

# Theory $|V_{xb}|$ determinations

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# Outline:

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$\delta V_{cb} / V_{cb} \sim 2 \%$$

- Semileptonic decays
  - Exclusive  $B \rightarrow D^{(*)} \ell \nu$   
 $\ell = e, \mu$
  - Inclusive  $B \rightarrow X_c l \nu$

$$\delta V_{ub} / V_{ub} \sim 6-15 \%$$

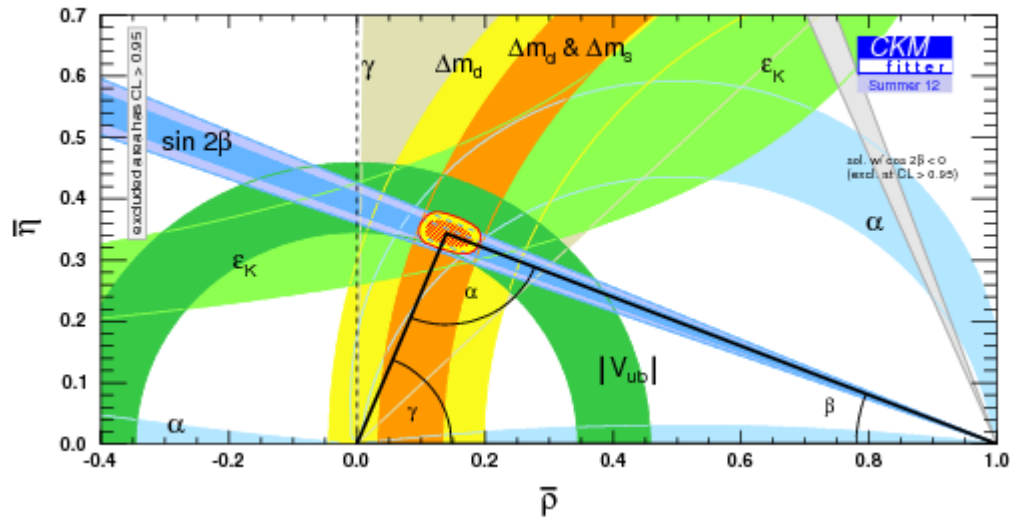
- Semileptonic decays
  - Exclusive  $B \rightarrow \pi \ell \nu$
  - Inclusive  $B \rightarrow X_u l \nu$ ,

$$\delta V_{tb} / V_{tb} \sim 5-8 \%$$

- $\sigma_{t\bar{t}}$ , single top

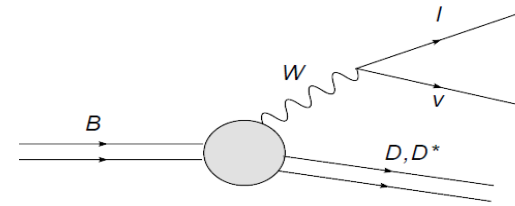
- Leptonic decay  $B \rightarrow \tau \nu$

# Vcb



- ❖ normalizes the whole unitarity triangle
- ❖ input for NP sensitive other estimates

# Exclusive decays $B \rightarrow D^{(*)} \ell \nu$



$$\frac{d\Gamma}{d\omega} (B \rightarrow D^* (D)) \propto |V_{cb}|^2 (\omega^2 - 1)^{1/2 (3/2)} F(\omega, \theta)^2 (G(\omega)^2)$$

$$\omega = \frac{p_{D^{(*)}} \cdot p_B}{m_B m_{D^{(*)}}}$$

1) Data for  $|V_{cb}| |\mathcal{G}(\omega)|$  and  $|V_{cb}| |\mathcal{F}(\omega)|$

taken at  $\omega \neq 1$  due to kinematics

← Data from LEP, CLEO, Babar and Belle reduction both syst (max  $\frac{1}{2}$ ) and stat errors (almost 1/10)

2) Results extrapolated at non-recoil point  $\omega=1$

$$F(\omega = 1) = G(\omega = 1) = 1 \quad \text{Heavy flavour limit}$$

+

$$+ c(\alpha) + C\left(\frac{1}{m}\right) + C\left(\frac{1}{m^2}\right) + \dots$$

3) Nonperturbative th evaluation of form factor at non-recoil point

4)  $|V_{cb}|$  extraction

# B → D\* ℓ ν

Preferred

Theory

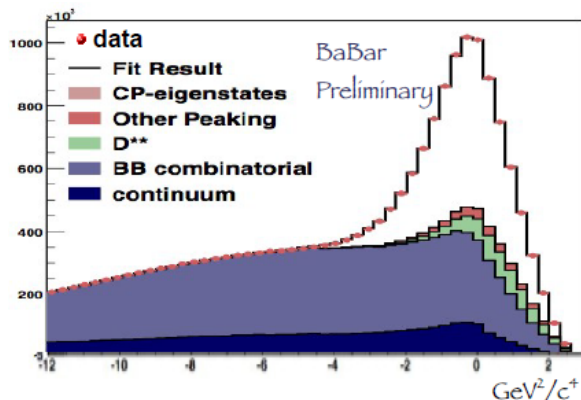
- ✓ less suppressed at zero recoil:  $(\omega^2-1)^{1/2}$  (rather than  $(\omega^2-1)^{3/2}$ )
- ✓ vanishing corrections order  $1/m$  (Luke's theorem)

Experiment

- ✓ cleaner signal  $D^* \rightarrow D \pi$  (slow pion)

