

# Lepton Flavour Violation and the Flavour Puzzle

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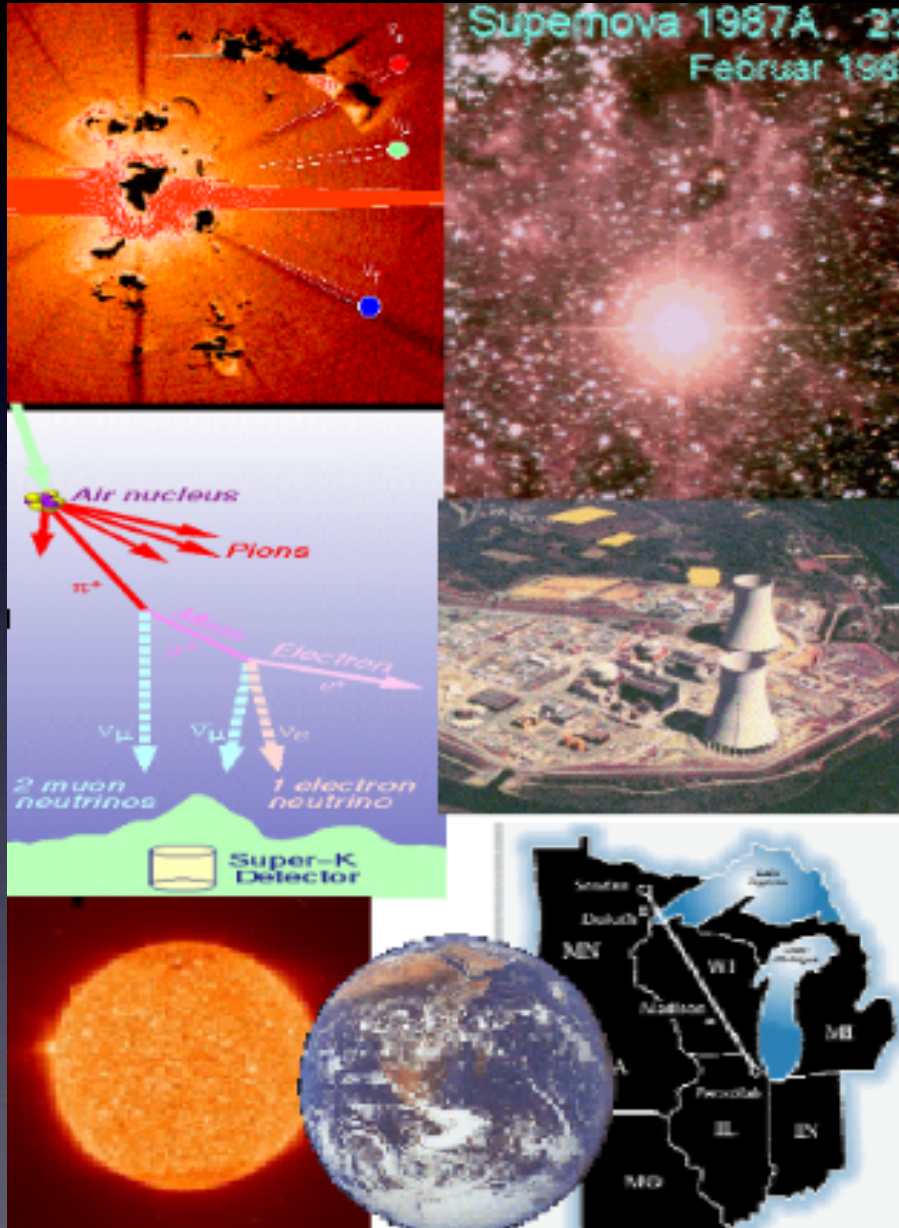


Beauty 2013, Bologna

April 9, 2013



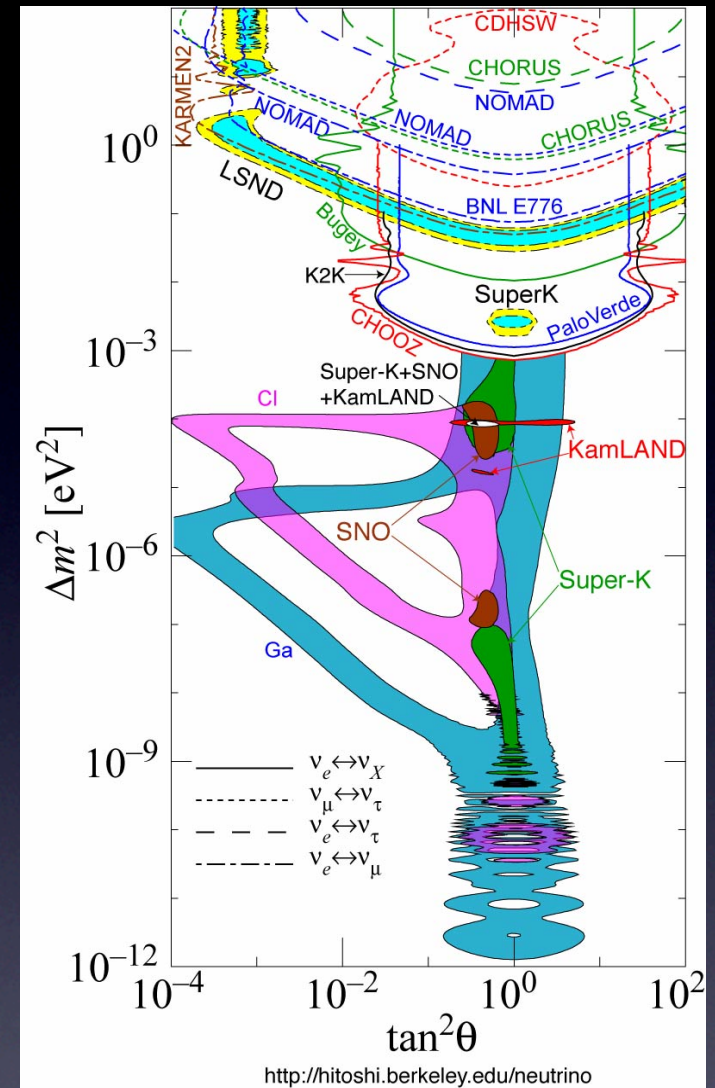
# LFV exists ...



Neutral LFV  
is observed!

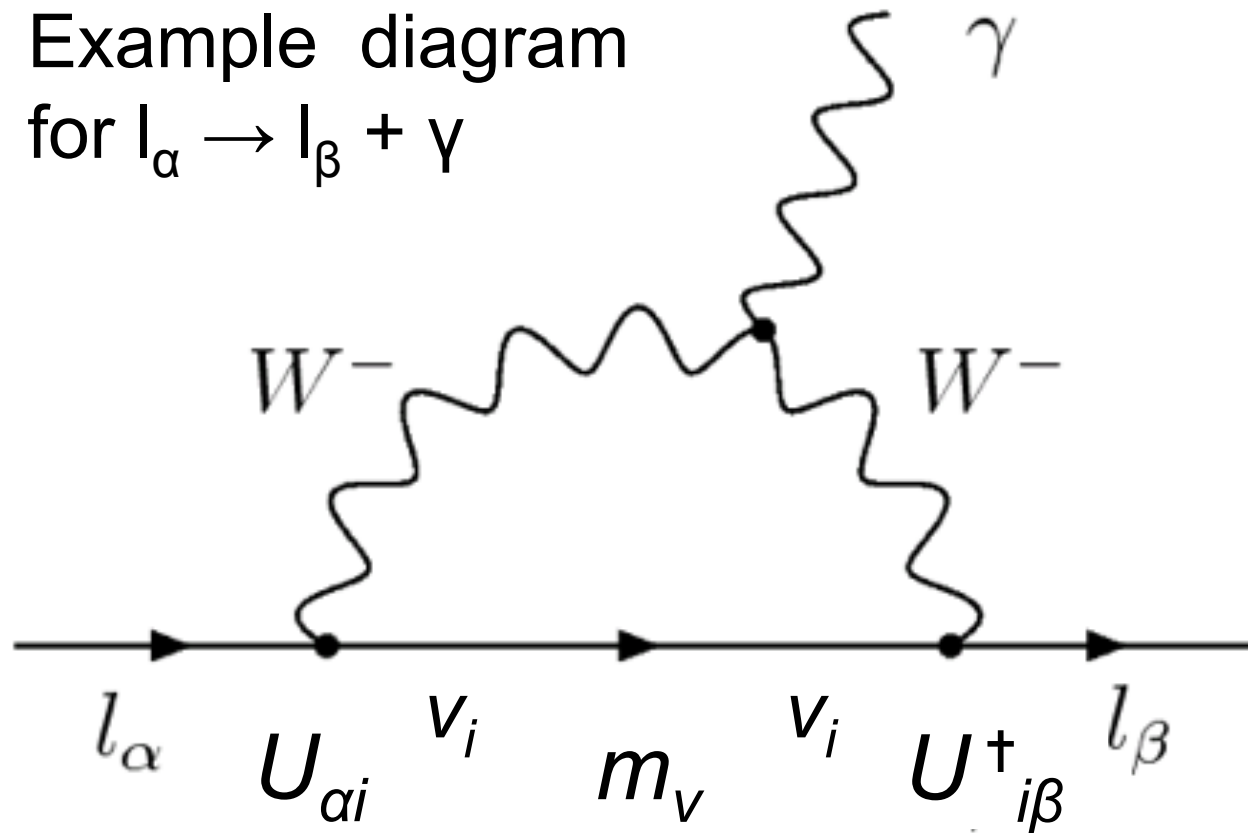
$\nu$  oscillations  
imply neutrino  
masses

$\Rightarrow$  also charged LFV exists ...



