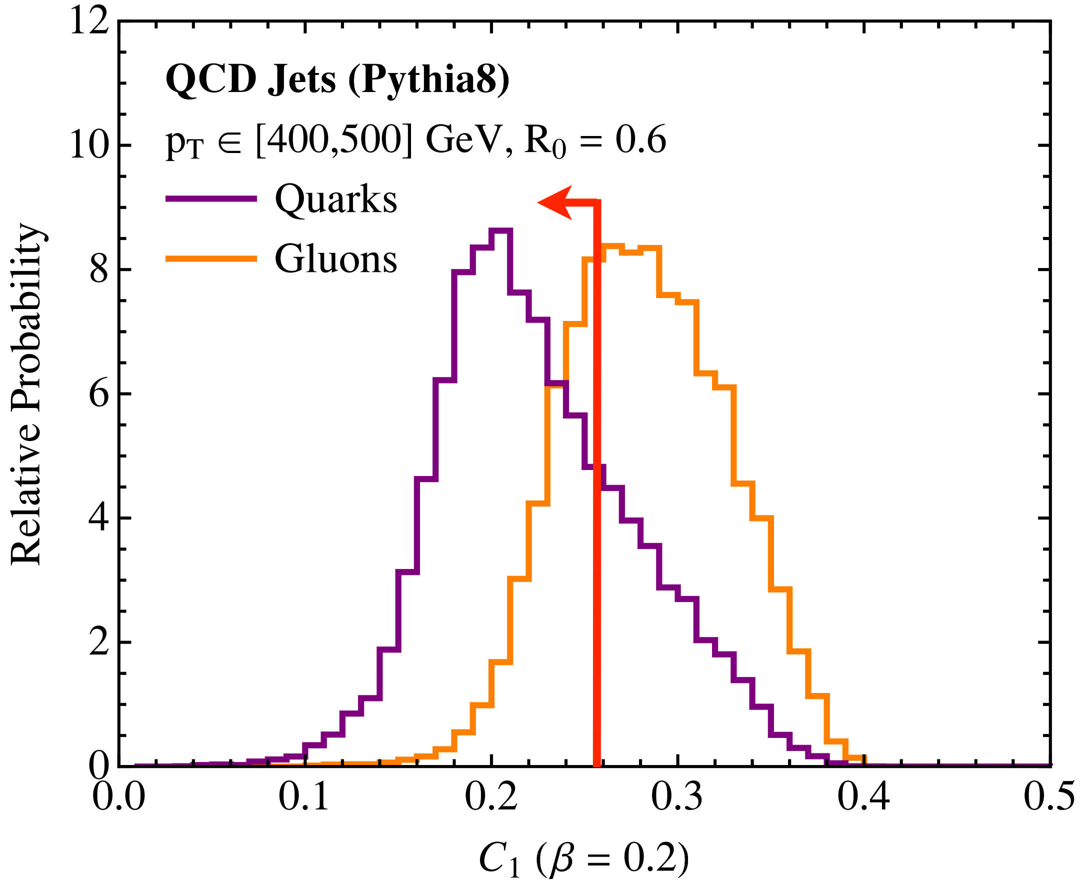
$$C_1^{(\beta)} = \frac{\sum\limits_{i,j\in J} p_{Ti} p_{Tj} R_{ij}^{\beta}}{\left(\sum\limits_i p_{Ti}\right)^2} \simeq \sum_{i\in J} z_i \theta_{iH}^{\beta} + \mathcal{O}(z^2)$$

$$z_i = \frac{p_{Ti}}{p_{TJ}}$$

soft-collinear emissions



Let's calculate the quark vs. gluon discrimination with C_1



Given a value of C_1, what fraction of quark jets or gluon jets lie to the left of that cut?

Compute the cumulative distribution of C_1 for a quark jet and gluon jet

Let's consider the double-logarithmic structure of C_1

NOTE: The "Cumulant" is **not** the "Cumulative Distribution"!

Double-logarithmic = Soft-collinear emissions

$$C_1^{(\beta)} = z\theta^\beta$$ (for one emission)

Phase space for soft-collinear emissions:

$$\int_0^1 \frac{d\theta}{\theta} \int_0^1 \frac{dz}{z} = \int_0^\infty d\log\frac{1}{\theta} \int_0^\infty d\log\frac{1}{z}$$

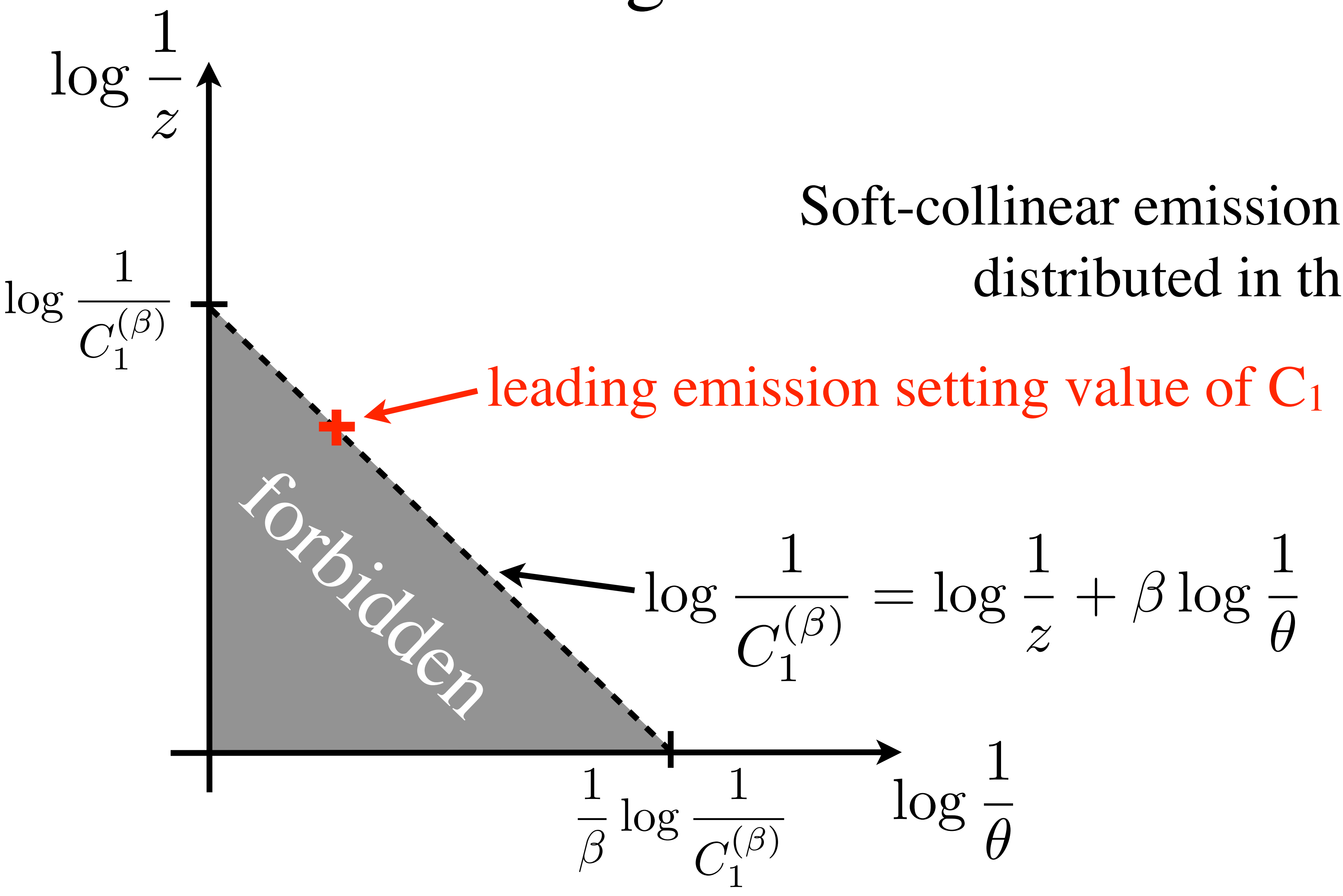
To this accuracy, the cumulative distribution is

$$P\left(x < C_1^{(\beta)}\right) = e^{-P_1\left(x > C_1^{(\beta)}\right)}$$

cumulative probability for an arbitrary number of emissions

probability for one soft-collinear emission

Double-logarithmic Resummation



Soft-collinear emissions are uniformly distributed in this plane

leading emission setting value of C_1

$$\log \frac{1}{C_1^{(\beta)}} = \log \frac{1}{z} + \beta \log \frac{1}{\theta}$$

Area of forbidden region = $\frac{\log^2 C_1^{(\beta)}}{2\beta} \propto P_1(x > C_1^{(\beta)})$