Dynamics decay  $\omega \neq 1$  may be parameterized by normalization and shape parameters  $\rho, R_1(1), R_2(1)$  (HQET)

Data fit  $|V_{cb}| |\mathcal{F}(1)| = (35.90 \pm 0.45) \times 10^{-3}$  HFAG 12 (more recent Belle 10)



Mangoni CKM 12

background events also involving higher mass charm states (via  $B \rightarrow D^{**} l \nu$ )

- ✓ BR for inclusive  $B \rightarrow X_c l \nu$  not saturated by sum of exclusive BR
- ✓  $\Gamma(B \rightarrow D^{**} (j_l = \frac{3}{2}) l \nu) \gg \Gamma(B \rightarrow D^{**} (j_l = \frac{1}{2}) l \nu)$   
th prediction not confirmed by data

**LATTICE STUDIES HAVE STARTED** (Atoui, Becirevic,<sup>5</sup>...12)

# $B \rightarrow D^* \ell \nu$

□ perturbative order: complete  $\alpha_s^2$  at zero recoil

Czarnecki 96

□ Power suppressions  $O(1/m_c^2)$

✓ Lattice

unquenched calculations

FNAL/MILC (from 2008)

$$\mathcal{F}(1) = 0.908 \pm 0.017$$

$$|V_{cb}| = (39.54 \pm 0.50_{\text{exp}} \pm 0.74_{\text{th}}) \times 10^{-3}$$

2010 update  
+ HFAG 2012

full MILC data set , reduced discretization effects,  $\delta|V_{cb}|$  down to 1.6%

Lahio 2012

✓ Non-lattice

zero recoil sum rules. Lattice budget error questioned (HQ masses, matching)

Gambino et al 2012

$$\mathcal{F}(1) = 0.86 \pm 0.02$$

$$|V_{cb}| = (41.6 \pm 0.6_{\text{exp}} \pm 1.9_{\text{th}}) \times 10^{-3} \text{ +HFAG 2012}$$

# $B \rightarrow D \ell \nu$

- Power suppression corrections to the unity limit at zero recoil

- ✓ **Lattice**

- unquenched calculations

$$\mathcal{G}(1) = 1.074 \pm 0.024$$

FNAL/MILC 05

Data fit: normalization and slope  $\rho^2$

$$|V_{cb}| |\mathcal{G}(1)| = (42.64 \pm 1.53) \times 10^{-3}$$

$$|V_{cb}| = (39.70 \pm 1.42_{\text{exp}} \pm 0.89_{\text{th}}) \times 10^{-3}$$

HFAG 12

- ✓ **Non-lattice**

- heavy quark expansion (BPS limit)

$$\mathcal{G}(1) = 1.04 \pm 0.02$$

Uraltsev 04

$$|V_{cb}| = (40.7 \pm 1.5_{\text{exp}} \pm 0.8_{\text{th}}) \times 10^{-3}$$

PDG 12

- form factor directly at non-zero recoil, avoiding extrapolation and reducing model dependence (**quenched**)

$$|V_{cb}| = (41.6 \pm 1.8 \pm 1.4 \pm 0.7_{FF}) \times 10^{-3}$$

Roma-TorVer 07  
+ Babar 09

- **non-recoil unquenched FNAL/MILC in progress**

Quio et al<sub>12</sub>

# Inclusive decays $B \rightarrow X_c l \nu$

$$\Gamma(B \rightarrow X_q l \nu) = \frac{G_F^2 m_b^5}{192\pi^3} |V_{qb}|^2 \left[ c_3 \langle O_3 \rangle + c_5 \frac{\langle O_5 \rangle}{m_b^2} + c_6 \frac{\langle O_6 \rangle}{m_b^3} + O\left(\frac{1}{m_b^4}\right) \right]$$

sum over all possible final states  $X_q$  (single and multi-particle)

no dependence on details of final state, quark-hadron duality generally assumed

OPE factorization of short and long distance dynamics

- ✓ Short distance: coefficients, perturbative
- ✓ Long distance: matrix elements, non-perturbative, HQET parameterization
- Common hadronic parameters in OPE to different inclusive B meson observables (spectra, moments): can be measured in experiments  $\implies$  global fit
- quark masses defined in a scheme (1S, kinetic, etc.); other hadronic parameters in consistent framework



## double series in $\alpha_s$ and $\Lambda/m_b$

### kinetic scheme

- $O(\alpha_s^2)$  corrections to leading term (parton model)+BLM terms  $\alpha_s^{n+1} \beta_0^n$   
[Melnikov , Czarnecki, Pak , Biswas , Gambino , ... ]
- $O(\Lambda/m_b^{2,3})$  known,  $O(\Lambda/m_b^{4,5})$  estimated  
[Gremm, Kapustin, Dassinger, Turczyk, Mannel , Gambino, Bigi, Uraltsev, Zwicky ...]
- $\alpha_s \frac{\mu_\pi^2}{mb}$  modest corrections to width and moments (2012 new analytical computation)  
[Becher, Boos, Lunghi, Alberti , Ewerth, Gambino, Nandi , ...]
- $\alpha_s \frac{\mu_G^2}{mb}$  **in progress** (known for inclusive radiative decays, about 20% in the rate)  
[Alberti, Ewerth, P. Gambino, S. Nandi, ...]
- $\log m_c, 1/m_c^2$  ... intrinsic charm estimates  
[Breidenbach, Feldmann, Mannel, Turczyk, Bigi, Mannel, Uraltsev, ...]

non trivial translation to other schemes  $\rightarrow$  e.g.  $1S \mu_\pi^2 = -\lambda_1 + O(\alpha_s), \dots$

[Hoang, Bauer, Ligeti, Luke, Manohar, Trott, ...]