# LFV in the SM + neutrino masses

Example diagram  
for  $l_\alpha \rightarrow l_\beta + \gamma$



E.g. in the SM  
+ d=5 operator

However, it is well known that the branching ratios are suppressed by  $(m_\nu/M_W)^4$  for unitary  $U$  ( $\leftrightarrow$  GIM mechanism) and thus unobservably small ...



***However, as soon as one extends the SM by a mechanism to generate the neutrino masses, charged LFV is typically induced at a much larger rate ... !***

# *(Some of) the pieces of the flavour puzzle*

I) The SM  
flavour puzzle

II) The neutrino  
flavour puzzle



III) The “new physics”  
flavour puzzle

IV) The puzzle of  
CP violation



# *(Some of) the pieces of the flavour puzzle*

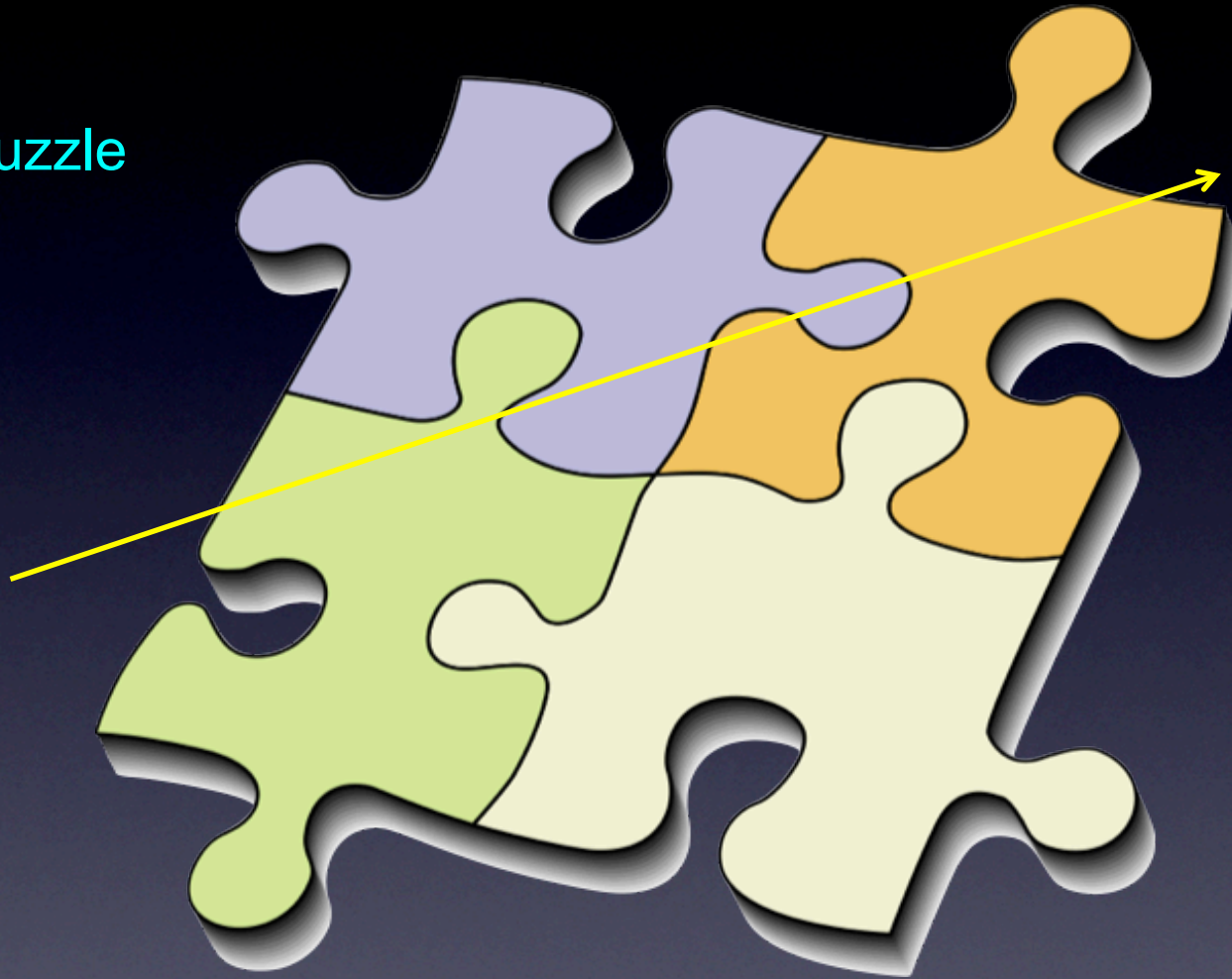
I) The SM  
flavour puzzle

II) The neutrino  
flavour puzzle

this talk

III) The “new physics”  
flavour puzzle

IV) The puzzle of  
CP violation

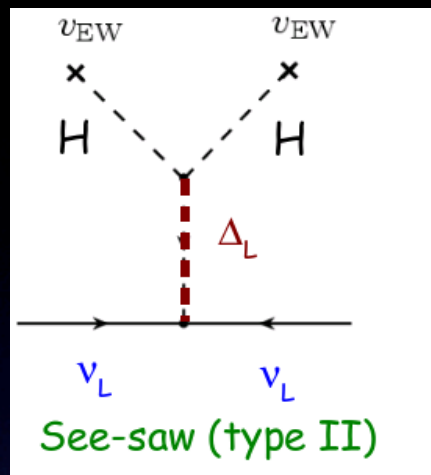
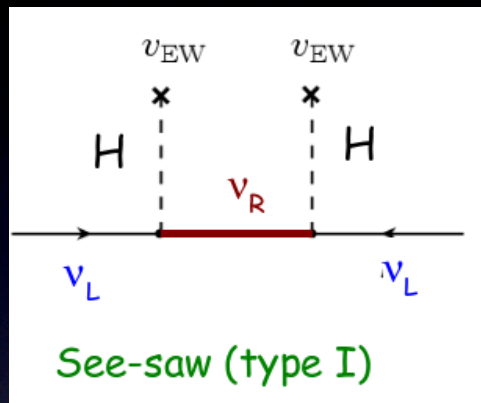


# *Overview: Two examples ...*

- Bottom-up example: LFV & non-unitarity of the leptonic mixing matrix
- Top-down example: LFV in SUSY GUT models of flavour

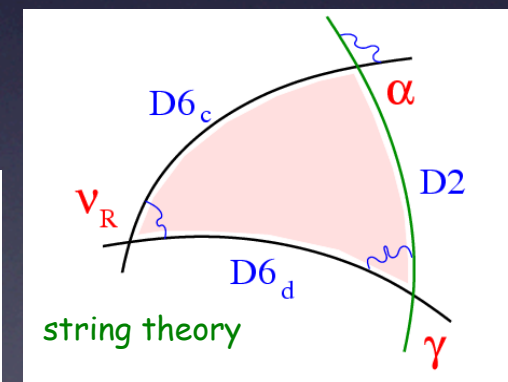
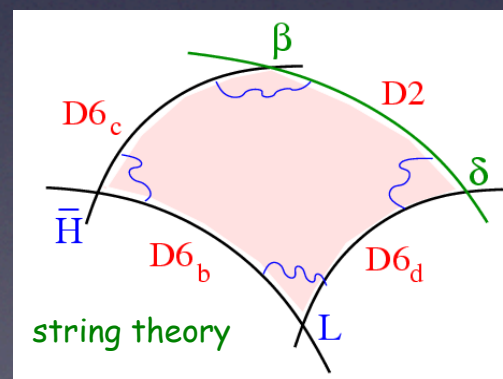
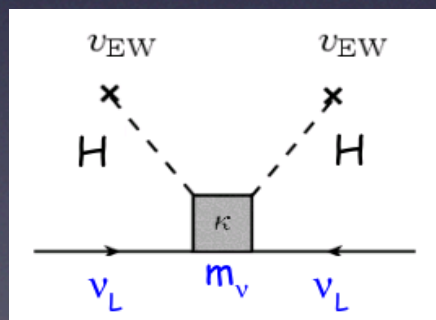
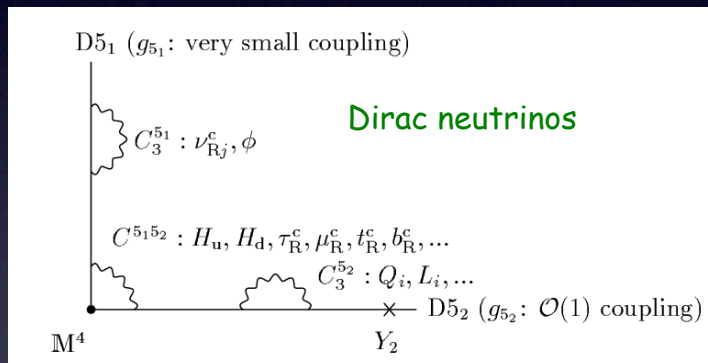
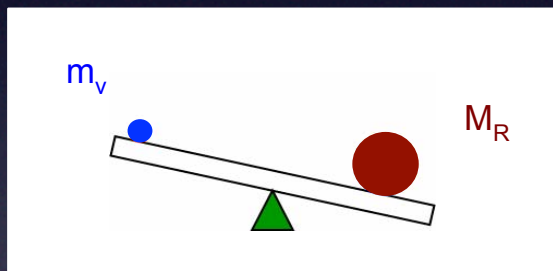
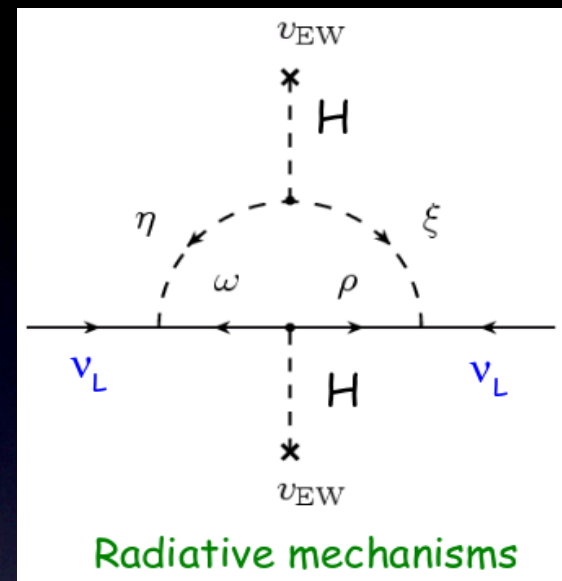


# Neutrino masses: How to extend the SM?



?