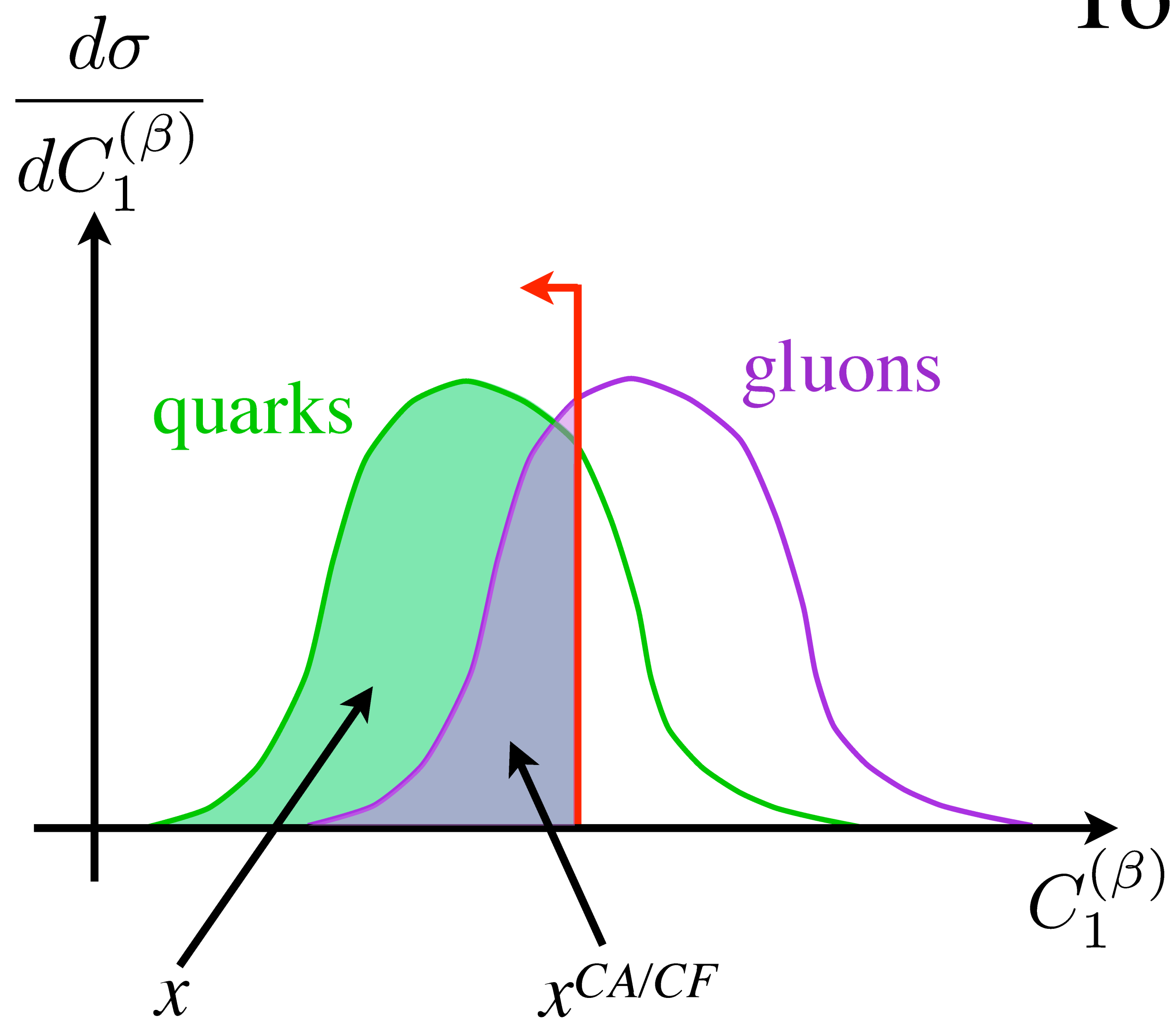
*to this accuracy, the jet radius is irrelevant

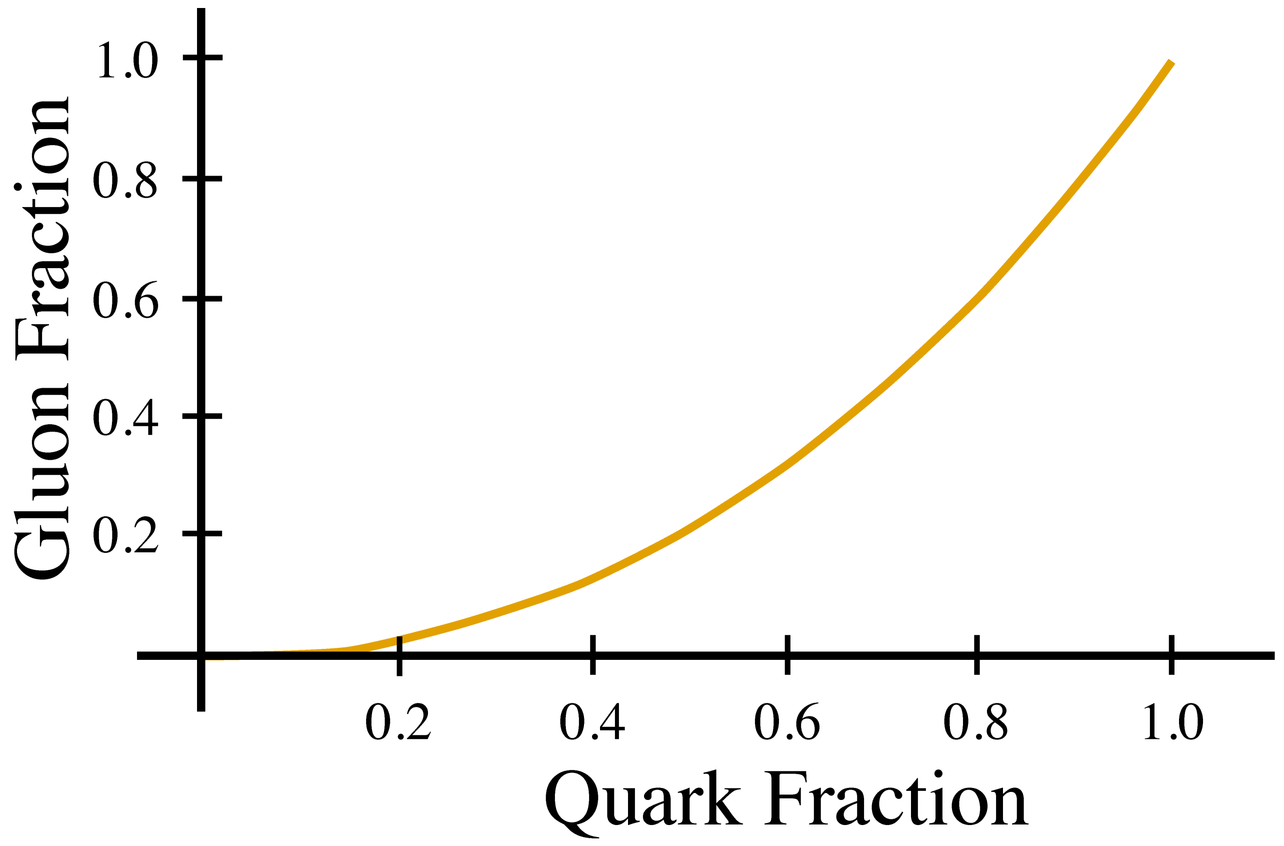
Double-logarithmic Resummation

$$\Sigma_q(C_1^{(\beta)}) = e^{-\frac{\alpha_s}{\pi}\frac{C_F}{\beta}\log^2 C_1^{(\beta)}}$$

$$\Sigma_g(C_1^{(\beta)}) = e^{-\frac{\alpha_s}{\pi}\frac{C_A}{\beta}\log^2 C_1^{(\beta)}}$$

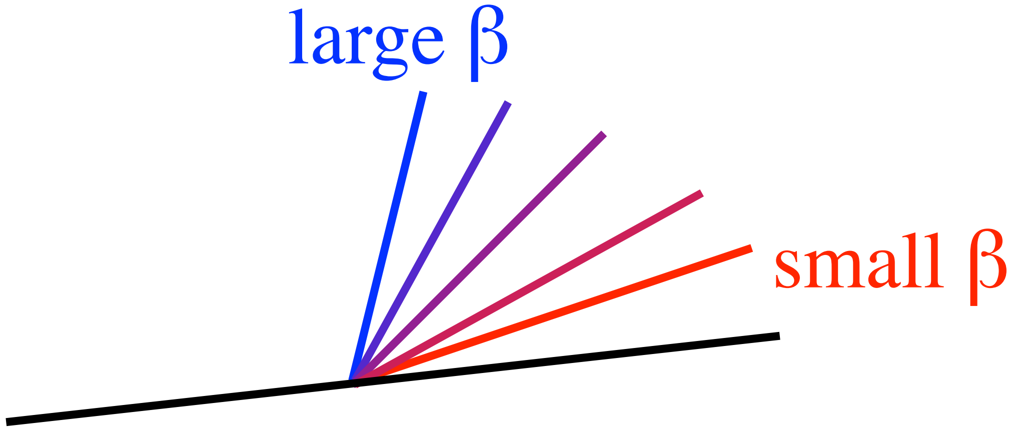
To LL accuracy: $\boxed{\Sigma_g = \Sigma_q^{\frac{C_A}{C_F}}}$