# Global fit results

global fit of  $|V_{cb}|, m_b$  and hadron parameters

width + hadron, lepton momenta: about 70 measurements available (80% from B factories)

Additional constraint to increase precision estimate in  $m_b$   
 photon energy moments in  $B \rightarrow X_s \gamma$ , or a precise constraint on  $m_c$

Constraint	$ V_{cb}  (10^{-3})$
$B \rightarrow X_s \gamma$	$41.94 \pm 0.43_{\text{fit}} \pm 0.59_{\text{th}}$
$m_c^{\overline{\text{MS}}}(3 \text{ GeV})$	$41.88 \pm 0.44_{\text{fit}} \pm 0.59_{\text{th}}$

kinetic scheme [HFAG 12](#)

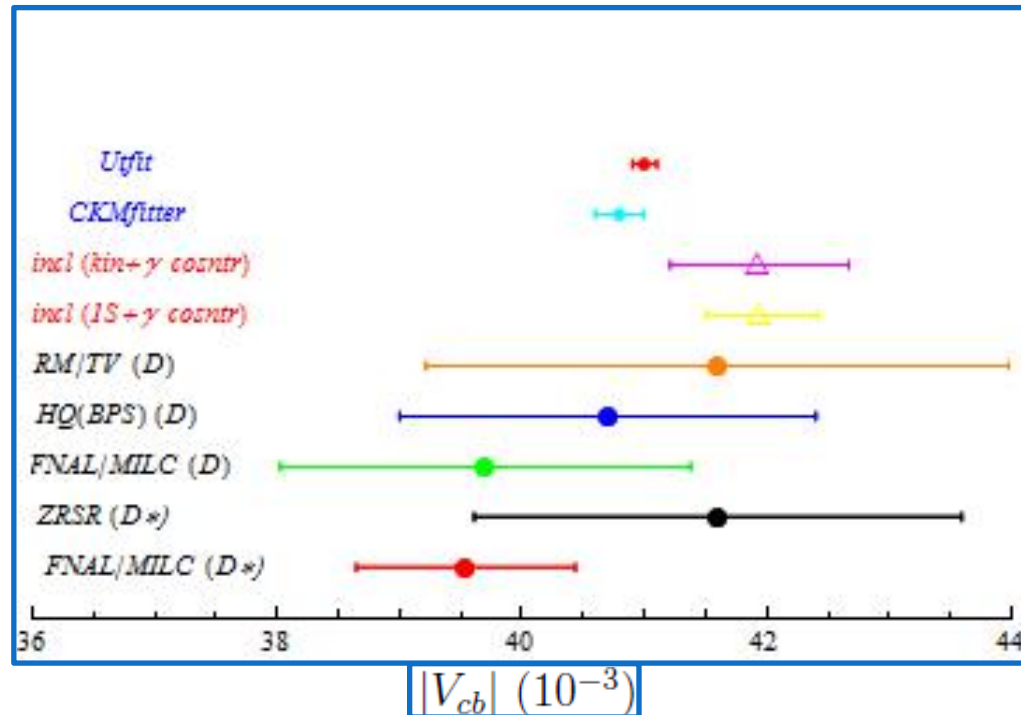
Constraint	$ V_{cb}  (10^{-3})$
$B \rightarrow X_s \gamma$	$41.96 \pm 0.45$
None	$42.37 \pm 0.65$

1S scheme  
 (no  $m_c$  dependence) [HFAG 12](#)

$$\frac{\delta V_{cb}}{V_{cb}} < 2\%$$

to compare with excl  $\sim 2\%$

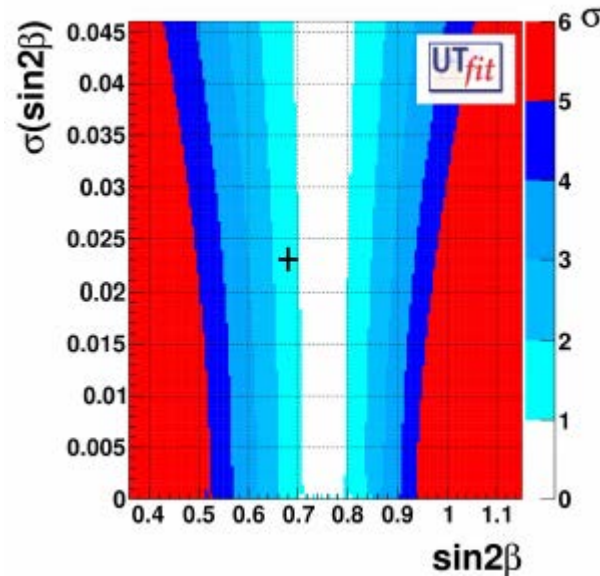
# Vcb results: Pick your one



## Future:

- CLEO  $\rightarrow$   $\approx$ 50-70 more stat B factories  $\rightarrow$   $\approx$ 50 more stat **Belle II**
- **LHCb**: about 1.2 million  $B \rightarrow D^* \mu \nu$  decay reconstructed in  $1 \text{ fb}^{-1}$ ; promising prospects for  $|V_{cb}|$  measurement

$$|V_{ub}|$$



- ❖ least known, but most studied CKM matrix element both theoretically and experimentally
- ❖ Connected to CP violation:  $\sin 2\beta$  from  $B \rightarrow J/\psi K$  compatibility strongly depends on input for  $|V_{ub}|$   
(see Silvestrini's talk)