... or something completely different



Effective theory:  
d=5 operator

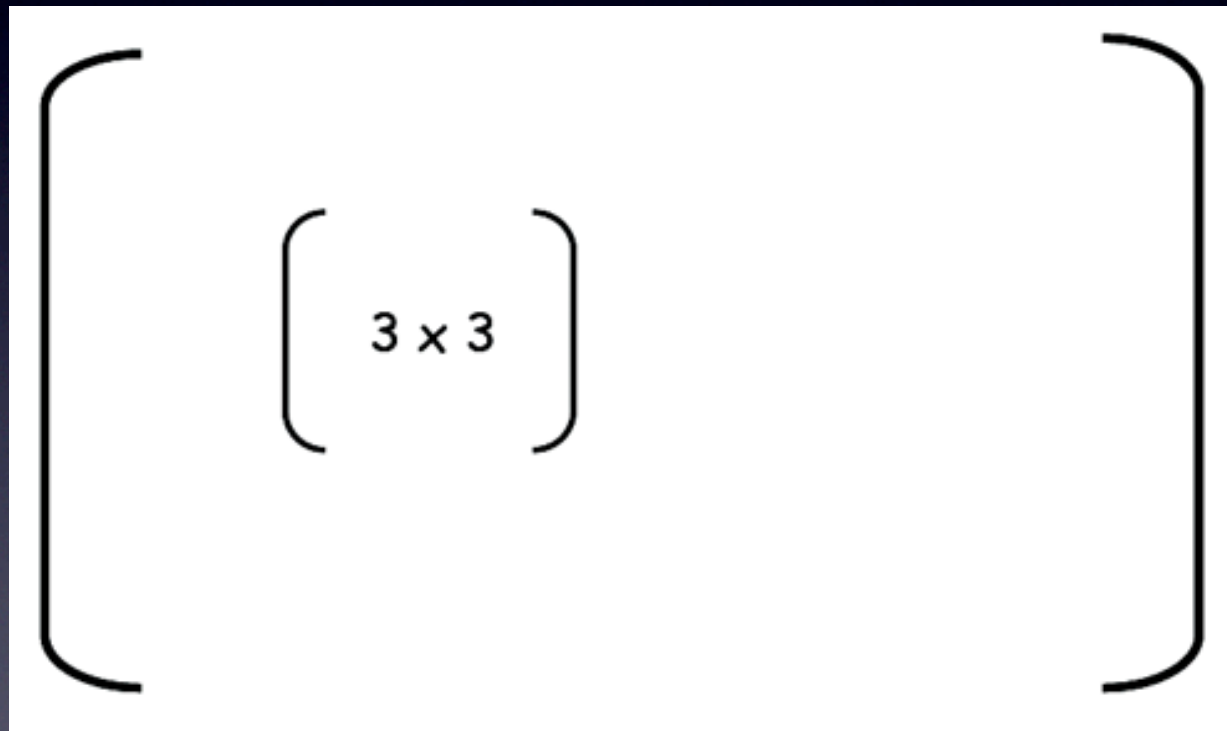
$$\delta\mathcal{L}^{d=5} = \frac{1}{2} c_{\alpha\beta}^{d=5} \left( \bar{L}^c_\alpha \tilde{\phi}^* \right) \left( \tilde{\phi}^\dagger L_\beta \right) + h.c.$$



***A comparatively model-independent  
consequence of new physics introduced to  
generate the observed neutrino masses:  
Non-unitarity of the leptonic mixing matrix ...***

# Non-unitary leptonic mixing

- Typical situation, intuitively:



(Effective) mixing matrix of light neutrinos is submatrix of a larger unitary mixing matrix (mixing with additional heavy particles)

Langacker, London ('88)

⇒  $U_{\text{PMNS}} \equiv N$  is non-unitary

Examples with possible large non-unitarity: 'inverse' seesaw or 'multiple' seesaw at TeV energies, SUSY with R-parity violation, large extra dimensions, ...



# Non-unitary leptonic mixing

- Lagrangian in the mass basis ...

kinetic term      neutrino mass term      charged current interaction      non-unitary mixing matrix N

$$\mathcal{L}^{eff} = \frac{1}{2} (\bar{\nu}_i i \not{\partial} \nu_i - \bar{\nu}_i^c m_i \nu_i + h.c.) - \frac{g}{2\sqrt{2}} (W_\mu^+ \bar{l}_\alpha \gamma_\mu (1 - \gamma_5) N_{\alpha i} \nu_i + h.c.) - \frac{g}{2 \cos \theta_W} (Z_\mu \bar{\nu}_i \gamma^\mu (1 - \gamma_5) (N^\dagger N)_{ij} \nu_j + h.c.) + \dots$$

+ modification in neutral current interaction in minimal schemes (MUV), to be explained later ...

# Non-unitary leptonic mixing

- ... now when we change to the flavour basis:

non-canonical kinetic terms

$$\begin{aligned} \mathcal{L}^{eff} = & \frac{1}{2} (i \bar{\nu}_\alpha \not{\partial} (NN^\dagger)^{-1}_{\alpha\beta} \nu_\beta) - \bar{\nu}^c_\alpha [(N^{-1})^t m N^{-1}]_{\alpha\beta} \nu_\beta + h.c.) \\ & - \frac{g}{2\sqrt{2}} (W_\mu^+ \bar{l}_\alpha \gamma^\mu (1 - \gamma_5) \nu_\alpha + h.c.) \\ & - \frac{g}{2 \cos \theta_W} (Z_\mu \bar{\nu}_\alpha \gamma^\mu (1 - \gamma_5) \nu_\alpha + h.c.) + \dots, \end{aligned}$$

Non-unitarity of the leptonic mixing matrix corresponds to non-canonical kinetic terms in the flavour basis!



# Non-unitary leptonic mixing

- There is a unique gauge invariant d=6 effective operator which leads to non-canonical kinetic terms only for the neutrinos:

$$\delta\mathcal{L}^{d=6} = c_{\alpha\beta}^{d=6} \left( \bar{L}_\alpha \tilde{\phi} \right) i\not{\partial} \left( \tilde{\phi}^\dagger L_\beta \right)$$

- After EW symmetry breaking it results in a non-unitary leptonic mixing matrix with:

$$|NN^\dagger - 1|_{\alpha\beta} = \frac{v^2}{2} |c^{d=6}|_{\alpha\beta}$$

De Gouvea, Giudice, Strumia, Tobe ('01),  
Broncano, Gavela, Jenkins ('02)

S.A., Biggio, Fernandez-Martinez, Gavela, Lopez-Pavon ('06)

- + modification of the NC interaction shown earlier ...

# Non-unitary leptonic mixing

- A minimal way to introduce neutrino masses and non-unitary leptonic mixing thus consists in adding a d=5 and a d=6 operator to the SM:

$$\mathcal{L}^{eff} = \mathcal{L}_{SM} + \delta\mathcal{L}^{d=5} + \delta\mathcal{L}^{d=6} + \dots$$

MUV scheme: Minimal Unitarity Violation  
S.A., Biggio, Fernandez-Martinez,  
Gavela, Lopez-Pavon ('06)

Neutrino masses  
(violates L)

$$\delta\mathcal{L}^{d=5} = \frac{1}{2} c_{\alpha\beta}^{d=5} \left( \overline{L^c}_\alpha \tilde{\phi}^* \right) \left( \tilde{\phi}^\dagger L_\beta \right) + h.c.$$

Non-unitarity  
(conserves L)

$$\delta\mathcal{L}^{d=6} = c_{\alpha\beta}^{d=6} \left( \overline{L}_\alpha \tilde{\phi} \right) i\cancel{\phi} \left( \tilde{\phi}^\dagger L_\beta \right)$$

not necessarily suppressed by the smallness of the neutrino masses

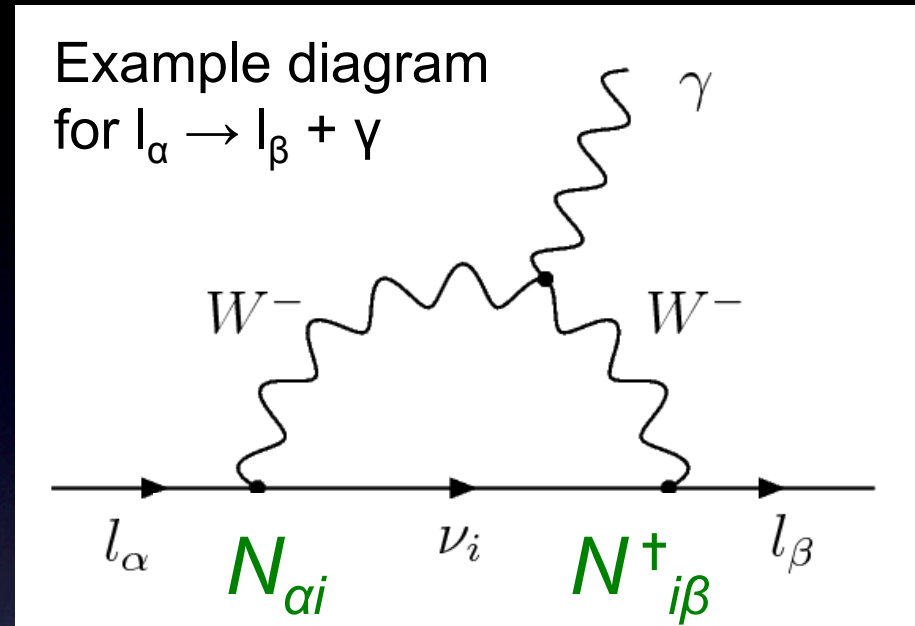


# Consequences of leptonic non-unitarity

- In the SM as an effective theory, the data should in principle be analyzed with a general, non-unitary leptonic mixing matrix  $N$  ...
- From neutrino oscillations alone, the general, non-unitary leptonic mixing matrix is quite poorly determined!
- However, leptonic non-unitarity gets constrained by various other physical processes ..., e.g. by
  - invisible Z decays
  - W decays
  - processes which are also used as universality tests
  - LFV processes