*This result holds for
many observables at LL

One higher order effect: hard collinear logs



$$C_1^{(\beta)} = \frac{\sum\limits_{i,j\in J} p_{Ti} p_{Tj} R_{ij}^{\beta}}{\left(\sum\limits_i p_{Ti}\right)^2}$$

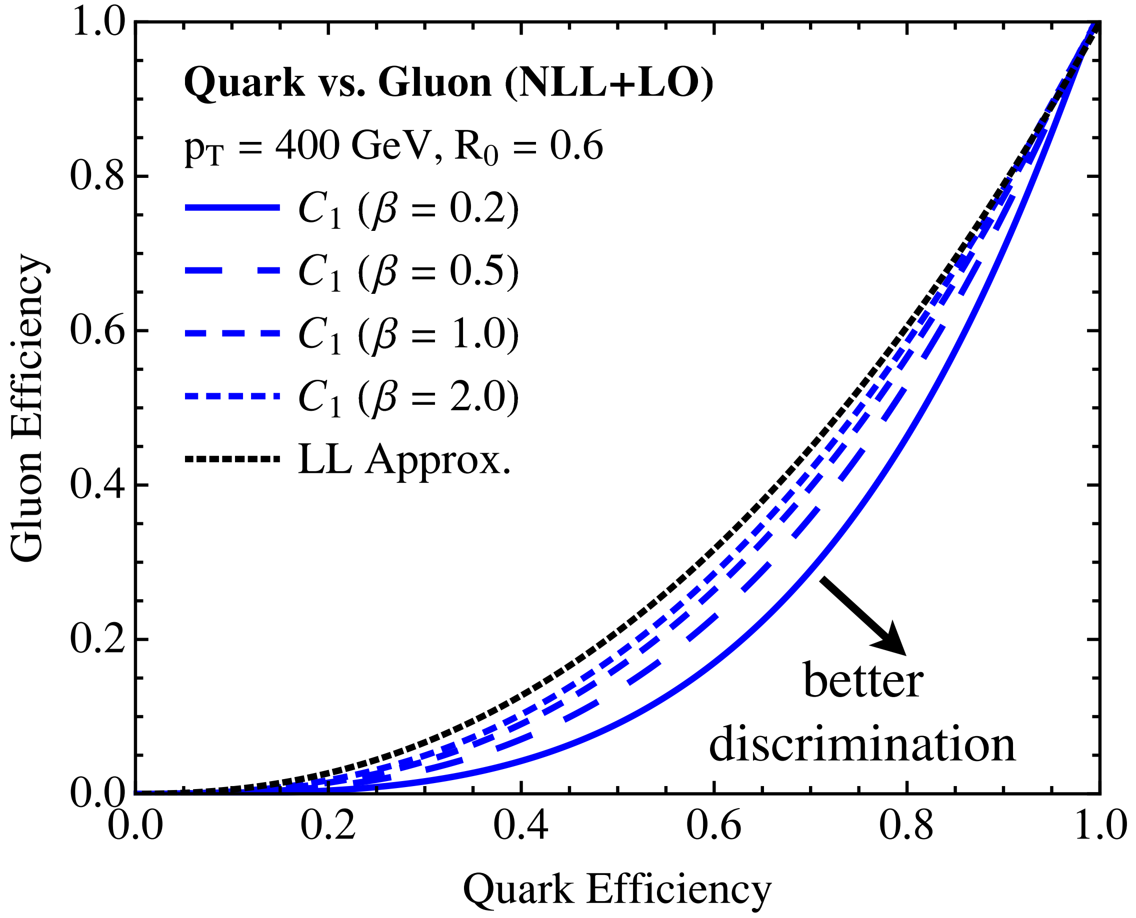
Large β: Dominated by soft radiation
Sensitive to total color of jet

Small β: Dominated by hard collinear radiation
Sensitive to color and spin of jet

Expect better quark/gluon discrimination at small β

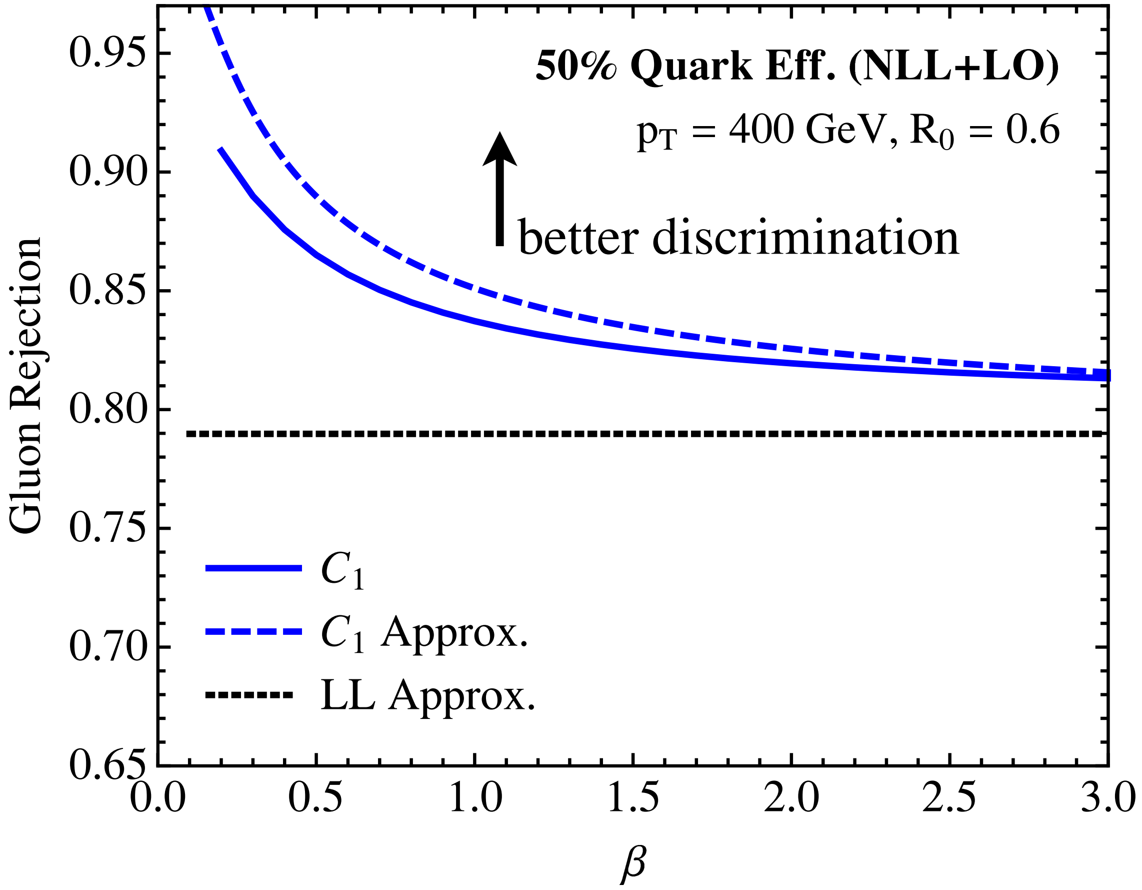
*For $n_F = C_A$, no improvement over LL discrimination

Next-to-Leading Log Discrimination



CAESAR

Banfi, Salam, Zanderighi 2004



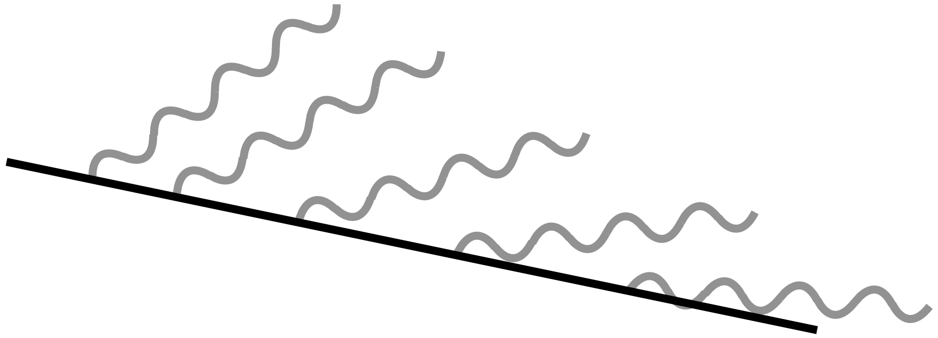
Several effects lead to better discrimination at small β:

- Hard collinear emissions
- Multiple soft emissions
- Running coupling

Isn’t this the same as angularities like thrust, broadening, etc?