# $|V_{ub}|$ exclusive determination

- Traditionally extracted by the decay  $B \rightarrow \pi \ell \nu$   
(only a single form factor in massless limit)

$$\frac{d\Gamma(\bar{B}^0 \rightarrow \pi^+ \ell \bar{\nu})}{dq^2} = \frac{G_F^2 |\vec{p}_\pi|^3}{24\pi^3} |V_{ub}|^2 |f_+(q^2)|^2$$

$$\langle \pi^+(p) | \bar{u} \gamma_\mu b | \bar{B}^0(p+q) \rangle = f_+(q^2) (2p_\mu + q_\mu)$$

Non-pert th predictions for  $f_+$  usually confined to regions of  $q^2$

## Complementarity

- ✓ Light Cone Sum Rules LCSR

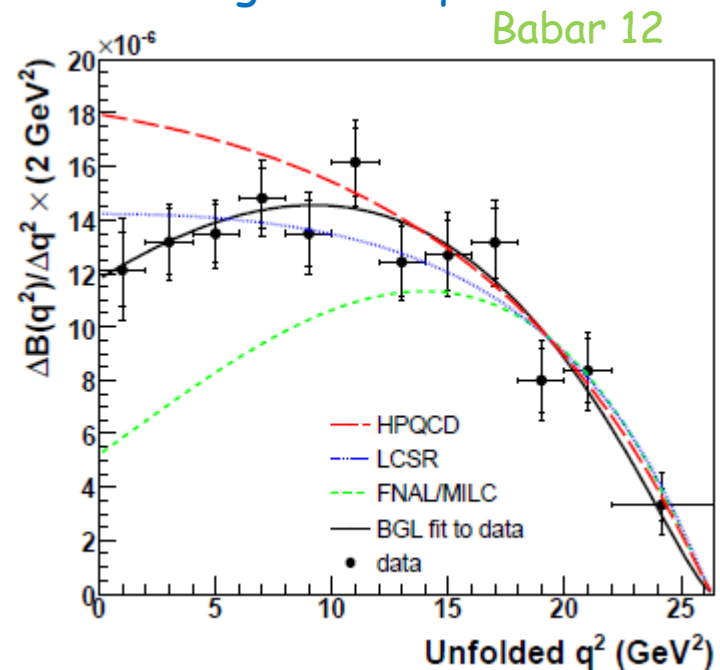
low  $q^2$  regions  $\sim < 16 \text{ GeV}$

(OPE near the light-cone)

- ✓ Lattice large  $q^2 \sim > 16 \text{ GeV}$

(to avoid large discretization errors)

Better fit with data



[Unquenched HPQCD 07, FNAL/MILC 09,  
LCSR Khodjamirian et al 11, BGL: Boyd, Grinstein, Lebed, (2 par)]

# Other exclusive semileptonic players

Babar 12 BF  $\times 10^{-4}$

$$B \rightarrow \pi \ell^+ \nu \quad 1.45 \pm 0.04 \pm 0.06$$

$$B^0 \rightarrow \pi^- \ell^+ \nu \quad 1.47 \pm 0.05 \pm 0.06$$

$$B^+ \rightarrow \pi^0 \ell^+ \nu \quad 0.77 \pm 0.04 \pm 0.03$$

$$B^+ \rightarrow \omega \ell^+ \nu \quad 1.19 \pm 0.16 \pm 0.09$$

$$B^+ \rightarrow \eta \ell^+ \nu \quad 0.38 \pm 0.05 \pm 0.05$$

$$B^+ \rightarrow \eta' \ell^+ \nu \quad 0.24 \pm 0.08 \pm 0.03$$

$$B(B^0 \rightarrow \rho^- \ell^+ \nu) = (1.75 \pm 0.15 \pm 0.27) \times 10^{-4}$$

Babar 11

$$B^0 \rightarrow \pi^+ \ell \nu \quad 1.49 \pm 0.09 \pm 0.08$$

$$B^+ \rightarrow \pi^0 \ell \nu \quad 0.80 \pm 0.08 \pm 0.04$$

$$B^0 \rightarrow \rho^+ \ell \nu \quad 3.17 \pm 0.27 \pm 0.18$$

$$B^+ \rightarrow \rho^0 \ell \nu \quad 1.86 \pm 0.10 \pm 0.09$$

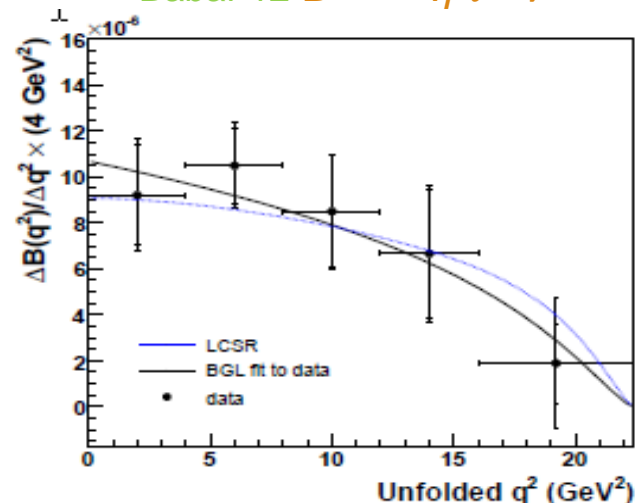
$$B^+ \rightarrow \omega \ell \nu \quad 1.09 \pm 0.16 \pm 0.08$$