# Constraints on leptonic non-unitarity

- Important part of the constraints stems from LFV  $\mu$  and  $\tau$  decays (and in the future maybe also from  $\mu \rightarrow 3e$  and/or from  $\mu \rightarrow e$  conversion in nuclei):



$$\frac{\Gamma(l_\alpha \rightarrow l_\beta \gamma)}{\Gamma(l_\alpha \rightarrow \nu_\alpha l_\beta \bar{\nu}_\beta)} = \frac{3\alpha}{32\pi} \frac{|\sum_k N_{\alpha k} N_{k\beta}^\dagger F(x_k)|^2}{(NN^\dagger)_{\alpha\alpha} (NN^\dagger)_{\beta\beta}}$$

irrelevant for unitary mixing matrix, but can lead to sizable Br's for non-unitary N!

$$F(x) \equiv \frac{10 - 43x + 78x^2 - 49x^3 + 4x^4 + 18x^3 \ln x}{3(x-1)^4}$$

where:

$$x_k \equiv m_k^2 / M_W^2$$

$m_k$ : light neutrinos' masses



# Constraints on leptonic non-unitarity

- LFV bounds result in strong constraints on the off diagonal elements

$$(N N^\dagger)_{\alpha\beta}$$

- In summary (from a global fit to all data), the constraints are:

$$|(N N^\dagger)_{\alpha\beta} - \delta_{\alpha\beta}| = \frac{v^2}{2} |c_{\alpha\beta}^{d=6,kin}| < \begin{pmatrix} 4.0 \cdot 10^{-3} & 1.2 \cdot 10^{-4} & 3.2 \cdot 10^{-3} \\ 1.2 \cdot 10^{-4} & 1.6 \cdot 10^{-3} & 2.1 \cdot 10^{-3} \\ 3.2 \cdot 10^{-3} & 2.1 \cdot 10^{-3} & 5.3 \cdot 10^{-3} \end{pmatrix}$$

← from  $\mu \rightarrow e \gamma$

Note: Latest MEG bounds  
not yet included ...

S.A., Biggio, Fernandez-Martinez, Gavela, Lopez-Pavon ('06)

S.A., Baumann, Fernandez-Martinez ('08)

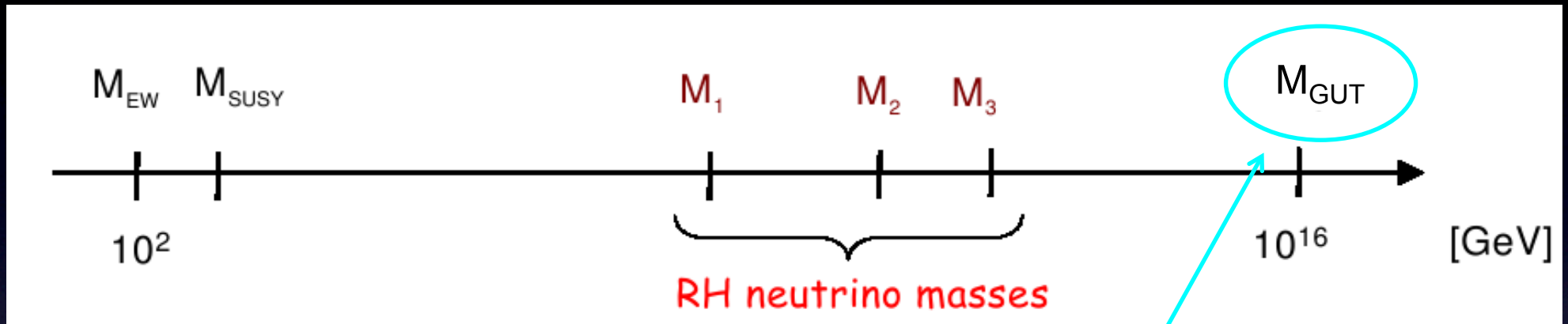
Now changing to a top-down motivated approach:

***In (supersymmetric) GUTs, neutrino masses are typically generated via the seesaw mechanism at high energies.***

***In SUSY GUT models of flavour, there are two effects inducing charged LFV ...***



For example: Scales in the type I seesaw scenario:



Scale where the model is defined

I) Non-universal soft SUSY breaking parameters (e.g. slepton masses) at high energies (= intrinsic non-universalities)

E.g.:

$$\tilde{m}_{LL}^{\text{High Scale}} = \begin{pmatrix} (m_{LL}^2)_{11} & (\Delta_{LL})_{12} & (\Delta_{LL})_{13} \\ (\Delta_{LL})_{21} & (m_{LL}^2)_{22} & (\Delta_{LL})_{23} \\ (\Delta_{LL})_{31} & (\Delta_{LL})_{32} & (m_{LL}^2)_{33} \end{pmatrix}$$

II) Non-universalities induced by RG effects from  $Y_\nu$

Borzumati, Masiero ('86), Hisano et al ('96)

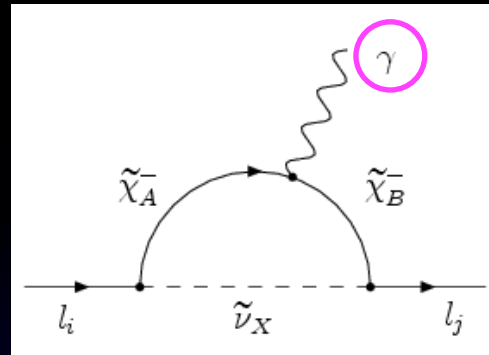
$$m_{\tilde{L}_{ij}}^2 = m_0^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \delta m_{\tilde{L}_{ij}}^2 \xleftarrow{\text{RG running}} m_{\tilde{L}_{ij}}^2 = m_0^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



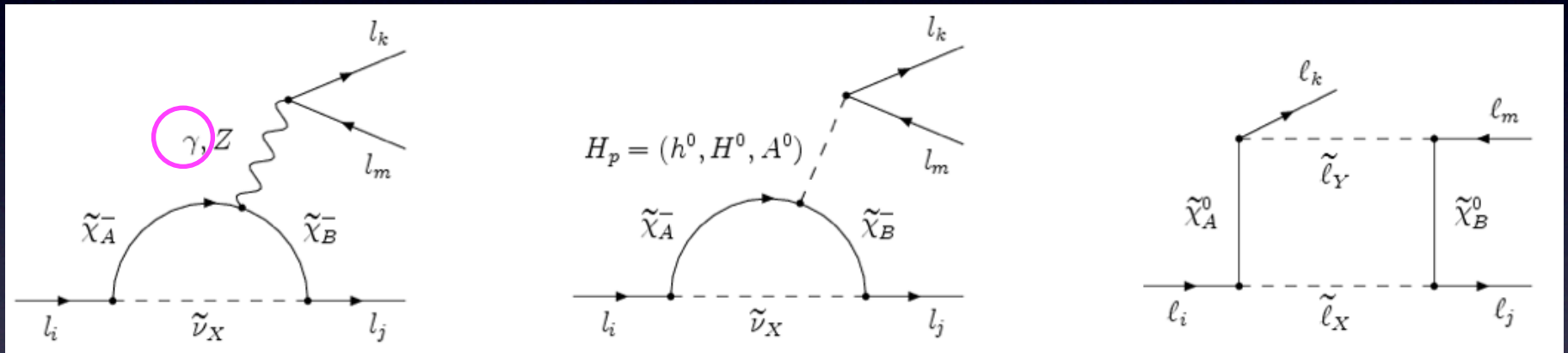
# LFV processes in SUSY extensions

- $l_\alpha \rightarrow l_\beta + \gamma$

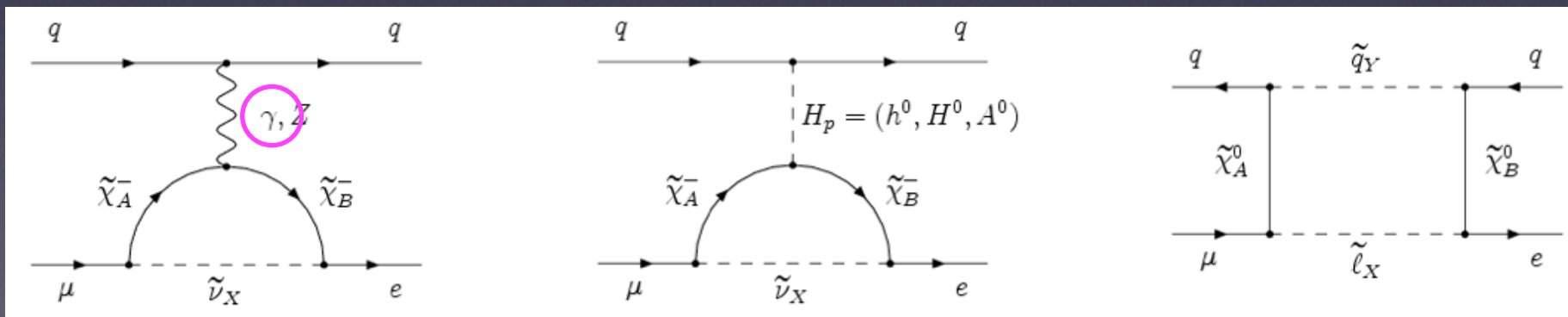
- $l_\alpha \rightarrow 3 l_\beta$



Remark: Typically close relations between the Br's for these processes if the  $\gamma$  diagrams dominate ...