$$C_1^{(\beta)} = \frac{\sum\limits_{i,j \in J} p_{Ti} p_{Tj} R_{ij}^{\beta}}{p_{T\text{jet}}^2}$$

identical to $\tau^{(\beta)}$ for $\beta > 1$ to NLL



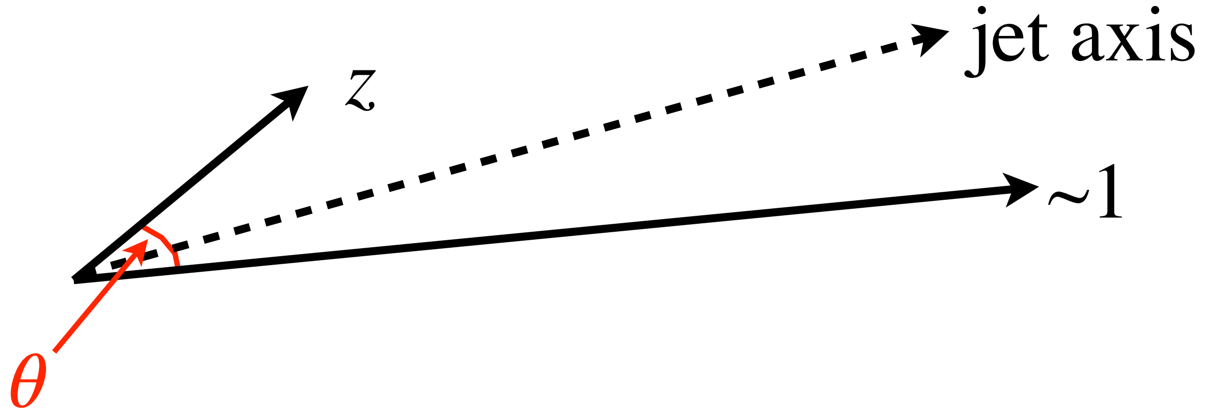
$$\tau^{(\beta)} = \frac{1}{p_{T\text{jet}}} \sum_{i \in J} p_{Ti} R_{iJ}^{\beta}$$

Important: C_1 is insensitive to recoil from soft, wide angle emission

Contrast C₁ to angularities $\tau^{(\beta)}$

$$\tau^{(\beta)} = \frac{1}{p_{TJ}} \sum_{i \in J} p_{Ti} R_i^\beta \simeq \sum_{i \in J} z_i \theta_i^\beta$$

angle from particle *i*
to jet axis



$$\tau^{(\beta)} \simeq \underset{\substack{\text{“recoil”} \\ \text{contribution}}}{z^\beta \theta^\beta} + \underset{\substack{\text{“direct”} \\ \text{contribution}}}{z \theta^\beta}$$

Value of β determines sensitivity to recoil

Contrast C_1 to angularities $\tau^{(\beta)}$

$$\tau^{(\beta)} \simeq \underbrace{z^\beta \theta^\beta}_{\text{"recoil" contribution}} + \underbrace{z\theta^\beta}_{\text{"direct" contribution}}$$

$\beta > 1: \quad \tau^{(\beta)} \to z\theta^\beta$ includes thrust/mass ($\beta = 2$)

$\beta = 1: \quad \tau^{(\beta)} \to 2z\theta$ broadening/width/girth

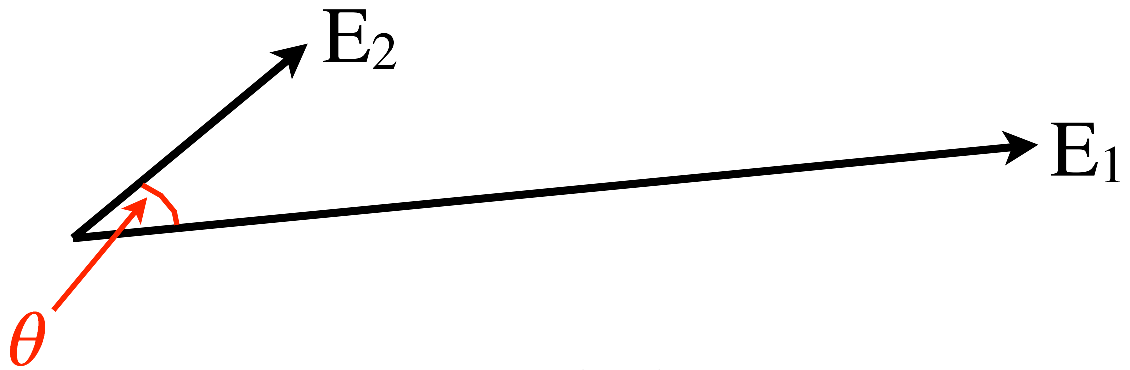
$\beta < 1: \quad \tau^{(\beta)} \to (z\theta)^\beta$ recoil-dominated angularities

*Angularities are only additive for $\beta > 1$

Contrast C_1 to angularities $\tau^{(\beta)}$

$$C_1^{(\beta)} = \frac{\sum\limits_{i,j\in J} p_{Ti} p_{Tj} R_{ij}^{\beta}}{\left(\sum\limits_i p_{Ti}\right)^2} \simeq \sum_{i\in J} z_i \theta_{iH}^{\beta}$$

angle from particle *i* to hard particle



$$C_1^{(\beta)} \simeq z\theta^{\beta}$$

for all β > 0
no recoil contribution

Insensitivity to recoil = more sensitivity to pattern of soft emissions

*C_1 is additive for all β > 0

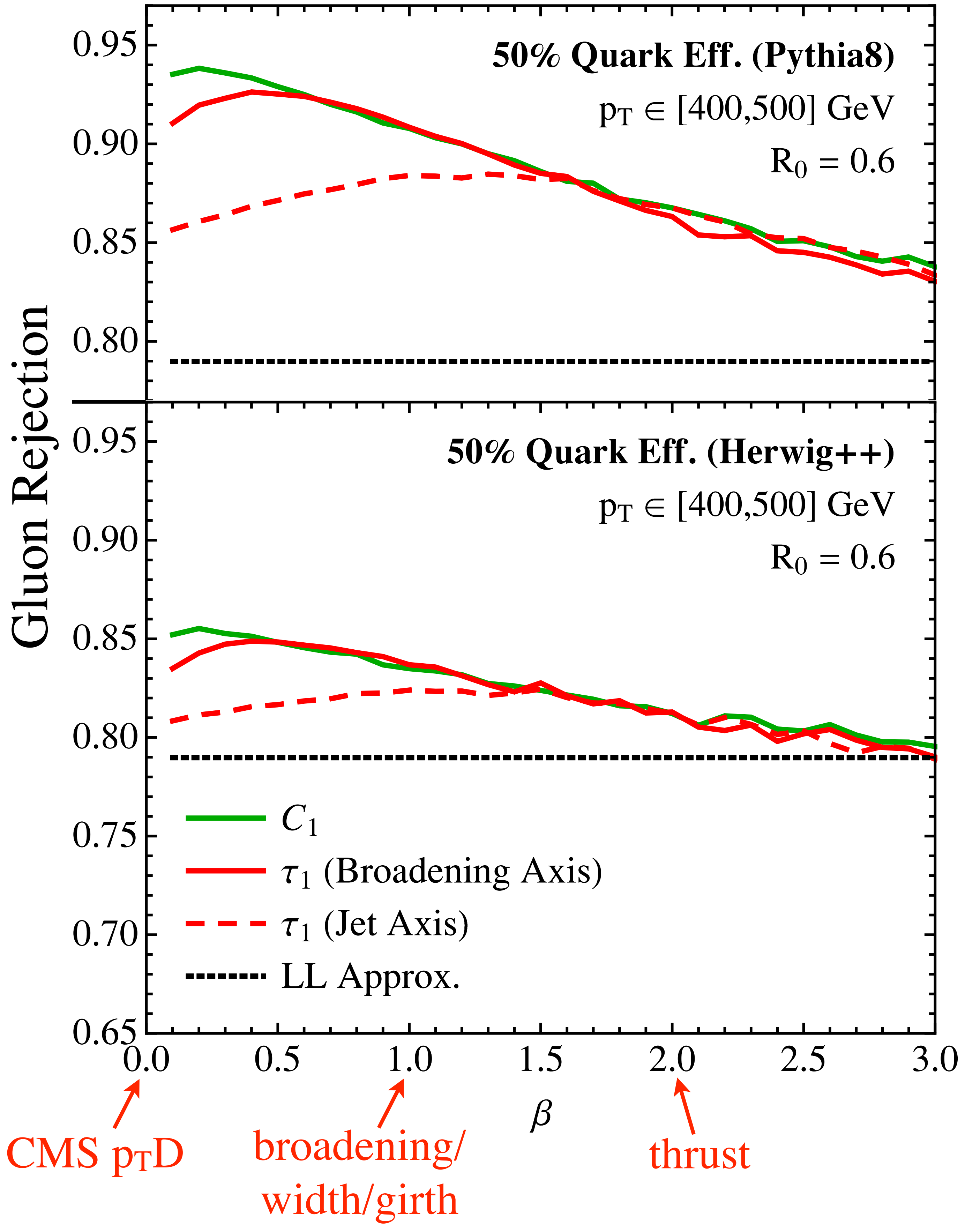
Monte Carlo Study

Expectations:

- Improved performance with recoil-free observables
- Improved discrimination at small β
- Identical performance at large β for most observables

Questions:

- Is Pythia or Herwig closer to data?
- Is the improved performance at small β observed in data?