$$B^+ \rightarrow \eta \ell \nu \quad 0.42 \pm 0.12 \pm 0.05$$

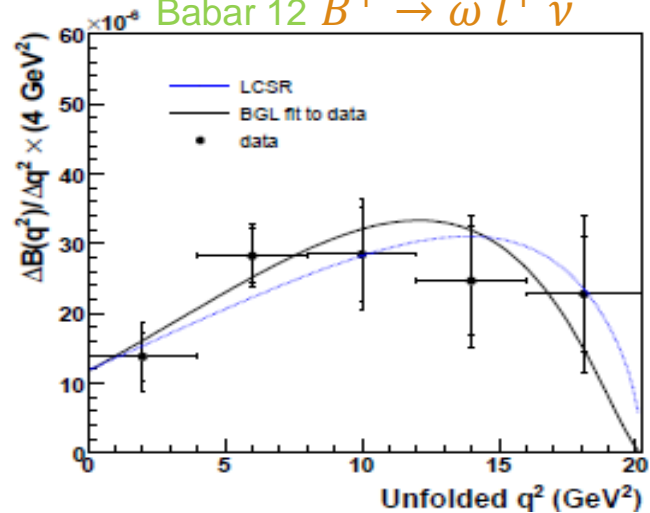
$$B^+ \rightarrow \eta' \ell \nu \quad < 0.57 \text{ at } 90\% \text{ CL}$$

Belle prelim results (ICHEP 12) BF  $\times 10^{-4}$

Babar 12  $B^+ \rightarrow \eta \ell^+ \nu$



Babar 12  $B^+ \rightarrow \omega \ell^+ \nu$



# Lattice

Unquenched results form factors for  $B \rightarrow \pi \ell \nu$

Fermilab/MILC and HPQCD collabs.  $\longrightarrow$  substantial agreement

Theory	$q^2$ (GeV <sup>2</sup> /c <sup>4</sup> )	$ V_{ub} $ ( $\times 10^{-3}$ )
HPQCD 06	$> 16$	$3.55 \pm 0.13^{+0.62}_{-0.41}$
FNAL 05	$> 16$	$3.78 \pm 0.14^{+0.65}_{-0.43}$
FNAL/MILC 09	all regions	$3.43 \pm 0.33$

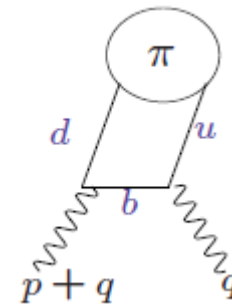
Belle 12

(see Gamiz's talk)

Theory	$q^2$ (GeV <sup>2</sup> /c <sup>4</sup> )	$ V_{ub} $ ( $\times 10^{-3}$ )
HPQCD 06	16-24.6	$3.47 \pm 0.10 \pm 0.08^{+0.60}_{-0.39}$
FNAL/MILC 09	16-24.6	$3.31 \pm 0.09 \pm 0.07^{+0.37}_{-0.30}$
FNAL/MILC 09	all regions	$3.43 \pm 0.33$

Babar 12

# Light Cone Sum Rules



- Correlation functions
- OPE near the light-cone

## Recent progress in pion distribution amplitudes

- NLO leading and LO high order twists

[Duplancic, Khodjamirian, Mannel, Melic, Offen, Wang, Ball, Jones...]

- latest update: leading-twist  $O(\alpha_s^2 \beta_0)$  corrections to  $f_+(q^2)$

$$|\vec{V}_{ub}| = (3.34 \pm 0.10 \pm 0.05 +_{-0.26}^{+0.29}) 10^{-3}$$

Babar 12+  
Bharucha 12

- From  $B^+ \rightarrow \omega l^+ \nu$

$$|\vec{V}_{ub}| = (3.20 \pm 0.21 \pm 0.12 +_{-0.32}^{+0.45}) 10^{-3}$$

Babar 12+  
Ball Zwicky 05

- From  $B^+ \rightarrow \rho l^+ \nu$

$$|\vec{V}_{ub}| = (2.75 \pm 0.24) 10^{-3}$$

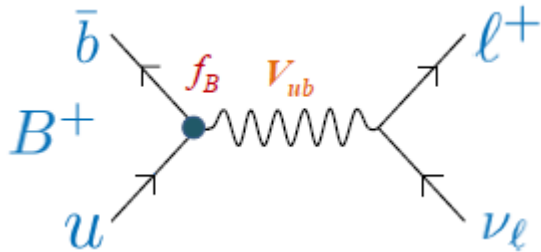
Babar 11+  
Ball Zwicky 05

- Future and in progress: extraction via  $\Lambda_b \rightarrow p l \nu$ , via  $B_s \rightarrow K l \nu$

Bharucha, Melic, Duplancic, ...