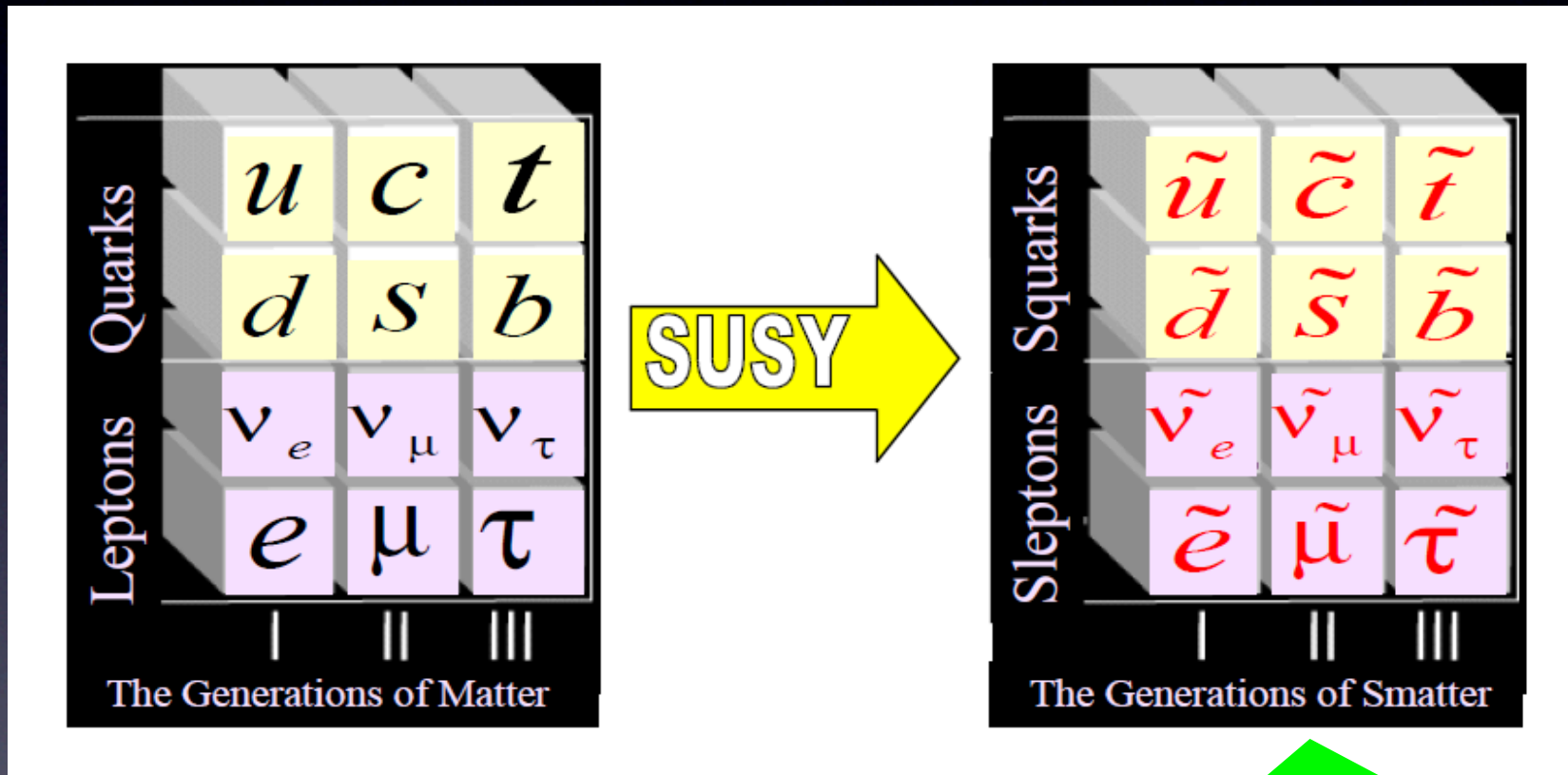


- $\mu \rightarrow e$  conversion in nuclei



# I) LFV from the model at high energies

- SUSY is broken: SUSY particles have their own flavour structure  
→ New sources of LFV!

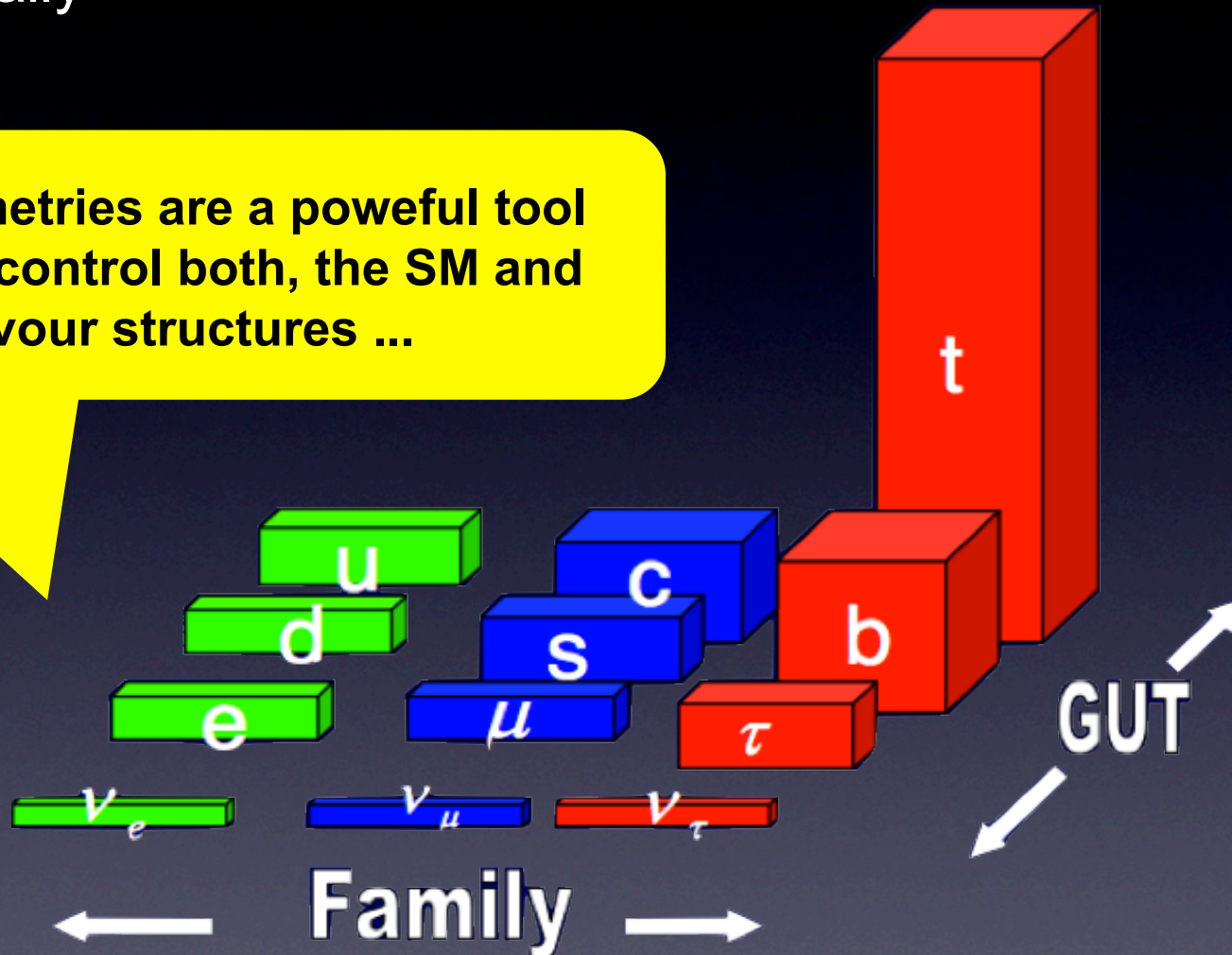


What can control the flavour structure of the SUSY particles?



- GUT symmetries unify “vertically”, family symmetries unify “horizontally”

Family symmetries are a powerful tool to constrain/control both, the SM and the SUSY flavour structures ...



# Family symmetries and the SUSY flavour structure

Particularly efficient: Non-Abelian family symmetries where all families are in 3 of  $G_{\text{Fam}}$ !

- Explain flavour structure in the SM, e.g.:

$$M_d \sim \begin{pmatrix} 0 & \epsilon_1 \epsilon_2 & \epsilon_1 \epsilon_2 \\ \epsilon_1 \epsilon_2 & \epsilon_2^2 & \epsilon_2^2 \\ \epsilon_1 \epsilon_2 & \epsilon_2^2 & \epsilon_3^2 \end{pmatrix} v_d$$

Abel, Khalil, Lebedev ('01)  
 Ross, Vives ('02),  
 Ross, Velasco-Sevilla, Vives ('04)  
 S.A., King, Malinsky ('07)  
 ...