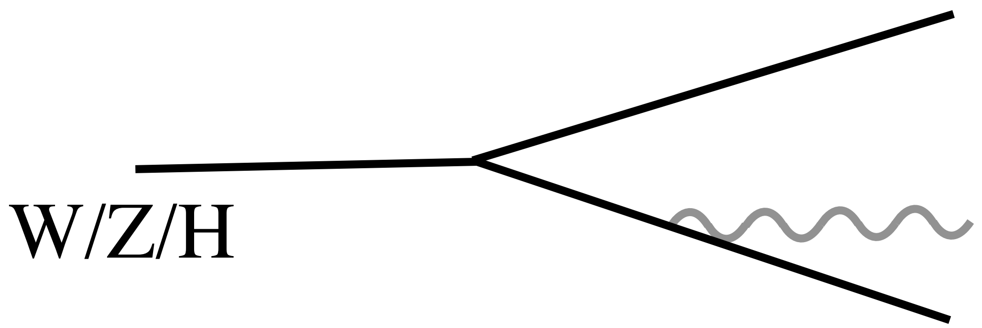
CMS p_TD

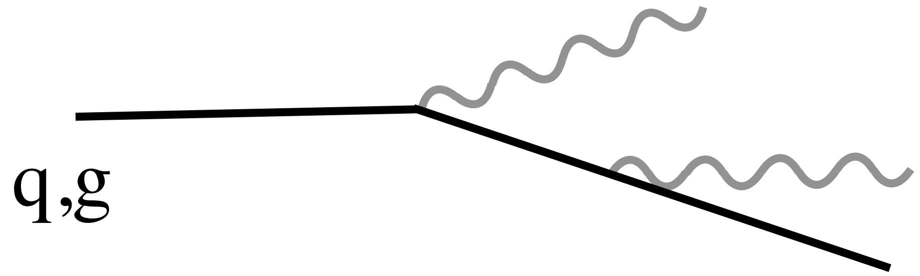
broadening/ width/girth

thrust

On to C$_2$ for W/Z/H discrimination

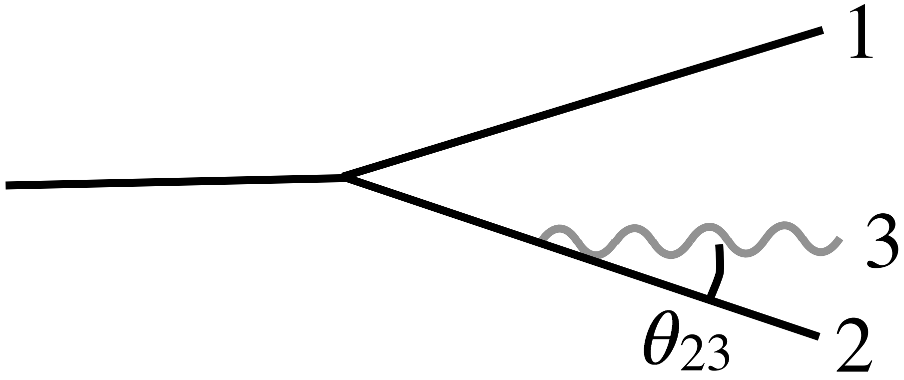
$$C_2^{(\beta)} = p_{TJ}\,\frac{\displaystyle\sum_{i<j<k\in J} p_{Ti}p_{Tj}p_{Tk}R_{ij}^{\beta}R_{ik}^{\beta}R_{jk}^{\beta}}{\left(\displaystyle\sum_{i<j\in J} p_{Ti}p_{Tj}R_{ij}^{\beta}\right)^2}$$





C$_2$ is sensitive to emissions about a two hard prongs

For emissions about two hard prongs:



$$\mathrm{ECF}(3,\beta) \simeq p_{T3}\theta_{23}^{\beta} \times p_{T1}p_{T2}R_{12}^{2\beta}$$

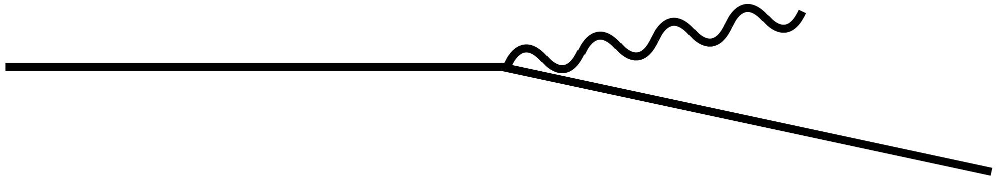
$$\mathrm{ECF}(2,\beta) \simeq p_{T1}p_{T2}R_{12}^{\beta}$$

$$\mathrm{ECF}(1,\beta) \simeq p_{T1} + p_{T2}$$

$$z_3 = \frac{p_{T3}}{p_{T1} + p_{T2}}$$

$$C_2^{(\beta)} = \frac{\mathrm{ECF}(3,\beta)\mathrm{ECF}(1,\beta)}{\mathrm{ECF}(2,\beta)^2} \simeq \underset{\uparrow\ \text{soft emission}}{z_3\theta_{23}^{\beta}} \times \underset{\uparrow\ \text{hard kinematics}}{\frac{\left(p_{T1} + p_{T2}\right)^2}{p_{T1}p_{T2}}}$$

$$C_2^{(2)} = p_{T\text{jet}} \frac{\displaystyle\sum_{i,j,k\in J} p_{Ti} p_{Tj} p_{Tk} R_{ij}^2 R_{jk}^2 R_{ik}^2}{\left(\displaystyle\sum_{i,j\in J} p_{Ti} p_{Tj} R_{ij}^2\right)^2}$$



identical to C-parameter in COM frame to $O(\alpha_s)$

$$C = \frac{3}{2} \frac{\sum_{i,j} |\mathbf{p}_i| \, |\mathbf{p}_j| \sin^2 \theta_{ij}}{\left(\sum_i |\mathbf{p}_i|\right)^2}$$

Important: C_2 is well-defined in any frame and so can be used at a hadron collider

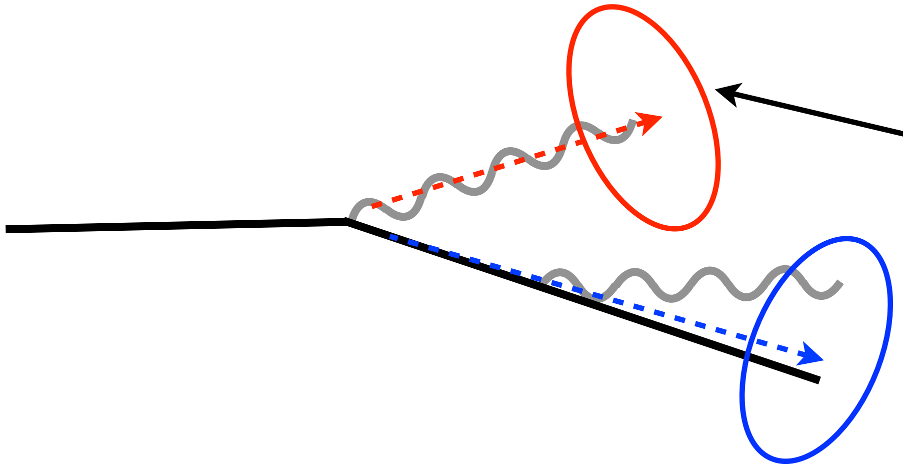
Contrast C_2 to N-subjettiness ratio τ_2/τ_1

N-subjettiness: **Always** partitions the jet into *N* subjets

$$\tau_N^{(\beta)} = \sum_i p_{Ti} \min\left\{R_{1,i}^\beta, R_{2,i}^\beta, \ldots, R_{N,i}^\beta\right\}$$

$$\tau_{N,N-1}^{(\beta)} = \frac{\tau_N^{(\beta)}}{\tau_{N-1}^{(\beta)}}$$

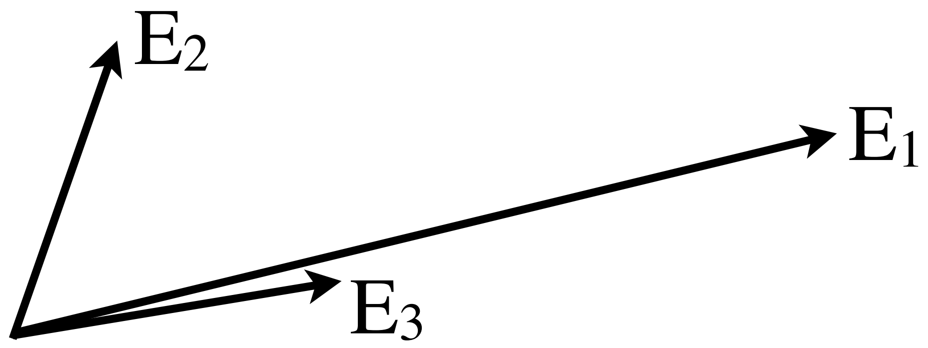
with some choice/procedure for determining *N* axes in the jet



τ_2 is ~independent of energy of this subjet

C_2: Sensitive to all soft and collinear singularities

Contrast C_2 to N-subjettiness ratio τ_2/τ_1



$$E_1 \gg E_2, E_3$$
$$\theta_{13} \ll \theta_{12} \simeq \theta_{23}$$

For this kinematic configuration (see back-up slides):