# Leptonic $B^+ \rightarrow \ell^+ \nu$

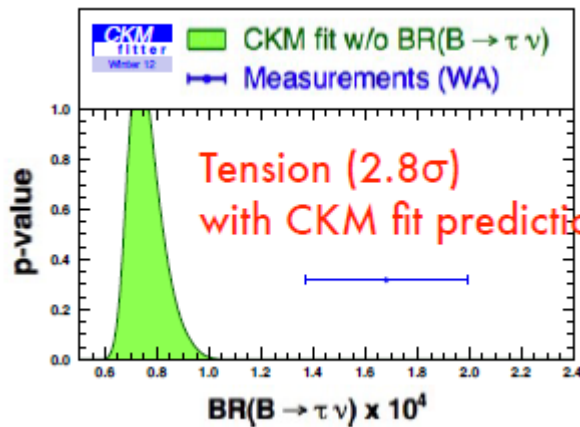


$$B(B^+ \rightarrow \ell^+ \nu_\ell) = \frac{G_F^2 m_B m_\ell^2}{8\pi} \left(1 - \frac{m_\ell^2}{m_B^2}\right)^2 f_B^2 |V_{ub}|^2 \tau_B$$

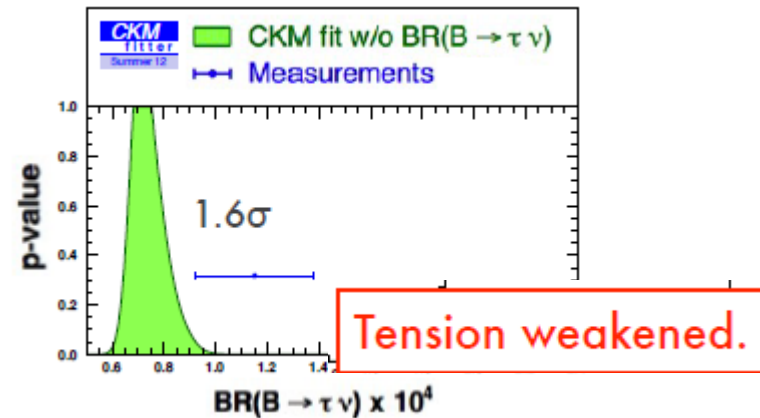
Helicity suppressed  
( $l=e, \mu$ )

✓ th clean process  
but depends on  $f_B$   
(lattice precisely det: see Gamiz talk)

$$\begin{aligned} \mathcal{B}(B \rightarrow e\nu)_{SM} &\sim 10^{-11} \\ \mathcal{B}(B \rightarrow \mu\nu)_{SM} &\sim 3.5 \times 10^{-7} \\ \mathcal{B}(B \rightarrow \tau\nu)_{SM} &\sim 10^{-4} \end{aligned}$$



After ICHEP 2012



Horii talk

# If there were NP....

Difficult to assess; a tree process

exchange of a new particle *without* NP effects in other observables

□ A charged scalar particle which couples proportionally to the masses of the fermions involved: a charged Higgs boson

✓ Consider also other constraints

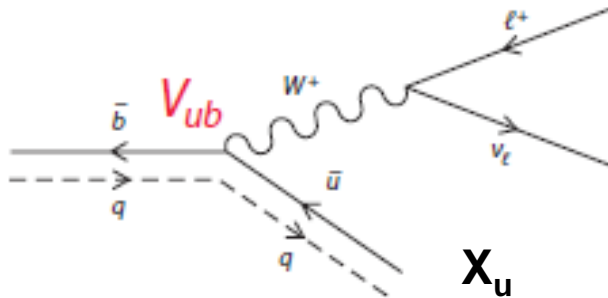
e.g. Type II 2HDM disfavoured by recent data on  $B \rightarrow D^{(*)} TV$

Compatible with Type III 2HDM

□ Other possibilities: (essentially changing  $|V_{ub}|$ ...)  
right-handed currents, NP in B mixings

Crivellin, Greub, Kokulu, 12 Lenz et al, 12 ....

# Inclusive $|V_{ub}|$



large  $b \rightarrow c$  background ( $|V_{cb}/V_{ub}|^2 \approx 100$ )

Need experimental phase space cuts to reduce background;  
in general

$$m_X \ll E_X$$

Phase space regions where OPE fails become dominant; new  
unwelcome effects (with respect to semileptonic  $b \rightarrow c$ ):

- Final gluon radiation strongly inhibited: soft and collinear singularities
- perturbative expansion of spectra affected by large logarithms

$$\alpha_s^n \log^{2n}(2 E_X/m_X)$$

to be resummed at all orders in PT

- non-perturbative effects related to a small vibration of the  $b$  quark in the  $B$  meson (Fermi motion) enhanced

- Experimental progress

- Belle results 09 access 90% data, claimed overall uncertainty of 7% on  $|V_{ub}|$
- More recent Babar similar data range

- Theoretical approaches (HAFG averages)