- Generate flavour structure of the SUSY particles:

$$\widetilde{M}_{dR} \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} m_0 + \begin{pmatrix} \epsilon_1^2 & \epsilon_1^2 & \epsilon_1^2 \\ \epsilon_1^2 & \epsilon_2^2 & \epsilon_2^2 \\ \epsilon_1^2 & \epsilon_2^2 & \epsilon_3^2 \end{pmatrix} m_0$$

SUSY flavour “problem”  
 can be resolved in SUGRA:  
 S.A., King, Ross,  
 Malinsky ('08)

Universality  
 (at LO) is  
 enforced  
 by the family  
 symmetry!

$$A_d \sim \begin{pmatrix} 0 & \epsilon_1 \epsilon_2 & \epsilon_1 \epsilon_2 \\ \epsilon_1 \epsilon_2 & \epsilon_2^2 & \epsilon_2^2 \\ \epsilon_1 \epsilon_2 & \epsilon_2^2 & \epsilon_3^2 \end{pmatrix} A_0$$

**SUSY flavour  
 structure related to  
 the one of the SM**



Altmannshofer, Buras, Gori, Paradisi, Straub ('09)

Flavour physics effects = "DNA" of flavour models

|  | AC   | RVV2 | AKM  | $\delta LL$ | FBMSSM | LHT  | RS   |
|--|------|------|------|-------------|--------|------|------|
| $D^0 - \bar{D}^0$                        | ★★★★ | ★    | ★    | ★           | ★      | ★★★★ | ?    |
| $\epsilon_K$                             | ★    | ★★★★ | ★★★★ | ★           | ★      | ★★   | ★★★★ |
| $S_{\psi\phi}$                           | ★★★★ | ★★★★ | ★★★★ | ★           | ★      | ★★★★ | ★★★★ |
| $S_{\phi K_S}$                           | ★★★★ | ★★   | ★    | ★★★★        | ★★★★   | ★    | ?    |
| $A_{CP}(B \rightarrow X_s \gamma)$       | ★    | ★    | ★    | ★★★★        | ★★★★   | ★    | ?    |
| $A_{7,8}(B \rightarrow K^* \mu^+ \mu^-)$ | ★    | ★    | ★    | ★★★★        | ★★★★   | ★★   | ?    |
| $A_9(B \rightarrow K^* \mu^+ \mu^-)$     | ★    | ★    | ★    | ★           | ★      | ★    | ?    |
| $B \rightarrow K^{(*)} \nu \bar{\nu}$    | ★    | ★    | ★    | ★           | ★      | ★    | ★    |
| $B_s \rightarrow \mu^+ \mu^-$            | ★★★★ | ★★★★ | ★★★★ | ★★★★        | ★★★★   | ★    | ★    |
| $K^+ \rightarrow \pi^+ \nu \bar{\nu}$    | ★    | ★    | ★    | ★           | ★      | ★★★★ | ★★★★ |
| $K_L \rightarrow \pi^0 \nu \bar{\nu}$    | ★    | ★    | ★    | ★           | ★      | ★★★★ | ★★★★ |
| $\mu \rightarrow e \gamma$               | ★★★★ | ★★★★ | ★★★★ | ★★★★        | ★★★★   | ★★★★ | ★★★★ |
| $\tau \rightarrow \mu \gamma$            | ★★★★ | ★★★★ | ★    | ★★★★        | ★★★★   | ★★★★ | ★★★★ |
| $\mu + N \rightarrow e + N$              | ★★★★ | ★★★★ | ★★★★ | ★★★★        | ★★★★   | ★★★★ | ★★★★ |
| $d_n$                                    | ★★★★ | ★★★★ | ★★★★ | ★★          | ★★★★   | ★    | ★★★★ |
| $d_e$                                    | ★★★★ | ★★★★ | ★★   | ★           | ★★★★   | ★    | ★★★★ |
| $(g-2)_\mu$                              | ★★★★ | ★★★★ | ★★   | ★★★★        | ★★★★   | ★    | ?    |

Table 8: "DNA" of flavour physics effects for the most interesting observables in a selection of SUSY and non-SUSY models ★★★★★ signals large effects, ★★ visible but small effects and ★ implies that the given model does not predict sizable effects in that observable.

# Recent analysis in a class of flavour models ...

- Model class:  $G_{\text{GUT}} = \text{SU}(5)$ ;  $G_{\text{Fam}} = \text{SO}(3)$ , spontaneously broken by flavour Higgs fields (in representations  $\mathbf{3}$  of  $\text{SO}(3)$ ) with vacuum expectation values pointing in the following flavour directions:

S.A., Calibbi, Maurer, Spinrath ('11)

$$\frac{\langle \phi_1 \rangle}{\Lambda} \sim \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \varepsilon_1 \quad \frac{\langle \phi_2 \rangle}{\Lambda} \sim \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \varepsilon_2$$

CP violation in the quark sector with a right angled UT (i.e. with  $\alpha = 90^\circ$ )

In leading order: Large "Tri-Bimaximal" mixing (in the neutrino-sector)

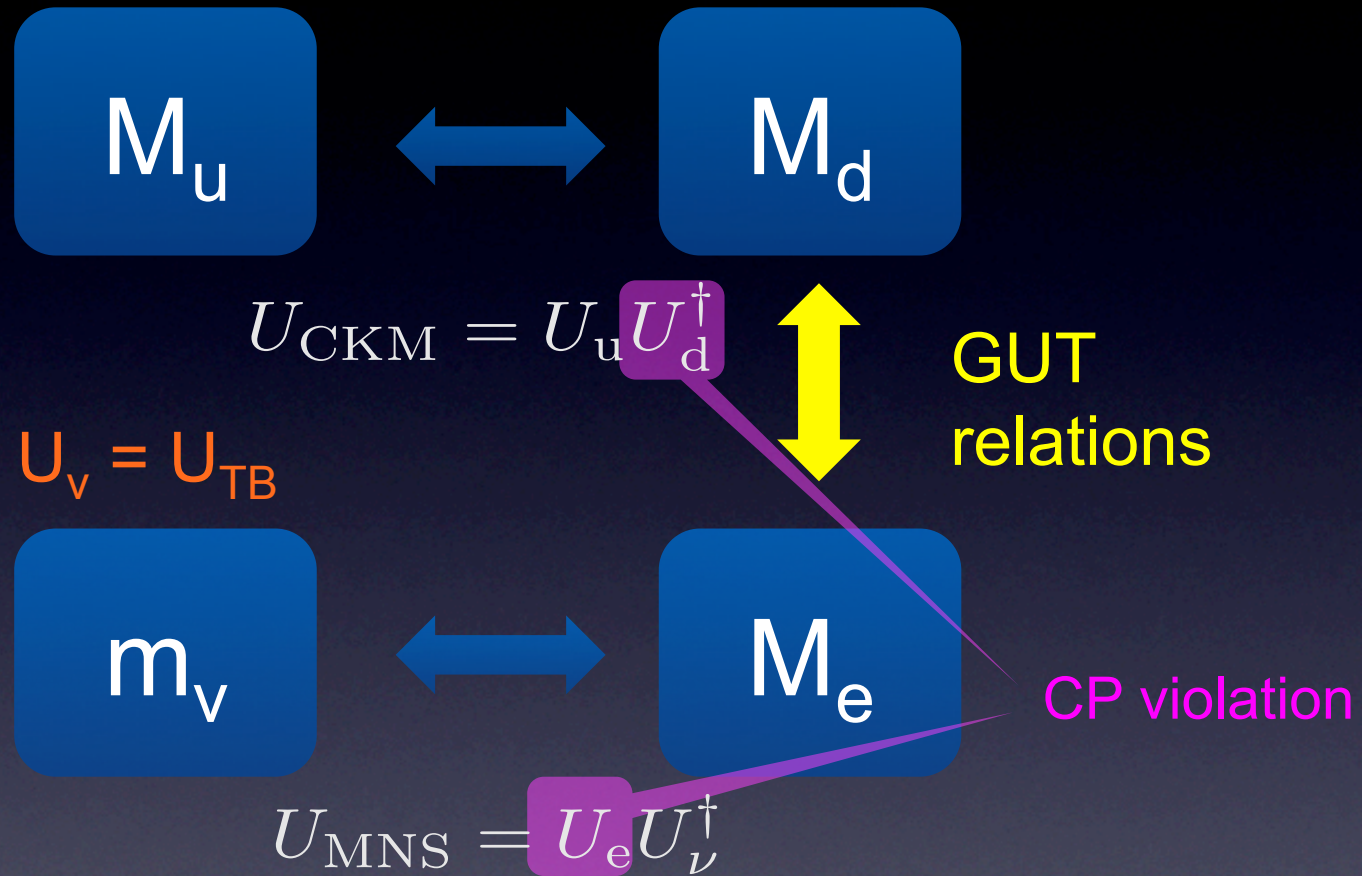
$$\frac{\langle \phi_3 \rangle}{\Lambda} \sim \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \varepsilon_3 \quad \frac{\langle \phi_4 \rangle}{\Lambda} \sim \begin{pmatrix} 0 \\ i \\ O(1) \end{pmatrix} \tilde{\varepsilon}_4$$

+ sequestering in the Kähler potential

$\phi_3$  and  $\phi_4$  in  $\mathbf{24}$  of  $\text{SU}(5) \Rightarrow$  GUT relations, e.g.  $m_\tau/m_b = 3/2$  and  $m_\mu/m_s = 9/2$