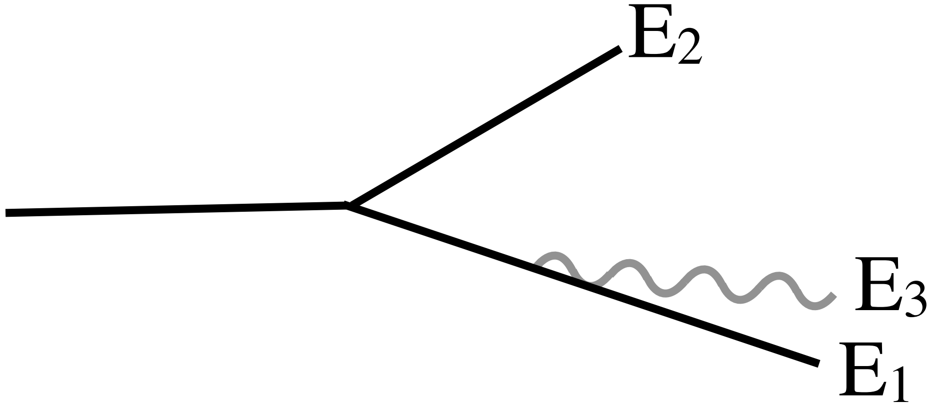
$$\boxed{C_2^{(\beta)} \simeq \tau_{2,1}^{(\beta)} \times (\theta_{12})^\beta} \longrightarrow$$

$C_2 > \tau_{2,1}$ in the presence of soft, wide angle radiation

Contrast C_2 to N-subjettiness ratio τ_2/τ_1

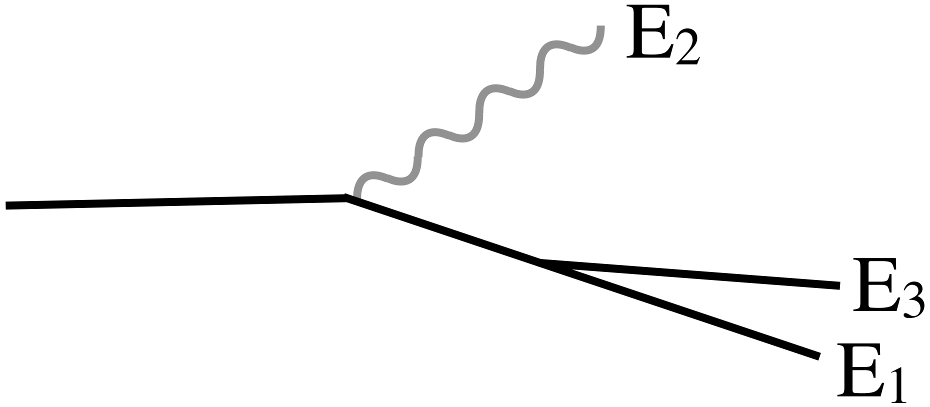
$$C_2^{(\beta)} \simeq \tau_{2,1}^{(\beta)} \times (\theta_{12})^{\beta}$$

Take $\beta = 2$ and fix the mass: $m^2 \simeq E_1 \max\left[E_2\,(\theta_{12})^2\,, E_3\,(\theta_{13})^2\right]$



$$C_2^{(2)} \simeq \tau_{2,1}^{(2)} \times \frac{m^2}{(E_1)^2}\frac{1}{z}$$

Greater weight to mass dominated by soft, wide angle emissions



C_2 independent of energy fraction of E$_3$

Phase space for wide angle emissions from signal jets decreases with boost

Let’s calculate the Z vs. QCD discrimination with C_2

Many challenges:

- Leading-order calculation at α_s^2

  Can use some tricks for signal calculation

  Feige, Schwartz, Stewart, Thaler 2012

  Can’t use same tricks for boosted background jets

- With no mass cut, C_2 is actually **not** IRC safe

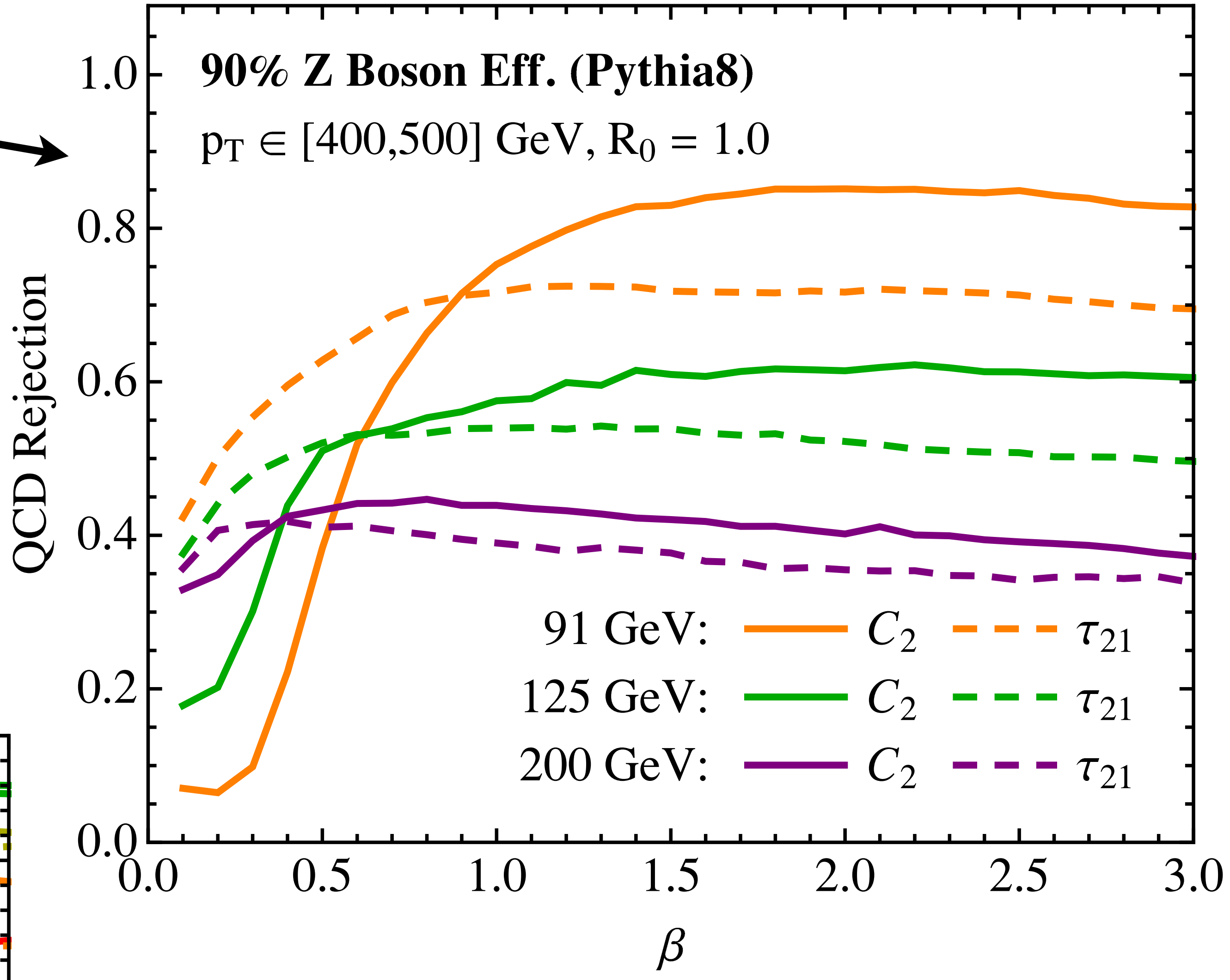
  Actually, **much** less of an issue than it sounds like; see Jesse’s talk