- predictions based on parameterizations of shape function, and OPE constraints

- BLNP

Bosch, Lange, Neubert, Paz

- GGOU

Gambino, Giordano, Ossola, Uraltsev

- predictions based on resummed pQCD

- DGE Dressed Gluon Exponentiation

Andersen, Gardi

- ADFR

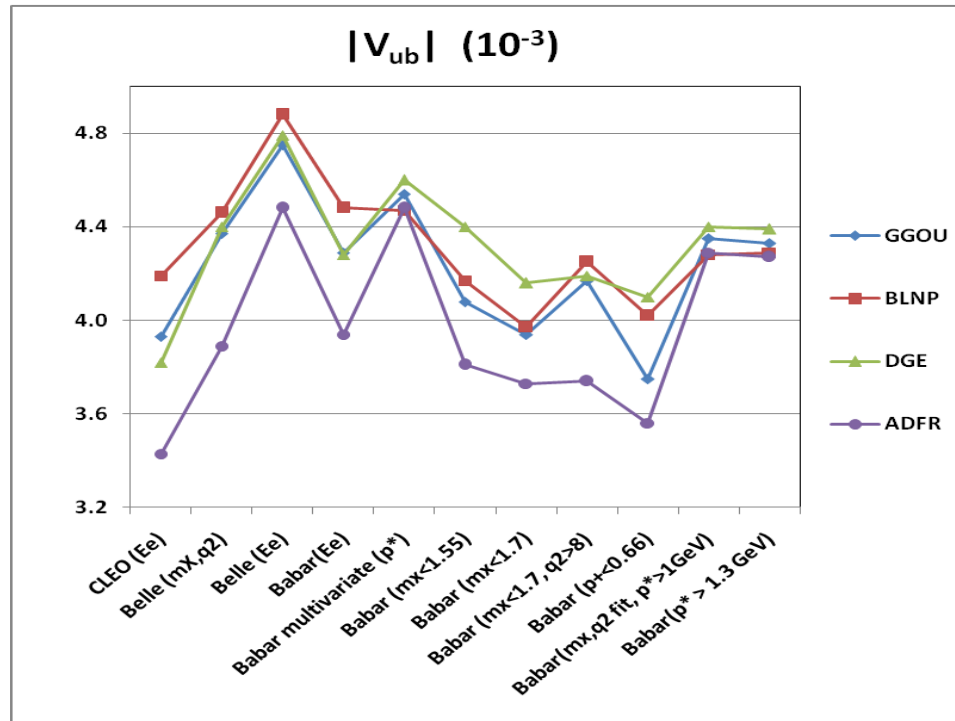
Aglietti, Di Lodovico, Ferrera, GR

- global fit of shape function,  $|V_{ub}|$  and  $m_b$  (also data on  $B \rightarrow Xs \gamma$ )

- SIMBA

Tackmann, Lacker, Ligeti, Stewart...

# Data fit experiment by experiment

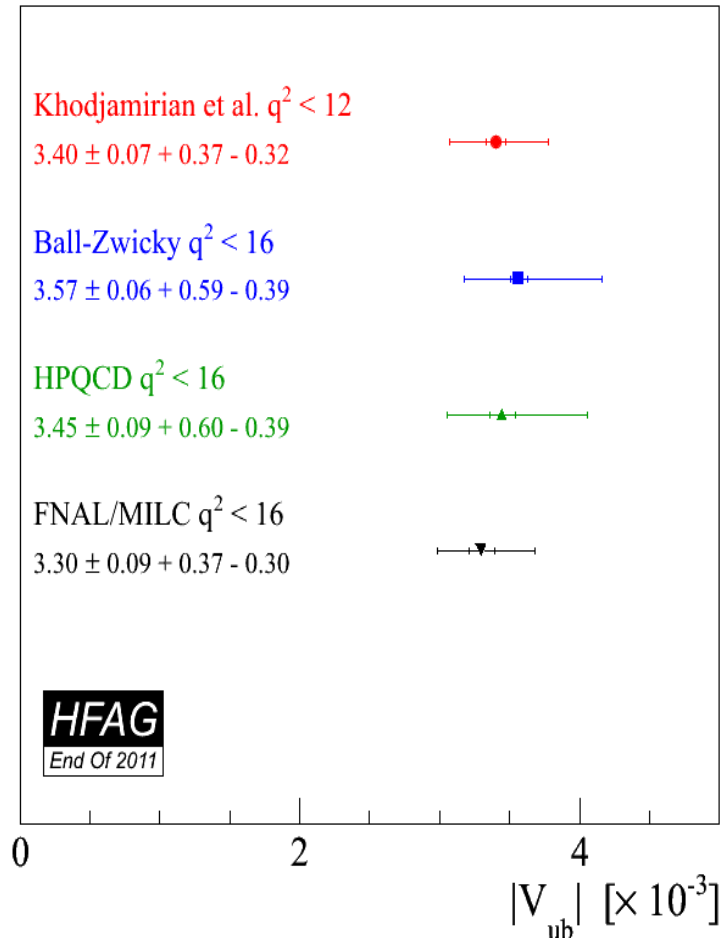


Data from HFAG (end of 2011)

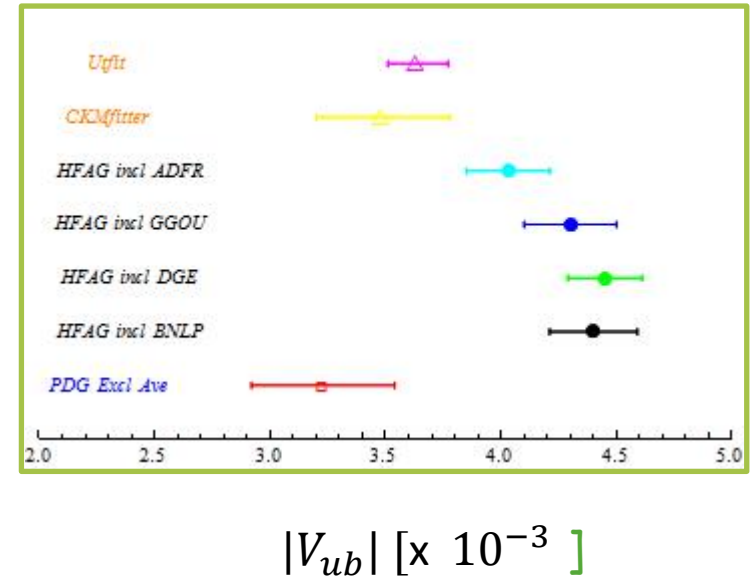
- Spread among calculations comparable to quoted theoretical (non-parametric) errors