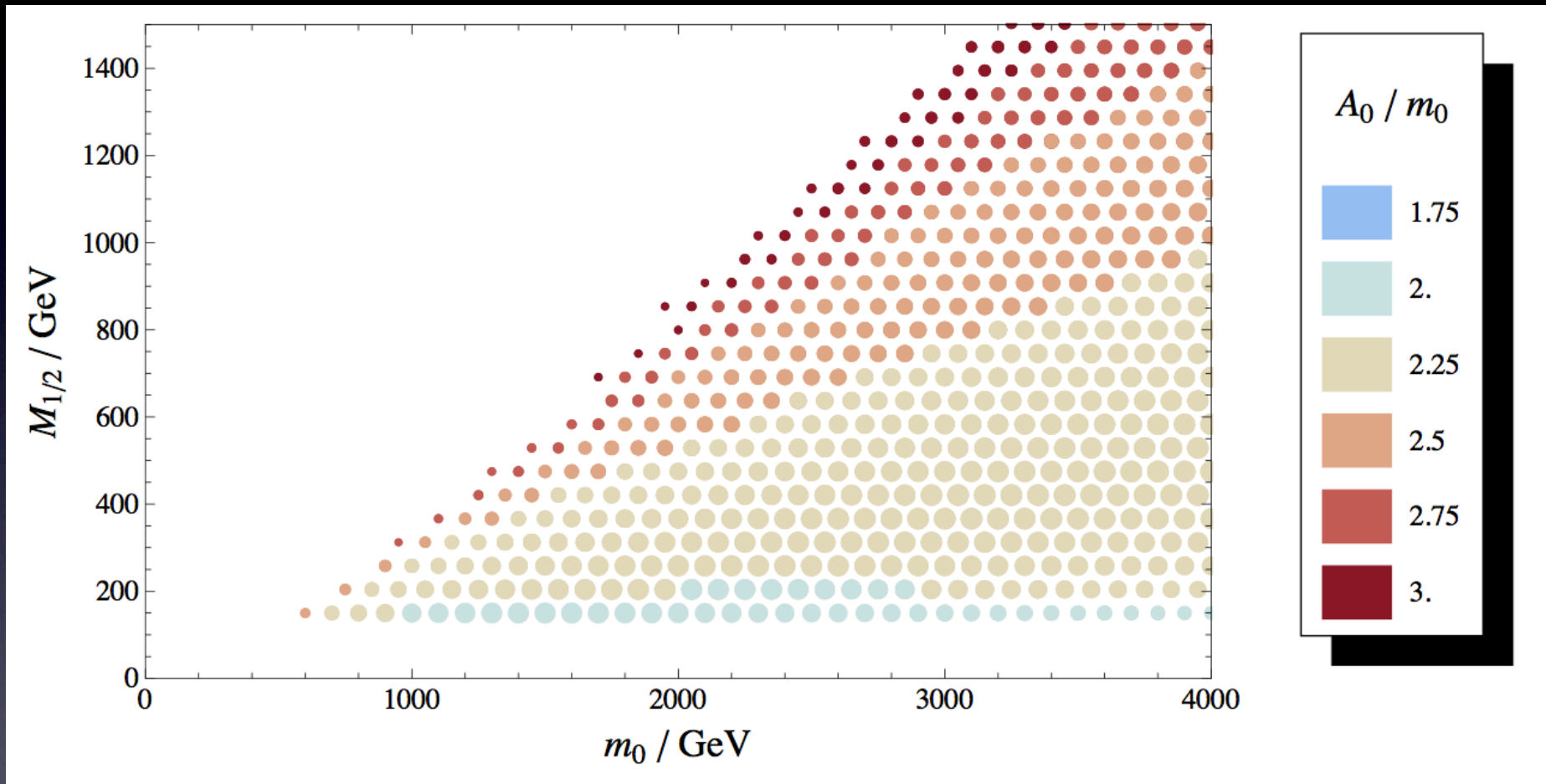
→ Quark and lepton flavour structure (including CP violation)



- ✓ Good fit to the experimental data; Predictions:  $\delta^{\text{MNS}} \sim \pm 90^\circ$ , SUSY spectrum, SUSY flavour structure; non-zero  $\theta_{13}^{\text{PMNS}}$  from charged lepton mixing effects

# Constraints on the SUSY spectrum

CMSSM-like (+ non-universalities)



S.A., Calibbi, Maurer, Spinrath ('11)

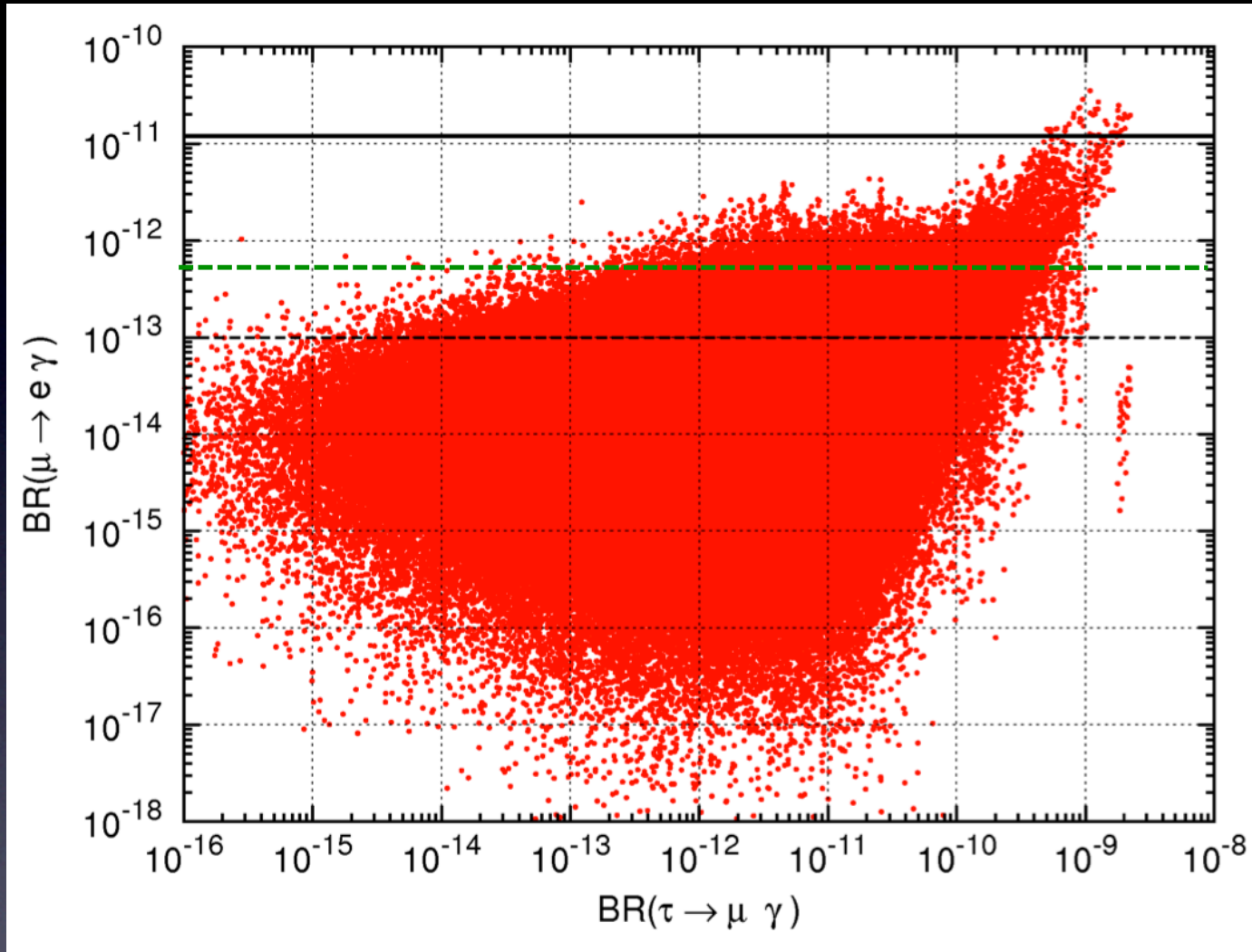
- Comparatively heavy SUSY preferred
- Higgs mass  $m_h \sim 125 \text{ GeV}$  can be accommodated



# Charged LFV in a SUSY GUT “toy model”

S.A., Calibbi,  
Maurer,  
Spinrath ('11)

Here: The  
intrinsic non-  
universalities at  
 $M_{\text{GUT}}$  are the  
dominant  
source of LFV!



MEG (2013):  
 $\text{Br}(\mu \rightarrow e \gamma)$   
 $< 5.7 \times 10^{-13}$   
(@ 90% CL)

Although flavour effects are suppressed by comparatively heavy SUSY:  
Nevertheless, charged LFV provides one of the most promising signals ...