  AL, Thaler 2013

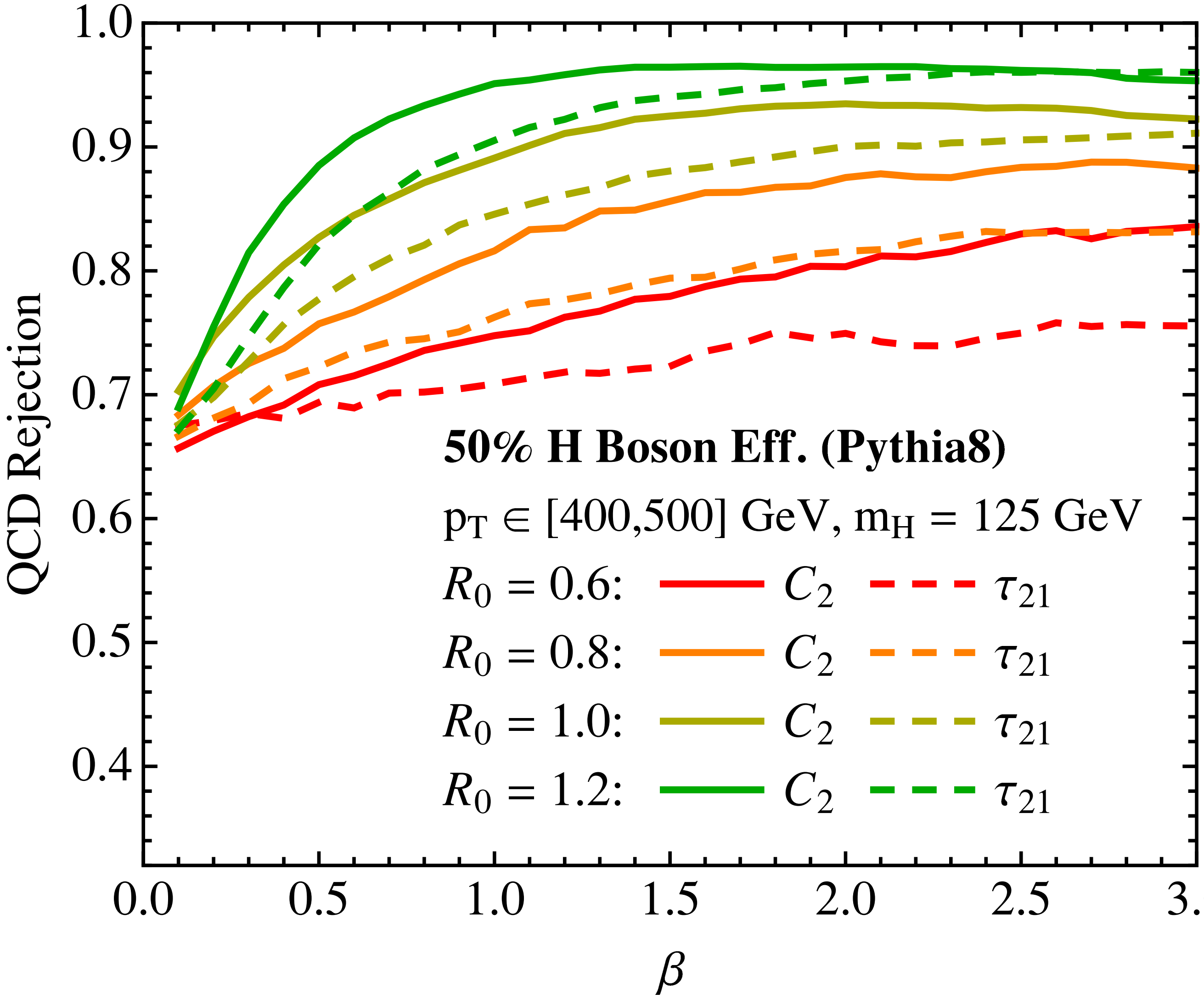
“Z” vs. QCD

Require mass of jet in a window about mass of boson

Rejection increases as boost increases because phase space for wide angle emissions in Z jet decreases



H vs. QCD



Require two b subjets

Dominant QCD background: g > b b

Rejection increases at larger β/jet radius because wide angle emissions are weighted more

On to C_3 for t discrimination

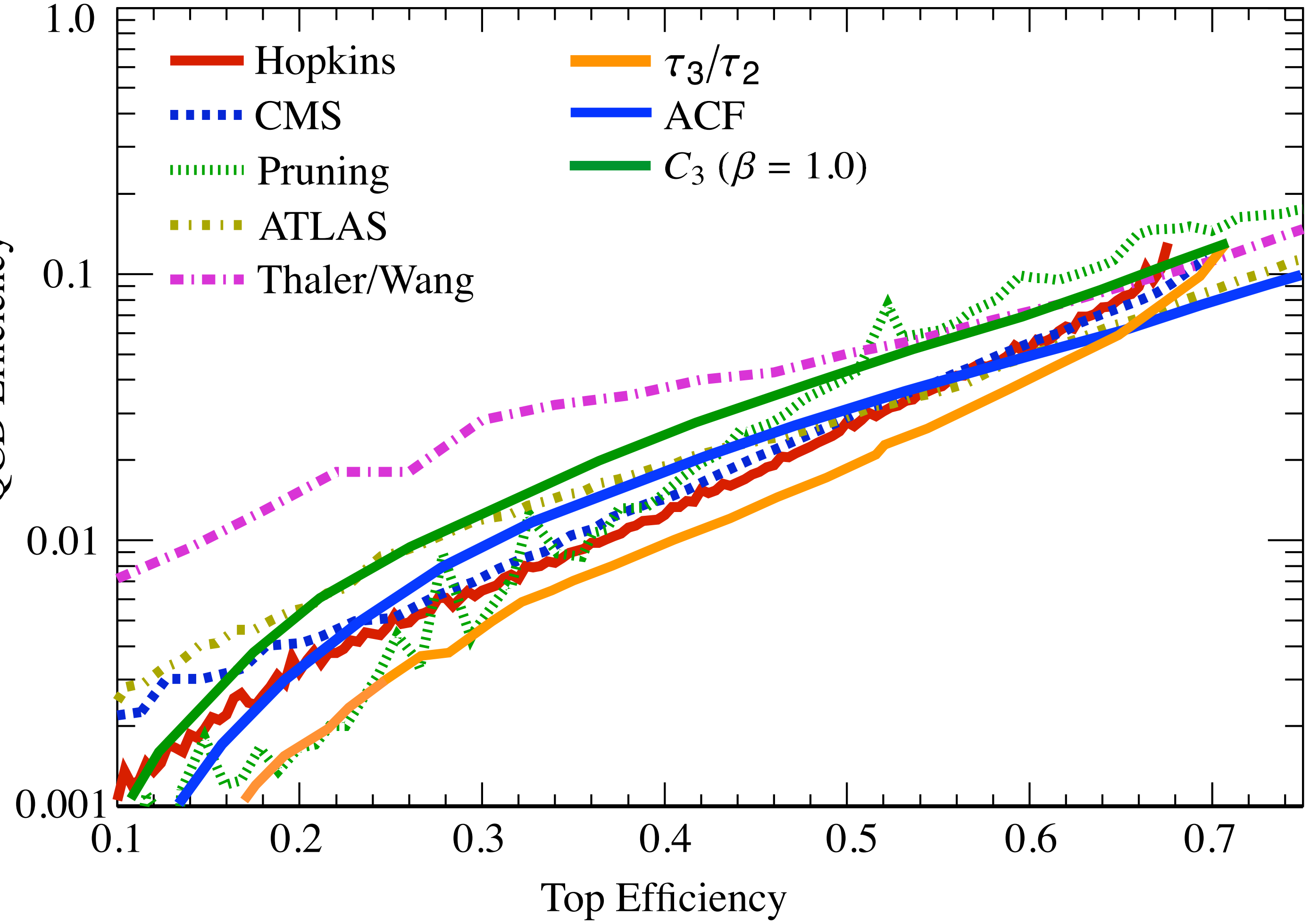
$$C_3^{(\beta)} = \frac{\mathrm{ECF}(4,\beta)\mathrm{ECF}(2,\beta)}{\mathrm{ECF}(3,\beta)^2}$$