# Results averages: Long lasting puzzle

## Exclusive

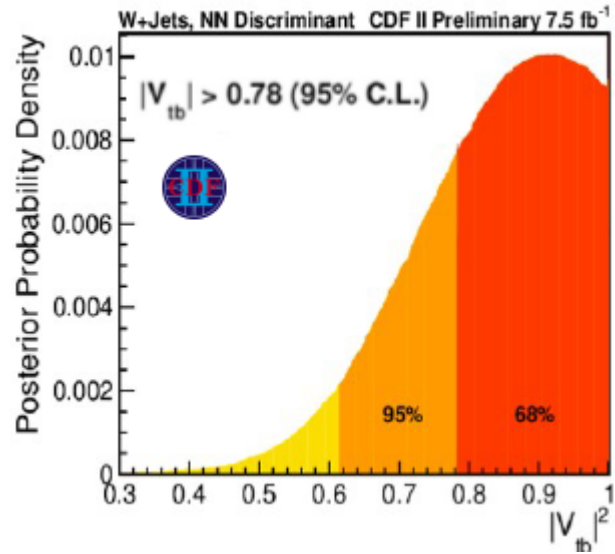


## Incl vs Excl vs indirect fit



At SuperFlavour factories  
 (75  $ab^{-1}$ ) errors expected to  
 reduce to 3 % (excl) 2% (incl)

# V<sub>tb</sub>



- ❖ Known by unitarity with great precision
- ❖ Let us meet directly: Beyond SM, beyond unitarity

□ In the standard model with 3 quark generations, the top quark is expected to decay to a  $W$  boson and a  $b$  quark roughly 99.8% of the time

□ The magnitude of  $|V_{tb}|$  is expected to be close to unity as a consequence of unitarity and of the measured values for the other CKM elements

indirect fit

$$|V_{tb}| = 0.999106 \pm 0.000024$$

Ufit 2013

Very recent CDF simultaneous measurement of ratio

$$\frac{\mathcal{B}(t \rightarrow Wb)}{\mathcal{B}(t \rightarrow Wq)} = \frac{|V_{tb}|^2}{|V_{tb}|^2 + |V_{ts}|^2 + |V_{td}|^2}$$

and top-quark-pair-production cross section  $\sigma_{t\bar{t}}$  (integrated luminosity of  $8.7 \text{ fb}^{-1}$ ), assuming  $|V_{tb}| > 0.89$

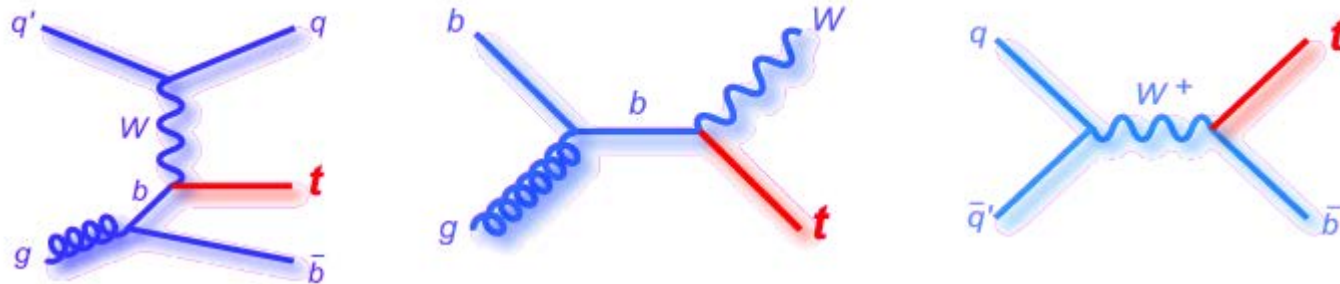
CDF 2013

$$|V_{tb}| = 0.97 \pm 0.05$$

Agreement with SM, and previous CDF, D0, CMS measurements  
 $|V_{tb}| = 0.89 \pm 0.07$  (PDG 2012 average (includes single top))



# Single top quark production cross section



possible to determine  $|V_{tb}|$  directly without assuming unitarity

First observation by **CDF** and **DO** in 2009;

With no assumptions on number of families or unitarity,  
but SM vertex +  $|V_{tb}|^2 \gg |V_{ts}|^2 + |V_{td}|^2$

$$|V_{tb}| = 0.92_{-0.08}^{+0.10} \pm 0.05 \text{ (th)}$$

CDF Moriond 13

**LHC** takes  
Tevatron legacy

**limits (unconstrained/constrained 95% C.L.):**

○ **CMS (8 TeV)**  $0.96 \pm 0.08_{(\text{exp})} \pm 0.02_{(\text{th})} / 81 < |V_{tb}| \leq 1$

○ **ATLAS (8 TeV)**  $1.04_{-0.11}^{+0.10} (\text{th} + \text{exp}) / 80 < |V_{tb}| \leq 1$

# If there were NP....

Relaxing unitarity (see also PMNS neutrino mixing matrix in Antush talk):

□ Here simplest way: adding fermions

✓ vector-like quarks (in many models, e.g. Randall-Sundrum or E6 GUTS)

Botella, Branco, Nebota 2012, Buras, Duling, Gori, 2009...

✓ more fermion generations

Lacker et al 2012,...

✓ Generally coupling affected at order 5%

✓ Actual measurements about 8% precision:  
should nail it

# Conclusions