***Even in the presence of a mechanism  
which enforces a universal flavour  
structure at high energies, there is still  
LFV induced by RG running***

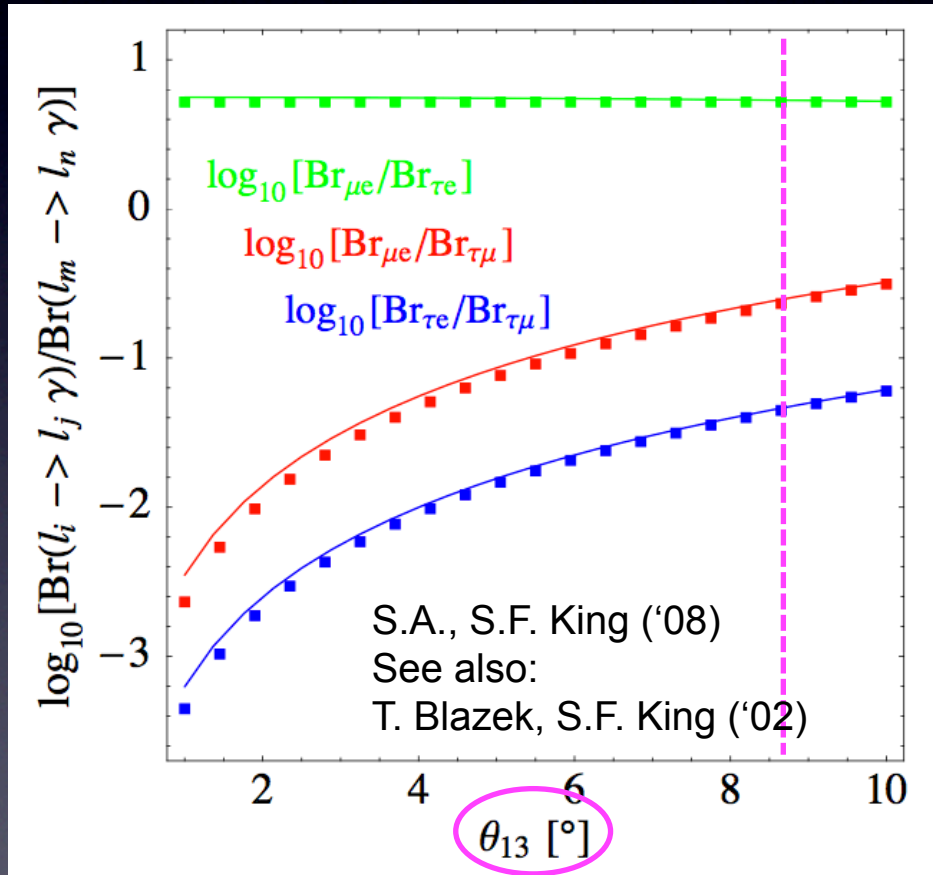
***→ In this case: LFV can offers a window  
into the flavour structure of the  
SUSY seesaw ...***

Borzumati, Masiero ('86), Hisano et al ('96), ...  
various works by many authors on this subject

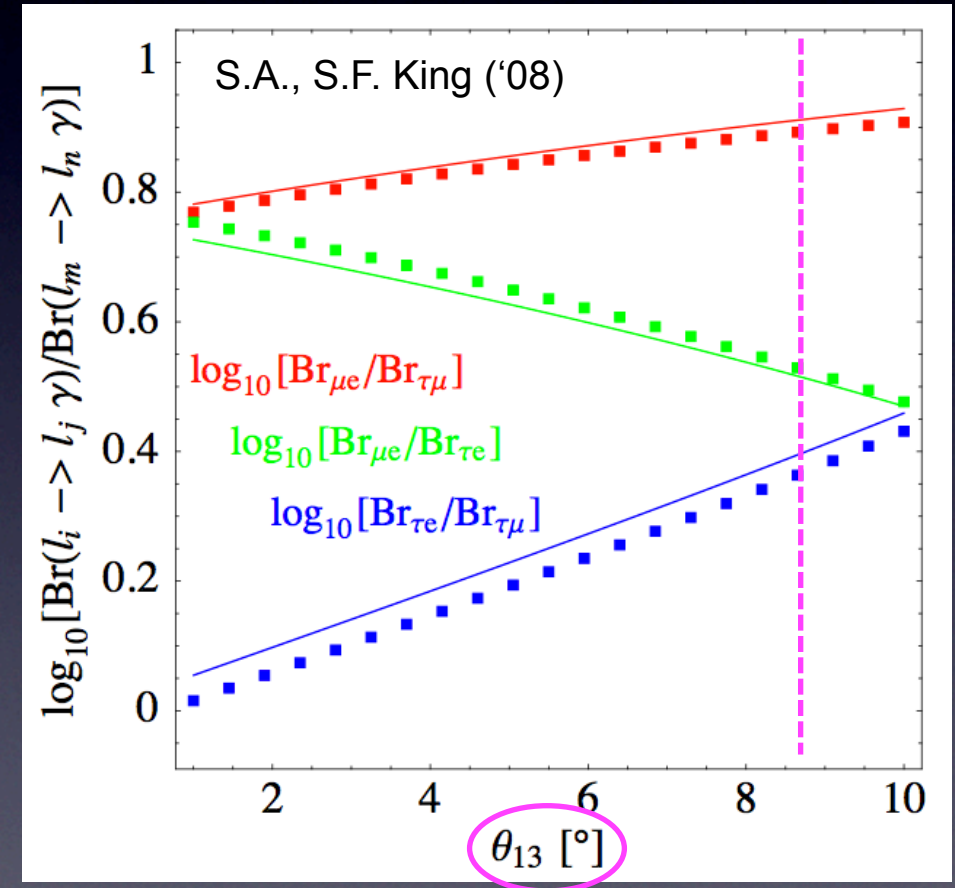


# Example: Classes of neutrino mass models predict very different ratios of Br's ...

## A: Heavy Sequential Dominance



## B: Intermediate Sequential Dominance



Note:  $\theta_{13}^{\text{PMNS}} = 8.6^\circ \pm 0.5^\circ$  has recently been measured!

T2K, Minos, DoubleCHOOZ, DayaBay, RENO

# Also, when constraints are imposed on the SUSY seesaw, e.g. from leptogenesis:

MEG (2013):  
 $\text{Br}(\mu \rightarrow e \gamma)$   
 $< 5.7 \times 10^{-13}$   
 (@ 90% CL)

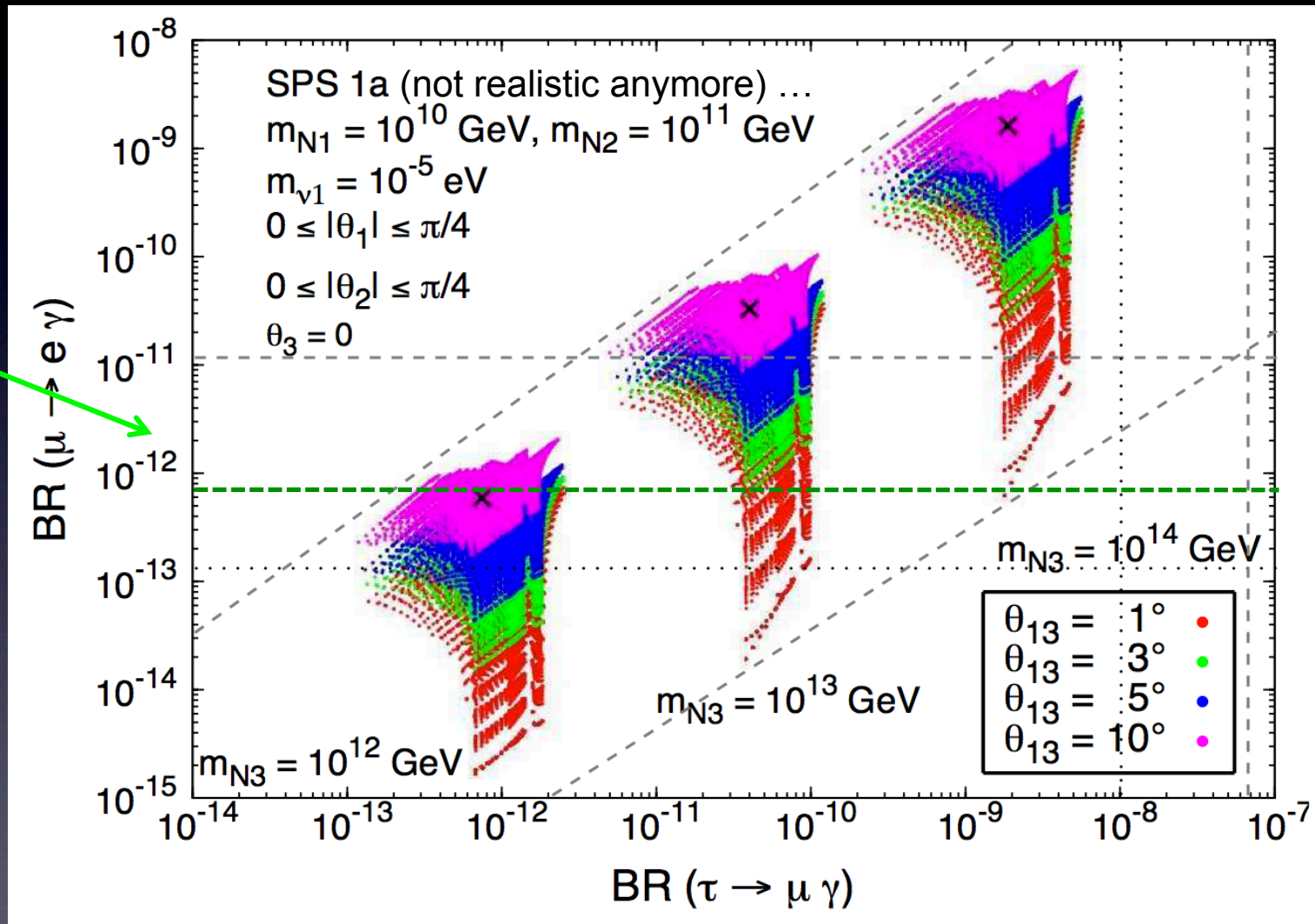


Figure from:  
 S.A., Arganda,  
 Herrero,  
 Teixeira ('06)

→ Correlations between observables

→ Constraints on seesaw parameters



# *Summary and concluding remarks*

- Charged LFV processes provide important channels to search for physics beyond the SM
- Many new physics scenarios receive strong constraints from/ predict observable rates for LFV processes
  - Bottom-up example: Strong constraints on the possible non-unitarity of the leptonic mixing matrix from LFV
  - Top-down example: LFV in SUSY GUT flavour models
- New insights expected from the future experimental results ... !

# Thanks for your attention!