Require mass of jet in a window about mass of top

C_3 is IRC unsafe even with a mass cut, so we apply a cut on C_2

Performance is significantly worse in comparison to C_1 and C_2

No analytic understanding of the discrimination whatsoever



Comparison to BOOST 2010

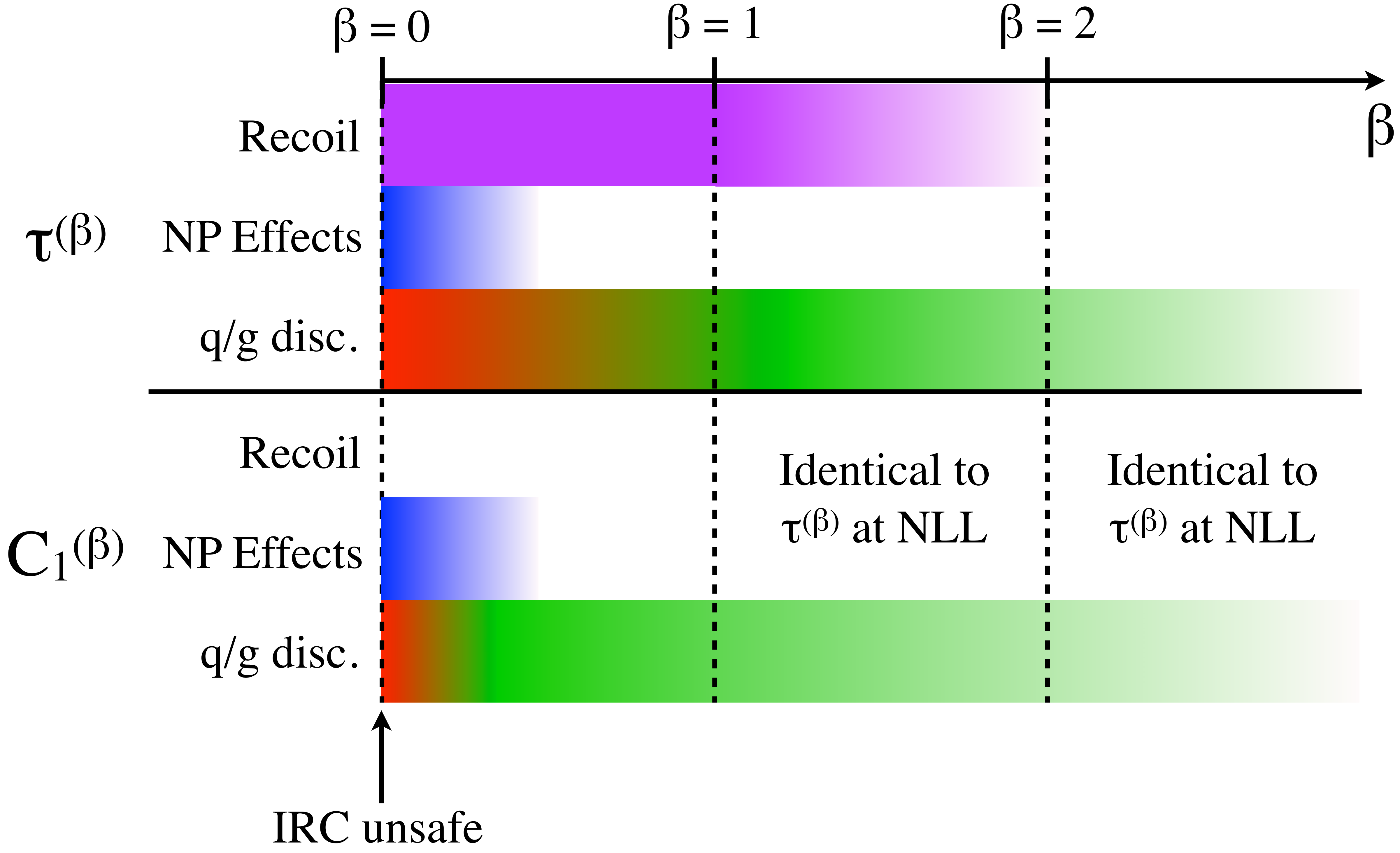
*BOOST 2011 used a W subjet tagger that would artificially improve performance

Conclusions

Energy correlation functions have nice properties:

- Sensitive to *N* subjet structure
- Simple and analytically calculable (for low-point correlators)
  - Computing C_2 (C_3, too?) is promising
- Recoil-free

Lessons: C_1 for quark vs. gluon discrimination



$\beta = 0$ $\beta = 1$ $\beta = 2$

β

$\tau^{(\beta)}$

Recoil

NP Effects

q/g disc.

$C_1^{(\beta)}$

Recoil

NP Effects

q/g disc.

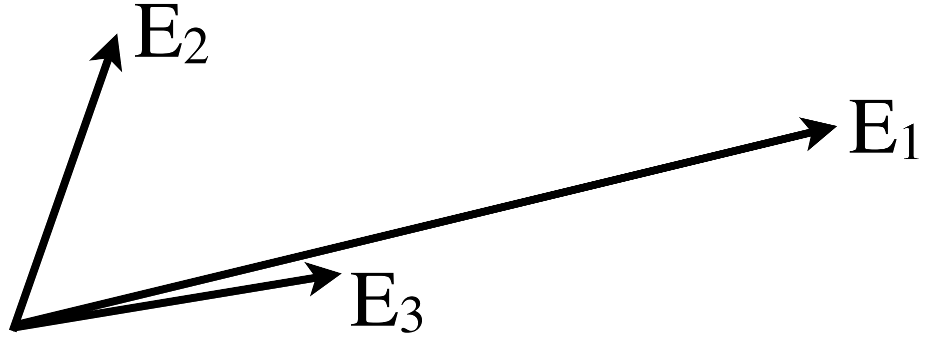
Identical to $\tau^{(\beta)}$ at NLL

Identical to $\tau^{(\beta)}$ at NLL

IRC unsafe

Back-up slides

Contrast C_2 to N-subjettiness ratio τ_2/τ_1

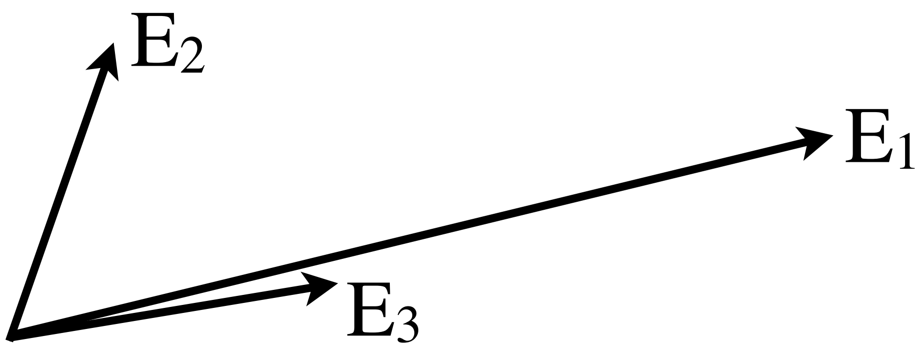


$$E_1 \gg E_2, E_3$$
$$\theta_{13} \ll \theta_{12} \simeq \theta_{23}$$

$$\begin{aligned} \mathrm{ECF}(1,\beta) &\simeq E_1 \\ \mathrm{ECF}(2,\beta) &\simeq E_1 \max\left[E_2\left(\theta_{12}\right)^{\beta}, E_3\left(\theta_{13}\right)^{\beta}\right] \\ \mathrm{ECF}(3,\beta) &= E_1 E_2 E_3 \left(\theta_{12}\theta_{23}\theta_{13}\right)^{\beta} \end{aligned}$$

$$C_2^{(\beta)} = \frac{\mathrm{ECF}(3,\beta)\mathrm{ECF}(1,\beta)}{\mathrm{ECF}(2,\beta)^2} \simeq \frac{E_2 E_3 \left(\theta_{12}\right)^{2\beta}\left(\theta_{13}\right)^{\beta}}{\max\left[E_2\left(\theta_{12}\right)^{\beta}, E_3\left(\theta_{13}\right)^{\beta}\right]^2}$$

Contrast C_2 to N-subjettiness ratio τ_2/τ_1



$$E_1 \gg E_2, E_3$$
$$\theta_{13} \ll \theta_{12} \simeq \theta_{23}$$

Choose subjet axes to lie along the hardest particle in the subjet

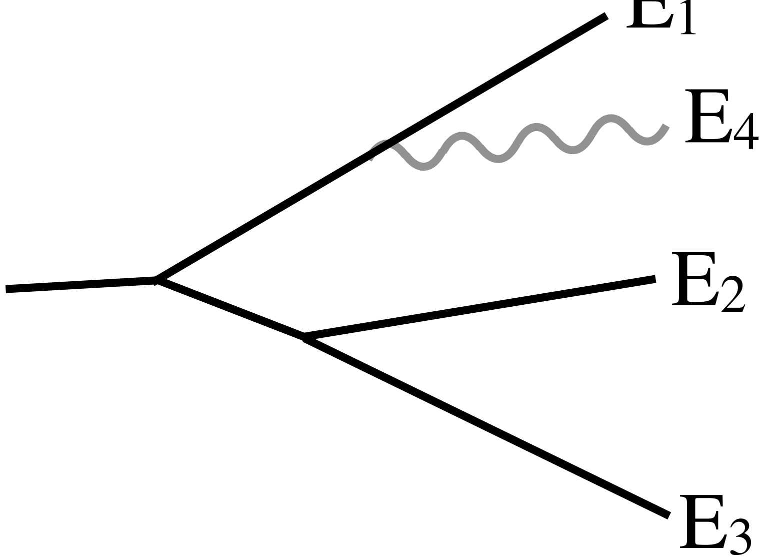
$$\tau_1^{(\beta)} \simeq \max\left[E_2(\theta_{12})^\beta, E_3(\theta_{13})^\beta\right]$$
$$\tau_2^{(\beta)} \simeq \min\left[E_2(\theta_{12})^\beta, E_3(\theta_{13})^\beta\right]$$

$$\tau_{2,1}^{(\beta)} = \frac{\tau_2^{(\beta)}}{\tau_1^{(\beta)}} \simeq \frac{\min\left[E_2\left(\theta_{12}\right)^\beta, E_3\left(\theta_{13}\right)^\beta\right]}{\max\left[E_2\left(\theta_{12}\right)^\beta, E_3\left(\theta_{13}\right)^\beta\right]}$$

On to C_3 for t discrimination

$$C_3^{(\beta)} = \frac{\mathrm{ECF}(4,\beta)\mathrm{ECF}(2,\beta)}{\mathrm{ECF}(3,\beta)^2}$$

Unlike C_1 and C_2, C_3 retains dependence on hard jet structure even for soft-collinear emission



$$C_3^{(\beta)} \simeq E_4\theta_{14}^{\beta} \times \frac{E_1E_2\theta_{12}^{\beta} + E_1E_3\theta_{13}^{\beta} + E_2E_3\theta_{23}^{\beta}}{E_1E_2E_3\theta_{23}^{\beta}}$$